Which policy-mix to mitigate the effects of financial heterogeneity in a monetary union?

Cristina Badarau-Semenescu* and Grégory Levieuge*

(Preliminary Draft – April 2010)

Abstract

This paper aims to study the suitable policy-mix for a monetary union like the euro area, in a context of financial heterogeneity. Relying on a DSGE model with empirically-justified heterogeneous bank capital channel, and considering financial shocks, the analysis leads to the following conclusions. A centralized monetary policy appears to be more advantageous for the union than an alternative inflation-divergences oriented policy. Besides, national budgetary policies can mitigate cyclical divergences. Nevertheless, the analysis of different policy-mixes shows that a cooperative regime between the members can be counterproductive. Its success depends in fact on the attachment of the collectivity to public spending stabilization in the Union. As this collective preference can reasonably be assumed as weak, a non-cooperative regime may be preferable. These results are finally discussed in light of the subprime mortgage crisis context.

Key words: euro area, financial heterogeneity, monetary/budgetary policy rules, DSGE model.

JEL classification: E44, E58, E63, C61

* Laboratoire d’Economie d’Orléans (LEO), UMR CNRS 6221, Rue de Blois, BP-6739, 45160 Orléans Cedex 2
Contacts: florina-cristina.semenescu@univ-orleans.fr, gregory.levieuge@univ-orleans.fr
1 Introduction

The structural heterogeneity of the European Monetary Union is largely documented and commented\(^1\). Its effects have given rise to numerous debates for at least two decades. Despite the attempts of convergence made by the national governments, recent studies\(^2\) conclude that the financial system is far from being integrated in Europe. More precisely, among the five main financial markets usually analyzed (money, government bond, corporate bond, banking credit and equity market), the banking markets appear as the most heterogeneous. Indeed, price differentials remain high compared to other monetary unions, and home biases in lending to and borrowing of small non-financial corporations and households are persistent\(^3\). According to Angeloni & al. (2003), the asymmetric information between lenders and borrowers in the European credit markets could partly explain these price differentials. Credit conditions would thus depend on the national firms’ and banks’ financial structures\(^4\).

Such a financial heterogeneity implies that a symmetric shock will have asymmetric effects in the member countries. This is true in particular for financial shocks, which have become recurrent over the last two decades (cf. the EMS crisis, the spillovers of the Asian financial crisis, the "dot-com bubble" boom and burst, etc.). As shown during the subprime mortgage crisis, banks constitute nowadays key actors for the transmission of financial shocks. Several recent contributions\(^5\) have highlighted the importance of the bank capital channel, according to which the banks’ balance sheet structures may act as an amplifier for the transmission of shocks to the real economy. According to this literature, the question of the banks’ financing is as problematic as the question of external financing for firms. Because of an agency problem between banks and their creditors, the formers bear an external financial premium which is negatively related to their capital ratio (and so is counter-cyclical). This external financing premium is ultimately passed on to the credit conditions to firms. Considering simultaneously the main factors underlying the bank capital channel, a preliminary empirical study by Badarau-Semenescu & Levieuge (2010) indicates that European countries are ought to

\(^2\) See Baele & al. (2004) and ECB (2008).
\(^3\) Compared to the credit market, the money market is found to be the most integrated being driven by the conduct of a single monetary policy for the euro area. In addition, equity and bond markets are more and more dependent on common news, instead of purely local risk factors.
\(^4\) National specificities in the firms’ and banks’ financial structures are documented for instance in Chatelain & al. (2003) or Ehrmann & al. (2003).
be more (Germany, Italy, Netherland) or less (Finland, France, Spain) sensitive to this mechanism\(^6\).

This raises the question of the appropriate macroeconomic policies in a context of heterogeneous monetary union facing recurrent financial shocks. Certainly, avoiding huge financial crisis requires adequate micro and macro-prudential measures (Levieuge 2009a). The reduction of financial heterogeneity also demands a convergence of structural policies. But both need time and strength of will to be implemented. It is thus worth examining the suitable mixing of the two main existing EMU policy tools: the common monetary policy led by an independent Central Bank and the budgetary policies conducted by national governments.

Since 2008, intensive debates have concerned the lack of coordination of economic stimulus plans inside the euro area, and the way the EMU-members could help the most affected countries. Discussions also exist about the appropriate design of monetary policy\(^7\). The aim of the present article is to determine which policy-mix arrangements are likely to mitigate the effects of financial asymmetries in a monetary union possibly hit by financial shocks.

To this end, we proceed to some policy experiments based on a Dynamic Stochastic General Equilibrium (DSGE) model for a monetary union gathering two countries with distinct banking structures (in line with the figure depicted in Appendix 1)\(^8\). As for the euro area, the monetary policy is supposed to be conducted by an independent Central Bank that must ensure the price-stability for the Union as a whole. Budgetary policies are decentralized and remain under the responsibility of the national governments. Different strategies for monetary policy (centralized vs. based on national information) and for budgetary policies (budgetary cooperation vs. autonomous conduct of the national budgetary policy) are studied. These policies are combined in a sequential game, with the Central Bank choosing first its strategy, and the national governments defining afterwards their (cooperative or non-cooperative) policies. Four policy-mixes are thus obtained. Their benefits are evaluated with respect to an exogenous social welfare criteria calculated as the average of national social loss functions within the Union.

\(^6\) The figure in the Appendix I summarizes the conclusion of this exam.

\(^7\) To this respect, the monetary policy tightening decided by ECB in summer 2008 had been widely criticized.

\(^8\) Previous examples of monetary policy analysis in two-country models with different financial systems are provides by Faia (2002) or Gilchrist & al. (2002). The last one is more close to our model, because it addresses the question of the monetary policy conduct in a monetary union. The authors settle for introducing asymmetric firms’ balance sheet channels within the union and analyze the transmission of technological shocks. We extend their study toward the consideration of the bank capital channel and we show how the model can be used to evaluate different policy-mix strategy in an asymmetric union.
It thus appears that a centralized monetary policy, seeking to stabilize the union-wide inflation rate, dominates a strategy that is simultaneously concerned by the stabilization of inflation divergences in the union. This is true whatever the budgetary regime, supporting the current orientation of the European Central Bank (ECB) policy for the euro area. Besides, national budgetary policies constitute relevant instruments (although insufficient) to fight the asymmetric transmission of shocks in a monetary union with financial heterogeneities. Nevertheless, the analysis of different policy-mixes shows that a cooperative regime between the members can be counterproductive. Its success depends in fact on the attachment of the collectivity to public spending stabilization in the Union. As this collective preference can reasonably be assumed as weak, a non-cooperative regime may be preferable.

The reminder of this paper is organized as follows. The second section introduces the baseline model. The third section verifies the dynamics of the model and discusses the role of the financial asymmetries for the transmission of shocks within the monetary union. The capacity of macroeconomic policies to mitigate the effects of financial structural heterogeneity is then analyzed in the fourth section of the paper. The last section formulates some concluding remarks.

2 The baseline model

The model used in this paper describes a two-country monetary union with financial heterogeneities introduced in the national banking structures, where the monetary policy decisions are delegated to a common Central Bank. It extends the reference financial accelerator model of Bernanke & al. (1999), by introducing the bank capital channel in the analysis, in line with the instructions provided by Sunirand (2003) and Levieuge (2009a).

2.1 An Overview

The behavior of five categories of national agents is considered for each country: entrepreneurs, households, retailers, banks and the government, to which we must add the role of the common Central Bank. Briefly, the financial mechanism of the model, written for a given country, relies on the following sequence: households lend money to banks, which in turn insure the financing of firms (wholesalers).

Wholesalers are risk neutral. To produce wholesale final goods for the period $t+1$, the representative firm $i$ buys, at the end of the period $t$, the capital $K_{t+1}^i$ at a price $Q_i$. The entrepreneur can not entirely self-finance its project. He thus uses to this aim the firm’s net wealth $NF_i$, and he borrows the remainder from a representative bank $j$. Debt contracts have one period maturity. An idiosyncratic risk $\omega_{t+1}$ affects the representative firm’s expected
Considering a costly state verification framework (Townsend, 1979) to introduce imperfections on the credit market, the realization of $\omega_{t+1}^i$ is private information. As a result, bank $j$ has to engage verification costs to reveal this value and, as for Williamson (1987), the verification procedure is open only if the barrower declares bankrupt. Moreover, because this conceptual context creates a wedge between the cost of internal and external financing, it motivates the self-participation of firms to the capital investment. The loan contracted by the firm $i$ from the bank $j$ is: 

$$i^tNF_{KQB} = i^tK_{t+1} + 1,$$

with $i^tNF$ the net wealth held and engaged in the capital investment by the firm, at the period $t$. Following Bernanke & al. (1999), a threshold value of $\omega_{t+1}^i$, noted $\omega_{t+1}^{iF}$, is defined such that it satisfies the relation:

$$\omega_{t+1}^{iF} R^{K}_{t+1} Q_{t+1} K^{i}_t = R^B_{i,t+1} B^i_t$$

(1)

where $R^B_{i,t+1}$ represents the non-default loan rate associated to the contract signed between the firm $i$ and the bank $j$, at the end of the period $t$. Given the predetermined threshold value $\omega_{t+1}^{iF}$, there are two possible situations for the following period: i) $\omega_{t+1}^{iF} \geq \omega_{t+1}^{iF}$, the realized return of the firm is sufficient to repay its debt to the bank, there is no bankruptcy and the firm obtain a benefit which is: $(i^tK_{t+1} - \omega_{t+1}^{iF} R^{K}_{t+1} Q_{t+1} K^{i}_t)$; or ii) $\omega_{t+1}^{iF} < \omega_{t+1}^{iF}$, the firm revenues are insufficient to fulfill the loan contract, it declares bankruptcy and is liquidated. The auditing cost the bank has to spend if the firm declares bankrupt is supposed to be proportional to the gross return to the firm’s investment: $\mu^B \omega_{t+1}^{iF} R^{K}_{t+1} Q_{t+1} K^{i}_t$, where $\mu^B$ is a factor of proportionality. The bank thus only receives: $(1 - \mu^B) \omega_{t+1}^{iF} R^{K}_{t+1} Q_{t+1} K^{i}_t$, after the verification procedure.

