Optimal (and simultaneous) Interest and Foreign Exchange feedback policies in a DSGE model for a small open economy
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\(^1\)The views expressed in this paper are the author’s and not necessarily those of the Central Bank of Argentina.
Optimal (and simultaneous) Interest and Foreign Exchange feedback policies in a DSGE model for a small open economy\textsuperscript{2}

1. Introduction

A distinguished scholar specialized in exchange rate policies and regimes states that ‘the overwhelming conventional view in the profession is that it is a mistake to try to manage exchange rates’ (J. Williamson (2007)), although he does not subscribe this view. After having for a long time recommended a basket, band, and crawl (BBC) regime, Williamson lately confesses to have converted to the cause of inflation targeting, but with some significant additional ingredients: ‘most of the time the only monetary policy objective that may merit consideration -other than inflation targeting- is the maintenance of a sufficiently competitive exchange rate to preserve the incentive to invest’... (in tradable sectors). He also argues that ‘the government can expect to reduce misalignments by a policy of intervention. The question is how those interventions should be structured: whether they should be ad-hoc or systematic and, if the latter, how the system should be designed.’

This paper attempts to deal with these issues in a novel way, integrating the usual ‘inflation targeting’ (or Taylor rule) approach with a policy of systematic intervention in the foreign exchange market.

Looking at the empirical literature, the usual conclusion is reflected in the recent IMF Regional Economic Outlook for the Western Hemisphere (IMF (2011)): ‘abundant liquidity in global markets and a high exposure to international capital movements have put foreign exchange intervention (FXI) at center stage of the policy debate in Latin America...The empirical literature (focused mostly on advanced economies) has failed to reach a conclusion about the effects of FXIs on exchange rates, frequently suggesting their absence....Still, many central banks appear to believe in the effectiveness of FXI and continue to pursue such policies.’

Indeed, the empirical literature, concerned mostly with advanced economies, has mainly focused on sterilized intervention, the probable cause being that it captures pure exchange rate policy, whereas unsterilized intervention represents a mix of exchange and monetary policy. But this slants the empirical enquiry, assuming a subordinate role for exchange market intervention, where the primary Central Bank policy has to do with ‘monetary’ policy (this meaning intervening in the ‘money’ market to determine the interest rate). On the other extreme, of course, are those who favor a nominal anchor based on the exchange rate.

In my view there is no justification for maintaining this binary vision of monetary policy: either an inflation target anchor or an exchange rate anchor. But there is a difficulty in trying to escape form this dichotomy in the absence of an adequate theoretical framework, at least in the DSGE modeling that central banks have been practicing recently. My hunch is that this absence is due to the pervasive preference of modelers (theoreticians) to ‘sweep under the rug’ some of the ‘nuts and bolts’ that are necessary to achieve a more general theory. Such ‘nuts and bolts’ as the Central Bank balance sheet and the financial assets and liabilities within it, are included in any IMF Article IV mission report pertaining to developing countries. However, when it comes to modeling they are simply omitted. What makes such

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an omission possible, of course, is that if we accept the dichotomy in question, an argument of system decomposability allows one to focus on the central block of equations. However, if we do not accept the dichotomy, the need for inclusion arises merely to ensure a consistent policy model.

This paper, and the model on which it is based, attempts to build such a consistent policy model. Using the model with various policy frameworks (simple policy rules, optimal simple policy rules, and optimal rules under commitment) and implementing it with Dynare, I find strong evidence that a proper systematic use by Central Banks (CBs) of small open economies (SOEs) of two feedback policy rules, one for the nominal interest rate and another for the rate of nominal depreciation, would outperform the ‘corner’ regimes of inflation targeting and exchange rate peg.

The basic difference between the model used here and the workhorse DSGE model of the profession is the inclusion of more detail in the modeling of money and the institutional structure that takes us closer to a formal representation of how most Central Banks (at least those in developing economies) really implement their interest and foreign exchange policies. The model used, ARGEMmin (a smaller version of two previous models: Escudé (2008) and Escudé (2009)), permits the simultaneous (i.e. within the same quarterly period) intervention in the foreign exchange (FX) and the domestic currency bond markets. The simultaneous use of two policy rules implies a generalization of standard open economy macroeconomic models that are limited to having either a Taylor rule for the interest rate with pure currency float or a pure pegged regime in which there is usually no feedback.

The fact that most CBs of developing economies intervene regularly in both markets should make this generalization of practical interest. And a model that only adds the essential features that are needed to include foreign exchange policy without excluding interest rate policy should help in obtaining intuition as to why the CB can better achieve its objectives, whatever they may be, by the use of two policy rules instead of one.

In the model used in this paper the household decision problem delivers the risk-adjusted uncovered interest parity equation (UIP). The endogenous risk premium function that rest of the world (RW) agents use to determine the interest rate at which they are willing to purchase the economy’s foreign currency bonds plays a fundamental role in the model’s dynamics of capital flows. The use of a risk premium for foreign debt has a long history in open economy macroeconomics (see e.g. Bhandari, Ul Haque and Turnovsky (1990)). In the DSGE strand, Schmitt-Grohé and Uribe (2003) note that the simplest small open economy (SOE) models

\[3^{\text{IMF (2011), for example, notes that ‘on average about on-third of the countries in the region (Latin America) intervened in any given day’. Indeed, their Table 3.1 (Stylized facts of FX Purchases, 2004-10) shows that Colombia and Peru intervened in 32 and 39% of working days, respectively. But this table also contains interesting information on other regions: in the same period, Australia and Turkey intervened in 62 and 66% of working days, respectively, while Israel intervened 24% of working days but with a cumulative intervention that represented 22.3% of GDP.}}\]

\[4^{\text{This differs from my two previous (and larger) models, where it was the decision of banks that delivered the model’s UIP equation.}}\]

\[5^{\text{The world riskfree interest rate, an exogenous liquidity/risk shock and the two components of the terms of trade are additional RW variables that impinge on the SOE of the model. In addition, there is a transitory productivity shock in the domestic sector and a government expenditure (as a ratio to GDP) shock.}}\]
with incomplete asset markets use the assumption that the subjective discount rate equals the average real interest rate and, hence, present equilibrium dynamics that have a random walk component. They present five alternative modifications that have been used to eliminate this random walk component and show that they have quite similar dynamics. Among these modifications is the complete assets market model (i.e., doing away with the incomplete asset markets assumption altogether) and, more relevant for this paper, the use of a risk premium function by which the interest rate on foreign funds responds to the amount of debt outstanding. In the latter variant, the steady state Euler and UIP equations give an equation such as 
\[ \beta (1 + i^*) \varphi_D (d) = \pi, \]
where \( i^* \) is the RW’s non-stochastic steady state (NSS) real interest rate, \( \pi \) is the SOE’s inflation rate, \( d \) is the SOE’s foreign debt and \( \varphi_D (\cdot) \) is a risk premium function. This equation then determines \( d \) as a function of model parameters, including those that define the risk premium function. Lubik (2007) adds that even if there is an exogenous risk premium function, to avoid the unit root problem it is necessary that it be fully internalized by the individual households, i.e., that each household take into account that other households’ decisions are the same as its own and, hence, that the risk premium it faces is a function of the aggregate (and not its individual) foreign debt. The only significant change that this paper presents with respect to the risk premium is that \( \varphi_D (\cdot) \) is a function of the foreign debt to GDP ratio: \( ed/Y \) (where \( e \) is the SOE’s real exchange rate (RER) and \( Y \) is its GDP) and that there is an additional multiplicative shock \( \phi^* \) (giving \( \phi^* \varphi_D (\cdot) \)) that may represent either an exogenous component of the risk function or an international liquidity shock (or both).

Simply for convenience, I call the policy framework where the CB uses two simultaneous policy rules a Managed Exchange Rate (MER) regime. I explicitly include the instruments that the CB uses for its intervention in the two markets as well as the CB balance sheet that binds them. Hence, the CB balance sheet is one of the model equations. It has cash \( m_t \) and CB-issued domestic currency bonds \( b_t \) on the liabilities side and foreign currency reserves \( r_t \) on the asset side. To make sure that there are no loose ends, I explicitly consider the CB’s flow budget constraint and assume that the institutional framework is such that any ‘quasi-fiscal’ surplus (or deficit) is handed over (financed) period by period to the Treasury, defining ‘quasi-fiscal surplus’ as financial flows (specifically, those related to interest earned and capital gains on international reserves, and the interest paid on CB bonds) that could make the CB net worth different from zero. Hence, while there is overall fiscal consistency (since the Treasury is assumed to be able to collect enough lump-sum taxes each period to finance its expenditures in excess of the quasi-fiscal surplus), the CB has a constraint each period on its two instruments \( (r_t \text{ and } b_t) \): \( e_t r_t = m_t + b_t \), where \( e_t \) and \( m_t \) are the equilibrium real exchange rate (RER) and real cash held by households. One can say that this equation defines how much the CB ‘sterilizes’ (through the issuance of domestic currency bonds) any unwanted monetary effect of its simultaneous and systematic monetary and exchange policy. However, I avoid the expression ‘sterilized intervention’ (in the foreign exchange market) because it implicitly gives the exchange rate policy a subordinate role (the undesired effects of which must be ‘sterilized’ to avoid disrupting the monetary equilibrium that is achieved through the use of conventional monetary policy). Generality is best preserved treating both interventions in a symmetrical way,
neither of which ‘sterilizes’ the effects of the other. When the CB intervenes in both the money and foreign exchange market, it is subject to the set of constraints given by the equations of the model, among which is monetary equilibrium, and the assumed institutional constraint that the CB’s net worth is kept at zero. Notice that the latter can be expressed as an institutional constraint of the CB preserving a ‘full backing’ of its domestic currency liabilities with (the domestic currency value of) its foreign reserves. Clearly, other similar constraints could be used for the same purpose of endogenizing the CB’s ‘sterilization’ policy. The one I use has the virtue of simplicity. The important point is that the overall means that the CB has available to use trying to achieve its objectives be made explicit. To further ensure consistency, the model includes the balance of payments (where both household foreign debt and CB reserves play relevant roles) and the fiscal equation.

First, I study the functioning of simple rules by looking at the effects of the policy rule coefficients on the variability of the main endogenous variables. I show that the basic ingredients of the usual workhorse model are preserved, such as the ‘Taylor Principle’ and Woodford’s generalization stressing the important role of interest rate inertia. I also show that there are clear benefits in reducing the variability of some of the target variables by being able to intervene simultaneously in the two markets. Second, I study optimal simple rules by obtaining the policy rule coefficients that minimize a CB quadratic objective function for different sets of weights (that define CB ‘styles’) and the three alternative exchange regimes. Third, I obtain the optimal policy rules under commitment in a linear-quadratic stochastic optimal control framework with perfect information (as in Svensson and Woodford (2002)) and show that there are indeed gains from using these two simultaneous policy rules instead of only one of the ‘corner’ regimes.

The rest of the paper has the following structure. Section 2 sets up the model, Section 3 shows the numerical results obtained using Dynare, and Section 4 concludes. Appendix I shows how the model parameters and the non-stochastic steady state (NSS) were jointly calibrated. Finally, Appendix 2 shows some a selection of the impulse response functions for the optimal simple rules and the optimal policy under commitment.

2. The model
2.1. Households
2.1.1 The household optimization problem

Infinitely lived identical households consume a CES bundle of domestic and imported goods and hold financial wealth in the form of domestic currency cash ($M_t$) and peso denominated one period nominal bonds issued by the CB ($B_t$) that pay a nominal interest rate $i_t$. They also issue one period foreign currency bonds ($D_t$) in the international capital market that pay a nominal (foreign currency) interest rate $i^F_t$. I assume that the CB fully and credibly insures investors in CB bonds, so the domestic currency nominal rate is considered riskless. However, foreign investors are only willing to hold the SOE’s foreign currency bonds if they receive a risk premium over the international riskless rate $i^*$. Since I do not model the RW, the premium function is exogenously given. It has an exogenous stochastic and time-varying component $\phi_t^*$ (that can represent general liquidity conditions in the
international market) as well as an endogenous (more country risk-related) component \( \tau_D(.) \) that is an increasing convex function of the aggregate foreign debt to GDP ratio. Individual households are assumed to fully internalize the dependence of the interest rate they face on the aggregate (instead of individual) foreign debt based on to their knowledge that all households are (at least in this aspect) identical. The foreign currency gross interest rate households face is:

\[
1 + i_D^t = (1 + i_t^*)\phi_t^\tau_D(\gamma_t^D),
\]

where

\[
\begin{align*}
\gamma_t^D &= S_t D_t = e_t d_t, \\
e_t &= \frac{S_t P_t^*}{P_t}, \\
d_t &= \frac{D_t}{P_t^*}.
\end{align*}
\]

\( \gamma_t^D, e_t, \) and \( d_t \), are the foreign debt to GDP ratio, the real exchange rate, and real foreign debt (in terms of foreign prices), respectively, \( S_t \) is the nominal exchange rate, \( P_t \) is the domestic goods price index, \( P_t^* \) is the price index of the goods the SOE imports, and \( Y_t \) is GDP. I assume that the gross risk premium function is increasing and convex \((\tau_D > 1, \tau_D' > 0 \text{ and } \tau_D'' > 0)\).

The household holds cash \( M_t \) because doing so it economizes on transaction costs. I assume that transaction frictions result in a loss of purchasing power (through the non-utility generating consumption of domestic goods) when households purchase consumption goods, and that this cost can be ameliorated using cash.\(^6\) To purchase consumption goods \( C_t \) households must spend \( \tau_M(\gamma_t^M) P_t C_t \) where \( C_t \) is a consumption index, and \( P_t C_t \) is the price index of the consumption bundle. All price indexes are in monetary units. The gross transactions cost function \( \tau_M(\gamma_t^M) \) is assumed to be a decreasing and convex function \((\tau_M > 1, \tau_M' < 0, \tau_M'' > 0)\) of the cash/consumption ratio \( \gamma_t^M \):

\[
\gamma_t^M = \frac{M_t}{P_t C_t} = \frac{m_t}{p_t C_t},
\]

where

\[
\begin{align*}
p_t^C &= \frac{P_t^C}{P_t}, \\
m_t &= \frac{M_t}{P_t}
\end{align*}
\]

are the relative price of consumption goods and real cash.

The representative household maximizes an inter-temporal utility function which is additively separable in (constant relative risk aversion subutility functions of) goods \( C_t \) and labor \( N_t \):

\[
E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi^N N_{t+j}^{1+\sigma^N} \right\},
\]

where \( \beta \) is the intertemporal discount factor, \( \sigma^C \), and \( \sigma^N \) are the constant relative risk aversion coefficients for goods and labor, respectively, and \( \xi^N \) is a parameter.

\(^6\)The introduction of money is similar to the theoretical treatment in Montiel (1999), and also to the numerically implemented Schmitt-Grohé and Uribe (2004). It differs from the latter in that instead of defining velocity I use its inverse (the cash/consumption ratio), and I use a different specification of the transactions cost function.
The household receives income from profits, wage, and interest, and spends on consumption, interest, and taxes. Its nominal budget constraint in period $t$ is:

$$\tau_M(\gamma^M_t) P_t^C C_t + M_t + B_t - S_t D_t = W_t N_t + \Pi_t - Tax_t$$

$$+ M_{t-1} + (1 + i_{t-1}) B_{t-1} - (1 + i^D_{t-1}) S_{t-1}$$

where $i_t$ is the interest rate that CB bonds pay each quarter, $W_t$ is the nominal wage rate, $\Pi_t$ is nominal profits, and $Tax_t$ is lump sum taxes net of transfers. Introducing (1) in (6) and dividing by $P_t$, the real budget constraint is:

$$\tau_M(\gamma^M_t) P_t^C C_t + m_t + b_t - e_t d_t = w_t N_t + \frac{\Pi_t}{P_t} - tax_t + \frac{m_{t-1}}{\pi_t}$$

$$+ (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} - (1 + i^*_{t-1}) \phi^{-1}_{t-1} \pi^*_t e_t \frac{d_{t-1}}{\pi^*_t}$$

where

$$b_t \equiv \frac{B_t}{P_t}, \quad w_t \equiv \frac{W_t}{P_t}, \quad tax_t \equiv \frac{Tax_t}{P_t}, \quad \pi_t \equiv \frac{P_t}{P_{t-1}}, \quad \pi^*_t \equiv \frac{P^*_t}{P_{t-1}}$$

are the real stock of domestic currency bonds, the real wage (in terms of domestic goods), real lump sum tax collection, and the gross rates of quarterly inflation for domestic goods and foreign goods, respectively.

The household chooses the sequence $\{C_{t+j}, m_{t+j}, b_{t+j}, d_{t+j}, N_{t+j}\}$ that maximizes (5) subject to its sequence of budget constraints (7) (and initial values for the predetermined variables). The Lagrangian is hence:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}}{1 - \sigma^C} - \xi^N N_{t+j} \frac{1 + \sigma^N}{1 + \pi^N_t} + \lambda_{t+j} \left\{ w_{t+j} N_{t+j} + \frac{\Pi_{t+j}}{P_{t+j}} + \frac{m_{t-1+j}}{\pi_{t+j}} \right\} 

+ (1 + i_{t+j}) \frac{b_{t-1+j}}{\pi_{t+j}} - (1 + i^*_t) \phi^{-1}_{t-1+j} \pi^*_{t+j} \frac{e_{t+j} d_{t-1+j}}{Y_{t-1+j}} \right\}$$

$$- \tau_M \left( \frac{m_{t+j}}{P_t^C C_{t+j}} \right) \phi^C_{t+j} C_{t+j} - m_{t+j} - b_{t+j} + e_{t+j} d_{t+j} - tax_{t+j}$$

where $\beta^j \lambda_{t+j}$ are the Lagrange multipliers, and can be interpreted as the marginal utility of real income.\(^7\)

The first order conditions for an optimum are the following:

\begin{align*}
C_t & : \quad C_t^{-\sigma^C} = \lambda_t p_t^C \varphi_M (m_t/p_t^C C_t) \quad (9) \\
m_t & : \quad \lambda_t \left[ 1 + \tau^*_M \left( m_t/p_t^C C_t \right) \right] = \beta E_t (\lambda_{t+1}/\pi_{t+1}) \quad (10) \\
b_t & : \quad \lambda_t = \beta (1 + i_t) E_t (\lambda_{t+1}/\pi_{t+1}) \quad (11) \\
d_t & : \quad \lambda_t e_t = \beta (1 + i^*_t) \phi^{-1}_{t+1} \varphi_D (e_{t+1} d_t/Y_t) E_t (\lambda_{t+1} e_{t+1}/\pi^*_t) \quad (12) \\
N_t & : \quad \xi^N N_t^{\sigma^N} = \lambda_t w_t \quad (13)
\end{align*}

\(^7\)There is also a no-Ponzi game condition that I omit for simplicity and yields the transversality condition $\lim_{t \to -\infty} \beta^j d_t = 0$ that prevents households from incurring in Ponzi games.
Notice that in (9) and (12) the auxiliary functions $\varphi_M$ and $\varphi_D$ have been introduced merely to obtain a more compact notation:

$$
\varphi_D (\gamma^D) \equiv \tau_D (\gamma^D) + \gamma^D \tau'_D (\gamma^D), \\
\varphi_M (\gamma^M) \equiv \tau_M (\gamma^M) - \gamma^M \tau'_M (\gamma^M).
$$

Combining (10) and (11) implicitly gives the demand for cash as a function of the nominal interest rate and consumption expenditure:

$$
\tau'_M \left( \frac{m_t}{p^C_t C_t} \right) = 1 - \frac{1}{1 + i_t},
$$

Inverting $-\tau'_M$ gives the explicit demand function for cash as a vehicle for transactions (or 'liquidity preference' function):

$$
m_t = \mathcal{L} \left( 1 + i_t \right) p^C_t C_t,
$$

where $\mathcal{L}(.)$ is defined as:

$$
\mathcal{L} \left( 1 + i_t \right) \equiv \left( -\tau'_M \right)^{-1} \left( 1 - \frac{1}{1 + i_t} \right),
$$

and is strictly decreasing, since:

$$
\mathcal{L}' \left( 1 + i_t \right) = \left[ -\tau''_M \mathcal{L} \left( 1 + i_t \right) \right] \left( 1 + i_t \right)^2 < 0.
$$

Under the assumption that the Central Bank always satisfies cash demand, from now on I call (15) the money market clearing condition.

