Discount factor shocks and labor market dynamics

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Abstract

In this paper we investigate the role played by the discount factor shock for the labor market dynamics. Movements in the discount factor can be used as a proxy for variations in financial risks, especially the expected payoff from employment. Through the lens of an otherwise standard matching model we show that the model is able to generate substantial volatility in unemployment and vacancies as compared to output without any need for wage rigidities. Using Bayesian methods to estimate the structural model, we found that the bulk of variations in unemployment and vacancies is mainly explained by disturbances pertaining to the discount factor rather than to productivity. In addition, the contribution of the discount factor shock to the labor market volatility has increased significantly since the eighties. These results are robust to alternative forms of wage rigidities.

Keywords: Search and matching, discount factor shock, Bayesian estimation, unemployment volatility puzzle.

JEL Classification: E3, J6

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1 Introduction

The ability of the search and matching model (pioneered by Diamond Mortensen and Pissarides, DMP hereafter) to reproduce the cyclical behavior of unemployment, vacancies and the labor market tightness has received an important attention. Shimer (2005) and Hall (2005) argued that the canonical model is clearly unable to generate the observed fluctuations. The reason is that wages absorb most of the variations coming from productivity shocks. An impressive body of research has stemmed from this "*unemployment-volatility puzzle*", claiming that a modification in internal propagation mechanisms is needed to reduce the sensitivity of wages to TFP shock. Given the emphasis put on productivity shocks, there is surprisingly little research on its contribution to the variance of the labor market. Most of this literature has considered that labor market fluctuations are solely driven by productivity shocks, leaving no explanatory power to alternative sources of disturbances. This paper contributes to the understanding of labor market fluctuations through the lens of an otherwise matching model by considering an additional source of business cycle fluctuations: variations in the discount factor. We argue that disturbances pertaining to the discount factor provide an important source of propagation and are more likely to explain the dynamics of the labor market.

This puzzling issue has led to an important literature trying to modify the matching model using wage rigidities (Shimer (2005), Hall (2005), Gertler et al. (2008), Hall & Milgrom (2008)), small surplus calibration (Hagedorn & Manovskii (2008)), workers and jobs heterogeneity (Krause & Lubik (2006), Chassamboulli (2013)), alternative forms of hiring costs (Yashiv (2006), Fujita & Ramey (2007), Rotemberg (2008), Pissarides (2009)) counter-cyclical payroll taxes (Burda & Weder (2010)), etc. The list is far from being exhaustive. All the aforementioned specifications have attempted, directly or indirectly, to prevent wages from adjusting rapidly. A notable exception is the study of Di Pace & Faccini (2012) that introduces deep habits in the matching model and shows that the endogenous counter-cyclical mark-ups amplify the response of labor market variables to technology shocks. Most of these studies[1] assumed that labor market fluctuations are solely driven by the popular productivity shock. However, many economists and institutions have cast some doubts on the movements of productivity as a main driver for business cycle fluctuations, especially over the last three recessions in the US. Figures 1 displays the cyclical component of the labor market tightness against the productivity. While the co-movements are fairly close until the mid-eighties, the synchronization falls dramatically thereafter, requiring an unrealistic number of lag periods to prop-

[1]With few exceptions like Rotemberg (2008) who uses changes in market power as a source of business fluctuations but it still make real wages less procyclical. Faccini & Ortigueira (2010) assume that investment-specific technology fuel up the cycles.

agate the productivity shock into the economy. The substantial decline of the correlation between labor productivity and output or labor inputs since 1984 is also well documented by Barnichon (2010b) and Galí & van Rens (2014). In this context, it seems hard to reconcile the equilibrium unemployment theory with empirical evidence.

Our starting point is that hirings can be viewed as an asset characterized by some initial costs and some expected returns. Employers and employees simply compare the payoff from an employment relationship to an alternative asset whose return is determined by the discount factor. Fluctuations in the discount factor make employment a risky asset. The discount rate expresses the difference between the remuneration of the risk free bonds and risky bonds also known as the risk premium. In a paper closely related this one, Hall (2014) wonders what force depresses the payoff to job creation in recession. He noticed that a rise in the discount rate has similar effects to an increase in financial risks. It makes employers less prone to invest in any type of investments, including job creation. A rise in the risk premium reduces the expected payoff from hiring a new worker because the real interest rate is simply the rate at which firms discount their future profit streams. The fall in the expected value of a filled job lowers firms' jobs openings which, in turn, increases aggregate unemployment. His intuition corroborates the idea of Yashiv (2000) and Merz & Yashiv (2007) who found a strong interaction between firms' hiring behavior and the volatility of financial assets.