Banks operate in a perfectly competitive environment and are also risk neutral. In the reference model of Bernanke & al. (1999), as banks’ portfolios are infinitely large, the idiosyncratic risk $\omega^i$ is completely diversified. Households are thus sure to benefit from a riskless return when they lend to banks. There is no need for them to monitor banks and there is no need for banks to hold inside capital. But, if it is assumed that banks’ loan portfolios are of finite size, the risk associated with firms’ investment projects is partly transferred to banks’ balance sheets, and finally to households. Such an approach allows avoiding the less realistic idea that banks never default and that the financial intermediation can be conducted without capital. So, a new agency problem occurs between banks and households, in addition to the agency problem between entrepreneurs and banks.

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9 $\omega_{t+1}^i$ is a random variable that follows a log-normal distribution of mean $-\sigma^2/2$ and standard deviation $\sigma$, independent and identically distributed among firms and in time. It fulfills all general condition for the existence of the financial contract (see Bernanke & al, 1999; Sunirand, 2003; or Leveugue, 2009, for example).
The costly state verification procedure reveals information only to the agent paying the auditing cost (see also Krasa & Villamil, 1992). As the return of investment of the representative firm remains private information, also is the return on the loans portfolio of the bank. If a bank goes bankrupt, a household who borrowed funds to it has to start a costly auditing procedure to observe the true value of the bank’s lending return. As in Krasa & Villamil (1992), households perform the role of ‘monitoring the monitors’. This implies that banks also face an external finance premium in obtaining funds from households, encouraging them to accumulate inside capital.

However, with the assumption of banks’ loan portfolios of finite size, the aggregation is more difficult and it directly depends on the distribution of risky portfolio in each bank. To maintain the model tractability, we follow hereafter Sunirand (2003) when supposing that a bank can only participate to the investment projects of one firm. In such a way, the idiosyncratic risk is fully diversified at the aggregate level, but not at the bank level. This assumption also allows us to simplify hereafter the previous notations, by renouncing to indices \( i \) and \( j \).

Thus, at the period \( t \), a representative bank which lends \( B_t = Q_t K_{t+1} - NF_t \) to a representative firm uses for that its inside accumulated capital \( (NB_t) \) and other funds raised from a representative households \( (A_t) \), amounting to: \( A_t = Q_t K_{t+1} - NF_t - NB_t \). Thus, a bank is involved in two successive financial contracts. On the one hand, its relationship with the entrepreneur conducts to a contract in which the bank acts as a lender. Given the previous information relative to the risk associated to the firm’s project and the fact that the bank should afterwards collect sufficient funds from households to finance the firm, the terms of this contract come from the maximization of the entrepreneur’s expected benefit, subject to the participation condition for the bank. The solutions of this program give the firm demand for capital, the value \( \omega \), and the value \( \omega \) that defines the threshold under which the realization of \( \omega \) cause a bank failure. The non-default loan rate associated to the contract between the entrepreneur and the bank \( (R_{t+1}^{a}) \) is then easily obtained from (1).

On the other hand, in a second time, the banker directly interacts with a household in order to obtain funds necessary to the financing of the firm. In this relationship, the bank acts as a borrower and the lender (household) has no information on the loan portfolio of the bank. He forms his expectations on the basis of the average return for banks in the economy. For firms,

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10 See Sunirand (2003) for more details on this subject.
11 See also Chen (2001) for an equivalent situation, when one bank can lend to several firms, but the return on firms’ investment projects is perfectly correlated within a bank, but i.i.d across banks.
12 This threshold \( \omega_{t+1}^{a} \) must fulfill the condition: \( (1 - \mu^{a}) \omega_{t+1}^{a} Q_{t+1} K_{t+1} = R_{t+1}^{a} A_t \), where \( R_{t+1}^{a} \) defined the highest return that can be demanded by the household when lending to the bank (supposing that this later is informed on the risk degree of the bank’s loans portfolio).
each bank are now subject to an idiosyncratic risk, noted $\varepsilon_{t+1}$, supposed to follow a log-normal distribution similar to that of $\omega_{t+1}$. The realization of $\varepsilon_{t+1}$ represents private information, and the bank could be encouraged to not publicly announce it, in a default situation. To obtain the real value of $\varepsilon_{t+1}$, the household have to pay an auditing cost $\left(\mu^A R^A_{t+1} B_t\right)$ proportional to the gross return of the bank’s loans portfolio. The intermediation activity of banks to be justified in the model, the monitoring procedure must be more costly for households than for banks (which are specialized in this kind of operations). The terms of the financial contract concluded between the bank and a household are obtained by maximizing the expected bank’s benefit, subject to the household participation constraint. The solution of the program allows determining $B_t$, a threshold value $\bar{e}_{t+1}$ under which the bank goes bankrupt, and the non-default rate $\left(R^A_{t+1}\right)$ on the funds raised from the household at the end of period $t$, by the use of the following condition:

$$\varepsilon_{t+1} R^A_{t+1} B_t = R^A_{t+1} A_t \quad (2).$$

If $\varepsilon_{t+1} \geq \bar{e}_{t+1}$, bank’s revenues are sufficient to fulfill its commitments towards the household. Otherwise, the bank goes bankrupt and the household must support the monitoring cost in order to recuperate the maximum of his rights, amounting to: $\left(1 - \mu^A\right)R^A_{t+1} B_t$.

Households are infinitely-lived in the model. They consume retail goods, work in wholesale enterprises and receive profits from retailers (that they are owners). Their saving consists in securities issued by banks (see $A_t$, in the previous relations, for the period $t$), or in other financial investments remunerated at the risk-free interest rate. Households are neutral to idiosyncratic risk hitting firms and banks, but they are averse to aggregate risk, which means that the aggregate risk inherent to the activity of firms will be borne by firms and banks. The mechanism which protects households from the aggregate risk is the following. Both the non-default lending rate $\left(R^A_{t+1}\right)$ and the non-default interest rate on bank securities $\left(R^A_{t+1}\right)$ are predetermined at the end of period $t$. So, if at the end of the period $t+1$, the effective return on non-idiosyncratic component of firms’ investments is lower than expected, households will be compensated with the higher non-default interest rate on bank securities.

### 2.2 Partial equilibrium on the credit market

To characterize the partial equilibrium on the credit market, the mathematical approach is quite similar to that used in Bernanke & al. (1999) and Sunirand (2003). Firstly, the solution of
the optimization program defining the relationship of a bank with a corresponding entrepreneur (borrower) is resumed in the following relation\textsuperscript{13}:

\[ S_t^F = \Psi_F \left[ k_t^F \right], \text{ where } S_t^F = E_t \left[ \frac{R_{t+1}^K}{R_{t+1}^F} \right], \frac{\partial \Psi_F}{\partial k_t^F} > 0 \text{ and } k_t^F = \frac{Q_tK_{t+1}}{NF_t + NB_t}, \]

In a logarithmic form, \( S_t^F \) simply defines to the firm’s external finance premium in the model, i.e. the difference between the net return on the firm’s physical capital required by the bank \( (r_t^K = R_t^K - 1) \) and the risk-free interest rate \( (r_t^F = R_t^F - 1) \).

Unlike the referential model by Bernanke & al. (1999), the firm’s external finance premium does not only depends on the firm’s financial position \( \left( \frac{Q_tK_{t+1}}{NF_t} \right) \), but also on the inside capital of its bank \( \left( NB_t \right) \). All things being equal, a low level of the firm’s net wealth \( \left( NF_t \right) \) induces a higher cost of the external finance. Moreover, it also depends on the bank’s financial situation. The lending interest rate required to the firm by a bank with low level of inside capital would be higher than that expected from another bank with better financial position (in (3), \( S_t^F \) negatively depends on \( NB_t \)). This fact clearly shows the internalization of the external financing cost for banks, by the entrepreneurs. The deterioration of the banks’ balance sheets, inducing higher cost for their external finance, will also reflect a tightening of the lending conditions to firms. It is the bank capital channel manifestation, discussed in the previous paragraph.

This mechanism becomes more evident after the resolution of the second optimization program corresponding to the relationship between bank (borrower) and household (lender) on the credit market. Indeed, the relation (4) resumes the terms of this second financial contract, by defining the bank’s external finance premium \( \left( S_t^B \right) \).

\[ S_t^B = \Psi_B \left[ k_t^B \right], \text{ where } S_t^B = \frac{R_t^B}{R_t^F}, \frac{\partial \Psi_B}{\partial k_t^B} > 0 \text{ and } k_t^B = \frac{B_t}{NB_t}, \]

As expected, the non-default return on the bank’s loans portfolio, required by the household \( \left( R_t^B \right) \), is higher than the risk-free interest rate. The amplitude of the gap only depends in (4) on the bank’s financial leverage, defined here by the inside capital on loans ratio. The financial health of the intermediary impacts on the cost of its external financing that will be finally transferred to firms.