Using (9) to eliminate $\lambda_t$ from (11) yields a version of the classical Euler equation that reflects the additional influence of the use of money on transactions costs:

$$
\frac{C_t - \sigma^C}{\varphi_M \left( \frac{m_t}{p^C_t C_t} \right)} = \beta \left( 1 + i_t \right) E_t \left( \frac{C_{t+1} - \sigma^C}{\varphi_M \left( \frac{m_{t+1}}{p^C_{t+1} C_{t+1}} \right)} \frac{1}{\pi^C_{t+1}} \right),
$$

where $\pi^C_t \equiv P_t^C / P_{t-1}^C$ is the gross rate of inflation of the basket of consumption goods and I have used the identity:

$$
\frac{p^C_t}{p^C_{t-1}} = \frac{\pi^C_t}{\pi_t}
$$

(based on the definition of $p^C_t$ in (4)) to eliminate the rate of inflation for domestic goods.

The definition of the RER in (2) gives the following identity:

$$
\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi^*_t}{\pi_t},
$$

where $\delta_t \equiv S_t / S_{t-1}$ is the rate of nominal depreciation of the domestic currency. Hence, (12) may be written as:

$$
1 = \beta \left( 1 + i^*_t \right) \varphi_D \left( \frac{e_t d_t}{Y_t} \right) E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{\delta_{t+1}}{\pi_{t+1}} \right).
$$
Also, multiplying both sides of (11) by $\delta_{t+1}$ and applying the expectations operator gives:

$$E_t \delta_{t+1} = \beta (1 + i_t) E_t \left( \frac{\lambda_{t+1} \delta_{t+1}}{\lambda_t / \pi_{t+1}} \right).$$

The last two equations yield the risk-adjusted uncovered interest parity equation:

$$1 + i_t = (1 + i_t^2) \phi_t \varphi_D \left( \frac{e_t d_t}{Y_t} \right) E_t \delta_{t+1}. \quad (20)$$

Finally, eliminating $\lambda_t$ from (13) gives the household’s labor supply:

$$N_t = \left( \frac{w_t}{e^N p_t^C C_t^D \varphi_M (m_t / p_t^C C_t)} \right)^{\frac{1}{\sigma^C}}. \quad (21)$$

### 2.1.2 Domestic and imported consumption

The consumption index used in the household optimization problem is a constant elasticity of substitution (CES) aggregate consumption index of domestic ($C_t^D$) and imported ($C_t^N$) goods:

$$C_t = \left( a_D \frac{1}{\sigma^C} (C_t^D)^{\frac{\sigma^C - 1}{\sigma^C}} + a_N \frac{1}{\sigma^C} (C_t^N)^{\frac{\sigma^C - 1}{\sigma^C}} \right)^{\frac{1}{\sigma^C}}, \quad a_D + a_N = 1. \quad (22)$$

$\sigma^C (\geq 0)$ is the elasticity of substitution between domestic and imported goods. Total consumption expenditure is:

$$P_t^C C_t = P_t^D C_t^D + P_t^N C_t^N, \quad (23)$$

where $P_t^N$ is the domestic currency price of imported goods. Then minimization of (23) subject to (22) for a given $C_t$, yields the following relations:

$$P_t = P_t^C \left( \frac{C_t^D}{a_D C_t} \right)^{-\frac{1}{\sigma^C}} \quad (24)$$

$$P_t^N = P_t^C \left( \frac{C_t^N}{a_D C_t} \right)^{-\frac{1}{\sigma^C}}. \quad (25)$$

Introducing these in (22) yields the consumption price index:

$$P_t^C = \left( a_D (P_t)^{1-\sigma^C} + a_N (P_t^N)^{1-\sigma^C} \right)^{\frac{1}{1-\sigma^C}}. \quad (26)$$

Dividing (26) through by $P_t$ yields a relation between the relative prices of consumption and imported goods (both in terms of domestic goods):

$$p_t^C = \left( a_D + (1 - a_D) (P_t^N)^{1-\sigma^C} \right)^{\frac{1}{1-\sigma^C}}, \quad (27)$$

where

$$p_t^N \equiv \frac{P_t^N}{P_t}.$$
For simplicity, I assume that the Law of One Price holds. Hence, the domestic price of (the aggregate of) imported goods is simply:

\[ P_t^N = S_t P_t^* \]

This implies that the domestic relative price of imports is simply the RER:

\[ p_t^N = \frac{P_t^N}{P_t} = \frac{S_t P_t^*}{P_t} = e_t. \] (28)

Hence, the relative price of the consumption bundle (27) is:

\[ p_t^C = \left( a_D + (1 - a_D) e_t^{1-\theta_C} \right)^{\frac{1}{1-\theta_C}}. \] (29)

(24) and (25) show that \( a_D \) and \( a_N = 1 - a_D \) in (22) are directly related to the shares of domestic and imported consumption in total consumption expenditures. In fact, the shares are: \(^8\)

\[ \frac{C_t^D}{p_t^C C_t} = a_D \frac{1}{(p_t^C)^{1-\theta_C}} \] (30)

\[ \frac{p_t^N C_t^N}{p_t^C C_t} = (1 - a_D) \left( \frac{e_t}{p_t^C} \right)^{1-\theta_C} \] (31)

I assume throughout that there is a bias for domestic goods, i.e., \( a_D > 1/2 > a_N \), and that \( \theta^C > 1 \).

\( C_t^D \) is a CES aggregate of an infinite number of domestic varieties of goods, each produced by a monopolist under monopolistic competition:

\[ C_t^D = \left( \int_0^1 C_t^D(i) \frac{\theta-1}{\theta-1} d\hat{i} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \] (32)

where \( \theta \) is the elasticity of substitution between varieties of domestic goods in household expenditure.

Conditions (24), and (25) are necessary for the optimal allocation of household expenditures across domestic and imported bundles of goods. Similarly, for the optimal allocation across varieties of domestic goods within the first of these classes, use of (32) yields the following necessary conditions:

\[ P_t(i) = P_t \left( \frac{C_t^D(i)}{C_t^D} \right)^{\frac{1}{\theta}}. \]

\(^8\)In the Cobb-Douglas case (\( \theta^C = 1 \)) the shares are \( a_D \) and \( a_N = 1 - a_D \) (and hence are time invariant). But in this case the relative demand of domestic to imported goods is independent of \( p_t^N \) (and hence, the RER), which is something not too desirable. With \( \theta^C > 1 \) an increase in the relative price of imported goods increases the relative demand for domestic goods.
2.2. Firms

2.2.1 The representative final goods firm

There is perfect competition in the production (or bundling) of final domestic output \( Q_t \), with the output of intermediate firms as inputs. A representative final domestic output firm uses the following CES technology:

\[
Q_t = \left( \int_0^1 Q_t(i)^{\theta \sigma} di \right)^{\frac{\theta}{\sigma - 1}}, \quad \theta > 1
\]  
(33)

where \( Q_t(i) \) is the output of the intermediate domestic good \( i \). The final domestic output representative firm solves the following problem each period:

\[
\max_{Q_t(i)} P_t \left( \int_0^1 Q_t(i)^{\theta \sigma} di \right)^{\frac{\theta}{\sigma - 1}} - \int_0^1 P_t(i) Q_t(i) di,
\]  
(34)

the solution of which is the demand for each type of domestic good (as an input):

\[
Q_t(i) = Q_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta}.
\]  
(35)

Introducing (35) in (33) and simplifying, it is readily seen that the domestic goods price index is:

\[
P_t = \left( \int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.
\]  
(36)

Also, introducing (35) into the cost part of (34) yields:

\[
\int_0^1 P_t(i) Q_t(i) di = P_t Q_t.
\]

2.2.2 The monopolistically competitive firms

A continuum of monopolistically competitive firms produce the intermediate domestic goods (that the final goods producer bundles) using homogenous labor, with no entry or exit. The production function of each firm is:

\[
Q_t(i) = \epsilon_t N_t(i)
\]  
(37)

where \( \epsilon_t \) is an industry-wide transitory productivity shock.

Since \( N_t(i) \) is firm \( i \)'s labor demand, using (37) and (35) and integrating yields aggregate labor demand:

\[
N_t^L = \int_0^1 N_t(i) di = \int_0^1 \frac{Q_t(i)}{\epsilon_t} di = \frac{1}{\epsilon_t} \int_0^1 Q_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di = \frac{Q_t}{\epsilon_t} \Delta_t
\]  
(38)

where (as in Schmitt-Grohé and Uribe (2004) and (2007)) I defined a measure of price dispersion at period \( t \):

\[
\Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di \geq 1.
\]
Notice that $\Delta_t = 1$ when all prices are the same and greater than one otherwise.\(^9\)

Equating labor supply (21) and demand (38) gives the labor market equilibrium real wage (in terms of domestic goods):

$$w_t = \xi^N \left( \frac{Q_t}{\epsilon_t} \Delta_t \right)^{\sigma^N} p_t^C C_t^{\sigma_C} \varphi_M \left( m_t/p_t^C C_t \right)$$ (39)

Each firm’s cost is $W_t N_t(i) = (W_t/\epsilon_t) Q_t(i)$. Hence, its marginal cost is $W_t/\epsilon_t$ and its real marginal cost (in terms of domestic goods) is:

$$mc_t = \frac{w_t}{\epsilon_t}.$$ (40)

Notice that all firms face the same marginal cost. Therefore, using (39) to eliminate $w_t$, real marginal cost can be written as:

$$mc_t = \frac{\xi^N}{\epsilon_t} \left( \frac{Q_t}{\epsilon_t} \Delta_t \right)^{\sigma^N} p_t^C C_t^{\sigma_C} \varphi_M \left( m_t/p_t^C C_t \right),$$ (41)

showing that increases in price dispersion raise the real marginal cost of firms. This is due to the positive effect of increased price dispersion on aggregate labor demand and, given the level of supply, on the real wage. Furthermore, tighter monetary conditions increase marginal cost because an increase in $i_t$ makes households economize on cash, lowering $m_t/p_t^C C_t$. Because $\varphi_M' = -\gamma M^{-\rho} < 0$, this has a positive effect on $\varphi_M$, lowering labor supply and hence increasing the equilibrium real wage.

### 2.2.3 The dynamics of inflation and price dispersion

Firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1 - \alpha$ of being able to set the optimum price for its specific type of good. The firms that can’t optimize must leave the same price they had last period. The pricing problem of firms that get to optimize is:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j N_{t+j} Q_{t+j}(i) \left\{ \frac{P_t(i)}{P_{t+j}} - mc_{t+j} \right\}$$ (42)

\(^9\)That $\Delta_t \geq 1$ is neatly proved by Schmitt-Grohé and Uribe (2007). Define $v_{i,t} = (P_t(i)/P_t)^{1-\theta}$. Dividing (36) by $P_t$ yields:

$$1 = \left( \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di \right)^{\frac{1}{1-\theta}} = \left( \int_0^1 v_{i,t} di \right)^{\frac{1}{1-\theta}}.$$

Hence,

$$1^{-\theta} = 1 = \left( \int_0^1 v_{i,t} di \right)^{\frac{\theta}{1-\theta}} \leq \int_0^1 (v_{i,t})^{\frac{\theta \cdot \gamma}{1-\theta}} di = \Delta_t,$$

where use is made of Jensen’s inequality.
subject to the demand they will face until they can again optimize:

\[ Q_{t+j}(i) = Q_{t+j} \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta}. \]  \hspace{1cm} (43)

\( \Lambda_{t,t+j} \) is the pricing kernel used by domestic firms for discounting, which, since firms are owned by households and respond to their preferences, is equal to households’ intertemporal marginal rate of substitution in the consumption of domestic goods between periods \( t+j \) and \( t \):

\[ \Lambda_{t,t+j} \equiv \beta \frac{U_{CD,t+j}}{U_{CD,t}}. \]

Notice that the marginal utility of consuming domestic goods can be obtained from the marginal utility of consuming the aggregate bundle of (domestic and imported) goods. Specifically:

\[ U_{CD,t} = U_{C,t} \frac{dC_t}{dC_{t,j}} = U_{C,t} \theta_{CD} \left( \frac{C_D}{C_t} \right)^{-\frac{1}{\theta}} = C_t^{-\sigma_C} \frac{P_t}{P_{t+j}} = \frac{1}{p_t C_t^{\sigma_C}}, \]

where the second equality if obtained by differentiating (22) with respect to \( C_t \), and the third comes from using (24). Hence, the pricing kernel of domestic firms is:

\[ \Lambda_{t,t+j} \equiv \beta \frac{p_t C_t^{\sigma_C}}{p_{t+j} C_{t+j}^{\sigma_C}}. \]  \hspace{1cm} (44)

Introducing (43) and (44) in (42) (and eliminating irrelevant multiplying terms that refer to time \( t \)) gives

\[ \max_{P_t(i)} E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j} C_{t+j}^{\sigma_C}} \left\{ \left( \frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} - mc_{t+j} \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta} \right\}. \]

Since by symmetry all optimizing firms make the same decision I call the optimum price \( \tilde{P}_t \) and drop the firm index. Hence, the firm’s first order condition is the following:

\[ 0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j} C_{t+j}^{\sigma_C}} \left( \frac{P_{t+j}}{P_t} \right)^{\theta} \left\{ \tilde{p}_t \tilde{P}_t - \frac{\theta}{\theta - 1} mc_{t+j} \right\}. \]  \hspace{1cm} (45)

where \( \tilde{p}_t \equiv \tilde{P}_t/P_t \) is the relative price of firms that optimize and the general price level (which includes both optimizers and non-optimizers). In the Calvo setup, because optimizers (and hence non-optimizers) are randomly chosen from the population, the average price in \( t - 1 \) of non-optimizers (which must keep their price constant) is equal to the overall price index in \( t - 1 \) no matter when they optimized for the last time. Hence, (36) implies the following law of motion for the aggregate domestic goods price index:

\[ P_t^{1-\theta} = \alpha (P_{t-1})^{1-\theta} + (1 - \alpha) \tilde{P}_t^{1-\theta}. \]  \hspace{1cm} (46)
Dividing through by $P_{t-1}^1$ and rearranging yields the relative price of optimizers as an increasing function of the inflation rate:

$$\tilde{p}_t = \left( \frac{1 - \alpha \pi_{t-1}^2}{1 - \alpha} \right) \tilde{p}_{t-1} \equiv \tilde{p}(\pi_t).$$

(47)

Hence, using this in (45) gives the (non-linear) Phillips equation that determines the dynamics of domestic inflation:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{P_{t+j}^C C_{t+j}^C} \left( \frac{P_{t+j}}{P_t} \right)^\theta \left\{ \tilde{p}(\pi_t) \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta - 1} mc_{t+j} \right\}.$$

(48)

In order to implement the Phillips equation in Dynare I express this in a recursive (nonlinear) form. To simplify notation, define the marginal utility of domestic output ($A_t$) and the expected discounted marginal revenue ($\Gamma_t$) and marked-up marginal cost ($\Psi_t$):

$$A_t \equiv \frac{Q_t}{P_t^C C_t^C},$$

$$\Gamma_t = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\theta-1},$$

$$\Psi_t = \frac{\theta}{\theta - 1} E_t \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^\theta mc_{t+j}$$

and express (48) as:

$$\tilde{p}(\pi_t) \Gamma_t = \Psi_t.$$

Now write $\Gamma_t$ and $\Psi_t$ recursively as follows:

$$\Gamma_t = A_t + E_t \sum_{j=1}^{\infty} (\beta \alpha)^j A_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\theta-1}$$

$$= A_t + \beta \alpha E_t \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j+1} \left( \frac{P_{t+j+1}}{P_{t+1}} \tilde{\pi}_{t+1} \right)^{\theta-1}$$

$$= A_t + \beta \alpha E_t \tilde{\pi}_{t+1}^{\theta-1} E_{t+1} \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j+1} \left( \frac{P_{t+j+1}}{P_{t+1}} \right)^{\theta-1}$$

$$= A_t + \beta \alpha E_t \tilde{\pi}_{t+1}^{\theta-1} \Gamma_{t+1}$$

$$\Psi_t = \frac{\theta}{\theta - 1} A_t mc_t + E_t \left( \frac{\theta}{\theta - 1} \right) \sum_{j=1}^{\infty} (\beta \alpha)^j A_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^\theta mc_{t+j}$$

$$= \frac{\theta}{\theta - 1} A_t mc_t + \beta \alpha E_t \left( \frac{\theta}{\theta - 1} \right) E_{t+1} \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j+1} \left( \frac{P_{t+j+1}}{P_{t+1}} \tilde{\pi}_{t+1} \right)^\theta mc_{t+1+j}$$

$$= \frac{\theta}{\theta - 1} A_t mc_t + \beta \alpha E_t \tilde{\pi}_{t+1}^{\theta-1} E_{t+1} \sum_{j=0}^{\infty} (\beta \alpha)^j A_{t+j+1} \left( \frac{P_{t+j+1}}{P_{t+1}} \right)^\theta mc_{t+1+j}$$

$$= \frac{\theta}{\theta - 1} A_t mc_t + \beta \alpha E_t \tilde{\pi}_{t+1}^{\theta-1} \Psi_{t+1}.$$
Hence, the complicated Phillips equation (with infinite summations) is transformed into three simple nonlinear equations:

\[ \Gamma_t = A_t + \beta \alpha \pi_t^{\theta-1} \Gamma_{t+1} \]
\[ \Psi_t = \frac{\theta}{\theta - 1} A_t m c_t + \beta \alpha \pi_t^{\theta} \Psi_{t+1} \]
\[ \Psi_t = \Gamma_t \tilde{\mu}(\pi_t) \]

Notice that collapsing the log-linear approximations of these equations yields the usual log-linearized Phillips equation:

\[ \tilde{\pi}_t = (1 - \beta \alpha) \left(1 - \frac{\alpha}{\alpha}\right) m c_t + \beta \alpha \pi_t^{\theta} \tilde{\pi}_{t+1} \]

\[ \Delta_t \] is an additional variable in the model, which hence needs an additional equation. A recursive equation for the dynamics of this variable is now derived in three steps. First, separate the set of non-optimizing firms \( N \) from the set of optimizing firms \( O \) and notice that in a given period the latter all set the same price \( P_t \) and have mass \( 1 - \alpha \):

\[ \Delta_t \equiv \int_{i \in N} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di + \int_{i \in O} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di = \alpha \Delta_t^N + (1 - \alpha) \tilde{\pi}_t^{-\theta} \quad (50) \]

where I defined the equivalent measure of price dispersion for non-optimizers:

\[ \Delta_t^N \equiv \int_{i \in N} \frac{1}{\alpha} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di. \]

Second, write \( \Delta_t^N \) recursively using the fact that non-optimizers maintain in \( t \) the same price as in \( t - 1 \):

\[ \Delta_t^N \equiv \int_{i \in N} \frac{1}{\alpha} \left( \frac{P_{t-1}(i)}{P_{t-1}} \right)^{-\theta} di = \pi_t^\theta \int_{i \in N} \frac{1}{\alpha} \left( \frac{P_{t-1}(i)}{P_{t-1}} \right)^{-\theta} di = \pi_t^\theta \Delta_{t-1}^N \]

and use this and (47) in (50) to get:

\[ \Delta_t = \alpha \pi_t^\theta \Delta_{t-1}^N + (1 - \alpha) \tilde{\pi}_t^{-\theta}. \]

Finally, since non-optimizers (as well as optimizers) are selected randomly from the set of all firms, the dispersion of non-optimizers in \( t - 1 \) is equal to the dispersion of the population: \( \Delta_{t-1}^N = \Delta_{t-1} \). The new model equation is therefore:

\[ \Delta_t = \alpha \pi_t^\theta \Delta_{t-1} + (1 - \alpha) \tilde{\pi}_t^{-\theta}. \quad (51) \]

A log-linear approximation of this equation is simply:

\[ \tilde{\Delta}_t = \alpha \pi_t^\theta \tilde{\Delta}_{t-1}. \]

Hence, if in the NSS there is price stability and hence no price dispersion, a log-linear approximation of the model will not give any dynamics for \( \tilde{\Delta}_t \) if initially there is no price dispersion (see Schmitt-Grohé and Uribe (2007)). Since in this paper I do not go beyond a log-linear approximation of the model and wish to see the dynamics of price dispersion in IRFs (that show the responses of the log-linear deviations of the variables from the NSS values to shocks when they are initially at the NSS), in Appendix I I calibrate a NSS with non-zero inflation.
2.3. Foreign trade, the public sector, and the balance of payments

Firms in the export sector use domestic goods and ‘land’ (representing natural resources) to produce an export commodity. Land is assumed to be fixed in quantity, hence generating diminishing returns. I assume that the export good is a single homogenous primary good (a commodity). Firms in this sector sell their output in the international market at the foreign currency price $P_t^X$. They are price takers in factor and product markets. The price of primary goods in terms of the domestic currency is merely the exogenous international price multiplied by the nominal exchange rate: $S_t P_t^X$.