Figure 2 displays the cyclical component of the tightness against that of the discount factor calculated using the S&P 500[2]. The co-movement between the tightness and our measure of the discount factor are remarkably close. In this paper we do not explain what exacerbates the uncertainty of financial markets. There is an abundant literature on this topic. We simply assume that a shock on the discount rate is a simple proxy for frictions in financial markets. We try to understand how the risk translates into the labor market and how firms react to changes in future flows of profits. Our analysis departs from Hall (2014) in several aspects. First, we focus on the unemployment volatility puzzle rather than the interactions between labor and financial market. We wonder if the model still requires a high level of wage rigidities to explain the large volatility of unemployment and vacancies. Second, we investigate the respective role of the two shocks in shaping the cyclical behavior of unemployment and vacancies in an estimated model. We wonder which shock mainly governs the labor market fluctuations and whether the model predicts a decline in the contribution of the productivity shock over time. To answer these questions, we use a very standard matching model with a time-varying discount factor and productivity. We estimate the model with Bayesian techniques and perform several numerical exercises in order to highlight the mechanisms.

[2]See appendix C for data description and methodology

Our results are as follows. We show that the model fits the business cycle features of key macroeconomic variables reasonably well and provides an important propagation mechanism. The estimated shocks series are remarkably close to the ones observed in the data, suggesting an adequate transmission channel. The introduction of the discount factor shock generates a strong relative volatility of the variables as compared to the productivity shock. The former is crucial to solve the unemployment-volatility puzzle. The model's ability to generate large fluctuations does not rely on any form of wage rigidities or on a specific calibration *a la* Hagedorn & Manovskii (2008), the latter being rejected by our estimation. The bulk of variations in unemployment and vacancies is mainly explained by disturbances pertaining to the discount factor. The productivity shock accounts for half of the output variations. Furthermore, we document an increasing contribution of the discount factor shock to the macroeconomic variables. This provides an appealing justification for the decline in the contribution of the productivity shock. These results are robust to alternative forms of wage rigidities and wage settings.

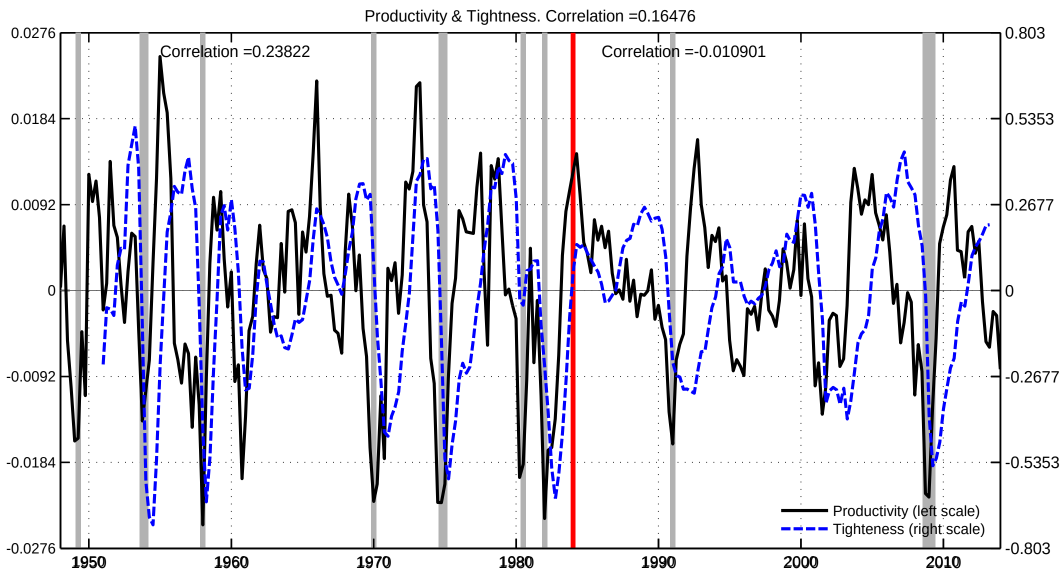

Figure 1: ***Cyclical component of Productivity and the labor market tightness.*** *Gray shaded areas are NBER recession dates. The vertical red line is the date (1984Q4) at which the correlation between labor productivity and the tightness falls.*