\textsuperscript{13} Details on the explicit form of all optimization programs and their resolution are available on request in a separate Technical Appendix. See also Levieuge (2009b).
The firm’s net wealth mainly comes from the accumulated benefits from a period to another, i.d. the value of the firm \( VF_t \), it is assumed that the entrepreneur also acts as employee on the labor market and perceives the wage \( WF_t \), which is added in the firm’s net wealth.

\[ NF_t = \gamma^t [VF_t + WF_t] \quad (5) \]

The coefficient \( \gamma^t \) in the equation (5) corresponds to the survival probability of the firm at the period \( t \). As in all financial accelerator models, it is supposed here that a constant proportion \( \{ 1 - \gamma^t \} \) of firms leave the market each period. When living the market, the entire net wealth is used to consume final goods \( CF_t \):

\[ CF_t = (1 - \gamma^t) [VF_t + WF_t] = \frac{1 - \gamma^t}{\gamma^t} NF_t \quad (6) \]

Besides, it can be shown that the value of the firm \( VF_t \) is given by the gross return on capital after the repayment of the debt and the associated interests to the lender:

\[ VF_t = Q_{i,t} R^K_t \left[ R^{i'} + \frac{\mu^B G(\overline{w}^F) + \mu^A (1 - \mu^B) G(\overline{w}^B)}{Q_{i,-1} K_i - NF_{i-1}} Q_{i,-1} R^K_t \right] B_{i,t} \quad (7) \]

In (7), \( \frac{\mu^B G(\overline{w}^F) + \mu^A (1 - \mu^B) G(\overline{w}^B)}{Q_{i,-1} K_i - NF_{i-1}} Q_{i,-1} R^K_t \) defines the external finance premium supported by the firm, and \( \left[ R^{i'} + \frac{\mu^B G(\overline{w}^F) + \mu^A (1 - \mu^B) G(\overline{w}^B)}{Q_{i,-1} K_i - NF_{i-1}} Q_{i,-1} R^K_t \right] \) can be replaced by \( S_{i,t} R^{i'} \).

The bank inside capital comes also from the accumulated benefits of the intermediation activity, i.d. the intrinsic value of the bank \( VB_t \), and from small transfers \( T^b_t \) received from the banks that are supposed to leave the market in a proportion \( \{ 1 - \gamma^B \} \) each period\(^{14}\). If the bank leaves the market at the period \( t \), a small part of its inside capital \( T^b_t \) is transferred to survival banks, the rest being used to consume final goods \( CB_t \). Relations (8) and (9) describe this behavior:

\[ NB_t = \gamma^B VB_t + T^b_t \quad (8) \]

\(^{14}\) In line with other financial accelerator models, this assumption is also used to insure the tractability of the model, namely to give the possibility to new banks to have initial inside capital, necessary for the access to external financing.
\[ CB_t = (1 - \gamma^B) (1 - t^B) VB_t = \frac{(1 - \gamma^B) (1 - t^B)}{\gamma^B (1 - t^B) + t^B} NB_t \]  

(9)

Similar to the relation (7) written for the firm, the bank’s value \( VB_t \) takes the form:

\[ VB_t = R^B_t B_{t-1} \left[ R^f_t + \frac{\mu^A (1 - \mu^B) G(\bar{\omega}^B)}{Q_{t-1} K_t - NF_{t-1} - NB_{t-1}} Q_{t-1} R^K_t K_t \right] A_{t-1} \]  

(10)

where \( \frac{\mu^A (1 - \mu^B) G(\bar{\omega}^B)}{Q_{t-1} K_t - NF_{t-1} - NB_{t-1}} Q_{t-1} R^K_t K_t \) is an expression of the bank’s external finance premium and \( \left[ R^f_t + \frac{\mu^A (1 - \mu^B) G(\bar{\omega}^B)}{Q_{t-1} K_t - NF_{t-1} - NB_{t-1}} Q_{t-1} R^K_t K_t \right] \) is to be replaced by \( S^K_t R^f_t \), for \( S^K_t \) given in (4).

Finally, with a constant return to scale assumption for the Cobb-Douglas technology used by firms and with an equivalent condition defined for the banks’ activity, the previous individual equations remain unchanged after aggregation.\(^{15} \)

2.3 General equilibrium

The partial equilibrium solved for the credit market in embedded in a dynamic stochastic general equilibrium model of a two-country monetary union. Apart from these financial imperfections, the DSGE model is standard. In each country, firms use labor and capital (partially financed by debt) to produce wholesale final goods, in perfectly competitive markets. Retailers buy wholesale goods from the producers and retail them in a monopolistic competition market. They slightly differentiate the output they purchase with no costs and their presence allows introducing nominal rigidities in the model, in line with the Calvo (1983) pricing assumption. Households and firms (producers of wholesale final goods) purchase CES aggregates of the retail products and transform then in consumption goods or in investment goods (used as capital in the production process, after some costly internal adjustments). Because the model consists of a two-country monetary union, domestic households from a given state simultaneously consume domestic goods and goods produced in the other country of the union.

Each country is inhabited by a continuum of infinitely-lived agents represented by the unit interval. These agents choose consumption \((C)\) and leisure \((L)\) and determine the working period \((H = 1 - L)\) remunerated at a real rate \(W\). The one period utility function is given by:

\(^{15} \) See Bernanke & al. (1999) or Sunirand (2003) for more details on the aggregation procedure.
$$U(C_t, H_t) = \frac{\sigma_c}{\sigma_c - 1} C_t^{\sigma_c - 1} - \frac{\sigma_h}{\sigma_h + 1} H_t^{\sigma_h + 1}$$ \hspace{1cm} (11),

with $\sigma_c$, the consumption intertemporal elasticity of substitution, and $\sigma_h$, the elasticity of the disutility associated to labour.

The consumption is a composite index which depends on the consumption of goods domestically produced and produced in the other country of the union. The origin of goods is indexed by 1 and 2, while $C$ and $C^*$ denote the aggregated consumption in the first and in the second country of the union, respectively. $\gamma \in [0,1]$ represents the relative preference for the consumption of domestic produced goods, in each country.

$$C = \frac{C_1^\gamma C_2^{1-\gamma}}{\gamma'(1-\gamma)^{-\gamma}}; \quad C^* = \frac{(C_1^*)^\gamma (C_2^*)^{1-\gamma}}{\gamma'(1-\gamma)^{-\gamma}}$$ \hspace{1cm} (12)

Price indexes for the two countries are respectively defined by: $P = P_1^\gamma P_2^{1-\gamma}$ and $P^* = (P_2^*)^\gamma (P_1^*)^{1-\gamma}$, and the law of one price is supposed to hold.

*Households* choose a sequence of consumption, labour, bank securities and other financial investment at the risk-free interest rate, which maximizes an intertemporal utility function, based on (11), subject to the following budget constraint:

$$P_t C_t + P_t D_t + A_t \leq P_t W_t H_t + A_{t+1} R_t^A_t + P_t D_{t+1} R_t^f_t - T_t + \Pi_t$$ \hspace{1cm} (13)

In (13), $R_t^A_t = 1 + r_t^A$ and $R_t^f_t = 1 + r_t^f$ denote respectively the gross returns of the two alternative financial investments for households, $T_t$ represents lump sum taxes and $\Pi_t$ are the dividends received from the ownership of retail firms. Symmetric constraint applies in the second country of the union, and the first order conditions associated to $C_t, D_t, A_t$ and $H_t$ appear in the following table:

**Table 1. First order conditions for the households’ optimization**

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_t = \frac{1}{P_t} C_t^{-1} \sigma_c$</td>
<td>$\lambda_t^* = \frac{1}{P_t^<em>} (C_t^</em>)^{-1} \sigma_c$</td>
</tr>
<tr>
<td>$0 = \lambda_t - \beta R_t^{f_t} E_t [\lambda_{t+1}] E_t \left[ \frac{P_{t+1}}{P_t} \right]$</td>
<td>$0 = \lambda_t^* - \beta R_t^{f_t} E_t [\lambda_{t+1}^<em>] E_t \left[ \frac{P_{t+1}^</em>}{P_t^*} \right]$</td>
</tr>
<tr>
<td>$0 = \lambda_t - \beta R_t^{A_t} E_t [\lambda_{t+1}]$</td>
<td>$0 = \lambda_t^* - \beta R_t^{A_t} E_t [\lambda_{t+1}^*]$</td>
</tr>
<tr>
<td>$H_t = (\lambda_t P_t W_t)^{\sigma_h}$</td>
<td>$H_t^* = (\lambda_t^* P_t^* W_t^*)^{\sigma_h}$</td>
</tr>
</tbody>
</table>
Moreover, the condition \((R_{i+1}^f)E_t\left[\frac{P_{r+1}}{P_t}\right] = (R_{i+1}^*)E_t\left[\frac{P_{r+1}^*}{P_t^*}\right]\) is fulfilled into the union, allowing to write:

\[C_t = C_t^*(\Theta_t)^\nu_t\]  

(14),

where \(\Theta_t = \frac{P_t^*}{P_t}\) is an expression of the bilateral terms of trade.