Let the production function employed by firms in the export sector be the following:

$$X_t^* = (Q_t^X)^{b_A} Y_t^{1-b_A}, \quad 0 < b_A < 1,$$

where $Q_t^X$ is the amount of domestic goods used as input in the export sector and $Y_t$ is real GDP. These firms maximize profit $S_t P_t^X X_t^* - P_t Q_t^X$ subject to (52). In terms of domestic goods, they maximize:

$$\Pi_t^X = e_t p_t^* \left( Q_t^X \right)^{b_A} Y_t^{1-b_A} - Q_t^X$$

where I defined the SOE’s external terms of trade (XTT):

$$p_t^* = \frac{P_t^X}{P_t^*},$$

where $P_t^*$ is the price index of the foreign currency price of the SOE’s imports. Notice that the XTT is a ratio of two price indexes determined in the RW. Hence, the follow identity relates the rates of foreign inflation of exported and imported goods to the XTT (giving the dynamics of the XTT):

$$\frac{p_t}{p_{t-1}} = \frac{\pi_t^X}{\pi_t^*}, \quad \text{where} \quad \pi_t^X \equiv \frac{P_t^X}{P_{t-1}^X}.$$

The first order condition for profit maximization yields the export sector’s (factor) demand for domestic goods:

$$Q_t^X = (b_A e_t p_t^*) \frac{1}{1-b_A} Y_t$$

(53)

Also, inserting the factor demand function in the production function shows that optimal exports vary directly with the product of the RER and the XTT and GDP:

$$X_t^* = (b_A e_t p_t^*) \frac{1}{1-b_A} Y_t.$$

(54)

The real value of exports in terms of domestic goods is:

$$X_t = \frac{S_t P_t^X X_t^*}{P_t} = e_t p_t^* X_t^* = e_t p_t^* \left( b_A e_t p_t^* \right) \frac{1}{1-b_A} Y_t = \kappa_X \left( e_t p_t^* \right) \frac{b_A}{1-b_A} Y_t$$

(55)

where for simplicity of notation I define:

$$b_X \equiv \frac{1}{1-b_A}, \quad \kappa_X \equiv \left( b_A \right) \frac{b_A}{1-b_A}.$$
Notice that, using this notation, intermediate output in the export sector (53) can be written as:

\[ Q_t^X = (b^A)^{\frac{1}{1+b^X}} (e_t p_t^* )^{b^X} Y_t = b^A \left( (e_t p_t^* )^{b^X} Y_t \right)^{\frac{1}{1+b^X}} \]

Government expenditure is assumed to be a time-varying and stochastic fraction \( \overline{G} \) of private consumption expenditure. Define the gross government expenditure fraction as: \( G_t \equiv 1 + \overline{G} \). Hence, using (30) and (55), GDP in terms of domestic goods is:

\[ Y_t = \tau_M \left( \gamma_t^M \right) G_t p_t^C C_t + X_t - (1 - a_D) e_t^{1-\theta_C} \tau_M \left( \gamma_t^M \right) G_t (p_t^C)^{\theta_C} C_t \]

(56)

\[ = a_D \tau_M \left( \gamma_t^M \right) G_t (p_t^C)^{\theta_C} C_t + X_t. \]

In the domestic goods market, the output of domestic firms \( Q_t \) must satisfy final demand from households (including the resources for transactions), the government, and the export sector:\(^{10}\)

\[ Q_t = a_D \tau_M \left( \gamma_t^M \right) G_t (p_t^C)^{\theta_C} C_t + Q_t^X = Y_t - (1 - b^A) X_t. \]

The public sector includes the Government and the CB. The latter issues currency \( M_t \) and domestic currency bonds \( B_t \), and holds international reserves \( R_t \) in the form of foreign currency denominated riskless bonds issued by the RW. I assume that CB bonds are only held by domestic residents. The (flow) budget constraint of the CB is:

\[ M_t + B_t - S_t R_t = M_{t-1} + (1 + i_{t-1}) B_{t-1} - (1 + i_{t-1}^*) S_{t-1} R_{t-1} \]

(57)

\[ = [M_{t-1} + B_{t-1} - S_{t-1} R_{t-1}] - QF_t. \]

where

\[ QF_t = i_{t-1}^* S_{t-1} R_{t-1} + (S_t - S_{t-1}) R_{t-1} - i_{t-1} B_{t-1} \]

\[ = \left[ i_{t-1}^* (1 - 1/\delta_t) \right] S_{t-1} R_{t-1} - i_{t-1} B_{t-1} \]

is the CB’s quasi-fiscal surplus, which includes interest earned and capital gains on international reserves minus the interest paid on its bonds. I assume that the CB transfers its quasi-fiscal surplus (or deficit) to the Government every period. Hence, its net wealth is constant. Furthermore, assuming for convenience that the CB’s net worth is zero, the following holds for all \( t \):

\[ M_t + B_t - S_t R_t = M_{t-1} + B_{t-1} - S_{t-1} R_{t-1} = 0. \]

(58)

The CB supplies whatever amount of cash is demanded by households, and can influence these supplies by changing \( R_t \) or \( B_t \), i.e. intervening in the foreign exchange market or in the domestic currency bond market. In terms of domestic goods, the CB balance, for all \( t \), is:

\[ m_t + b_t = e_t r_t. \]

(59)

\(^{10}\)Notice that rearranging the third equality shows that GDP is the sum of the outputs of the domestic and export sectors, minus the intermediate use of domestic goods in the export sector.

\[ Y_t = Q_t + X_t - b^A X_t. \]
This equation provides a constraint on the CB’s ability to simultaneously intervene in the foreign exchange market (through sales and purchases of foreign reserves $r_t$) and in the domestic bonds market (through sales and purchases of domestic currency CB bonds $b_t$).\footnote{It is obviously unnecessary to restrict the CB net wealth to zero. Any fixed number would do. Moreover, there is clearly the possibility of adding a degree of freedom for a more general model in which the CB net wealth can vary stochastically or even be used as an additional control variable. The latter would require additional modeling, such as market perceptions of CB risk. For my purpose of modeling the simultaneous use of the interest rate and the rate of nominal depreciation as control variables, the simplest assumption of zero CB net wealth is sufficient.}

The Government spends on goods, receives the quasi-fiscal surplus (or finances the deficit) of the CB, and collects taxes. I assume that fiscal policy consists of an exogenous autorregressive path for real government expenditures as a (gross) fraction of private consumption ($G_t$) and collecting whatever lump-sum taxes are needed to balance the budget each period. The Public Sector flow budget constraint is hence:

$$T_{axt} = \overline{G_t} P_t^C C_t - QF_t.$$  \hspace{1cm} (60)

So in real terms:

$$\text{tax}_t = \overline{G_t} P_t^C C_t - qf_t,$$  \hspace{1cm} (61)

$$qf_t = \left(1 + i_{t-1}^* - 1/\delta_t\right) \frac{e_t r_{t-1}}{\pi_t^*} - ((1 + i_{t-1}) - 1) \frac{b_{t-1}}{\pi_t}.$$  \hspace{1cm} (62)

Inserting

$$Y_t = w_t N_t + \frac{\Pi_t}{P_t},$$

in the household budget constraint (7) and consolidating the household, CB and government budget constraints yields the balance of payments equation:

$$r_t - d_t = CA_t + r_{t-1} - d_{t-1},$$

where the current account (in foreign currency) is:

$$CA_t = \left(1 + \frac{i_{t-1}^*}{\pi_t^*} - 1\right) r_{t-1} - \left[1 + \frac{i_{t-1}^*}{\pi_t^*} \phi_{t-1} \tau_D \left(\frac{e_{t-1} d_{t-1}}{Y_{t-1}}\right) - 1\right] d_{t-1} + TB_t,$$

and (using (31), (56) and (55)) the trade balance (in foreign currency) is:

$$TB_t = \frac{1}{e_t} \left[ X_t - e_t \tau_M \left(\gamma_t^M\right) G_t C_t \right] = \frac{1}{e_t} \left[ X_t - (1 - a_D) e_t^{1-\theta} \tau_M \left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta} C_t \right] = \frac{1}{e_t} \left[ X_t - \frac{1 - a_D}{a_D} e_t^{1-\theta} \left(Y_t - X_t\right)\right] = \frac{1}{e_t} \left[ (p_t^C)^{1-\theta} X_t - (1 - a_D) e_t^{1-\theta} Y_t\right].$$

2.4. Monetary and exchange rate policy

In this paper the CB uses policy rules to stabilize the SOE’s macroeconomy. These rules may be 1) simple and with exogenous coefficients 2) simple and with endogenous (optimal) coefficients, or 3) endogenous and optimal under commitment (and full information). In case 1), the simple interest rate rule is a feedback rule, and the simple rule for nominal depreciation may or may not involve feedback. In case
2), the CB is assumed to minimize a weighted average of the squared deviations of some of the endogenous variables (from their NSS values). In case 3), the CB is assumed to minimize an expected discounted quadratic loss function of some of the endogenous variables. Furthermore, in any of these cases, the CB can operate under one of three alternative monetary regimes. I use the expression ‘monetary regime’ broadly. It expresses the combination of the CB’s operating procedures concerning the issuance of (base) money, and the intervention it may have in the bond and FX markets to influence the nominal interest rate and the rate of nominal currency depreciation. As shown below, in this paper ‘monetary’ policy (in the narrow sense) is passive, being money issuance whatever is needed to balance the money market once the other two policies are defined. For convenience, the three alternative monetary regimes are denominated: I) a managed exchange rate (MER) regime, II) a floating exchange rate (FER) regime, and III) a pegged exchange rate (PER) regime.

In the MER regime, the CB, through its regular and systematic interventions in the domestic currency bond (or ‘money’) market and in the foreign exchange market, aims for the achievement of two operational targets: one for the interest rate $i_t$, and another for the rate of nominal depreciation $\delta_t$. When there are simple policy rules (cases 1) and 2)), the CB can respond to deviations of the consumption inflation rate ($\pi^C_t$) from a target ($\pi^T$) which is the NSS value of this variable, and to deviations of detrended GDP and the RER from their respective NSS levels. The rate of nominal depreciation also responds to the deviations of the international reserves (IRs) to GDP ratio from a long run target ($\gamma^R$). Variables without a time subscript denote non-stochastic steady state values. There may be history dependence (or inertia) in one or both of the two feedback rules through the presence of the lagged operational target variable. The simple rules are the following:

$$\frac{1 + i_t}{1 + i} = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{h_0} \left( \frac{\pi^C_t}{\pi^T} \right)^{h_1} \left( \frac{Y_t}{Y} \right)^{h_2} \left( \frac{e_t}{e} \right)^{h_3}$$

$$\frac{\delta_t}{\delta} = \left( \frac{\delta_{t-1}}{\delta} \right)^{k_0} \left( \frac{\pi^C_t}{\pi^T} \right)^{k_1} \left( \frac{Y_t}{Y} \right)^{k_2} \left( \frac{e_t}{e} \right)^{k_3} \left( \frac{e_tR_t/Y_t}{\gamma^R} \right)^{k_4}$$

where $h_1 \neq 0$ and $k_4 \neq 0$ and variables without time subscripts denote NSS values. The first of these is used in the MER and FER regimes, and the second is used in the MER and PER regimes. In a floating exchange rate regime (FER), the CB abstains from intervening in the foreign exchange market. Hence, the international reserves that appear in its balance sheet remain constant. For simplicity, I assume that they remain constant at the NSS value $r$ of the general model (with MER regime). In a pegged exchange rate regime (PER), the CB abstains from intervening in the domestic currency bond market. Hence, its stock of bonds remains constant, and I assume that they remain at the NSS value $b$ of the MER regime. In both of the corner cases, one of the policy rules is dropped and one of the endogenous variables is turned into an exogenous parameter. But there is an alternative way of thinking about this issue which is more illuminating, particularly in an optimal control framework.

The FER and PER regimes are simply very special cases (‘corner regimes’) of the MER regime. In the case of optimal feedback policy rules (case 3) above) this
means that the optimal rule under any one of the ‘corner’ regimes cannot dominate the optimal rule under the MER regime. I argue below that one can define these regimes as cases in which the CB imposes an additional restriction on itself (‘ties its hands’) and relinquishes its use of one of its ‘control’ variables. Hence that variable turns into a ‘non-control’ variable.\footnote{I hesitate to use the term ‘state variable’ because in this model both \(i\) and \(\delta\) are non-predetermined (or jump) variables and it is usual to call predetermined variables ‘state variables’.}

To obtain a generalization of the standard DSGE monetary policy model, I specify the instruments that the CB uses when it intervenes in each of the two markets and include them in the model. The CB purchases or sells domestic currency bonds, and thus changes its stock of bonds \(b_t\), to intervene with high frequency in this market in order to attain its operational target for the interest rate as determined by (62).\footnote{Notice that this high-frequency action may be modeled in different ways. But in the quarterly frequency of the model the instruments, operational target variables, and the rest of the model variables are related through the model equations that any higher frequency model must respect if it is designed to be consistent with the quarterly model.} And it purchases or sells foreign exchange to intervene in the foreign exchange market, thereby changing its stock of international reserves \(r_t\), in order to attain its operational target for the rate of nominal depreciation as determined by (63). While at high frequency (hours, days, weeks) the CB is active changing \(b_t\) and/or \(r_t\), at low frequency (quarters in this paper) these variables passively adapt to accommodate \(i_t\) and \(\delta_t\) as given by the feedback policy rules and the rest of the model equations.

To represent the constraints that the CB faces it is necessary to broaden the usual policy model to include the CB balance sheet (59) and its arrangement with the rest of the government (Treasury) as to the use of the fiscal dimension of the CB’s flow budget constraint (which I called CB quasi-fiscal surplus \(q_{ft}\) above). By assuming, as I do here, that the CB’s arrangement with the Treasury is that it hands over its quasi-fiscal surplus (or receives automatic finance for its quasi-fiscal deficit) period by period, the CB balance sheet equation is maintained period by period in the sense that the CB’s net worth is constant. This can be seen as a simple device for defining the CB’s ‘sterilization’ policy, i.e. the value of \(b_t\), given the values of \(m_t\) (‘determined’ by money market balance), and the values of \(e_t\) and \(r_t\), the latter influenced by the intervention in the FX market). But it is probably more adequate to think more symmetrically that (59) imposes a constraint on the simultaneous use of \(b_t\) and \(r_t\). From this vantage point, one can think of the ‘corner’ regimes as the imposition of an additional constraint (instead of the dropping of an endogenous variable). In the case of the FER regime, the additional constraint is \(r_t = r\). And in the case of the PER regime, the additional constraint is \(b_t = b\). In terms of an optimal control framework, any one of the ‘corner’ regimes imposes an additional constraint on the policymaker and, simultaneously, converts one of the ‘controls’ (\(\delta_t\) in the case of the FER regime and \(i_t\) in the case of the PER regime) into a non-control variable. Hence, it quite evident that the MER regime cannot be inferior to any of the two ‘corner’ regimes. With the same loss function and the same (basic) model equations and endogenous variables, but with one additional constraint (equation) and one less ‘control’, the expected discounted loss cannot be lower. Indeed, I show below that it is quite higher in all of the cases I found
interesting to consider.

The policy framework in this paper is one in which monetary growth is passive. Indeed, defining the rate of money growth \( \mu_t \equiv M_t/M_{t-1} \), (15) and (17) imply:

\[
\mu_t = \pi_t^C \frac{\mathcal{L}(1 + i_t)}{\mathcal{L}(1 + i_{t-1})} \left[ \beta (1 + i_t) \frac{\varphi_M(\mathcal{L}(1 + i_t))}{\varphi_M(1 + i_{t-1})} \right] \frac{1}{\varphi_M^C}. \tag{64}
\]

Hence, under the MER or FER regimes, achieving the operational target for the nominal interest rate bearing in mind the need to balance the money market implies a definite rate of money growth that varies directly with the consumption inflation rate and a complex factor that only depends on the current and lagged interest rate. However, (64) is equally valid under the PER regime, where there is no CB policy rule for the interest rate.

2.5. The nonlinear system of equations

In this section I put together the model equations for simple feedback rules in a MER regime.