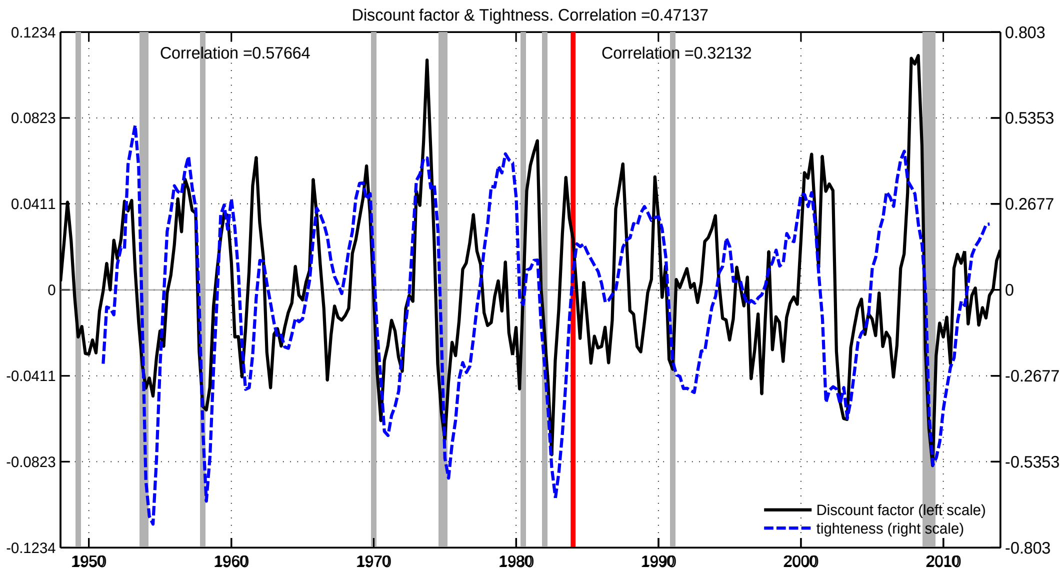

Figure 2: ***Cyclical component of the discount factor and the labor market tightness.*** *Gray shaded areas are NBER recession dates. The vertical red line is the date (1984Q4) at which the correlation between labor productivity and the tightness falls.*

The rest of the paper is organized as follows. Section 2 is devoted to the presentation of the dynamic matching model. Section 3 addresses estimations and simulations of the structural model. Section 4 investigates several robustness tests. Section 5 concludes.

2 The model

We use a discrete time version of the standard matching model where the economy is populated by homogeneous workers and firms. We focus on workers flows between employment and unemployment. Separations are exogenous. Labor is the only input into the production process and it may be adjusted through the extensive margin (employment). Wages are set according to a Nash bargaining process.

2.1 Matching

A job may either be filled and productive, or unfilled and unproductive. Workers are identical and they may either be employed or unemployed. The number of matches, m_t, is given by the following Cobb-Douglas matching function:

$$m_t = \chi j_t^{\gamma} v_t^{1-\gamma} \leq \min(j_t, v_t) \quad (1)$$

where $v_t \geq 0$ denotes the total mass of vacancies, $j_t \geq 0$ represents the mass of searching workers. The matching function (1) is increasing and concave in its two arguments and homogenous of degree 1. A vacancy is filled with probability $q_t = m_t/v_t$ and the job finding probability is $f_t = m_t/j_t$. The labor market tightness is equal to $\theta_t = f_t/q_t$ Total employment is n_t and the number of job seekers is defined by $j_t = 1 - (1-s)n_{t-1}$. The labor force is constant and equal to one such that end-of-period unemployment is $u_t = 1 - n_t$. The employment law of motion is given by:

$$n_t = (1-s)n_{t-1} + m_t \quad (2)$$

which involves that hirings are immediately productive.

2.2 Representative household

The representative household maximizes aggregate consumption[3] c_t:

$$\max_{\Omega_t^H} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) (c_t + (1-n_t)b) \quad (3)$$

subject to (i) the budget constraint:

$$c_t = w_t n_t + \Pi_t \quad (4)$$

(ii) the job seekers j_t definition and (iii) the law of motion of employment:

$$n_t = (1-s)n_{t-1} + f_t j_t \quad (5)$$

β_t represents a discount factor shock with $\beta_0 = \beta$. w_t is the wage level. Π_t represents profits from holding shares in firms. The representative household derives a constant utility b from unemployment (leisure and home production). Prices are normalized to 1. The representative household chooses the set of processes $\Omega_t^H = \{c_t, n_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{w_t, f_t\}_{t=0}^{\infty}$, so as to maximize their intertemporal utility. The optimality conditions of the household's problem defines the marginal value of employment for a worker:

$$\varphi_t = (w_t - b) + \mathbb{E}_t \beta_{t+1}(1-s)(1-f_{t+1})\varphi_{t+1} \quad (6)$$

[3]In order to fully concentrate our attention on the pure effects of the discount factor, we consider a linear utility function. In addition, we present the model resulting from a dynamic program. The equilibrium conditions are identical to the ones obtained by writing the Bellman equations directly.