Wholesale producers combine labour and capital with a Cobb-Douglas constant return to scale technology:

\[Y_t = a_t K_t^{\alpha_t} L_t^{1-\alpha_t} \text{ and } Y_t^* = a_t^* (K_t^*)^{\nu_t} (L_t^*)^{1-\alpha_t}\]  

(15),

with \(a_t\), an exogenous productivity factor that follows a standard autoregressive process in the model: \(a_t = \rho_a a_{t-1} + \varepsilon_a\), where \(\varepsilon_a\) defines a productivity shock, with zero mean and unit variance. The labour input in (15) is a composite index of households labour \((H_t)\) and entrepreneurial labour \((H_t^E)\): \(L_t = H_t^{\alpha_t} (H_t^E)^{1/\alpha_t}\). As briefly introduced previously, we assume here that, in addition to operating firms, entrepreneurs supplement their income by acting as suppliers on the labour market. They are remunerated on this market at a rate \(W^F\), and the total entrepreneurial labour is normalized to unity. This assumption allows the wholesale producers to have baseline revenue to borrow immediately; otherwise, they should face unrealistically high external finance premium level.

In each country, the investment \((I_t)\) is supposed to concern domestic produced goods. The accumulation of physical capital is introduced by the standard equation:

\[K_{t+1} = (1 - \delta)K_t + I_t\]  

(16),

where \(\delta\) is the depreciation rate of the capital.

It is also assumed that there are some internal capital adjustment costs \(\Phi(\cdot)\), given by:

\[\Phi(I_t, K_t) = \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \text{ for } \phi > 0\]  

(17)

Noting \(\rho_t = \frac{P_t}{P_{r+1}}\) the relative price of wholesale goods produced in the first country of the union, \(Q_t\) the Lagrange multiplier associated to the process of capital accumulation, and given

\[\text{Taking it as a composite index of goods produced in the two countries of the union, similar to the consumption index, would not significantly change the results of the model.}\]
the term of trade \( \frac{P_1}{P_2} = \frac{P_1^*}{P_2^*} = \Theta_t \), the maximization of the expected discounted sum of domestic firms operating income flows give the first order conditions relative to \( H_t, H_t^F, I_t \) and \( K_{t+1} \) respectively, reported in the table 2. The first two conditions define the labour demands. The third defines the Tobin’s \( Q \) ratio. The last relation represents the expected gross return to holding a unity of capital from \( t \) to \( t+1 \). At the optimum, the firms’ demand for capital insures the equality between the expected marginal cost for the external financing and the expected marginal return on capital.

### Table 2. First order conditions for firms’ optimization

<table>
<thead>
<tr>
<th>Country 1(*)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_t(\Theta_t)^{1-\gamma} \Omega(1-\alpha) \frac{Y_t}{H_t} = W_t ; \rho_t(\Theta_t)^{1-\gamma}(1-\Omega)(1-\alpha) \frac{Y_t}{H_t^F} = W_t^F ; Q_t = 1 + \frac{\partial \Phi()}{\partial I_t} )</td>
<td></td>
</tr>
</tbody>
</table>

\( E_t[K_{t+1}^T] = \frac{1}{Q_t} E_t \left[ \rho_{t+1}(\Theta_{t+1})^{1-\gamma} \alpha \frac{Y_{t+1}^F}{K_{t+1}^F} - \frac{\phi}{2} (\delta^2 - \left( \frac{I_{t+1}}{K_{t+1}} \right)^2) + (1-\delta)Q_{t+1} \right] \)

(*) For the second country of the union the first order conditions are symmetric, except for the exponent of \( \Theta_t \), which becomes \( (\gamma - 1) \) instead of \( (1-\gamma) \).

Retailers are represented by firms, held by households, which purchase wholesale goods and retail them afterwards. Their main role in the model is to differentiate final goods. They also serve to introduce price inertia. Following Calvo (1983), it is assumed that a retailer changes his price with probability \( 1 - \varsigma \), in a given period. Subsequently, the retailer pricing behavior leads to the following ‘new Phillips curves’ in the two countries of the union:

\[
\hat{\pi}_{1,t} = \beta E_t[\hat{\pi}_{1,t+1}] + \kappa \hat{\phi}_t \text{ and } \hat{\pi}_{2,t} = \beta E_t[\hat{\pi}_{2,t+1}] + \kappa \hat{\phi}_t^* \tag{18}
\]

where \( \pi_{1,t} = \log(P_{1,t} / P_{1,t-1}) \) and \( \pi_{2,t} = \log(P_{2,t} / P_{2,t-1}) \) give the inflation rates calculated in the domestically priced goods for the two countries, \( \kappa = \frac{(1-\varsigma)(1-\varsigma\beta)}{\varsigma} \), \( \rho, \rho^* \) are respectively the real marginal cost for a representative retailer in each country, and \( \hat{x} \) defines, for all \( x_t \), the deviation of a variable \( x_t \) from its steady-state.

Financial imperfections are then introduced in the present general equilibrium model, with regards to equations (3) to (10), after aggregation. Because the firms’ investment in new capital is conditional to the external financing, the demand for capital in the economy depends on the tightness of the constraints imposed on the credit market. Imperfections on this market make the cost of external financing more sensitive to the financial situation of agents, amplifying the
transmission of shocks on the real economy, as we will see in the following sections of the paper. Before, note that the equilibrium relations on the national goods markets are:

\[
Y_t = \Theta_t^{1-\gamma} C_t^\gamma (\gamma + (1-\gamma) \Theta_t^{1-\sigma_Y}) + I_t + G_t + CF_t + CB_t
\]  
(19)

\[
Y_t^* = (\Theta_t^{1-\gamma})^* C_t^{*\gamma} [(1-\gamma) + \Theta_t^{1-\sigma_Y} \gamma] + I_t^* + G_t^* + CF_t^* + CB_t^*
\]  
(19’).

while the national labour markets equilibrium implies:

\[
\left( H_t \right)^{\sigma_i+1} \sigma_i = \left( C_t^* \right) ^ {1-\sigma_i} \rho_i \left( \Theta_t \right)^{1-\gamma} \Omega (1-\alpha) Y_t
\]  
(20)

\[
\left( H_t^* \right)^{\sigma_i+1} \sigma_i = \left( C_t^* \right) ^ {1-\sigma_i} \rho_i^* \left( \Theta_t \right)^{1-\gamma} \Omega (1-\alpha) Y_t^*
\]  
(20’).

National governments are responsible for the budgetary policy. They decide the level of government expenditures, financed by lump-sum taxes. Equations (21) and (21’) define active budgetary policy conducted by governments:

\[
\hat{g}_t = \rho_{g} \hat{g}_{t-1} + \rho_{x} \hat{x}_t + \rho_{y} \hat{y}_t + \varepsilon_t
\]  
(21)

\[
\hat{g}_t^* = \rho_{g}^* \hat{g}_{t-1} + \rho_{x}^* \hat{x}_t^* + \rho_{y}^* \hat{y}_t^* + \varepsilon_t^*
\]  
(21’),

where \( \rho_{g}, \rho_{g}^*  < 1, \rho_{x}, \rho_{x}^* < 0 \) represent the reaction coefficients of the budgetary policy to national inflation deviation from the steady-state, \( \rho_{y}, \rho_{y}^* < 0 \) are the coefficients of reaction to the output-gap deviation from the steady-state, and \( \varepsilon_t, \varepsilon_t^* \) are random shocks with zero mean and unit standard deviation. If \( \rho_{x}, \rho_{x}^* = 0 \) and \( \rho_{y}, \rho_{y}^* = 0, \) there is no active policy in the economy, and government expenditures follows a standard autoregressive process.

Finally, the common Central Bank conducts a common monetary policy rule (with respect to the union-wide inflation):

\[
\hat{r}_t^n = \beta_0 \hat{r}_{t-1}^n + (1-\beta_0) \beta \hat{x}_t^{\text{UM}} + \varepsilon_t
\]  
(22),

where \( \hat{x}_t^{\text{UM}} = \frac{1}{2} (\hat{x}_t + \hat{x}_t^*) \). The \( \beta > 0 \) coefficient corresponds to the reaction of the monetary policy to the union-wide inflation deviation from the steady-state level; \( \beta_0 \in (0;1) \) is the smoothing coefficient of the nominal interest rate; and \( \varepsilon_t \) is a random variable of zero average and standard deviation equal to 1, corresponding to an exogenous monetary shock (see Clarida & al., 1998).

Definition of the financial shock
In previous equations, \( Q_t \) represents the fundamental value of the firms’ physical capital, given by the actualized amount of dividends to be obtained by the firms’ shareholders. We now allow for the possibility that the market value of the capital, denoted hereafter by \( Q^m_t \), differ temporarily from its fundamental value \( Q_t \), because of a temporary financial shocks \( \varepsilon_q \), such that:

\[
Q^m_t = Q_t + \varepsilon_q
\]

with \( \varepsilon_q \) a random variable of zero average. If the shock arises in \( t \), it affects the market value \( Q^m_t \) of the capital only at this period; afterwards, starting from the \( t+1 \) period, the equality between \( Q^m_t \) and \( Q_t \) holds again\(^{17}\). Hence, in case of financial shock, the fundamental return on the physical capital given in Table 2 becomes an abnormal return on capital given by:

\[
R_{t-1}^{km} = \frac{\rho_{t} \left( \Theta_{t} \right)^{-\gamma} \alpha \frac{Y_t}{K_t} - \frac{\phi}{2} \left( \delta^2 - \left( \frac{I_t}{K_t} \right)^2 \right) + (1 - \delta)Q^m_t}{Q_{t-1}}
\]

Then, \( Q^m_t \) replaces \( Q_t \) in the equations (3), (4), (5) and (8), respectively defining the dynamics of firms’ net worth, banks’ net worth, and the subsequent external finance premiums. So, when \( Q^m_t > Q_t \), the firms’ and banks’ net values increase without any rational justification. The seeming improvement of their balance sheet allows them to obtain better conditions for external financing, stimulating the national investment and output (and inversely in case of adverse financial shock).