Consumption Euler:

\[
\frac{C_t - \sigma^C}{\varphi_M \left( m_t / (p_t^C C_t) \right)} = \beta (1 + i_t) E_t \left( \frac{C_{t+1} - \sigma^C}{\varphi_M \left( m_{t+1} / (p_{t+1}^C C_{t+1}) \right)} \frac{1}{\pi_{t+1}^C} \right) \tag{65}
\]

Risk-adjusted uncovered interest parity:

\[
1 + i_t = (1 + i_t^*) \phi_t^* \varphi_D \left( e_t d_t / Y_t \right) E_t \delta_{t+1} \tag{66}
\]

Phillips equations:

\[
\Gamma_t = \frac{Q_t}{p_t^C C_t} + \beta \alpha E_t \pi_{t+1}^{\theta - 1} \Gamma_{t+1} \tag{67}
\]

\[
\Psi_t = \frac{\theta}{\theta - 1} \frac{Q_t}{p_t^C C_t^\sigma^C} mc_t + \beta \alpha E_t \pi_{t+1}^{\theta} \Psi_{t+1} \tag{68}
\]

\[
\Psi_t = \widetilde{p}(\pi_t) \Gamma_t \tag{69}
\]

Dynamics of price dispersion:

\[
\Delta_t = \alpha \pi_t^\theta \Delta_{t-1} + (1 - \alpha) \left( \frac{1 - \alpha \pi_{t-1}^{\theta - 1}}{1 - \alpha} \right) \frac{\theta}{\varphi_M} \tag{70}
\]

Exports:

\[
X_t = \kappa_X (e_t p_t^x)^{b_X} Y_t \tag{71}
\]

Trade Balance:

\[
TB_t = \frac{1}{a_D e_t} \left[ (p_t^C)^{1-\sigma^C} X_t - (1 - a_D) e_t^{1-\sigma^C} Y_t \right] \tag{71}
\]

Current Account:

\[
CA_t = \left( \frac{1 + i_t^* - 1}{\pi_t^*} \right) r_{t-1} - \left[ \frac{1 + i_{t-1}^* \pi_t^* \varphi_{t-1} \tau_D \left( e_t d_{t-1} / Y_{t-1} \right)}{\pi_t^*} - 1 \right] d_{t-1} + TB_t. \tag{72}
\]

\[14\text{See Olivera (1970).}\]
Balance of Payments: 
\[ r_t - d_t = CA_t + r_{t-1} - d_{t-1} \] 
(73)

Real marginal cost: 
\[ mc_t = \frac{w_t}{\epsilon_t} \] 
(74a)

Labor market clearing:
\[ w_t = \xi^N p_t^C C_t^\sigma^C \varphi_M \left( \frac{m_t}{p_t^C C_t} \right) N_t^\sigma^N \]

Hours worked:
\[ N_t = \frac{Q_t}{\epsilon_t} \Delta_t \]

Domestic goods market clearing:
\[ Q_t = Y_t - (1 - b^4) X_t \] 
(77)

GDP:
\[ Y_t = a_D \tau_M \left( \frac{m_t}{p_t^C C_t} \right) G_t \left( p_t^C \right)^{\sigma^C} C_t + X_t \]
(78)

Consumption relative price:
\[ p_t^C = \left( a_D + (1 - a_D) e_t^{1-\sigma^C} \right)^{\frac{1}{1-\sigma^C}} \] 
(79)

Money market clearing:
\[ m_t = \mathcal{L} (1 + i_t) p_t^C C_t, \]
(80)

CB balance sheet:
\[ b_t = e_t r_t - m_t \]
(81)

Consumption inflation:
\[ \frac{\pi_t^C}{\pi_t} = \frac{p_t^C}{p_{t-1}^C} \] 
(82)

Real Exchange Rate:
\[ \frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^*}{\pi_t} \] 
(83)

External terms of trade:
\[ \frac{p_t^*}{p_{t-1}^*} = \frac{\pi_t^* X}{\pi_t^*} \] 
(84)

Tax collection:
\[ tax_t = \mathcal{O}_t p_t^C C_t - qf_t \] 
(85)

Quasi-fiscal surplus:
\[ qf_t = (1 + i_{t-1}^* - 1/\delta_t) \frac{e_t p_{t-1}^*}{\pi_t^*} - ((1 + i_{t-1}) - 1) \frac{b_{t-1}}{\pi_t} \]

Interest rate feedback rule:
\[ \frac{1 + i_t}{1 + i} = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{h_0} \left( \frac{\pi_t^C}{\pi_t} \right)^{h_1} \left( \frac{Y_t}{\bar{Y}} \right)^{h_2} \left( \frac{e_t}{e} \right)^{h_3} \]
(86)
Nominal depreciation feedback rule:

\[
\frac{\delta_t}{\delta_t} = \left( \frac{\delta_{t-1}}{\delta_t} \right)^k_0 \left( \frac{\pi^C_t}{\pi_t} \right)^k_1 \left( \frac{Y_t}{Y_t} \right)^k_2 \left( \frac{e_t}{\epsilon_t} \right)^k_3 \left( \frac{e_t r_t / Y_t}{\gamma^R} \right)^k_4 \tag{87}
\]

Notice that I am not constraining \( b_t \) nor \( r_t \) to be non-negative, which may be realistic constraints for any Central Bank. Negative international reserves would mean borrowing from abroad and, in the context of this model, would require a risk premium as in the case of households. And many Central Banks are institutionally constrained in lending to the non-financial private sector, making \( b_t \) non-negative.

Here, I assume that the Central Bank’s target for reserves \( R \) is sufficiently high and the household’s steady state demand for cash is sufficiently low to ensure that these non-negativity constraints hold for all \( t \) and all relevant stochastic shocks.\(^{15}\)

In addition to these equations there are those that are subject to stochastic shocks, most of which are simple AR(1) processes. The external terms of trade (XTT) is a particularly important external effect for most SOE’s. This justified giving the calibration of its components a careful treatment. As a working hypothesis, I assumed that the inflation rates for imported and exported goods are interrelated in such a way that a shock to one of them affects the other through the dynamics of the XTT (which is the ratio of the two corresponding foreign price levels). Hence, I assumed:

\[
\pi_t^X = \left( \pi_{t-1}^* \right)^{\rho_t^X} \left( \pi^X_{t} \right)^{1-\rho_t^X} \left( p_{t-1}^* \right)^{\alpha^X_{t}} \exp \left( \sigma^X_{t} \varepsilon^X_{t} \right), \tag{88}
\]

\[
\pi_t^* = \left( \pi_{t-1}^{*} \right)^{\rho_t^*} \left( \pi^*_{t} \right)^{1-\rho_t^*} \left( p_{t-1}^* \right)^{\alpha^*_{t}} \exp \left( \sigma^*_{t} \varepsilon^*_{t} \right),
\]

\[
p_t^* = p_{t-1}^* \left( \frac{\pi_t^*}{\pi_t} \right)^{\beta^*_{t}}.
\]

Notice that if the two price indexes are non-stationary, this implies that they are cointegrated. The XTT variable \( p_t^* \) plays the role of a cointegration error term, \( \alpha_{t+X} \leq 0, \alpha_{t+} > 0 \) are the speeds of adjustment and \( (1, -\beta_{t+}) \) plays the role of a cointegrating vector, with \( \beta_{t+} = 1 \) as in the identity (84).\(^ {16}\)

The equations subject to exogenous shocks are hence the following (where the NSS values \( \epsilon_t, \pi_t^*, \pi_t^X \) are assumed equal to one):

Productivity shock

\[
\epsilon_t = (\epsilon_{t-1})^{\rho_{t}} \exp \left( \sigma^+ \varepsilon_t^+ \right)
\]

Government expenditure shock

\[
G_t = (G_{t-1})^{\rho_{t}} G^{1-\rho_{t}} \exp \left( \sigma^G \varepsilon_t^G \right)
\]

Riskfree interest rate shock

\[
1 + i_t^* = \left( 1 + i_{t-1}^* \right)^{\rho^*} (1 + i^*)^{1-\rho^*} \exp \left( \sigma_{t+}^* \varepsilon_{t+}^* \right)
\]

\(^{15}\)In the parent model ARGEM, it is banks that invest in domestic currency bonds and usually Central Banks do have the institutional ability to assist banks, though usually with limitations.

\(^{16}\)In Appendix I I estimate these equations using data for Argentina and find evidence for the cointegration hypothesis with an additional influence of \( \pi_{t+X}^* \) on \( \pi_t^* \), as in the equation below.
Financing liquidity shock

\[ \phi_t^* = (\phi_{t-1}^*)^{\rho_x^*} (\phi_x^*)^{1-\rho_x^*} \exp\left(\sigma_x^\phi \varepsilon_t^\phi\right) \]

Exports inflation shock

\[ \pi_{tX}^* = (\pi_{t-1X}^*)^{\rho_x^*X} (\pi_x^X)^{1-\rho_x^*X} (p_{t-1}^*)^{\alpha_x^*} \exp\left(\sigma_x^\pi \varepsilon_t^\pi\right) \]

Imported inflation shock

\[ \pi_t^* = (\pi_{t-1}^*)^{\rho_x^*} (\pi_x^*)^{1-\rho_x^*} (p_{t-1}^*)^{\alpha_x^*} (\pi_{t-1X}^*)^{\rho_x^*XN} \exp\left(\sigma_x^\pi \varepsilon_t^\pi\right). \]

2.6. Functional forms for auxiliary functions

I use the following functional forms for the endogenous risk premium and transactions costs functions:\(^{17}\)

\[ \tau_D (\gamma_t^D) \equiv 1 + \frac{\alpha_1}{1 - \alpha_2 \gamma_t^D}, \quad \alpha_1, \alpha_2 > 0, \quad (89) \]

\[ \tau_M (\gamma_t^M) \equiv 1 + \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}}, \quad \beta_1, \beta_2, \beta_3 > 0 \quad (90) \]

which, according to definitions (14), give:

\[ \varphi_D (\gamma_t^D) = 1 + \frac{\alpha_1}{(1 - \alpha_2 \gamma_t^D)^2}, \quad (91) \]

\[ \varphi_M (\gamma_t^M) = 1 + \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}} \left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M}\right). \]

For calibrations it is convenient to define the net functions:

\[ \tau_D^N (\gamma_t^D) = \tau_D (\gamma_t^D) - 1, \quad \varphi_D^N (\gamma_t^D) = \varphi_D (\gamma_t^D) - 1 \quad (92) \]

\[ \tau_M^N (\gamma_t^M) = \tau_M (\gamma_t^M) - 1, \quad \varphi_M^N (\gamma_t^M) = \varphi_M (\gamma_t^M) - 1. \]

The elasticities of \(\tau_D^N\) and \(\tau_M^N\) will be used below in calibrations and are, respectively:

\[ \varepsilon_D^N (\gamma_t^D) \equiv \frac{\alpha_2 \gamma_t^D}{1 - \alpha_2 \gamma_t^D} \quad (93) \]

\[ \varepsilon_M^N (\gamma_t^M) \equiv \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M}. \quad (94) \]

\(^{17}\)In calibrating the model parameters I found it important to include a third parameter in the transactions cost function. Otherwise I could not obtain realistic money demand interest elasticities, and the variability of the instruments was systematically excessive.
3. Numerical solution in Dynare

A detailed calibration of the parameters and derivation of the NSS values of the endogenous variables can be found in Appendix 1. In this section I explore the policy parameter ranges that guarantee the Blanchard-Kahn (BK) stability conditions using the non-policy parameter values in the table below, as well as the resulting volatilities (standard deviations) for the main endogenous variables in the model. The following table summarizes the calibrated values of the main model parameters, and contains some comparisons with parameter values used in other SOE models.\(^{18}\)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>This paper</th>
<th>G-M</th>
<th>De P</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta) Intertemporal discount factor</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>(\sigma^C) Relative risk aversion for goods</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\sigma^N) Relative risk aversion for labor</td>
<td>0.5</td>
<td>3</td>
<td>0.47</td>
</tr>
<tr>
<td>(\alpha) Probability of not adjusting price</td>
<td>0.66</td>
<td>0.75</td>
<td>0.66</td>
</tr>
<tr>
<td>(\theta) E.S. between domestic goods</td>
<td>6</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>(\theta^C) E.S. domestic vs. imported goods</td>
<td>1.5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(a_D) Coef. for share of domestic goods</td>
<td>0.86</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>(b^A) Coef. in production function for commodities</td>
<td>0.5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon^D_\varphi) Elasticity of risk function in UIP (\varphi_D(\epsilon d/Y))</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\varepsilon_L) Elasticity of (L(1+i))</td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The standard errors and persistence parameters used for the six shock variables are given in the table below. They were calibrated taking into account the available time series for Argentina and the RW during the period 1994.1-2009.2: public consumption to GDP in the case of \(G^G\), imported and exported goods inflation as they conform Argentina’s XTT, in the cases of \(\pi^*\) and \(\pi^{**}\), Libor 3 months in the case of \(\phi^\ast\), and balance of payments information on private sector foreign debts and interest payments as well as my own calculation of the spread over Libor 3 months, in the case of \(\phi^\ast\). The only cases in which I took the standard deviations exactly according to the data are the cases of \(\pi^*, \pi^{**}\), and \(\phi^\ast\). The rest were calibrated taking both the data (except for \(\phi^\ast\)) and the resulting theoretical standard deviation and variance decomposition for GDP with a baseline calibration of the two policy rules \((h_1 = 0.8, h_2 = 0.8, k_4 = -0.8, \text{ and the rest of the coefficients zero})\). This implied diminishing the observed standard deviation of \(G\) (from 0.054 in a simple AR(1) estimation from which I did use the persistence parameter \(\rho^G\)), which seemed to weigh too heavily in the volatility of \(Y\), and increasing the standard deviation of \(\phi^\ast\) (from 0.0034), which seemed not to weigh enough. The value of \(\sigma^\ast\) was chosen so that the resulting theoretical standard deviation of \(Y\) was similar to the data for detrended and s.a. GDP for Argentina leaving out the crisis years 2001/2002.

\(^{18}\)‘E.S.’ denotes ‘elasticity of substitution’, G_M stands for Galí and Monacelli (2005), and De P for De Paoli (2006).
4.1. Preliminary exploration of the effects of the simple policy rule coefficients

I first study some of the general stability properties of the model in relation to the parameters of the two simple policy rules in the MER regime. The coefficients on the policy rules not explicitly mentioned below are made equal to zero. When I say that a particular configuration of parameters gives stability I mean that all the requirements for determinacy and non-explosiveness are met, including the rank condition. In particular, there are no unit roots. A very general result is that if $k_4 = 0$ the model has a unit root for any value of the remaining coefficients. Hence, from now on $k_4$ will always be different from zero. Let $k_4 = -0.8$ until further notice. Notice that with a negative values for $k_4$, when there are insufficient reserves, and hence, $e_t r_t / Y_t < \gamma R$, i.e. $\tilde{e}_t + \tilde{r}_t - \tilde{Y}_t < 0$, the CB tends to depreciate the currency (more than in the NSS), for which it purchases foreign exchange:

$$
\tilde{\delta}_t = k_4 \left( \tilde{e}_t + \tilde{r}_t - \tilde{Y}_t \right) > 0.
$$

1) First I look at a very streamlined policy with no inertia, an interest rate policy rule that only responds to inflation, and a nominal depreciation policy rule that only responds to the deviation from the long run target for international reserves. Hence, in (86) and (87) only $h_1$ and $k_4 = -0.8$ are non-zero. There is stability as long as $h_1$ is greater than one: the classical Taylor Principle. Otherwise there is indeterminacy.

2) Next I introduced interest rate inertia, letting $h_0$ become positive. For example, if $h_0 = 0.2$, then $h_1$ must be at least 0.81 (to two decimal points) for stability. Otherwise there is indeterminacy. The following table shows that gradually raising $h_0$ lowers the minimum value of $h_1$ required for stability:

<table>
<thead>
<tr>
<th>$h_0$</th>
<th>$h_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.01</td>
</tr>
<tr>
<td>0.2</td>
<td>0.81</td>
</tr>
<tr>
<td>0.4</td>
<td>0.61</td>
</tr>
<tr>
<td>0.6</td>
<td>0.41</td>
</tr>
<tr>
<td>0.8</td>
<td>0.21</td>
</tr>
<tr>
<td>1.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

As the table shows, there is an ‘inertia-inflation-responsiveness frontier’ for the interest rate policy rule that is downward sloping and linear. $h_0 + h_1$ must be greater than one (but $h_1$ must necessarily be positive) for stability. This is precisely the content Woodford’s Proposition 2.8 (see Woodford (2003), page 96). Hence, a ‘Taylor Principle’ holds in the model, in the (restricted) sense that a minimum of interest rate response to inflation is required for stability, given the value of $h_0$. But there is also what might be called a ‘Woodford Principle’, in the sense that, given a positive value of $h_1$, there is a minimum value for $h_0$ that is required for stability. It is interesting to observe that the ‘inertia-inflation-responsiveness frontier’ (IIRF) is valid for any values of the remaining coefficients in the two policy rules as long as $k_4$ is negative. That is, below this frontier there is no way to stabilize the economy using interest rate responsiveness to other model variables, or depreciation rate responsiveness to any variable. Furthermore, it is robust to any other negative
value of $k_4$. Hence, it is nice to see that the present generalization of the standard monetary policy framework in DSGE models maintains some of the key ingredients in the more limited model.

On the other hand, if $k_4$ has low positive values there is a reversal in the Taylor and Woodford principles: stability requires that $h_0 + h_1$ be less than one. Remarkably, a policy with a positive $k_4$ less than 0.24 (to two digits) and all the other coefficients in both policy rules equal to zero is stable. Although it is difficult to obtain intuition with this result, since positive values of $k_4$ seem to be at odds with common sense, a positive $k_4$ will come up below as an optimal value for simple policy rules in the MER regime for a central bank with certain preferences. This seems to indicate that the model (with the MER) is considerably more complex (and richer) than the standard model with either of the two corner regimes (FER or PER).

3) Next I looked a little closer into the effects of changing one of the two critical coefficients $h_0$ and $h_1$ on the standard deviations of some of the endogenous variables Central Banks typically care for. First I take a fixed value of $h_0$ starting on the IIRF and find the volatilities (standard deviations) for increasing values of $h_1$. The results are in the table below, where the minimum value in each row is highlighted in bold and the maximum is in italics. The ratio between the maximum and minimum volatility, showing the effectiveness in changing the volatility, is also shown in the last column. It is interesting to see that some of the volatilities of variables of interest decrease steadily (inflation -$piC$ in the Dynare file-, price dispersion -$DeltaP$ in the Dynare file-, RER, TB, Utility, $d$) while others increase steadily ($C$, real interest rate, $r$), and still others at first diminish, reach their minimum, and then increase ($Y$, $Q$, $mc$, $N$, $b$). Maximum volatilities are almost always in the extremes, but minimum volatilities are more scattered.

Although attention is usually focused on the volatility of $Y$, it is $C$ and $N$ that enter the aggregate utility of households, and their volatilities respond quite differently to increases in $h_1$. Indeed, while the volatility of $C$ increases steadily with $h_1$, that of $N$ falls up to $h_1 = 2$ and then starts to increase. The volatility of period utility (Utility), however, steadily diminishes as $h_1$ increases, as does the volatility of inflation. Also, bearing in mind that $N$ is the product of $Q$ and $\Delta$ (divided by $\epsilon$), we see that the volatility of $Q$ reaches a minimum at $h_1 = 1.5$ while that of $\Delta$ steadily diminishes.

---

19 Amato and Laubach (2003) do a similar analysis for the case of sticky prices and wages when only an interest rate rule is used.
The remaining rows in this table focus on the volatility of the CB intermediate targets and instruments, and the volatility of the rest of the financial stock variables. While the volatility of $i$ is non-monotonic ($ii$ in the Dynare file), decreasing at first and then increasing, the volatility of $\delta$ (delta in the Dynare file) is steadily decreasing. Furthermore, the volatility of the variables that the CB actually uses on a day by day basis, $b$ and $r$ have quite different behaviors. The volatility of $b$ varies in the opposite direction to $i$ as $h_1$ increases. The highest volatility of $b$ is achieved for $h_1 = 0.8$, where the lowest volatility of $i$ is attained. Hence, it seems that to achieve a low volatility of the operational target it is necessary to use the instrument in a very volatile way. The volatility of $r$ varies in the opposite direction to $\delta$ as $h_1$ increases, reaching its maximum where the volatility of $\delta$ reaches its minimum. Hence, in general a higher volatility of the instrument is necessary to achieve a lower volatility of the corresponding operational target.

The next table shows a similar exercise except that $h_1$ is now fixed and it is $h_0$ that increases. The volatilities of $\pi^C$, $\Delta$, $Y$, $Q$, $N$, and $Utility$ are highest for the lowest value of $h_0$, fall to a minimum and then start increasing. As in the previous table, the volatilities of $C$ and the real interest rate increase steadily. But now the volatility of $e$ and $TB$ increase steadily as $h_0$ increases. But (as in the previous case) they vary little. As to the intermediate targets and the instruments, the volatility of $i$ steadily falls, as the volatility of the corresponding instrument $b$, steadily rises. In order to implement an increasing ‘inertia’ for the operational target for the interest rate, the CB must use its instrument $b$ with moderately higher volatility. And, as one would expect, it is very efficient in reducing the volatility $i$: the ratio between the highest and lowest volatilities of $i$ is 700%, while the equivalent for $b$ is only 9%. On the other hand, it is clear that $h_0$ is not efficient for reducing the volatility of $\delta$.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>30</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h_0=0.4</td>
<td>h_1=0.61</td>
<td>h_1=0.8</td>
</tr>
<tr>
<td>$\pi C$</td>
<td>1.0150</td>
<td>0.0190</td>
<td>0.0155</td>
</tr>
<tr>
<td>DeltaP</td>
<td>1.0051</td>
<td>0.0882</td>
<td>0.0048</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.4430</td>
<td>0.0742</td>
<td>0.0720</td>
</tr>
<tr>
<td>$Q$</td>
<td>1.3153</td>
<td>0.0638</td>
<td>0.0619</td>
</tr>
<tr>
<td>$C$</td>
<td>1.3108</td>
<td>0.0236</td>
<td>0.0272</td>
</tr>
<tr>
<td>real_i</td>
<td>1.0101</td>
<td>0.0111</td>
<td>0.0131</td>
</tr>
<tr>
<td>mc</td>
<td>0.8302</td>
<td>0.0166</td>
<td>0.0135</td>
</tr>
<tr>
<td>$e$</td>
<td>0.5951</td>
<td>0.0496</td>
<td>0.0494</td>
</tr>
<tr>
<td>$TB$</td>
<td>0.0082</td>
<td>0.0608</td>
<td>0.0604</td>
</tr>
<tr>
<td>$N$</td>
<td>1.3220</td>
<td>0.0731</td>
<td>0.0663</td>
</tr>
<tr>
<td>Utility</td>
<td>-2.2744</td>
<td>0.0574</td>
<td>0.0551</td>
</tr>
<tr>
<td>$ii$</td>
<td>1.0253</td>
<td>0.0153</td>
<td>0.0148</td>
</tr>
<tr>
<td>delta</td>
<td>1.0150</td>
<td>0.0711</td>
<td>0.0695</td>
</tr>
<tr>
<td>$b$</td>
<td>0.0722</td>
<td>0.0160</td>
<td>0.0162</td>
</tr>
<tr>
<td>$r$</td>
<td>0.3152</td>
<td>0.0432</td>
<td>0.0439</td>
</tr>
<tr>
<td>$d$</td>
<td>1.2125</td>
<td>0.1020</td>
<td>0.1016</td>
</tr>
<tr>
<td>$m$</td>
<td>0.1154</td>
<td>0.0032</td>
<td>0.0032</td>
</tr>
</tbody>
</table>
4) Now I concentrate on what happens when the CB changes the speed with which it seeks to attain its long run target for international reserves through its nominal depreciation response. For this I keep $h_0$ and $h_1$ constant at values in the interior of the IIRF ($h_0 = 0.4$ and $h_1 = 0.8$) while $k_4$ gets increasingly negative, starting from -0.1.

Most of the variables of interest have minimum volatilities for $k_4$ in the range $-0.1 / -0.7$. On the other hand, $\Delta$ and $mc$, have lowest volatilities at $k_4 = -5$. The opposite effects of increments in $k_4$ on inflation and price dispersion are noteworthy, especially because it is $\Delta$ that appears in the utility function (through $N$). Intervening more strongly in the foreign exchange market so as to attain the CB reserves target, while increasing the volatility of inflation, decreases the volatility of price dispersion.

The lowest volatilities of the operational targets are reached at $k_4 = -0.4$ (the case of $i$) or $k_4 = -0.1$ (the case of $\delta$). Relatively large volatilities of the instruments
are necessary to achieve low volatilities of the operational targets. Clearly, the policy of responding to deviations from the long run target for CB reserves with the operational target for nominal depreciation has a significant stabilizing role.

5) To get a feeling for the range within I could move each policy rule coefficient, I started from a baseline calibration for the coefficients in the two policy feedback rules well within the IIRF frontier and looked for the intervals within which each of the coefficients could be moved individually (leaving the rest at the baseline value) without impairing stability. I restricted my search to two decimal points accuracy and only checked for parameter values below 10 in absolute value. The following was the baseline calibration for this exercise:

Baseline calibration

\[
\begin{array}{cccccccc}
  h_0 & h_1 & h_2 & h_3 & k_0 & k_1 & k_2 & k_3 & k_4 \\
  0.8 & 0.8 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.8
\end{array}
\]

The results for the three policy regimes are in the table below. Starting with the general MER regime, both of the inertial coefficient intervals of stability are quite wide, both going into high superinertial levels (of 10 and 4.48 for the interest rate and depreciation rate rules, respectively). Because unity is included in the feasible intervals for \( h_0 \) and \( k_0 \), one or both of the simple policy rules can be implemented as the feedback response of the first difference (in the interest rate or the depreciation rate) to the various arguments on the r.h.s. In the case of the interest rate rule, there were no upper bounds for the reactions to inflation or the RER, but, perhaps surprisingly, there was an upper bound of only 1.04 for the response to GDP. There is much more room for an accommodating policy of diminishing the interest rate when GDP is high (up to -3.69). In the case of the nominal depreciation rule, there were no upper or lower bounds for the reactions to inflation or GDP, and an upper bound of 9.12 for the reaction to the RER. In the case of \( k_4 \), the only restriction is that it must be outside of a small interval around zero, which is mostly on the positive side. The fact that there is a comparatively low upper bound for the interest rate response to GDP while there is no bound for the nominal depreciation response is quite interesting, since the stabilization of GDP is, of course, of primary interest in most CBs (along with the stabilization of inflation).

Policy rule coefficients stability ranges

<table>
<thead>
<tr>
<th>Interest rate feedback rule:</th>
<th>MER</th>
<th>FER</th>
<th>PER</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_0 \in )</td>
<td>[0.21,10]</td>
<td>[0.21,10]</td>
<td></td>
</tr>
<tr>
<td>( h_1 \in )</td>
<td>[0.21,10]</td>
<td>[0.21,10]</td>
<td></td>
</tr>
<tr>
<td>( h_2 \in )</td>
<td>[-3.69,1.04]</td>
<td>[-3.54,1.03]</td>
<td></td>
</tr>
<tr>
<td>( h_3 \in )</td>
<td>[-8.14,10]</td>
<td>[-6.89,4.63]</td>
<td></td>
</tr>
<tr>
<td>Nom. depreci. feedback rule:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_0 \in )</td>
<td>[-4.55,4.48]</td>
<td></td>
<td>[-1.32,0.67]</td>
</tr>
<tr>
<td>( k_1 \in )</td>
<td>[-10,10]</td>
<td>[-10,10]</td>
<td>[-1.16,1.67]</td>
</tr>
<tr>
<td>( k_2 \in )</td>
<td>[-10,10]</td>
<td>[-10,10]</td>
<td>[-1.77,2.82]</td>
</tr>
<tr>
<td>( k_3 \in )</td>
<td>[-10,9.12]</td>
<td>[-10,9.12]</td>
<td></td>
</tr>
<tr>
<td>( k_4 \in )</td>
<td>[-10,-0.01] ( \cup ) [0.23,10]</td>
<td>[-0.95,2.44]</td>
<td></td>
</tr>
</tbody>
</table>
The FER regime shows stability ranges very similar to those of the first policy rule of the MER regime. There is some narrowing of the ranges in the cases of \( h_2 \) and \( h_3 \). The narrowing of the range of stability is more significant in the case of the PER regime, especially in the cases of \( k_2 \), \( k_3 \), and \( k_4 \). However, in the PER regime the stability range for \( k_4 \) includes 0, indicating that the need to respond to a target for international reserves is only valid in the MER regime.

Leaving behind the baseline reference, it is interesting to verify that in the FER case there is stability when all the coefficients are zero (\( k_j = 0 \), \( j = 0, 1, 2, 3, 4 \)). In this case the policy rule is to intervene in the FX market sufficiently to maintain the nominal exchange rate fixed at the existing level, letting the economy run its course, and not worrying about international reserves.\(^{20}\)

6) The relatively narrow range of stability for the coefficient on the interest rate response to GDP deviations (\( h_2 \)) in the MER case, along with the boundless range of stability for the corresponding coefficient in the second policy rule (\( k_2 \)), naturally raises the question of the effects of the latter coefficient on the volatilities. The table below shows these effects. All the variables reach minimum volatilities for non-positive values of \( k_2 \). And for a number of very significant variables such as \( \pi^C \), \( \Delta \), \( Y \), \( Q \), \( C \), \( N \), and Utility, the minimum is reached for highly negative values of \( k_2 \) (-10 or -8). Indeed, the lowest volatility of \( Y \), \( Q \), \( N \), and Utility is lower than the lowest volatility they achieve, respectively, in any of the analogous tables above. Hence, reducing the rate of nominal depreciation (or perhaps even appreciating the currency) when GDP is above its NSS level has a very important stabilizing role for most of the variables of interest. Notice that this implies using the international reserves instrument with high volatility.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>k_2=10.0</th>
<th>k_2=8.0</th>
<th>k_2=6.0</th>
<th>k_2=4.0</th>
<th>k_2=2.0</th>
<th>k_2=0.0</th>
<th>k_2=2.0</th>
<th>k_2=4.0</th>
<th>k_2=6.0</th>
<th>max/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^C )</td>
<td>1.0150</td>
<td>0.0119</td>
<td>0.0117</td>
<td>0.0116</td>
<td>0.0117</td>
<td>0.0120</td>
<td>0.0128</td>
<td>0.0147</td>
<td>0.0183</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td>DeltaP</td>
<td>1.0051</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0013</td>
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<td>0.0013</td>
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<td>1.08</td>
</tr>
<tr>
<td>Y</td>
<td>1.4430</td>
<td>0.0596</td>
<td>0.0645</td>
<td>0.0669</td>
<td>0.0700</td>
<td>0.0741</td>
<td>0.0800</td>
<td>0.0886</td>
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</tr>
<tr>
<td>Q</td>
<td>1.3153</td>
<td>0.0542</td>
<td>0.0572</td>
<td>0.0605</td>
<td>0.0628</td>
<td>0.0660</td>
<td>0.0704</td>
<td>1.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.3108</td>
<td>0.0318</td>
<td>0.0319</td>
<td>0.0322</td>
<td>0.0325</td>
<td>0.0332</td>
<td>0.0346</td>
<td>0.09</td>
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<td></td>
<td></td>
</tr>
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<td>0.0146</td>
<td>0.0144</td>
<td>0.0145</td>
<td>0.0148</td>
<td>0.0158</td>
<td>0.0180</td>
<td>0.0226</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>mc</td>
<td>0.8302</td>
<td>0.0112</td>
<td>0.0111</td>
<td>0.0109</td>
<td>0.0109</td>
<td>0.0110</td>
<td>0.0114</td>
<td>0.0122</td>
<td>1.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>0.5951</td>
<td>0.0509</td>
<td>0.0503</td>
<td>0.0495</td>
<td>0.0494</td>
<td>0.0499</td>
<td>0.0511</td>
<td>0.0539</td>
<td>0.0592</td>
<td>1.20</td>
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</tr>
<tr>
<td>TB</td>
<td>0.0082</td>
<td>0.0658</td>
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<td>0.0604</td>
<td>0.0600</td>
<td>0.0613</td>
<td>0.0655</td>
<td>0.0745</td>
<td>0.0907</td>
<td>1.51</td>
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</tr>
<tr>
<td>N</td>
<td>1.3220</td>
<td>0.0545</td>
<td>0.0554</td>
<td>0.0576</td>
<td>0.0590</td>
<td>0.0608</td>
<td>0.0631</td>
<td>0.0662</td>
<td>0.0706</td>
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</tr>
<tr>
<td>Utility</td>
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<td>0.0492</td>
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<td>0.0519</td>
<td>0.0532</td>
<td>0.0549</td>
<td>0.0572</td>
<td>0.0607</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.0253</td>
<td>0.0114</td>
<td>0.0114</td>
<td>0.0113</td>
<td>0.0114</td>
<td>0.0116</td>
<td>0.0121</td>
<td>0.0130</td>
<td>0.0146</td>
<td>1.29</td>
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</tr>
<tr>
<td>delta</td>
<td>1.0150</td>
<td>0.0669</td>
<td>0.0664</td>
<td>0.0659</td>
<td>0.0665</td>
<td>0.0687</td>
<td>0.0737</td>
<td>0.0843</td>
<td>0.1048</td>
<td>1.59</td>
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</tr>
<tr>
<td>b</td>
<td>0.0722</td>
<td>0.0908</td>
<td>0.0732</td>
<td>0.0369</td>
<td>0.0198</td>
<td>0.0167</td>
<td>0.0355</td>
<td>0.0616</td>
<td>0.0940</td>
<td>5.62</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>0.3152</td>
<td>0.1622</td>
<td>0.1340</td>
<td>0.0778</td>
<td>0.0538</td>
<td>0.0448</td>
<td>0.0629</td>
<td>0.0988</td>
<td>0.1467</td>
<td>3.62</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1.2125</td>
<td>0.1061</td>
<td>0.1059</td>
<td>0.1055</td>
<td>0.1051</td>
<td>0.1045</td>
<td>0.1037</td>
<td>0.1034</td>
<td>0.1057</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>0.1154</td>
<td>0.0028</td>
<td>0.0028</td>
<td>0.0028</td>
<td>0.0028</td>
<td>0.0028</td>
<td>0.0029</td>
<td>0.0029</td>
<td>0.0030</td>
<td>0.0031</td>
<td>1.11</td>
</tr>
</tbody>
</table>

\(^{20}\)However, one must bear in mind that here the nominal and real exchange rates are (in spirit) multilateral. If we modeled a multicountry RW, the nominal exchange rate would be the domestic currency price of a basket of the nominal exchange rates of the SOE’s trade partners, with weights equal to the shares in trade. Hence, our peg is completely different from pegging against the currency of a country with which only a small part of the SOE’s trade is done (as was the case of Argentina’s ill fated ‘Convertibility’).
4.2. Optimal simple rules

In view of these results, it is worthwhile to enquire what the optimal simple policy rules coefficients are, when using some ad-hoc criterion.

1) First I used Dynare’s osr (optimal simple rule) command to obtain the policy coefficients that minimized the variance of Utility (with a coefficient of 100 in the loss function)\(^{21}\).

\[
\arg\min_{h_i,k_t} \{\omega_U Var(\text{Utility}_t)\} \tag{95}
\]

\[
= \arg\min_{h_i,k_t} \lim_{\beta \to 1} E_0 \sum_{t=1}^{\infty} (1 - \beta^t) \beta^t \{\omega_U (\text{Utility}_t - \text{Utility})^2\} \tag{96}
\]

I obtained the following optimal coefficients for the two simple policy rules (rounding off to two digits except for \(k_4\)):

<table>
<thead>
<tr>
<th>(h_0)</th>
<th>(h_1)</th>
<th>(h_2)</th>
<th>(h_3)</th>
<th>(k_0)</th>
<th>(k_1)</th>
<th>(k_2)</th>
<th>(k_3)</th>
<th>(k_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54</td>
<td>-1.34</td>
<td>0.37</td>
<td>-0.15</td>
<td>-0.92</td>
<td>-0.54</td>
<td>-2.14</td>
<td>-0.74</td>
<td>-0.006</td>
</tr>
</tbody>
</table>

Note that the sum of \(h_0\) and \(h_1\) is above one, but \(h_0\) is very superinertial while \(h_1\) is very negative.\(^{22}\) Also, the optimal value of \(k_2\) is -2.14. These values are in accordance with what was obtained above. Under our simple rules, and assuming that the policymakers wish to reduce the variance of the utility of households, it is optimal to react strongly to inflation in the interest rate rule and strongly to GDP in the nominal depreciation rule. A deviation of 1% in inflation above its target value here commands a reduction of 1.3 p.p. in the interest rate (assuming it was at the NSS level the previous period) and a reduction of 0.5 p.p. in the rate of nominal depreciation (assuming it was at the NSS level the previous period). And a deviation of 1% in GDP above its NSS value commands an increase in the interest rate of 0.4 p.p. and a reduction of 2.1 p.p. in the rate of nominal depreciation. On the other hand, a deviation of 1% in the RER above its NSS value commands a reduction in the interest rate of 0.2 p.p. and a reduction in the rate of nominal depreciation of 0.7 p.p. The latter seems quite natural: if the currency is weak in real terms (\(e\) is high), depreciate less. Finally, it is optimal to make strong use of policy inertia in the case of the interest rate, even though no CB preference for such policies has been assumed, and to have a negative inertia coefficient (-0.9) for the nominal depreciation rule.

The following table shows the standard deviations of the main endogenous variables when using these optimal simple rules.

\(^{21}\)The use of large coefficients in the objective function is motivated by the need to have osr effectively search the parameter space before settling on the optimal coefficients. When I used low coefficients (in the order of 1) the search was very short and I had to iterate the command (after putting the resulting coefficients as the initial ones) many times before converging to the truly optimal ones. Looking into Dynare’s forum I found that someone had suggested having very large coefficients as a way of inducing an effective search. And indeed this was very helpful.

\(^{22}\)Some of these values are outside the stability ranges shown in the table above. However, that range was obtained keeping all the other coefficients at their baseline levels, which is clearly not the case here. Dynare’s osr search changes direction whenever it goes into parameter values that do not comply with the Blanchard and Kahn conditions. In this particular case, the search engine tended to obtain minor gains is loss with exceedingly high coefficients (in absolute value, in the thousands) when using some initializations. I ignored such gains and kept to the more moderate coefficient values.
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>Std.Dev</th>
<th>Std.Dev./Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>piC</td>
<td>1.015</td>
<td>0.046</td>
<td>0.05</td>
</tr>
<tr>
<td>DeltaP</td>
<td>1.005</td>
<td>0.033</td>
<td>0.03</td>
</tr>
<tr>
<td>Y</td>
<td>1.443</td>
<td>0.091</td>
<td>0.06</td>
</tr>
<tr>
<td>Q</td>
<td>1.315</td>
<td>0.094</td>
<td>0.07</td>
</tr>
<tr>
<td>C</td>
<td>1.311</td>
<td>0.085</td>
<td>0.06</td>
</tr>
<tr>
<td>real_i</td>
<td>1.010</td>
<td>0.022</td>
<td>0.02</td>
</tr>
<tr>
<td>mc</td>
<td>0.830</td>
<td>0.092</td>
<td>0.11</td>
</tr>
<tr>
<td>e</td>
<td>0.595</td>
<td>0.081</td>
<td>0.14</td>
</tr>
<tr>
<td>TB</td>
<td>0.008</td>
<td>0.161</td>
<td>19.57</td>
</tr>
<tr>
<td>N</td>
<td>1.322</td>
<td>0.099</td>
<td>0.07</td>
</tr>
<tr>
<td>Utility</td>
<td>-2.274</td>
<td>0.022</td>
<td>-0.01</td>
</tr>
<tr>
<td>ii</td>
<td>1.025</td>
<td>0.041</td>
<td>0.04</td>
</tr>
<tr>
<td>delta</td>
<td>1.015</td>
<td>0.060</td>
<td>0.06</td>
</tr>
<tr>
<td>b</td>
<td>0.072</td>
<td>1.539</td>
<td>21.32</td>
</tr>
<tr>
<td>r</td>
<td>0.315</td>
<td>2.582</td>
<td>8.19</td>
</tr>
<tr>
<td>d</td>
<td>1.213</td>
<td>0.176</td>
<td>0.14</td>
</tr>
<tr>
<td>m</td>
<td>0.115</td>
<td>0.008</td>
<td>0.07</td>
</tr>
</tbody>
</table>

First, notice how small the standard deviation of Utility is. While the four tables above all had standard deviations above 0.0486, the osr routine reduced it to 0.022. Second, it is noteworthy that minimizing the volatility of Utility actually implies having substantial volatilities in many of the variables that ad-hoc CB loss functions usually try to minimize. While the highest s.d. of consumer inflation in the above four tables was 0.019, the s.d. is 0.046 when this optimal simple policy rule is used. The contrast with the price dispersion variable is similar. The highest above was 0.008 and now it is 0.033, two and a half times higher.