Finally, the model is log-linearized around its steady state. The calibration for the parameters and the variables (or ratio) at their steady-state is made according to the references found in the literature for the euro area. Ratios such as capital/GDP, investment/GDP or total consumption expenses/GDP are all compatible with the estimations made for the euro area by Fagan & al. (2001). Moreover, it is realistically supposed that banks have a lower probability of default than firms and that the ratio \( \frac{NB}{B} \) belongs to the interval \([0.1,0.2]\).\(^{18}\) Finally, the probability for a bank to leave the credit market is lower than for firms, and the audit is more costly for households than for banks (what justifies the presence of banks in the economy). The calibration for the baseline model is detailed in appendix 3.

\(^{17}\) Then, the financial shock corresponds to a one-period financial bubble, whereas Bernanke & Gertler (1999) and Levieuge (2009) simulate an exogenous multi-period one. The aim here is not to reproduce the effects of a long-lasting financial bubble, but simply to adequately insert financial shocks in the model.

\(^{18}\) See, for example, the numerical values used by Sunirand (2003) and Levieuge (2009) in models with the bank capital channel, calibrated for the euro area.
3 Financial asymmetries and transmission of shocks inside the union

In accordance with empirical evidence and referring to the analysis summed up in Appendix 1, financial structural heterogeneity is now introduced in the model, by assuming that the banks financial leverage at their steady-state and the sensibility coefficient of the banks’ external finance premium to their financial structure ($\psi^*_B$) are not similar in the two countries. We then analyze the sensitivity of the national dynamics to the degree of the union’s financial heterogeneity. We finally discuss the cost associated to the participation of a given country to such an asymmetric monetary union.

3.1 Transmission channels of shocks and dynamics of the model

We assume that the banking system in country 2 is better capitalized than in country 1
\[
\left(\frac{NB^*}{B^*} = 0.2 > 0.15 = \frac{NB}{B}\right).
\]
Moreover, country 1 is characterized by an external finance premium for banks that is more sensitive to changes in their leverage, compared to country 2\(^{19}\). Concretely, $\psi^*_B = 0.002$ and $\psi^2_B = 0.001$ are chosen for the baseline calibration. To well distinguish the role of the asymmetric bank capital channel in the union, we consider that both countries are identical in the firm-side (i.e. firms’ leverage and sensitivities for firms’ external finance premium are the same). So, besides the symmetrical financial accelerator related to the firms’ financial situation, we expect to obtain an additional and asymmetrical financial accelerator due to the banking sectors heterogeneity.

This is verified in the Figure 1, which represents the dynamics of the two member countries of the union following a restrictive common monetary shock.\(^{20}\) As a whole, the response functions have the expected profile. With respect to the calibration, it unsurprisingly appears that banks’ external finance premiums are more reactive to the shock in country 1, where credit conditions for firms are more severely tightened. Then demand for capital, investment and output decline. Finally, the drop of aggregate demand leads to a further decrease of the inflation rate in country 1, compared with country 2.

The (heterogeneous) bank capital channel is then potentially very powerful. Despite the low calibrated values for the elasticity of banks’ finance premium to their respective balance

\(^{19}\) A lower capitalization ratio at the steady state endogenously implies a higher monitoring cost (see Appendix 3). As the latter leads to the definition of an external financial premium in such a CSV framework, assuming simultaneously a lower capitalization and a higher elasticity to banks’ leverage to characterize the most affected country is logical. All the same, both asymmetries will be analyzed separately straight afterwards.

\(^{20}\) Simulations are implemented with Dynare. See Adjemian & Al. (2009). Note that the Blanchard-Kahn conditions are satisfied; the model has a unique and stable trajectory.
sheet structures, the asymmetries in the national reaction to shock are nevertheless large. The investment and the output decreases are almost 60% more important in country 1, compared with country 2, and the inflation decrease is 30% higher.

Large asymmetrical responses are observed whatever the shock (technological, budgetary or financial). Typically, the figure 2 depicts the impulse response functions for the two countries of the union, face to an unexpected fall in the market value of the physical capital ($Q_m$). This shock negatively affects the agents’ net worth, their financial position and the external finance premiums they must bear. Once again country 1 is more affected than country 2. Two factors contribute to these dissimilar national adjustments. First, national banks face higher external finance premium in country 1 because of their deeper financial fragility and because of the higher sensitivity coefficient $\psi^t_B$ to their balance sheet structure. The cost of the firms’ external finance is subsequently higher, reducing the incentive to invest and the aggregate demand in turn. As a result, inflation falls more in country 1.

*Figure 1. Impulse response functions to a monetary shock*

---

Country 2: Weak bank capital channel ($NB/B = 0.2; \psi^t_B = 0.001$)

Country 1: High bank capital channel ($NB/B = 0.15; \psi^t_B = 0.002$)

17
Second, as the Central Bank reduces the common nominal interest rate accordingly to the average inflation rate, the real interest rate increases more in country 1 than in country 2. In the absence of union, the national Central Bank of the country 1 would have cut its policy rate more than would have done a common Central Bank (with average objectives). This reinforces the adverse effects of the initial shock, as the higher the real interest rate, the lower the investment incentives for firms, the lower the present consumption incentives, and the lower the aggregate demand. Subsequently, the investment drop is more than 100% higher in country 1, and inflation and output divergences are important within the union.

Figure 2. Impulse response functions to a negative financial shock

---

Country 2: Weak bank capital channel \( \left( NB / B = 0.2; \psi \right) \)

Country 1: High bank capital channel \( \left( NB / B = 0.15; \psi \right) \)

3.2 Sensitivity of the economies to the degree of financial heterogeneity

The more heterogeneous the union is, the larger the effect of financial asymmetries on the transmission of shocks. Two sources of financial asymmetries are successively analyzed, following a negative financial shock.
Firstly, the figure 3 illustrates the sensitivity of the economies to differences solely in terms of national banking systems leverage. While the elasticity coefficient for the banks’ external finance ($\psi_B^*$) is fixed to 0.002 for the two countries, the national banking systems leverage take the value 0.1 for country 1, and varies within the interval $[0.1, 0.2]$ for country 2. The national differential in the banks’ and firms’ finance premiums are depicted in the first two graphs. It clearly appears that the higher heterogeneity between national banks’ financial structures, the higher external financial premiums differential (for banks and firms). Consequently, as illustrated by the last graph of the figure 3, which represents the variance of inflation and output differentials under the four scenarios considered, higher financial heterogeneity implies significant higher inflation and output divergences among member countries. The figure in Appendix 2 echoes this numerical experiment. It is shown that the common equity to risk-weighted asset ratios were reduced in 2007-2008 in Europe and the United State, with a deeper decline for the latter. As a result, the AA-rated bank bond index spreads relative to government bonds – a proxy for the external financial premium embedded in the model – increased in both regions, but far more in the United States. The same empirical evidence might be noted among the EMU members.

**Figure 3. Impact of the banking system leverage asymmetry on the model dynamics**

---

Inflation and output divergences in the union

- $\sigma$ (inflation differential) - left side
- $\sigma$ (output differential) - right side
Secondly, the figure 4 illustrates the increasing divergences implied by growing differences in terms of elasticity of the finance premium for banks to their financial structure. It is now assumed that the $\frac{NB}{B}$ ratios are identical and equal to 0.15 in the two countries, while $\psi_B^*$ is fixed to 0.001 for country 2, and varies within the interval $[0.001,0.003]$ for country 1. Differentials are represented in absolute value. Once again, higher heterogeneity in the sensitivity of the national banks’ premiums to their balance sheet structure is associated to more asymmetric transmission of the financial shock inside the union, and to higher inflation and output divergences among member countries.

**Figure 4. Impact of the sensitivity coefficients heterogeneity on the model dynamics**

![Graph showing EFP differential for banks and firms](image)

3.3 The costs of a heterogeneous monetary union

As briefly evoked, it can be demonstrated that the conduct of a single monetary policy for the (financially asymmetric) union as a whole worsens the cyclical divergences.

When considering a symmetric monetary shock in the baseline model, preliminary simulations indicate that the reaction of the output of the country 1 (stronger affected by the bank capital channel) is instantly 60% higher than in country 2. In contrast, if each country were supposed to conduct autonomously its monetary policy, the output response in the country 1
would be only 20\% higher than in country 2.\textsuperscript{21} In other words, a common monetary policy in an asymmetric union implies a \textit{stabilization bias}.

\textit{Figure 5} illustrates the rationale for the stabilization bias in a context of negative and symmetric financial shock. As a common monetary policy seeks to stabilize the average inflation in the union, the interest rate cut is more important in the union than what a national monetary policy would implied for country 2 (which is by definition less sensitive to shocks). Subsequently, this economy benefits from a lower real interest rate, which mitigates the decline of investment and output, and immunizes it to shocks as a whole. On the contrary, the participation to the asymmetric monetary union implies more serious reactions to shocks (compared to the national conduct of monetary policy) for the country with high bank capital channel. Thus, a single monetary policy that only reacts to average variables of an asymmetric union worsens the cyclical divergences among member countries.