The volatility of the two instruments is very high indeed (21.3 times its mean for $b$ and 8.2 times its mean for $r$), pointing to the fact that in actual practice there may be constraints not imposed in this exercise that prevent the attainment of such optimal simple policy rules. But the high volatility of the instruments is not surprising since the attainment of optimal results can imply responding very strongly in order to achieve the operational targets for $i$ and $\delta$.

2) Few CBs actually use models in which the explicit goal of the policymaker has to do with household utility. This is probably due to the fact that most models misrepresent reality in way that CBs cannot take for granted: they assume homogenous households (except possibly for the heterogeneity derived from wage setting in a monopolistically competitive setting). The usual target variables of CB loss functions are inflation and GDP, and there is usually some explicit distaste for excessive movement in the operational target variable (the interest rate). This, of course, also brushes away, though in a different way, the incidence of CB actions on different sectors of the economy and different factor incomes. However, whereas aggregate household utility is an abstract concept because it is known that the model is misspecified in the dimension of household heterogeneity, variables like inflation, GDP, or the RER, have clear empirical counterparts. Hence, I now repeat the above exercise assuming that the CB minimizes a linear combination of the variances of its target variables.

$$\arg \min_{h_i, k_i} \{ \omega_{\pi} Var(\pi_t^C) + \omega_Y Var(Y_t) + \omega_e Var(e_t) + \omega_r Var(r_t) + \omega_{\Delta i} Var(\Delta_i) + \omega_{\Delta s} Var(\Delta s) \}$$
Aside from the usual terms (with weights $\omega_\pi$, $\omega_Y$, $\omega_\Delta$), this loss function also allows for CB preferences with respect to the variances of the RER, of the CBs IRs, and of changes in the rate of nominal depreciation (with weights $\omega_e$, $\omega_r$, $\omega_{\Delta\delta}$). I define six different CB styles (A-F) according to the combinations of weights in each. In all of them I have given the same weights to the changes in each of the operational targets (50), and avoided zeros giving a weight of 1 to target variables with little importance. In style A only inflation matters and in style B only GDP matters. Both matter equally in style C. In style D the RER matters as much as inflation and GDP, and in style E IRs matter as much as inflation and GDP. Finally, in style F inflation, GDP, the RER and the IRs all equally matter.

<table>
<thead>
<tr>
<th>Weights</th>
<th>Central Bank Styles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>$\omega_{\pi}$</td>
<td>100</td>
</tr>
<tr>
<td>$\omega_Y$</td>
<td>1</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>1</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>1</td>
</tr>
<tr>
<td>$\omega_{\Delta}$</td>
<td>50</td>
</tr>
<tr>
<td>$\omega_{\Delta\delta}$</td>
<td>50</td>
</tr>
</tbody>
</table>

With Dynare’s osr command I obtained the following optimal simple policy rules for each of the CB styles in each of the interest and exchange regimes:

The inertial coefficient for the interest rate is always high in both the MER regime, and very superinertial in all styles except A, in which only inflation matters. In the MER regime, the interest rate response to inflation deviations is greater than one in styles A and C, in both of which inflation matters. Under styles B and D, the optimal policy rules in the MER regime present those strange situations in which there is a counterintuitive response of the rate of nominal depreciation to deviations from the IRs target ($k_4$ is positive). In both $h_0$ is superinertial and $h_1$ is negative. Furthermore, in both cases the Taylor and Woodford principles do not hold, since $h_0 + h_1$ is less than one, and yet there is stability. In both cases the interest rate response to GDP is high (4.3 and 2.6, respectively) and the depreciation rate response to to inflation and GDP are highly negative.

In the FER regime, central banks of style A practically respond only to inflation, with a coefficient greater than two. However, in all the rest of the styles, $h_0$ is superinertial and $h_1$ is negative. The Taylor and Woodford principles hold in these cases. And in the PER regime, all the coefficients are negative except for $k_3$, which is positive for styles B, D, E, and F. Hence, high inflation and high GDP imply lowering the rate of nominal depreciation (or appreciating).
The following table shows the standard deviations of the main endogenous variables in each regime and CB style, as well as the total and relative losses. As expected, the loss is always lowest with the MER regime. For CB styles A and B the losses with the FER and PER regimes are between six and eleven times higher than with the MER regime. In style C, where both inflation and GDP matter, the losses in the FER and PER regimes are 3 and 2.4 times the loss with the MER regime. The differences in the losses are lowest with CB styles E and F (where IRs matter). But they still have losses that are between 34% and 65% greater than in the MER regime.

It should be emphasized that the PER regime here is not the usual pegged exchange rate. The simple rule in the PER regime includes the typical peg, which has no feedback. But this section shows that it is optimal to operate the PER regime with feedback. And the coefficients are in general quite high. Hence, it is optimal to operate a very active peg for any of the CB styles.

<table>
<thead>
<tr>
<th>OPTIMAL SIMPLE POLICY RULES</th>
</tr>
</thead>
<tbody>
<tr>
<td>STANDARFIRST DEVIATION A</td>
</tr>
<tr>
<td>piC 1.015 0.006 0.016 0.011 0.070 0.157... 6.60 6.45 9.08 2.99 2.37 3.45 2.72 1.65 1.47 1.55 1.34</td>
</tr>
<tr>
<td>DeltaP 1.005 0.004 0.007 0.007 0.029 0.022... 0.037 0.017 0.019 0.029 0.157 0.045 0.033 0.039 0.017 0.037 0.017 0.037 0.039 0.018</td>
</tr>
<tr>
<td>Y 1.443 0.043 0.070 0.078 0.012 0.014... 0.012 0.027 0.027 0.027 0.027 0.027 0.027 0.027 0.027 0.027 0.027 0.027 0.027</td>
</tr>
<tr>
<td>Q 1.316 0.033 0.029 0.074 0.015 0.009... 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012 0.012</td>
</tr>
<tr>
<td>C 1.312 0.037 0.039 0.074 0.066 0.069... 0.033 0.033 0.033 0.033 0.033 0.033 0.033 0.033 0.033 0.033 0.033 0.033</td>
</tr>
<tr>
<td>real, 1.010 0.011 0.035 0.052 0.017 0.035... 0.028 0.028 0.028 0.028 0.028 0.028 0.028 0.028 0.028 0.028 0.028 0.028</td>
</tr>
<tr>
<td>mc 0.830 0.026 0.025 0.069 0.079 0.092... 0.045 0.045 0.045 0.045 0.045 0.045 0.045 0.045 0.045 0.045 0.045 0.045</td>
</tr>
<tr>
<td>e 0.594 0.038 0.052 0.048 0.037 0.052... 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044</td>
</tr>
<tr>
<td>TB 0.007 0.060 0.057 0.058 0.052 0.068... 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039</td>
</tr>
<tr>
<td>N 1.322 0.063 0.064 0.079 0.072 0.138... 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037</td>
</tr>
<tr>
<td>Utility -2.273 0.050 0.055 0.054 0.074 0.105... 0.037 0.041 0.041 0.041 0.041 0.041 0.041 0.041 0.041 0.041 0.041 0.041</td>
</tr>
<tr>
<td>l 1.025 0.011 0.036 0.048 0.076 0.136... 0.039 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.029</td>
</tr>
<tr>
<td>delta 1.015 0.029 0.081 0.058 0.066 0.166... 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044 0.044</td>
</tr>
<tr>
<td>b 0.072 0.106 0.021 0.000 0.116 0.022... 0.000 0.163 0.163 0.163 0.163 0.163 0.163 0.163 0.163 0.163 0.163 0.163</td>
</tr>
<tr>
<td>r 0.316 0.197 0.000 0.042 0.196 0.000... 0.035 0.282 0.282 0.282 0.282 0.282 0.282 0.282 0.282 0.282 0.282 0.282</td>
</tr>
<tr>
<td>d 1.214 0.135 0.081 0.099 0.123 0.072... 0.094 0.144 0.144 0.144 0.144 0.144 0.144 0.144 0.144 0.144 0.144 0.144</td>
</tr>
<tr>
<td>m 0.115 0.003 0.006 0.011 0.014 0.019... 0.007 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037 0.037</td>
</tr>
<tr>
<td>Loss 0.68 0.87 0.53 0.10 0.65 0.91... 0.11 0.12 0.09 0.09 0.12 0.09 0.09 0.12 0.09 0.09 0.12 0.09</td>
</tr>
<tr>
<td>Relative Loss 10.88 6.60 6.45 9.08 2.99 2.37... 3.45 2.72 1.65 1.47 1.55 1.34</td>
</tr>
</tbody>
</table>

4.3. Optimal policy rules under commitment

In this section I use Dynare’s ‘ramsey’ command to obtain the optimal rules under commitment, i.e., the rules that yield the minimum expected value of the discounted ad-hoc loss function:

\[ L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} e^{-\gamma t} L_t, \]  
(97)

conditional on the information at \( t = t_0 \), where \( L_t \) is given by:

\[ L_t = \omega_\pi (\pi^C_t - \pi^T_t)^2 + \omega_Y (Y_t - Y)^2 + \omega_e (e_t - e)^2 + \omega_r (r_t - r)^2 + \omega_{\Delta} (\Delta i_t)^2 + \omega_{\Delta} (\Delta \delta_t)^2, \]  
(98)

given initial conditions for the predetermined values, and subject to all the model equations (except the simple policy rules). I maintain the same definition of CB styles as in the previous section. In the table below I report the standard deviations of the main variables as well as the expected loss for the alternative CB styles (A-F) and the alternative policy regimes (MER, FER, PER).
As expected, the MER regime always dominates the two ‘corner’ regimes. Under CB styles A, B, and C, the losses under the FER and PER regimes are between 265\% and 390\% higher than under the MER regime. Under CB styles E and F, where IRs matter for the CB, the losses under the FER and PER regimes are ‘only’ 17/18\% higher. In CB style B, in which only GDP matters, the FER regime achieves a significantly lower cost than the PER regime. In CB styles A, C, and D, it is the PER regime that is second best. And in CB styles E and F, the ‘corner’ regimes obtain losses that are approximately the same.

The two tables below show the optimal policy rules for the three alternative regimes. The operational target variables are linear functions of 9 non-shock predetermined variables (the interest rate, the nominal depreciation rate, the IRs, the RER, GDP, foreign debt, price dispersion, the relative price of consumption goods, and the external terms of trade), the 6 shock variables, and the Lagrange multipliers corresponding to the 5 equations with forward-looking terms (the UIP equation, the two dynamic Phillips equations, the consumption Euler equation, and the real interest rate equation). In all of the CB styles there is substantial inertia in the interest rate rule (between 0.1 and 0.69) and in the nominal depreciation rule (between 0.1 and 0.6). This is hardly surprising since all these CB styles have been defined to show a significant preference for policy inertia. What is perhaps surprising is the dispersion in the inertial coefficients, given that they all have the same weight for preference for inertia (50). The coefficients on the Lagrange multipliers are relatively small, so the policy rules (for the rest of the variables) do not vary much from quarter to quarter when these effects are cumulated. The largest of these coefficients correspond to the Lagrange multipliers for the Phillips equations under CB style B, where only GDP matters.
## OPTIMAL POLICY RULES UNDER COMMITMENT

### MER

<table>
<thead>
<tr>
<th>STYLES:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.025</td>
<td>1.015</td>
<td>1.025</td>
<td>1.015</td>
<td>1.025</td>
<td>1.015</td>
</tr>
<tr>
<td>ii(-1)</td>
<td>0.689</td>
<td>0.511</td>
<td>0.551</td>
<td>0.325</td>
<td>0.387</td>
<td>0.106</td>
</tr>
<tr>
<td>delta(-1)</td>
<td>0.011</td>
<td>0.358</td>
<td>0.325</td>
<td>0.604</td>
<td>0.106</td>
<td>0.302</td>
</tr>
<tr>
<td>r(-1)</td>
<td>-0.021</td>
<td>-0.070</td>
<td>0.044</td>
<td>-0.044</td>
<td>0.022</td>
<td>-0.073</td>
</tr>
<tr>
<td>e(-1)</td>
<td>0.029</td>
<td>-0.564</td>
<td>0.507</td>
<td>-0.484</td>
<td>0.372</td>
<td>-0.685</td>
</tr>
<tr>
<td>Y(-1)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>d(-1)</td>
<td>0.021</td>
<td>0.070</td>
<td>-0.044</td>
<td>0.044</td>
<td>0.022</td>
<td>0.073</td>
</tr>
<tr>
<td>DeltaP(-1)</td>
<td>0.009</td>
<td>0.007</td>
<td>-0.019</td>
<td>0.016</td>
<td>-0.009</td>
<td>0.031</td>
</tr>
<tr>
<td>PC(-1)</td>
<td>-0.177</td>
<td>0.364</td>
<td>0.008</td>
<td>0.012</td>
<td>0.119</td>
<td>0.195</td>
</tr>
<tr>
<td>pStar(-1)</td>
<td>-0.017</td>
<td>-0.182</td>
<td>0.194</td>
<td>-0.184</td>
<td>0.158</td>
<td>-0.235</td>
</tr>
<tr>
<td>z_piStar(-1)</td>
<td>-0.004</td>
<td>-0.106</td>
<td>0.073</td>
<td>-0.071</td>
<td>0.036</td>
<td>-0.123</td>
</tr>
<tr>
<td>z_piStarX(-1)</td>
<td>-0.021</td>
<td>-0.126</td>
<td>0.110</td>
<td>-0.106</td>
<td>0.069</td>
<td>-0.163</td>
</tr>
<tr>
<td>z_G(-1)</td>
<td>0.012</td>
<td>-0.052</td>
<td>0.154</td>
<td>-0.124</td>
<td>0.115</td>
<td>-0.162</td>
</tr>
<tr>
<td>z_epsilon(-1)</td>
<td>-0.029</td>
<td>-0.031</td>
<td>0.066</td>
<td>-0.053</td>
<td>0.018</td>
<td>-0.114</td>
</tr>
<tr>
<td>z_iStar(-1)</td>
<td>0.045</td>
<td>0.169</td>
<td>-0.117</td>
<td>0.117</td>
<td>-0.071</td>
<td>0.178</td>
</tr>
<tr>
<td>z_phiStar(-1)</td>
<td>0.035</td>
<td>0.120</td>
<td>-0.075</td>
<td>0.075</td>
<td>-0.038</td>
<td>0.124</td>
</tr>
<tr>
<td>mult_8(-1)</td>
<td>0.000</td>
<td>0.004</td>
<td>0.003</td>
<td>0.006</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>mult_15(-1)</td>
<td>0.005</td>
<td>0.026</td>
<td>0.123</td>
<td>0.165</td>
<td>0.016</td>
<td>0.021</td>
</tr>
<tr>
<td>mult_16(-1)</td>
<td>-0.003</td>
<td>0.038</td>
<td>0.161</td>
<td>0.216</td>
<td>0.020</td>
<td>0.025</td>
</tr>
<tr>
<td>mult_22(-1)</td>
<td>0.007</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>mult_30(-1)</td>
<td>0.001</td>
<td>-0.002</td>
<td>-0.004</td>
<td>-0.005</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>eps_epsilon</td>
<td>-0.036</td>
<td>-0.039</td>
<td>0.083</td>
<td>-0.066</td>
<td>0.022</td>
<td>-0.143</td>
</tr>
<tr>
<td>eps_G</td>
<td>0.014</td>
<td>-0.061</td>
<td>0.181</td>
<td>-0.145</td>
<td>0.135</td>
<td>-0.191</td>
</tr>
<tr>
<td>eps_iStar</td>
<td>0.037</td>
<td>0.151</td>
<td>-0.111</td>
<td>0.111</td>
<td>-0.074</td>
<td>0.161</td>
</tr>
<tr>
<td>eps_phiStar</td>
<td>-0.033</td>
<td>-0.114</td>
<td>0.075</td>
<td>-0.075</td>
<td>0.041</td>
<td>-0.120</td>
</tr>
<tr>
<td>eps_piStar</td>
<td>0.025</td>
<td>-0.255</td>
<td>0.121</td>
<td>-0.123</td>
<td>0.029</td>
<td>-0.257</td>
</tr>
<tr>
<td>eps_piStarX</td>
<td>-0.052</td>
<td>-0.306</td>
<td>0.269</td>
<td>-0.259</td>
<td>0.168</td>
<td>-0.398</td>
</tr>
</tbody>
</table>
In order to study the sensitivity of expected discounted loss under Ramsey to different parameter values, I first obtained the structural parameter ranges under which (under a MER regime and Ramsey optimal policy rules) there is stability. Starting from the baseline parameter combination that has been used above, I varied each parameter individually until stability was impaired. The following table shows that there are remarkably wide ranges within which the parameters can be moved. Obviously, in some cases I did not bother to find the actual extremes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline value</th>
<th>Stability range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^T$</td>
<td>1.015</td>
<td>&lt;0.9 to 1.07</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>&lt;0.8 to &gt; 0.9999</td>
</tr>
<tr>
<td>$\sigma^C$</td>
<td>1.5</td>
<td>0.01 to 50</td>
</tr>
<tr>
<td>$\sigma^N$</td>
<td>0.5</td>
<td>0.01 to 50</td>
</tr>
<tr>
<td>$a_D$</td>
<td>0.86</td>
<td>0.35 to 0.99</td>
</tr>
<tr>
<td>$\theta$</td>
<td>6</td>
<td>1.01 to 27</td>
</tr>
<tr>
<td>$\theta^C$</td>
<td>1.5</td>
<td>0.01 to 0.99 and 1.01 to 50</td>
</tr>
<tr>
<td>$b^A$</td>
<td>0.5</td>
<td>0.01 to 0.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.66</td>
<td>0.01 to 0.91</td>
</tr>
<tr>
<td>$\epsilon_{(D)}^{\varphi}$</td>
<td>2</td>
<td>0.01 to 10000</td>
</tr>
<tr>
<td>$\epsilon_{(L)}$</td>
<td>1.02</td>
<td>0.3 to 100</td>
</tr>
<tr>
<td>$\gamma^D$</td>
<td>0.5</td>
<td>0.01 to 50</td>
</tr>
</tbody>
</table>

The degree of price stickiness in the New Keynesian Phillips equation is usually considered an important factor concerning the desirability of alternative exchange...
regimes. The following table shows the losses under each CB style and exchange rate regime for six alternative degrees of price stickiness, which go from practically no price stickiness (\(\alpha=0.01\)) to very high price stickiness (\(\alpha=0.90\)). As expected, for each CB style and value of \(\alpha\), the MER regime does better and in most cases much better. CB styles E and F are the ones for which the advantage of the MER regime is smallest, especially when there is little price stickiness: for \(\alpha=0.01\) and \(\alpha=0.10\), the PER regime has a loss which is only 3% higher than in the MER regime. This is probably because the CB preference for stabilizing IRs makes it behave similarly in MER and PER regimes. However, for CB styles A, B, and C, the corner regimes have losses between 20% and 248% higher. The highest relative advantage for the MER regime is obtained for high degrees of price stickiness. In general, the PER regime is second best for low degrees of price stickiness (\(\alpha \leq 0.30\)). For \(\alpha=0.50\), the FER regime is second best only for CB style B. And for higher values of \(\alpha\), the FER regime is second best for CB styles A, B, E, and F. Another interesting feature is that the losses are not always strictly increasing with \(\alpha\). For example, under CB style B and regime MER, the loss reaches a peak for \(\alpha=0.30\) and then diminishes up to \(\alpha=0.70\). For the same CB style, however, but regimes FER and PER, the loss does increase monotonously with \(\alpha\). But for CB style A, these regimes reach a peak at \(\alpha=0.70\), while the MER regime has its loss increasing monotonously throughout.

Summing up, with or without price stickiness there is a gain from intervening in the FX market in the sense that the CB can better stabilize its target variables. The advantage is greater when the CB does not have a strong preference for the stabilization of its international reserves and the degree of price stickiness is around 0.70 (CB style A) or 0.90 (CB style B).

<table>
<thead>
<tr>
<th>STYLE</th>
<th>LOSS</th>
<th>RELATIVE LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha=0.01</td>
<td>alpha=0.10</td>
<td>alpha=0.30</td>
</tr>
<tr>
<td>MER</td>
<td>FER</td>
<td>PER</td>
</tr>
<tr>
<td>A</td>
<td>78.2</td>
<td>144.1</td>
</tr>
<tr>
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</tr>
<tr>
<td>C</td>
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<td>211.3</td>
</tr>
<tr>
<td>D</td>
<td>162.1</td>
<td>242.8</td>
</tr>
<tr>
<td>E</td>
<td>196.2</td>
<td>211.3</td>
</tr>
<tr>
<td>F</td>
<td>225.2</td>
<td>242.8</td>
</tr>
</tbody>
</table>

4. Policy and capital flows in the SOE
To obtain some intuition as to what factors may account for the lower losses in the MER regimes, let us take the log-linear approximations of the UIP equation and the simple policy rules equations under the MER regime:

\[
\tilde{t}_t = E_t \hat{d}_{t+1} + \tilde{\alpha}_t + \alpha \tilde{v}_t + \varepsilon_d \left( \tilde{d}_t + \tilde{c}_t - \hat{Y}_t \right)
\]

\[
\hat{t}_t = h_{0} \tilde{\alpha}_{t-1} + h_{1} \tilde{\alpha}_t + h_{2} \tilde{Y}_t + h_{3} \tilde{c}_t
\]

\[
\hat{\alpha}_t = k_{0} \tilde{\alpha}_{t-1} + k_{1} \tilde{\alpha}_t + k_{2} \tilde{Y}_t + k_{3} \tilde{c}_t + k_{4} \left( \tilde{r}_t + \tilde{e}_t - \hat{Y}_t \right)
\]
Leading the third equation, subtracting the resulting equation from the second equation, and using the first gives the following:

\[
\varepsilon_D^\varphi \left( \hat{a}_t + \hat{c}_t - \hat{Y}_t \right) + \left( \hat{h}_t^* + \hat{\phi}_t^* \right) = \left( h_0 \hat{a}_{t-1} - k_0 \delta_t \right) + \left( h_1 \hat{\pi}_t^C - k_1 E_t \hat{\pi}_{t+1}^C \right) \\
+ \left( h_2 \hat{Y}_t - k_2 E_t \hat{Y}_{t+1} \right) + \left( h_3 \hat{e}_t - k_3 E_t \hat{e}_{t+1} \right) - k_4 \left( E_t \hat{e}_{t+1} + E_t \hat{e}_{t+1} - E_t \hat{Y}_{t+1} \right).
\]

On the l.h.s. is the log-linear deviation (from the NSS) of the risk/liquidity premium that foreign investors use to determine the interest rate at which they lend to the SOE households. On the r.h.s. is a complex term that exclusively depends on the log-linear deviations of the target variables the CB uses for its policy rules and the coefficients in the simple policy rules. The CB policy rules have the effect of modifying the effects that the risk/liquidity premium of foreign investors has on some important variables as a response to shocks. The constraints that the respective corner regimes impose, imply that the CB has less leeway to affect international capital flows in the direction that may help it stabilize the economy according to its preferences (or style). Under the FER regime, in which all the \( k_i \) are zero, the equation reduces to

\[
\varepsilon_D^\varphi \left( \hat{a}_t + \hat{c}_t - \hat{Y}_t \right) + \left( \hat{h}_t^* + \hat{\phi}_t^* \right) = h_0 \hat{a}_{t-1} + h_1 \hat{\pi}_t^C + h_2 \hat{Y}_t + h_3 \hat{e}_t
\]

and under the PER regime, in which all the \( h_i \) are zero, it becomes

\[
\varepsilon_D^\varphi \left( \hat{a}_t + \hat{c}_t - \hat{Y}_t \right) + \left( \hat{h}_t^* + \hat{\phi}_t^* \right) = -k_0 \hat{\delta}_t - E_t \left[ k_1 \hat{\pi}_{t+1}^C + k_2 \hat{Y}_{t+1} + k_3 \hat{e}_{t+1} + k_4 \left( \hat{e}_{t+1} - \hat{Y}_{t+1} \right) \right].
\]

In both of these corner cases, the CB affects capital inflows (increases in \( d \)) and outflows (reductions in \( d \)) through its interest rate or exchange rate policy, respectively. Note that in the particular PER regime in which there is no feedback, the r.h.s. of the last equation is simply \(-k_0 \delta_t \), and in the fixed exchange rate policy it reduces to zero. However, the flexibility it achieves by using two simultaneous policy rules generates gains that, at least for the most usual CB styles, can be substantial. Such gains have been measured above, in the context of this particular model, as the reductions in expected loss obtained from using the MER regime instead of any of the corner regimes.

I illustrate this with IRFs corresponding to a positive shock to \( \phi_t^* \) (i.e., an adverse liquidity/risk shock) in the case of optimal simple policy rules and CB styles A and B. The first (second) set of 3 graphs below shows IRFs for each of the three policy regimes when they use the optimal simple rules found in the table above for style A (B).

Notice that under the MER regime, and in both CB styles, the optimal policy achieves a much smaller increase in the real interest rate and in the RER, than any of the corner regimes. More specifically, under style A, the overall effect on consumption inflation is much smaller than in the corner regimes. And under style B, the overall effect on \( Y \) is much smaller under the MER regime.\(^{23}\) Hence, a

\(^{23}\)Notice that the expansionary effect on output is not realistic. This is due to the fact that in this simple model the effect of the RER on exports is contemporaneous instead of lagged (as in the more elaborate model in Escudé (2009)). Hence, the expansionary effect on exports more than compensates for the contractionary effect on consumption.
stronger stabilization is achieved in the variable that is most important for the CB under each style.
We can conclude that a policy of systematically intervening in the foreign exchange market through a feedback rule is a valuable complement to any interest rate policy rule framework, and that there are good reasons for defending a managed exchange rate regime as the baseline in any SOE modeling framework. Furthermore, it seems reasonable to put the burden of proof on those SOE CBs that exercise 'corner' policies (i.e., that have either pure floats or pure pegs) in explaining why in their particular economies (which includes the stochastic properties of the shocks they are subject to), and under their particular preferences, there is nothing (or not
much) to be gained from using two forms of intervention (and hence two control variables) instead of one.

5. Conclusion
This paper tries to bridge the gap between the fact that many central banks systematically intervene in the foreign exchange market and the absence of any generally accepted model for the representation of this practice. It builds a model and a policy framework in which the CB can simultaneously intervene in the foreign exchange and bond markets, varying its outstanding bond liabilities and reserve assets in order to achieve two operational targets: one for the interest rate and another for the rate of nominal depreciation. For this, the DSGE model includes financial variables and institutional practices (‘nuts and bolts’ of central banking) that are left out of the modeling when only the extreme policy regimes of a pure float or a pure peg are considered, but cannot be left out when trying build a more general model. The resulting model has a core that is little more than the typical DSGE workhorse of the profession, but extends it in directions which allow for a richer policy framework. The model parameters and steady state values of endogenous variables are carefully calibrated, and implemented in Dynare. Three alternative regimes are considered: the general, two rules regime (denominated Managed Exchange Rate regime), and the two corner regimes of Floating Exchange Rate and Pegged Exchange Rate (both using a single feedback policy rule). Simple rules, optimal simple rules, and optimal rules in a linear-quadratic framework under commitment are considered. In both of the optimal rules frameworks it is shown that the use of two policy rules systematically outperforms any of the corner regimes (and for most central bank styles considered achieves substantially better results). The Appendix shows some of the IRFs generated by Dynare.

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Appendix 1. Calibration of parameters and derivation of the associated non-stochastic steady state

In this Appendix I obtain the non-stochastic steady state (NSS) values of the endogenous variables and the calibrated values for the model’s parameters. There are always many ways of doing this. I calibrate some of the parameters, some ratios and some NSS values of endogenous variables, and obtain the rest sequentially from the nonlinear equations so that a computer code can follow the same steps if one changes some of the calibrated values or estimates some of them from the data. The Dynare code follows these steps exactly.

A.1.1. Calibration of the components of the external terms of trade (XTT)

The terms of trade is a particularly important variable for any SOE. Hence, I made a preliminary investigation of the data pertaining to Argentina. To check (88) with the data, notice that the first two of these equations can be written in terms of the (logs of) price indexes:

\[
\begin{align*}
\Delta \log P^X_t &= \rho^X \Delta \log P^X_{t-1} + (1 - \rho^X) \Delta \log \pi^X + \alpha^X (\log P^X_{t-1} - \log P^N_{t-1}) + \sigma^X \varepsilon^X_t, \\
\Delta \log P^N_t &= \rho^N \Delta \log P^N_{t-1} + (1 - \rho^N) \Delta \log \pi^N + \alpha^N (\log P^X_{t-1} - \log P^N_{t-1}) + \sigma^N \varepsilon^N_t.
\end{align*}
\]

A quick estimation for cointegration of Argentina’s trade price indexes during 1993Q3-2009Q2 gave the following result (the notation should be obvious):
Although empirically I was not able to impose a coefficient of negative one for the second coefficient in the cointegrating relation, I do impose it in the calibration to be consistent with the definition of the terms of trade. I also ignore the small deterministic trend in the cointegrating relation, the two time dummies (first and fourth quarters of 2008) that made the residuals normal, homoscedastic and devoid of serial correlation, as well as the non-significant coefficients. Hence, I use the following specification in the model:

\[
\Delta \log P_t^{*X} = 0.41 \Delta \log P_{t-1}^{*X} + (1 - 0.41) \Delta \log \pi^{*X} - 0.25 \left( \log P_{t-1}^{*X} - \log P_{t-1}^{*N} \right) + 0.0424 \varepsilon_t^{\pi **}, \\
\Delta \log P_t^{*N} = 0.20 \Delta \log P_{t-1}^{*N} + (1 - 0.20) \Delta \log \pi^{*N} + 0.18 \left( \log P_{t-1}^{*X} - \log P_{t-1}^{*N} \right) + 0.18 \Delta \log P_{t-1}^{*X} + 0.0295 \varepsilon_t^{\pi *},
\]

Although empirically I was not able to impose a coefficient of negative one for the second coefficient in the cointegrating relation, I do impose it in the calibration to be consistent with the definition of the terms of trade. I also ignore the small deterministic trend in the cointegrating relation, the two time dummies (first and fourth quarters of 2008) that made the residuals normal, homoscedastic and devoid of serial correlation, as well as the non-significant coefficients. Hence, I use the following specification in the model:

\[
\Delta \log P_t^{*X} = 0.41 \Delta \log P_{t-1}^{*X} + (1 - 0.41) \Delta \log \pi^{*X} - 0.25 \left( \log P_{t-1}^{*X} - \log P_{t-1}^{*N} \right) + 0.0424 \varepsilon_t^{\pi **}, \\
\Delta \log P_t^{*N} = 0.20 \Delta \log P_{t-1}^{*N} + (1 - 0.20) \Delta \log \pi^{*N} + 0.18 \left( \log P_{t-1}^{*X} - \log P_{t-1}^{*N} \right) + 0.18 \Delta \log P_{t-1}^{*X} + 0.0295 \varepsilon_t^{\pi *},
\]

### Table: Vector Error Correction Estimates

<table>
<thead>
<tr>
<th>Cointegrating Eq:</th>
<th>CointEq1</th>
</tr>
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<tr>
<td>LPSTARKLEVEL(-1)</td>
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<tr>
<td>LPSTARNLEVEL(-1)</td>
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</tr>
<tr>
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<td>0.1263</td>
</tr>
<tr>
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</tr>
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<td>-0.0044</td>
</tr>
<tr>
<td>C</td>
<td>2.3074</td>
</tr>
</tbody>
</table>

### Error Correction:

\[
\begin{array}{ccc}
\text{CointEq1} & \Delta \text{(LPSTARKLEVEL)} & \Delta \text{(LPSTARNLEVEL)} \\
-0.25543 & 0.18115 & 0.09767 & 0.06597 \\
[-2.61520] & [2.74597] & [0.1263] & [0.10865] \\
0.40776 & 0.17699 & 0.1046 & 0.08965 \\
0.15719 & 0.20980 & 0.12046 & 0.08965 \\
-0.00273 & -0.0048 & -0.00273 & -0.0048 \\
0.00838 & 0.00566 & [0.32536] & [-0.87938] \\
0.00021 & 0.00105 & 0.00021 & 0.00105 \\
0.08543 & 0.0287 & 0.03638 & 0.02457 \\
-0.15245 & -0.12326 & 0.03296 & 0.02226 \\
\end{array}
\]

### Model Information:

- \( \text{R-squared} = 0.48888 \)
- \( \text{Adj. R-squared} = 0.43508 \)
- \( \text{Sum sq. resid} = 0.05778 \)
- \( \text{S.E. equation} = 0.03184 \)
- \( \text{Log likelihood} = 133.50707 \)
- \( \text{Akaike AIC} = -3.95335 \)
- \( \text{Schwarz SC} = -3.71722 \)
- \( \text{Mean dependent} = 0.00581 \)
- \( \text{S.D. dependent} = 0.04236 \)
- \( \text{Determinant resid covariance (dof adj.)} = 0.00000045 \)
- \( \text{Determinant resid covariance} = 0.00000035 \)
- \( \text{Log likelihood} = 293.76131 \)
- \( \text{Akaike information criterion} = -8.68004 \)
- \( \text{Schwarz criterion} = -8.14032 \)
where \( \beta_\pi = 1 \) is the coefficient in the cointegration relation and \( \rho^{**} = 0.18 \) is the coefficient for the effect of \( \Delta \log P_{t-1}^* \) on \( \Delta \log P_N^* \) (which did not appear in the original specification). Hence, the final specification of the XTT block is:

\[
\pi_t^X = (\pi_{t-1}^X)^{0.41} (\pi_t^*)^{1-0.41} (p_{t-1}^*)^{-0.25} \exp (0.0424 \varepsilon_t^{**}),
\]

\[
\pi_t^* = (\pi_{t-1}^*)^{0.20} (\pi_t^*)^{1-0.20} (p_{t-1}^*)^{0.18} (\pi_t^X)^{0.18} \exp (0.0295 \varepsilon_t^*),
\]

\[
p_t^* = \frac{\pi_t^*}{\pi_t^X}.
\]

A.1.2. The NSS relations between parameters and endogenous variables

Eliminating time indexes from the model equations and simplifying gives a set of nonlinear equations that involve both the parameters and NSS values of the endogenous variables. I assume that in the NSS \( \epsilon = 1 \). I also use the target value for the CB reserves ratio \( \gamma^R = er/Y \), the NSS household foreign debt ratio \( \gamma^D = ed/Y \) and money ratio \( \gamma^M = m/(p^C \pi) \). In some cases I divided the equation through by GDP.

**Consumption:**

\[
\frac{1 + i}{\pi^C} = \frac{1}{\beta} \quad (99)
\]

**Risk-adjusted uncovered interest parity:**

\[
1 + i = (1 + i^*) \phi^* \varphi D (\gamma^D) \delta \quad (100)
\]

**Phillips inflation equations:**

\[
\Gamma = Q/\left( p^C \pi^C \right)^{1 - \beta \alpha \pi^{\theta - 1}} \quad (101)
\]

\[
\Psi = \frac{\theta}{\theta - 1} \frac{mc}{\alpha \pi^\theta} \quad (102)
\]

\[
\frac{\Gamma}{\Psi} = \left( \frac{1 - \alpha \pi^{\theta - 1}}{1 - \alpha} \right)^{\frac{1}{\Gamma}} = \tilde{p}(\pi)^{-1} \quad (103)
\]

**Dynamics of price dispersion:**

\[
\Delta = \left( \frac{1 - \alpha}{1 - \alpha \pi^\theta} \right) \left( \frac{1 - \alpha \pi^{\theta - 1}}{1 - \alpha} \right)^{\frac{\theta}{\Gamma}}. \quad (104)
\]

**Exports:**

\[
X = \kappa_X (ep^*)^{b_X} Y \quad (105)
\]

**Trade Balance:**

\[
TB^e_Y = \frac{1}{a_D} \left[ (p_t^C)^{1 - \theta_c} X \left( \frac{1}{Y} - (1 - a_D) e_t^{1 - \theta_c} \right) \right] \quad (106)
\]

**Current Account:**

\[
CA^e_Y = \left( \frac{1 + i^*}{\pi^*} - 1 \right) \gamma^R - \left[ \frac{1 + i_{t-1}^*}{\pi_{t}^*} \phi^* \tau_D (\gamma^D) \right] \gamma^D + TB^e_Y \quad (107)
\]
Balance of Payments: \[ CA = 0 \] (108)

Real marginal cost: \[ mc = w \] (109)

Labor market clearing: \[ w = \xi^N p^C C^\sigma p^C \varphi_M (\gamma^M) N^\sigma \] (110a)

Hours worked: \[ N = Q \Delta \] (111a)

Domestic goods market clearing: \[ \frac{Q}{Y} = 1 - (1 - bA) \frac{X}{Y} \] (112)

GDP: \[ 1 = a_D \frac{\tau_M (\gamma^M)}{\tau_M (p^C)^{1 - \sigma}} G p^C C + X \] (113)

Consumption relative price: \[ p^C = \left( a_D + (1 - a_D) e^{1 - \sigma} \right)^{\frac{1}{1 - \sigma}} \] (114)

Money market balance: \[ m = L (1 + i) p^C C, \] (115)

CB balance sheet: \[ \frac{b}{Y} = \gamma^R - \gamma^M \frac{p^C C}{Y} \] (116)

Consumption inflation: \[ \pi^C = \pi \] (117)

Real Exchange Rate: \[ \delta \pi^* = \pi \] (118)

External terms of trade: \[ \pi^{*X} = \pi^* \] (119)

Tax collection: \[ \text{tax} = \bar{G} p^C C - qf \] (120)

Quasi-fiscal surplus: \[ qf = (1 + i^* - 1/\delta) \frac{er}{\pi^*} - ((1 + i) - 1) \frac{b}{\pi} \]

Interest rate feedback rule: \[ 1 = \left( \frac{\pi^C}{\pi^F} \right)^{h_1} \] (121)

Nominal depreciation feedback rule: \[ 1 = \left( \frac{\pi^C}{\pi^F} \right)^{k_1} \left( \frac{er/Y}{\gamma^R} \right)^{k_4} \] (122)
Exports inflation shock

\[ \pi^X = (p^*)^{\alpha_{\pi^*}} \left( \frac{1}{1-\rho_{\pi^*}} \right) \]  \hspace{1cm} (123)

Imported inflation shock

\[ \pi^* = (p^*)^{\alpha_{\pi^*}} \left( \frac{1}{1-\rho_{\pi^*}} \right) . \]  \hspace{1cm} (124)

I now show one way in which the NSS values of the model’s endogenous variables can be obtained sequentially, usually from the values of the model’s parameters (including the NSS values of exogenous variables), many of which are assumed given. In some cases, however, it is more enlightening to assume certain great ratios, or elasticities, or even the NSS values of some of the endogenous variables, and obtain the values of some of the parameters endogenously.