\textit{Figure 5. Centralized monetary policy and macroeconomic divergences}

![](image)

\begin{tabular}{l}
<table>
<thead>
<tr>
<th>Common monetary policy</th>
<th>National monetary policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbullet Country 2: weak bank capital channel</td>
<td>\textbullet Country 2: weak bank capital channel</td>
</tr>
<tr>
<td>\textbullet Country 1: high bank capital channel</td>
<td>\textbullet Country 1: high bank capital channel</td>
</tr>
</tbody>
</table>
\end{tabular}

\textsuperscript{21} The model then provides results that are quantitatively in accordance with \textit{Sunirand (2003)} and \textit{Levieuge (2009)} for a homogenous economy.
In this respect, we wonder in the following section whether the consideration of national information for the conduct of monetary policy is likely to mitigate cyclical disparities, and to this end, how national budgetary policies have to be combined.

4 Macroeconomic policies to mitigate the effects of financial heterogeneity

Starting from the empirical evidence mentioned in the introduction about the financial heterogeneity of the euro area, in a situation where financial shocks are not insignificant, this section aims to study the suitable macroeconomic policy-mix for a monetary union embedded in an institutional context based on the Treaty of Lisbon.

On the one hand, we consider an independent common Central Bank whose policy is responsible for the union-wide price stability and which does not cooperate with the national governments (in accordance with the Article 130 of the Treaty). On the other hand, budgetary policies are conducted by the national governments. Alternative strategies for the Central Bank (centralized vs. based on national information) and for governments (budgetary cooperation vs. autonomous conduct of national budgetary policies) are analyzed following a sequential game. The Central Bank chooses first its strategy. National governments observe the orientation of the monetary policy and define afterwards their policies. Simple monetary and budgetary rules are optimized and evaluated in terms of welfare gains, under each configuration.

As indicated in (22), the monetary policy rule links the short-term nominal interest rate to the union-wide inflation. But the coefficients $\beta_0, \beta_1$ are optimized under two alternative objectives. In a centralized strategy, the Central Bank stabilizes only the average inflation for the union, and is not concerned by national divergences. The loss function to be minimized is then given by:\footnote{Following Woodford (2003), the Central Bank loss function could be derived from the intertemporal utility function of the representative agent. Nevertheless, this is in effect not a result, but a hypothesis; Woodford assumes that the objective function of the Central Bank perfectly matches the objectives of the collectivity. This returns to neglect the vast and persuasive literature which indicates that the central bankers’ preferences depend on institutional and political matters, and not only on structural ones. See for instance the survey by Hayo & Hefeker (2008). So it is not less rigorous to directly refer to the actual conduct of the ECB to deduce its preferences. From this viewpoint, it can reasonably be asserted (de facto and de jure) that inflation stability is its single objective. With respect to empirical evidence, a penalty with regard to the interest rate volatility is also added in its objective function in order to reproduce the interest rate smoothing.}

\[
L^{BC} = \operatorname{var}(\hat{x}^{LM}) + \lambda_r \operatorname{var}((\Delta \hat{x})^n) \tag{25}
\]

where $\operatorname{var}(\hat{x})$ defines the second order moment for the $\hat{x}$ variable of the model, and
$\Delta \hat{r}^n = \hat{r}^n - \hat{r}_{t-1}^n$. $\lambda$, is the relative importance given by the monetary policy to the interest rate smoothing. Besides, a monetary strategy based on national information responds to the situation in which the Central Bank is simultaneously concerned by the union-wide inflation stabilization and by the stabilization of the inflation differentials inside the union (Badarau-Semenescu & al., 2009).

The loss function of the Central Bank then becomes:

$$L^{BC} = \text{var}(\hat{\pi}_{UM}) + \lambda \text{var}(\Delta \hat{r}^n), \text{ for } \hat{\pi}_{UM} = \frac{\hat{\pi}_t - \hat{\pi}_t^*}{2}$$

(26)

Now, Budgetary policy takes the form of active budgetary rules, as defined by (21) and (21'), whose coefficient $\rho_y$, $\rho_\pi$, and $\rho_\sigma$ have to be optimally chosen by each national government respectively. Again, two budgetary regimes alternatively considered. In a non-cooperative budgetary policy regime, which refers to an autonomous conduct of national policies, each government optimize the following loss function, considering as exogenous the public expenditures of the other country:

$$L^G = \lambda_\pi^G \text{var}(\hat{\pi}) + \lambda_y^G \text{var}(\hat{y}) + \lambda_g^G \text{var}(\hat{g})$$

(27),

where $\lambda_\pi^G$, $\lambda_y^G$ and $\lambda_g^G$ define the national preferences for inflation, output and public expenditures stabilization, respectively. In a cooperative budgetary policy regime, on the contrary, both governments are endowed by a unique cooperative loss function, calculated as the average of national loss functions:

$$L^{Coop} = \frac{1}{2} \left( L^G + L^{G^*} \right)$$

(28)

According to the new Treaty of Lisbon, entered into force on 1st December 2009, national governments have the autonomy in the conduct of the budgetary policy for their own country, but they must however respect a global orientation decided at the union-wide level. Such a global orientation could be interpreted in terms of common objectives. This implies that $\lambda_\pi^G$, $\lambda_y^G$ and $\lambda_g^G$ in (25) are identical for each national government. This institutional framework can be seen as an implicit coordination mechanism that affects not only the cooperative budgetary regime, but also the non-cooperative one.

---

Simple optimal rules for the monetary policy

The common Central Bank chooses the design of its monetary policy independently of the national governments actions. The results of the optimization procedure, under the two alternative strategies defined above, and considering both technological and financial shocks, are summarized in the table 3. As expected, the centralized monetary policy appears to be more reactive to symmetric shocks than a policy that takes into account the specific situation of member countries.

Table 3. Optimal coefficients for the monetary policy rule

<table>
<thead>
<tr>
<th>Optimal $\beta$ with centralized strategy</th>
<th>Optimal $\beta$ with strategy based on national information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.45704</td>
<td>1.43749</td>
</tr>
</tbody>
</table>

Simple optimal rules for budgetary policies

After the Central Bank announces the orientation of the monetary policy, national governments optimize their budgetary decisions. We seek to consider the reaction of the budgetary policies to symmetric (monetary, technological or financial) shocks that may hit the union. The results of the numerical optimization of the budgetary rules coefficients are summarized in Table 4 for the non-cooperative budgetary regime, and in Table 5 for cooperative national budgetary policies. In both cases, it is assumed that the Central Bank conducts a centralized policy.

It clearly appears from Table 4 that, whichever coefficients for the governmental loss functions, the coefficients for inflation and output stabilization are (as expected) negative in the budgetary rules. Moreover, taken in absolute value, these coefficients are systematically lower in country 2 than in country 1. Precisely, in the calibration, country 1 was supposed to be more sensitive to shocks. It thus needs more stabilization by the budgetary policy, and it is exactly what the government does by choosing higher corresponding coefficients in its budgetary rule. This means that, with a simple non-cooperative budgetary regime, national governments could play an active role in mitigating asymmetries in the transmission of shocks due to the structural heterogeneity of the union.

---

24 In line with Sauer & Sturm (2007), Fourçans & Vranceanu (2007) et Licheron (2009), $\beta_0$ equals 0.96. Optimization was made considering monetary, budgetary and technological shocks with unit standard deviation, and financial shocks with standard deviation equal to 0.2.

25 For asymmetric shocks, the situation reverses. They are better stabilized under a monetary policy oriented to reduce inflation divergences inside the union, than under a centralized monetary policy.

26 Results with an inflation divergences oriented monetary policy are qualitatively similar (see tables in Appendix 4).
Table 4. Non-cooperative budgetary rules coefficients with centralized monetary policy

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1.5; \lambda^G_g = 0.5$</td>
<td>$\rho_s = 0.2189$</td>
<td>$\rho_s^* = 0.1477$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.2022$</td>
<td>$\rho_y^* = -0.1727$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.0861$</td>
<td>$\rho_\pi^* = -0.7125$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_g = 0.5$</td>
<td>$\rho_s = 0.2368$</td>
<td>$\rho_s^* = 0.1720$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1355$</td>
<td>$\rho_y^* = -0.1155$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.7648$</td>
<td>$\rho_\pi^* = -0.5162$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_g = 0.1$</td>
<td>$\rho_s = 0.2175$</td>
<td>$\rho_s^* = 0.1623$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.6526$</td>
<td>$\rho_y^* = -0.5476$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -3.6283$</td>
<td>$\rho_\pi^* = -2.3157$</td>
</tr>
</tbody>
</table>

Unlike the non-cooperative regime, optimal cooperative budgetary rules are not consistent with the stabilization needs of member countries (Table 5). For example, the coefficient associated to the inflation gap in the country 2 budgetary rule is positive, corresponding to a definitely destabilizing effect of the government optimal actions in this country.