(121) implies \( \pi^C = \pi^T \) as long as \( h_1 \neq 0 \) (which I assume throughout). Inserting this in (122) and assuming \( k_4 \neq 0 \) (which I also assume throughout) implies that the CB’s target ratio of international reserves to GDP is attained in the NSS \((\epsilon \tau / Y = \gamma^R)\). And inserting it in (117) yields \( \pi = \pi^T \). (119), (123) and (124), along with the fact that \( \alpha_{\pi^*} \) and \( \alpha_{\pi^*} \) have opposite signs, imply that the XTT is \( p^* = 1 \). Hence, (118) implies \( \delta = \pi^T \). Summing up, we have:

\[ \pi = \delta = \pi^C = \pi^T, \quad \text{and} \quad \pi^* = \pi^X = p^* = 1. \]

Hence, (99) gives the nominal interest rate: \( 1 + i = \pi^T / \beta \).

I assume \( \beta = 0.99 \). For illustrative purposes I use as Argentina’s NSS GDP its 2010 level (at 2010 prices and in trillions of pesos): \( Y = 1.443 \). The gross exogenous risk/liquidity premium for households and the RW gross interest rate are assumed to be \( \phi^* = 1.005^{0.25} \) and \( 1 + i^* = 1.03^{0.25} \), respectively. Also, the ratio of foreign debt to GDP, cash to private consumption, and the Government to household consumption ratio are assumed to be \( \gamma^D \equiv ed/Y = 0.5, \gamma^M \equiv m/p^C = 0.095522 \) and \( g = 0.19 \).

The home bias parameter (or share of domestic goods in household consumption) is calibrated to \( \alpha_D = 0.86 \). The constant relative risk aversion for labor (which is also the inverse of the elasticity of labor supply with respect to the real wage) is \( \sigma^N = 0.5 \), and the constant relative risk aversion for consumption is \( \sigma^C = 1.5 \). Finally, I assume that NSS values for the elasticity of substitution for domestic goods is \( \theta = 6 \) and the elasticity of substitution between the bundles of domestic and imported goods is \( \theta^C = 1.5 \). Assuming that the exogenous parameter for exports demand is \( b_A = 0.5 \), yields \( b_X \equiv (1 - b_A)^{-1} = 2 \) and \( \kappa_X \equiv (b_A)^{b_A b_X} = 0.5 \).

I now focus on the NSS values of the remaining endogenous variables and parameters.

### A.1.2.1 The endogenous risk premium

Using (99), (117), and (118) in the UIP equation (100) gives the household foreign debt to GDP ratio as a function of parameters which we have already calibrated:

\[ \gamma^D \equiv \frac{ed}{Y} = \varphi^{-1}_D \left( \frac{1/\beta}{\phi^* (1 + i^*) / \pi^*} \right) = \varphi^{-1}_D \left( \frac{1}{\beta \phi^* (1 + i^*)} \right) \]
However, calculating this requires the values of the exogenous parameters $\alpha_1$ and $\alpha_2$ which help define the function $\varphi_D$. I now seek to calibrate them. I first express $\tau^N_D$ and $\varphi^N_D$ (see (92)) in terms of the elasticity $\varepsilon^N_D$ (see (93)):

$$
\tau^N_D (\gamma^D_t) = \alpha_1 \left[1 + \varepsilon^N_D (\gamma^D_t)\right],
\varphi^N_D (\gamma^D_t) = \alpha_1 \left[1 + \varepsilon^N_D (\gamma^D_t)\right]^2.
$$

Hence, if the NSS values of $\varepsilon^N_D$ and $\gamma^D$ are calibrated, (93) gives the value of $\alpha_2$:

$$
\alpha_2 = \frac{1}{\gamma^D \left(\frac{1}{\gamma^D} + 1\right)}.
$$

Also, using (125) and (92) in (100) yields:

$$
\varphi^N_D (\gamma^D) = \frac{1 + i}{(1 + i^*) \delta^* \phi} - 1 = \frac{1}{\beta (1 + i^*) \phi} - 1 = \alpha_1 \left(1 + \varepsilon^N_D\right)^2,
$$

which gives the value of $\alpha_1$:

$$
\alpha_1 = \frac{1}{\left(1 + \varepsilon^N_D\right)^2} \left(\frac{1}{\beta (1 + i^*) \phi} - 1\right) = \left(1 - \alpha_2 \gamma^D\right)^2 \left(\frac{1}{\beta (1 + i^*) \phi} - 1\right),
$$

where the second equality is derived from (126).

However, because of the critical role of $\varphi_D$ in the UIP equation (66) it is perhaps more convenient in calibrations to start with the value of the elasticity of $\varphi_D$, which I denote as $\varepsilon^\varphi_D$, along with $\gamma^D$, and derive the value of $\varepsilon^N_D$. It is straightforward to prove that $\varepsilon^\varphi_D$ and $\varepsilon^N_D$ are related by:

$$
\varepsilon^\varphi_D = \varepsilon^N_D \frac{2 \varphi^N_D}{1 + \varphi^N_D} = \varepsilon^N_D \frac{2}{\gamma^D} \left[1 - \beta (1 + i^*) \phi^*\right]
$$

Hence, using (126), (128) and (93):

$$
\alpha_2 = \frac{1}{\varepsilon^\varphi_D \left[1 - \beta (1 + i^*) \phi^*\right] + \gamma^D} \frac{1}{\alpha_1} = \left(1 - \alpha_2 \gamma^D\right)^2 \left(\frac{1}{\beta (1 + i^*) \phi^*} - 1\right).
$$

If, say $\varepsilon^\varphi_D = 2$, then

$$
\alpha_2 = \frac{1}{\left(1 - 0.99 \left(1.03^{0.25} \right) \right) 1.005^{0.25} + 0.5} = 1.9944
$$

$$
\alpha_1 = \left(1 - 1.9944 \times 0.5\right)^2 \left(\frac{1}{0.99 \left(1.03^{0.25} \right) \times 1.005^{0.25} - 1}\right) = 1.1092 \times 10^{-8}
$$

$$
\varepsilon^N_D = \varepsilon^\varphi_D \frac{\gamma^D}{2} \frac{1}{1 - \beta (1 + i^*) \phi^*} = 0.5 \frac{1}{1 - 0.99 \left(1.03^{0.25} \right) \times 1.005^{0.25}} = 353.92.
$$

and from (125):

$$
\tau_D = 1 + \tau^N_D = 1 + \left(1.1092 \times 10^{-8}\right) (1 + 353.92) = 1.0000039368
$$

$$
\varphi_D = 1 + \varphi^N_D = 1 + \left(1.1092 \times 10^{-8}\right) (1 + 353.92)^2 = 1.0013972.
$$
A.1.2.2 The balance of payments

Using the previous calibrations, (108) and (107) give the trade balance to GDP ratio necessary to sustain net interest payments abroad:

\[
TB_e = \left[ 1 + \left( \frac{1}{\pi^*} - 1 \right) D \right] \gamma^D - \left( \frac{1}{\pi^*} - 1 \right) \gamma^R \\
= 0.00337476
\]

Then, using (106), (105), and (114), one can obtain the RER necessary to generate this trade surplus:

\[
\lambda_X \left[ a_D + (1 - a_D) e^{1-\theta C} \right] - (1 - a_D) e^{1-\theta C} = a_D TB_e \frac{Y}{Y} = 0.00337476
\]

and hence the exports to GDP ratio and \( p_C \):

\[
\frac{X}{Y} = \lambda_X \left( e p^* \right)^{b_X} = 0.5 \left( 0.595055 \right)^2 = 0.177045,
\]

\[
p_C = (0.86 + (1 - 0.86) (0.595055)^{1.5})^{-1} = 0.921915
\]

A.1.2.3 The transactions cost function and money demand

The liquidity preference function (16) that results from the specification of the transactions cost function (90) is:

\[
\frac{m_t}{p_t^i C_t} \equiv \gamma_t^M = \mathcal{L} (1 + i_t) = \frac{1}{\beta_2} \left[ \left( \frac{\beta_1 \beta_2 \beta_3}{1 - 1 + i_t} \right)^{1/\beta_3 + 1} - 1 \right]. \tag{130}
\]

The elasticity of \( \mathcal{L} (1 + i_t) \) can be shown to be

\[
\varepsilon_{\mathcal{L}} (\gamma_t^M) = \frac{1}{\beta_3 + 1} \frac{1}{\beta_2 \beta_3} \left( \frac{\beta_1 \beta_2 \beta_3}{1 - 1 + i_t} \right)^{1/\beta_3 + 1}.
\]

Also, (130) yields the following relation

\[
\beta_1 = \left( \frac{1 + \beta_2 \gamma_t^M}{\beta_2 \beta_3} \right)^{\beta_3 + 1} \left( 1 - \frac{1}{1 + i_t} \right). \tag{132}
\]

Using the last two expressions in (94) gives:

\[
\tau_t^N (\gamma_t^M) = \left( 1 + \frac{1}{\beta_3} \right) \left( 1 - \frac{1}{1 + i_t} \right) \varepsilon_{\mathcal{L}} (\gamma_t^M) i_t \gamma_t^M, \tag{133}
\]
and using this in (14) yields:

\[
\varphi_M^N(\gamma_t^M) = \tau_M^N(\gamma_t^M) + \gamma_t^M \left(1 - \frac{1}{1 + i_t}\right) \tag{134}
\]

Since transaction costs are dependent on the inflation rate (through the nominal interest rate) I cannot calibrate the three parameters \(\beta_1, \beta_2, \text{and } \beta_3\) without first calibrating the inflation rate. Let us assume that the target inflation rate is \(\pi^T = 1.015\). Hence, the nominal interest rate is given by (99): \(1 + i = 1.015/0.99 = 1.0253\). Next, calibrate the value of the interest elasticity of money demand, say \(\varepsilon_L = 1.02\), and give a sufficiently high value to \(\beta_3\) (below I explain why it must be high), say 160, to obtain the NSS value of \(\tau_M^N\) through the NSS version of (133):

\[
\tau_M = 1 + \tau_M^N = 1 + \left(1 + \frac{1}{160}\right) \left(1 - \frac{0.99}{1.015}\right) \left(0.99 \times 1.015 - 1\right) \ast 1.02 \ast 0.095522 = 1.00006098
\]

and hence \(\varphi_M^N\) through the NSS version of (134):

\[
\varphi_M = 1 + \varphi_M^N = 1 + \tau_M^N + \gamma_t^M \left(1 - \frac{1}{1 + i}\right) \tag{135}
\]

\[
= 1 + \left(1 + \frac{1}{160}\right) \left(1 - \frac{0.99}{1.015}\right) \left(0.99 \times 1.015 - 1\right) \left(1 - \frac{0.99}{1.015}\right) \ast 0.095522 = 1.0024137
\]

Now use the NSS version of (131) to obtain

\[
\beta_2 = \frac{1}{\gamma_t^M \left(\frac{\beta_3}{\beta_3 + 1}\right) \varepsilon_L \varepsilon_i - 1} = \frac{1}{0.09553 \left(160 + 1\right) \ast 1.02 \left(\frac{1.015}{0.99} - 1\right) - 1} = 3.3266
\]

Notice that the choice of \(\beta_3\) must be sufficiently high to make \(\beta_2 > 0\).

\[
(\beta_3 + 1) \varepsilon_i - 1 > 0
\]

\[
\beta_3 > \frac{1}{\varepsilon_L \varepsilon_i - 1} = \frac{1}{1.02 \ast 0.0253} - 1 = 37.751
\]

Finally, use (132) to obtain the value of \(\beta_1\)

\[
\beta_1 = \frac{(1 + \beta_2^M)^{\beta_3+1}}{\beta_2^M \beta_3} \left(1 - \frac{\beta}{\pi}\right) = \frac{(1 + 3.3266 \ast 0.095522)^{160+1}}{3.3266 \ast 160} \left(1 - \frac{0.99}{1.015}\right)
\]

\[
= 9.072 \ast 10^{14}.
\]

Using (113), the consumption to GDP ratio is:
\[
\frac{p^C C}{Y} = \frac{(p^C)^{1-\theta^C}}{a_{D_M} (\gamma^M)} \left[ 1 - \kappa X (e p^*)^b X \right] \\
= \frac{(0.921915)^{1-1.5}}{0.86 \times 1.00006098 \times 1.19} (1 - 0.177045) = 0.837457
\]

Hence, \( C \) and \( Q \) can be obtained:

\[
C = \frac{p^C C}{Y} \frac{Y}{p^C} = 0.837457 \times \frac{1.443}{0.921915} = 1.3108
\]

\[
Q = \left[ 1 - (1 - b^A) \frac{X}{Y} \right] Y = [1 - (1 - 0.5) 0.177045] 1.443 = 1.31526.
\]

**A.1.2.4 Inflation, price dispersion and marginal cost**

(104) shows NSS price dispersion as a function of the NSS inflation rate. It is easy to check that this function has a local minimum at \( \pi = 1 \), where there is price stability and no price dispersion (\( \Delta = 1 \)). For \( \alpha = 0.66 \) and \( \theta = 6 \) it has the following graph:

\[
\Delta (\text{thick), } p_{\text{tilde}}
\]

The thin line shows the NSS relative price of optimizers (versus all price setters) \( \bar{p} \) as a function of the rate of inflation, i.e., the NSS version of (47). At the point of complete price stability (\( \pi = 1 \)) the relative price \( \bar{p} \) is one, i.e., all price setters set the same price and there is hence no price dispersion (\( \Delta = 1 \)).

(104) gives the value of price dispersion:

\[
\Delta (1.015) = \frac{1 - 0.66}{1 - 0.66 (1.015)^6} \left( \frac{1 - 0.66 (1.015)^{6-1}}{1 - 0.66} \right)^{\frac{6}{6-1}} = 1.0051
\]

Hence, (112) gives the value of hours worked:

\[
N = Q \Delta = 1.31526 \times 1.0051 = 1.32197,
\]

(101) gives the value of \( \Gamma \):

\[
\Gamma = \frac{Q / (p^C C^\theta^C)}{1 - \beta \alpha \pi^{\theta - 1}} = \frac{1.31526 / (0.921915 \times 1.3108^{1.5})}{1 - 0.99 \times 0.66 \times 1.015^{6-1}} = 3.210508,
\]
(103) gives the value of $\Psi$:

$$
\Psi = \Gamma \left( \frac{1 - \alpha}{1 - \alpha \pi^{q-1}} \right)^{\frac{1}{\pi - 1}} = 3.210 \, 508 \left( \frac{1 - 0.66}{1 - 0.66 \times 1.015^{6-1}} \right)^{\frac{1}{0.6 - 1}} = 3.316 \, 59,
$$

and (102) gives the value of $mc$:

$$
mc = \frac{\Psi}{\left( \frac{\theta}{\theta - 1} \frac{Q}{1 - \beta \alpha \pi^{q}} \right)} = 3.316 \, 59 \left( \frac{6 - 1.315 \, 26}{6 - 1 - 0.99 \times 0.66 \times 1.015^{6}} \right) = 0.830 \, 17.
$$

Finally, (109) and (110a) give the value of $\xi^N$:

$$
\xi^N = \frac{mc / p^C C^{\alpha} \varphi_M (m/p^C C) N^{\alpha N}}{0.830 \, 172 \, 187 \, 6 / (0.921915 \times 1.310 \, 8^{1.5} \times 1.0024137 \times 1.321 \, 97^{0.5})} = 0.520 \, 612.
$$

Finally, the NSS value of period aggregate utility is:

$$
\text{Utility} = \frac{1.310 \, 8^{1-1.5}}{1 - 1.5} - 0.520 \, 612 \frac{1.321 \, 97^{1+0.5}}{1 + 0.5} = -2.274 \, 414 \, 7.
$$

The fact that it is negative is irrelevant, since utility has only ordinal, not cardinal, significance.
Appendix 2. Impulse Response Functions for the MER regime
All shocks in the IRFs below are positive and of 1 standard deviation. Shock variables are in logs in the nonlinear model.

A.2.1 Optimal Simple Policy Rules
A.2.1.1 Central Bank style A

\[ \omega_x = 100, \quad \omega_Y = 1, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta \delta} = 50 \]

\[ h_0 \quad h_1 \quad h_2 \quad h_3 \quad k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \]

0.33 1.26 0.02 0.12 -0.03 -0.07 -0.08 -0.43 -0.08

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: $\phi^*$
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^{x}$
A.2.1.2 Central Bank style B

\[ \omega_x = 1, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta_i} = 50, \quad \omega_{\Delta\delta} = 50 \]

\[
h_0 \quad h_1 \quad h_2 \quad h_3 \quad k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4
\]

1.86  -1.01  4.34  -0.21  3.08  -3.92  -2.28  1.18  0.40

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: \( \phi^* \)
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^X$
A.2.1.3 Central Bank style C

\[ \omega_\pi = 100, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta \delta} = 50 \]

\[
\begin{align*}
& h_0 \quad h_1 \quad h_2 \quad h_3 \quad k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \\
& 1.63 \quad 1.92 \quad 1.43 \quad 0.82 \quad 0.44 \quad -1.31 \quad -0.12 \quad -0.91 \quad -0.06
\end{align*}
\]

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: $\phi^*$
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^X$
A.2.2 Optimal policy under commitment
The IRFs below correspond to the specified weights on the loss function:

A.2.2.1 Central Bank style A

\[ \omega_\pi = 100, \quad \omega_Y = 1, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_\Delta i = 50, \quad \omega_\Delta \delta = 50. \]

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: $\phi^*$
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^{*X}$
A.2.2.2 Central Bank style B

\[ \omega_\pi = 1, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta_i} = 50, \quad \omega_{\Delta_\delta} = 50. \]

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: $\phi^*$.
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^X$
A.2.2.3 Central Bank style C

\( \omega_\pi = 100, \quad \omega_Y = 100, \quad \omega_\epsilon = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta \delta} = 50. \)

Response to a positive shock to domestic sector productivity: \( \epsilon \)
Response to a positive shock to government expenditures: $G$
Response to a positive shock to the RW interest rate: $i^*$
Response to a positive shock to the SOE’s exogenous risk/liquidity premium: $\phi^*$
Response to a positive shock to imports inflation: $\pi^*$
Response to a positive shock to exports inflation: $\pi^X$