Table 5. Cooperative budgetary rules coefficients with centralized monetary policy

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1.5; \lambda^G_g = 0.5$</td>
<td>$\rho_s = 0.1779$</td>
<td>$\rho_s^* = 0.6051$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1901$</td>
<td>$\rho_y^* = -0.0632$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.5985$</td>
<td>$\rho_\pi^* = 0.2576$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_g = 0.5$</td>
<td>$\rho_s = 0.2079$</td>
<td>$\rho_s^* = 0.5625$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1237$</td>
<td>$\rho_y^* = -0.0459$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.4442$</td>
<td>$\rho_\pi^* = 0.2049$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_g = 0.1$</td>
<td>$\rho_s = 0.1929$</td>
<td>$\rho_s^* = 0.9225$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.5955$</td>
<td>$\rho_y^* = -0.0445$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.6366$</td>
<td>$\rho_\pi^* = 0.048$</td>
</tr>
</tbody>
</table>
At first glance, this result seems counter-intuitive. But following Badarau-Semenescu & al. (2009), it can easily demonstrated that the cooperative loss function $L^{Coop}$ can be alternatively written as:

$$L^{Coop} = \hat{\lambda}_x^G \text{var}(\hat{x}_{UM}) + \hat{\lambda}_y^G \text{var}(\hat{y}_{UM}) + \hat{\lambda}_g^G \text{var}(\hat{g}_{UM}) + \hat{\lambda}_x^G \text{var}(\hat{x}_{UM}) + \hat{\lambda}_y^G \text{var}(\hat{y}_{UM}) + \hat{\lambda}_g^G \text{var}(\hat{g}_{UM})$$  \hspace{1cm} (29)

This function implicitly incorporates centralized stabilization objectives and national divergences stabilization objectives. Since all governments accept to fight divergences in the union, one possible solution is that each economy makes an effort to reach the average performance of the union. This explains the positive sign of the national inflation stabilization in the budgetary rule of country 2 (which is supposed to be less affected by shocks). Consequently, the cooperative budgetary regime is not necessarily suitable in an asymmetric monetary union. The need for responding to divergences in inflation, output and public expenditures finally makes the individual stabilizations less satisfying than in a non-cooperative regime.

Figure 6. National responses to a restrictive monetary shock

(for $\hat{\lambda}_x = 1; \hat{\lambda}_y = 1.5; \hat{\lambda}_g = 0.5$)

Figure 6 illustrates this point, in case of restrictive and symmetric monetary shock. As the government in country 2 takes care of macroeconomic divergences in the union, its policy is not expansionist enough to duly stabilize its national output (otherwise it risks to exacerbate the divergences). Precisely, reaction to inflation divergences implies a reduction of public expenditures in country 2 simultaneously to an increase in country 1 (cf. $\hat{\rho}_x$ and $\hat{\rho}_x$ coefficients in...
Moreover, the reduction of output divergences implies a lower increase of public spending in country 2 relatively to country 1 (see $\rho_2$ and $\rho^*_2$ coefficients in table 5). Certainly, the budgetary response of country 2 to government spending divergences asks for an increase of national expenditures, but this pressure is insufficient to compensate the reaction to inflation divergences.

Consequently, as the global effect of these mixed forces finally leads to (excessively) moderate public expenditures (at the national level) in country 2, the country 1 in turn can not envisage implementing an ambitious stimulus scheme. Otherwise, it would be penalized by a growing public expenditures gap. In other words, in country 1, the lower stabilization of the national variables is explained by the reaction of the budgetary policy to government spending divergences. This reduces the amount of public expenditures in the cooperative regime, compared to the non-cooperative one, with consequently less stabilizing effect on the economy. All in all, country 1 does not significantly benefit from a cooperative regime, whereas the situation of country 2 is worsening (comparatively with non-cooperative regime).

**Table 6. Stabilization performance of a cooperative/non-cooperative regime**

<table>
<thead>
<tr>
<th>Financial shock</th>
<th>Monetary shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Country 1:</strong></td>
<td><strong>Country 1:</strong></td>
</tr>
<tr>
<td>$y_1: 1.02$</td>
<td>$\pi_1: 1.02$</td>
</tr>
<tr>
<td>$y_2: 1.06$</td>
<td>$\pi_2: 1.05$</td>
</tr>
<tr>
<td>$\bar{y}^{UM}: 0.98$</td>
<td>$\bar{\pi}^{UM}: 0.98$</td>
</tr>
<tr>
<td><strong>Inflation and output differentials:</strong></td>
<td><strong>Inflation and output differentials:</strong></td>
</tr>
<tr>
<td>$\pi_1: 1.003$</td>
<td>$\pi_1: 1.002$</td>
</tr>
<tr>
<td>$y_2: 1.15$</td>
<td>$\pi_2: 1.07$</td>
</tr>
<tr>
<td>$\bar{y}^{UM}: 0.65$</td>
<td>$\bar{\pi}^{UM}: 0.70$</td>
</tr>
</tbody>
</table>

*The numerical values in the table give the ratios between the standard deviation of the variable $x$ (computed for the first 5 periods after the shock) in the cooperative regime compared to the non-cooperative one: $\sigma^{Coop}_x/\sigma^{NCoop}_x$, for $x \in \{y_1, y_2, \pi_1, \pi_2, \bar{y}^{UM}, \bar{\pi}^{UM}\}$.

The last exam consists in analyzing the qualitative properties of four alternative policy-mixes (centralized/inflation divergences oriented with cooperative/non-cooperative budgetary policies), evaluated with respect to a union-wide social loss function, which corresponds to the average of national social loss functions:

$$EL = \frac{1}{2} \left[ \lambda_y^s \text{var}(\hat{y}) + \lambda^s \text{var}(\hat{y}) + \lambda_y^s \text{var}(\hat{g}) + \lambda^s \text{var}(\hat{g}) + \lambda_y^s \text{var}(\hat{\pi}) + \lambda^s \text{var}(\hat{\pi}) \right]$$

(30),

$\lambda_y^s, \lambda^s, \lambda_y^s$ are symmetric preferences for the stabilization of output gap, inflation and public expenditures in the national social loss functions. Two cases are considered. On the one hand, it is assumed that governments share the preferences of the society for inflation and output stabilization: $\lambda_y^s = \lambda_y^G$, for $x \in \{y, \pi\}$. On the other hand, it is worth considering that
governments are more concerned about the stabilization of public expenditures than the society. In extremis, society is assumed to not really care about the public spending stabilization, in which case: \( \lambda_s^g = 0 \).

The evaluations for these different policy-mixes are reported in Table 7. Three sets of social loss function’s coefficients are considered and reported in the first column. The second column compares the expected losses issued from alternative budgetary regimes, independently of the monetary policy design. The third column compares the expected losses issued from alternative monetary strategies for the Central Bank, independently of the budgetary regime.

Table 7. Expected social loss comparison for alternative policy-mixes

<table>
<thead>
<tr>
<th>Social loss function coefficients</th>
<th>( EL_{S-Coop}^{NC} / EL_{S-Coop}^{Coop} ) (whatever the monetary strategy)</th>
<th>( EL_{S-Coop}^{C} / EL_{S-Coop}^{C+Div} ) (whatever the budgetary regime)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_x^s = 1; \lambda_y^s = 1.5; \lambda_y^g = \lambda_x^g = 0.5 )</td>
<td>( EL_{S-Coop}^{NC} = 1.054 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.972 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
<tr>
<td>( \lambda_x^s = 0 &lt; \lambda_y^g = 0.5 )</td>
<td>( EL_{S-Coop}^{NC} = 0.979 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.973 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
<tr>
<td>( \lambda_x^s = 1; \lambda_y^s = 1; \lambda_y^g = \lambda_x^g = 0.5 )</td>
<td>( EL_{S-Coop}^{NC} = 1.039 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.972 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
<tr>
<td>( \lambda_x^s = 0 &lt; \lambda_y^g = 0.5 )</td>
<td>( EL_{S-Coop}^{NC} = 0.986 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.973 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
<tr>
<td>( \lambda_x^s = 1; \lambda_y^s = 1; \lambda_y^g = \lambda_x^g = 0.1 )</td>
<td>( EL_{S-Coop}^{NC} = 1.12 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.973 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
<tr>
<td>( \lambda_x^s = 0 &lt; \lambda_y^g = 0.1 )</td>
<td>( EL_{S-Coop}^{NC} = 0.902 \times EL_{S-Coop}^{Coop} )</td>
<td>( EL_{S-Coop}^{C} = 0.974 \times EL_{S-Coop}^{C+Div} )</td>
</tr>
</tbody>
</table>

* \( EL_{S-Coop}^{C} \) = expected social loss with a centralized monetary policy; \( EL_{S-Coop}^{C+Div} \) = expected social loss with monetary policy based on national information; \( EL_{S-Coop}^{NC} \) = expected social loss in a non-cooperative budgetary regime; \( EL_{S-Coop}^{Coop} \) = expected social loss in a cooperative budgetary regime.

When the social preferences for stabilization are the same as the governments’ preferences, the expected loss of a centralized monetary policy is systematically lower than in the alternative case where the Central Bank fights inflation divergences in the union. This indicates that a change in the monetary policy design, in favour of inflation divergences, is not suitable.27 Concerning budgetary regimes, under the assumption of identical social and

27 As discussed in Badarau-Semenescu & al. (2009) such modification would be beneficial only if it is supported by simultaneous consideration of output divergences in the union.
governmental stabilizing preferences in the union, the results favour the cooperative regime over the non-cooperative one. But the relative benefit of the cooperative regime comes only from the stabilization of public expenditures and the decrease of their divergences inside the union \( \lambda_g \neq 0 \). Indeed, the computation of the alternative social loss function solely defined in terms of inflation and output stabilization \( \lambda_g^S = 0 \) reasserts the superiority of the non-cooperative regime, as indicated in figure 6. Besides, considering that social preferences mostly concern inflation and output stabilization, whereas governments also care about public spending (level and dispersion) is plausible. Typically, such a difference in preferences could be explained by ad-hoc budgetary constraints like those imposed by the Stability and Growth Pact, which do not necessarily reflect the social preferences. In this case, a non-cooperative regime remains preferable.

5 Conclusions

Considering the well-documented financial heterogeneity of the euro area, paying attention to the bank capital channel (which has generated great interest for several years), and to financial shocks (which are now recurrent), this paper aims to study the suitable policy-mix in such an European context. The analysis relies on a dynamic stochastic general equilibrium model, calibrated in reference to previous studies for the euro area. This model generates conventional dynamics, but with a deeper amplification of shocks, because of the effects of financial accelerator and bank capital channel. Simulations indicate that structural and precisely financial asymmetries lead to striking cyclical divergences among members of the Union. This is true in case of financial shocks, what illustrates the diverging individual responses of European countries following the subprime mortgage crisis. In this vein, it is shown that the more financially heterogeneous the Union is, the larger the cyclical divergences. Moreover, the conduct of a single monetary policy for the Union as a whole seriously worsens these national divergences.

The normative conclusions are the following. Firstly, it appears that a centralized monetary policy dominates a strategy based on inflation divergences in the Union, whatever the budgetary regime. This conclusion confirms previous results in the literature, according to which the aversion of the common Central Bank to national divergences could be beneficial only if it focuses simultaneously on inflation and output (what is not the case for the ECB). Secondly, decentralized budgetary policies need to be more proactive in countries that are structurally more
sensitive to shocks (those where the bank capital channel is more powerful). In that case, budgetary policies can contribute to mitigate the effects of adverse shocks. Thirdly, a cooperative budgetary regime (defined as the average of the national objective functions) is likely to be counterproductive. Indeed, in such a regime, each country has to make a step toward a common target (partly defined as a combination of inflation, output and public spending divergences). If heterogeneity is important, this returns to be unsatisfactory for any country in the end. Finally, it is shown that if social preferences mostly concern inflation and output stabilization, whereas governments also care about public spending (because of the existence of an excessive deficits procedure for instance), then a non-cooperative solution (but with an implicit coordination mechanism implying similar objective functions in the member countries, what is an interpretation of the global common orientation promoted by the new Treaty of Lisbon) is clearly preferable.

Typically, in the context of diverging responses by European economies following the subprime mortgage crisis, a cooperative budgetary regime would have implied an insufficient reaction of national governments, with regards to what their own situation had required. Caring about inflation divergences between the members, the less affected countries like France for instance would have had to refrain from ambitious stimulus plan (compared to what should be nationally required). Otherwise, they would have been responsible for worsening macroeconomic divergences, what is inconsistent with “cooperation”. In the same way, the most affected countries, like Italy for example, would have been constrained to circumscribe their economic stimulus plan, in order to limit the public spending divergences. All in all, European countries would not have benefited from a cooperative budgetary regime, whatever their sensitivity to the financial shock.

Certainly, it would be worth assessing the potential advantages of budgetary federalism in such a context. But the European hesitations that have accompanied the Greek debt crisis tend to demonstrate that such an arrangement is far from being widely accepted. And anyway, monetary and budgetary arrangements are surely not enough to entirely annihilate the effects of adverse shocks (and in particular the effects of financial ones) in a heterogeneous Union. Efforts aimed to improve structural, and particularly financial, convergence are indispensable. On the other hand, micro and macro-prudential measures are essential to dampen the recurrence of financial shocks, or at least to mitigate their macroeconomic effects.
References


Treaty on the European Union and Treaty on the functioning of the European Union (Consolidated version), 6655/1/08 REV1, European Union Council, Bruxelles, 30 avril 2008
Appendix 1. The heterogeneity of the bank capital channel in Europe

Badarau-Semenescu & Levieuge (2010) evaluate the potential strength of the bank balance sheet channel in the European countries. Their analysis relies on national data collected for nine European states, from 1999 to 2007. Indexes of concentration and competitiveness in the banking market, banks’ balance sheet based structural indicators, indexes of the banking system profitability or liquidity, the importance of other financial markets (equity or corporate bonds markets) as substitute to the credit market, the existence of strong relationships among national banks, and the dependency of the domestic agents to the banking credit, are all considered in the study. After extraction and interpretation of the principal components, results are gathered and a cumulative score is calculated for each country. Those are represented in the following figure. A high positive (negative) score is associated to potentially high (weak) bank capital channel, comparatively to the union as a whole (which defines the origin ‘zero’).

The strength of the bank capital channel in euro area countries

Appendix 2. The financial structure and the external finance premium for banks: A comparative illustration United-States / euro area

Aggregated banks’ capital and external financial premiums (FMI)
### Appendix 3. Calibration of the DSGE model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value country 1</th>
<th>Value country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\sigma_c$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Elasticity of labour disutility</td>
<td>$\sigma_h$</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Part of retailers with unchanged prices on the period</td>
<td>$\zeta$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Capital contribution to GDP</td>
<td>$\alpha$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Part of entrepreneurial labour in total labour</td>
<td>$1 - \Omega$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Part of households labour in total labour</td>
<td>$\Omega$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Depreciation rate for capital</td>
<td>$\delta$</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Internal capital adjustment costs parameter</td>
<td>$\phi$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Part of inside capital transferts to survival banks</td>
<td>$t^B$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Banks external finance premium elasticity</td>
<td>$\psi^B$</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Firms external finance premium elasticity</td>
<td>$\psi^F$</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Part of foreign goods in national consumption</td>
<td>$1 - \gamma$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

#### Steady State: Exogenous fixed values

- Real marginal cost: $\rho = 1/1.1$ (country 1), $1/1.1$ (country 2)
- Banks inside capital/loans ratio: $NB/B = 0.15$ (country 1), $0.2$ (country 2)
- Firms net wealth/capital ratio: $NF/K = 0.4$ (country 1), $0.4$ (country 2)
- Public expenditures/GDP ratio: $G/P\text{IB} = 0.16$ (country 1), $0.16$ (country 2)
- Firms probability of default: $F(\omega^F) = 0.03$ (country 1), $0.03$ (country 2)
- Banks probability of default: $F(\omega^B) = 0.07$ (country 1), $0.07$ (country 2)
- Average external finance premium for firms: $r^K - r^F = 0.02$ (country 1), $0.02$ (country 2)

#### Steady State: Calculated values

- Auditing cost for banks: $\mu^B = 0.018$ (country 1), $0.077$ (country 2)
- Auditing cost for households: $\mu^A = 0.807$ (country 1), $0.545$ (country 2)
- Variance for the $\omega$ distribution: $\sigma = 0.2531$ (country 1), $0.2531$ (country 2)
- $\omega$ threshold value for banks: $\overline{\omega}^B = 0.52$ (country 1), $0.52$ (country 2)
- $\omega$ threshold value for firms: $\overline{\omega}^F = 0.6016$ (country 1), $0.6016$ (country 2)
- Banks probability to leave the market: $1 - \gamma^B = 0.01$ (country 1), $0.01$ (country 2)
- Firms probability to leave the market: $1 - \gamma^F = 0.017$ (country 1), $0.017$ (country 2)
- Capital/GDP ratio: $K/Y = 7.0549$ (country 1), $7.0549$ (country 2)
- Investment/GDP ratio: $I/Y = 0.2116$ (country 1), $0.2116$ (country 2)
- Banks consumption expenses/GDP: $CB/Y = 0.006$ (country 1), $0.008$ (country 2)
- Firms consumption expenses/GDP: $CF/Y = 0.048$ (country 1), $0.048$ (country 2)
- Households consumption expenses/GDP: $C/Y = 0.5735$ (country 1), $0.5501$ (country 2)
- Total consumption expenses/GDP: $(C + CF + CB)/Y = 0.628$ (country 1), $0.628$ (country 2)
Appendix 4. Budgetary policies optimization under inflation divergences-oriented monetary policy

Table 4.1 Optimal coefficients for non-cooperative budgetary policy rules

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1.5; \lambda^G_s = 0.5$</td>
<td>$\rho^*_s = 0.2204$</td>
<td>$\rho^*_s = 0.1483$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.2035$</td>
<td>$\rho_y = -0.1740$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.0849$</td>
<td>$\rho_\pi = -0.7107$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_s = 0.5$</td>
<td>$\rho^*_s = 0.2382$</td>
<td>$\rho^*_s = 0.1727$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1364$</td>
<td>$\rho_y = -0.1164$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.7639$</td>
<td>$\rho_\pi = -0.5149$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_s = 0.1$</td>
<td>$\rho^*_s = 0.2191$</td>
<td>$\rho^*_s = 0.1631$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.6578$</td>
<td>$\rho_y = -0.5522$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -3.6250$</td>
<td>$\rho_\pi = -2.3106$</td>
</tr>
</tbody>
</table>

Table 4.2 Optimal coefficients for cooperative budgetary policy rules

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1.5; \lambda^G_s = 0.5$</td>
<td>$\rho^*_s = 0.1810$</td>
<td>$\rho^*_s = 0.5964$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1915$</td>
<td>$\rho_y = -0.0654$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.5978$</td>
<td>$\rho_\pi = 0.2608$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_s = 0.5$</td>
<td>$\rho^*_s = 0.2110$</td>
<td>$\rho^*_s = 0.5355$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1246$</td>
<td>$\rho_y = -0.0476$</td>
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<tr>
<td></td>
<td>$\rho_\pi = -0.4436$</td>
<td>$\rho_\pi = 0.2075$</td>
</tr>
<tr>
<td>$\lambda^G_x = 1; \lambda^G_y = 1; \lambda^G_s = 0.1$</td>
<td>$\rho^*_s = 0.1974$</td>
<td>$\rho^*_s = 0.9098$</td>
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<td></td>
<td>$\rho_y = -0.5996$</td>
<td>$\rho_y = -0.0519$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.6349$</td>
<td>$\rho_\pi = 0.0553$</td>
</tr>
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