2.3 Firms

Firms chooses the set of processes $\Omega_t^F = \{v_t, n_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{w_t, q_t\}_{t=0}^{\infty}$ so as to maximize the following intertemporal profit function:

$$\max_{\Omega_t^F} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) (y_t - w_t n_t - \kappa v_t) \tag{7}$$

subject to the production function and the law of motion of employment:

$$\begin{aligned} y_t &= z_t n_t \\ n_t &= (1-s)n_{t-1} + q_t v_t \end{aligned}$$

Hiring is costly and incurs a cost κ per vacancy posted. z_t is an aggregate productivity shock. The optimality conditions of the above problem gives the job creation condition which equal expected surplus from a filled job μ_t to the average cost of search (κ/q_t):

$$\frac{\kappa}{q_t} = \mu_t \tag{8}$$

$$\mu_t = z_t - w_t + (1-s)\mathbb{E}_t \beta_{t+1} \mu_{t+1} \tag{9}$$

Combining the two gives the job creation condition:

$$\frac{\kappa}{q_t} = z_t - w_t + (1-s)\mathbb{E}_t \beta_{t+1} \frac{\kappa}{q_{t+1}} \tag{10}$$

2.4 Wages

The wages are determined every period through an individual Nash bargaining process between each worker and the large firm. They share the total surplus of the match. The standard optimality condition writes: $\xi \mu_t = (1-\xi)\varphi_t$ where $\xi \in [0,1]$ and $1-\xi$ denote the workers' and the firms' bargaining power respectively. Using Equations (6), (8) and (9), the wage is:

$$w_t = \xi \left(z_t + E_t \beta_{t+1} (1-s) \kappa \theta_{t+1} \right) + (1-\xi) b \tag{11}$$

It should be noted that the presence of the discount factor shock in the wage equation is inherited from the contemporaneous hirings assumption. Within the same period, firms pay the cost κ, post v_t vacancies and hire m_t new workers that are immediately productive. Since wages are bargained after that hiring expenses are settled, workers can not use them as a threat point[4] to get an higher share of the surplus. They can only bargain on the next period job posting costs. We relax the contemporaneous hirings assumption in the robustness analysis.

To close the model we define profits as $\Pi_t = y_t - w_t n_t - \kappa v_t$ which, combined with the household budget constraint (4) yields the following market clearing condition: $c_t = y_t - \kappa v_t$.

3 Results

3.1 Estimation method

We use Bayesian techniques to estimate the model's parameters and shock variances. The posterior density is evaluated using a random-walk Metropolis-Hastings algorithm, for which we generate 2 000 000 draws and we target an acceptance ratio of 0.3. We log-linearize the model around the deterministic steady state and apply the Kalman filter to evaluate the likelihood function. We combine the likelihood function with the prior distribution of the model parameters to obtain the posterior distribution as in Lubik & Schorfheide (2006), An & Schorfheide (2007) and Lubik (2009).

We set the steady state level of β and estimate the rest of the parameters. We adopt relatively loose priors for the model parameters except for the separation rate[5] (see Table 2). We assume a beta-distribution for share parameters defined on unit intervals and a gamma-distribution for positive-valued parameters. Our data set runs from 1948Q1 to 2014Q1. We have two observable variables: the unemployment rate and the real gross domestic product. We take log and pass the series through an HP-filter with smoothing parameter 1600 as in Lubik & Schorfheide (2004).

3.2 Prior and posteriors

We adopt a very standard calibration based on US data at quarterly frequencies to set the priors. We do not set prior parameters to make the model match the data along several moments. We choose to be agnostic and let the estimation determine the parameters' values. We set the steady state discount factor to 0.99 which gives an annual real interest rate of 4%. The US unemployment rate u is about 5.5% on average. We set the prior mean of the separation rate s to

[4] The first term on the right-hand side of Equation (11).

[5] Due to identification problems of this parameter we restrict the standard deviation to be equal to 0.01.

10% which match the monthly rate of 3.35% from the BLS. At the steady state, the number of matches must be equal to the number of separations: $m = sn$ with $n = 1 - u = 0.945$. We get the number of job seekers from the definition $j = 1 - (1 - s)n$ and the job finding rate from $f = m/j \simeq 63\%$. The rate at which a firm fills a vacancy is set to match the vacancy rate found in the data (BLS) of 3.5%. This one is defined as $\tilde{v}/(\tilde{v} + n)$ where $\tilde{v} = v - m$ is the end-of-period number of vacancies[6]. We deduce $v = m/q$ and set the prior of χ in such a way that $m = \chi j^{\gamma} v^{1-\gamma}$. The prior mean for the elasticity of the matching function w.r.t. the number of job seekers γ is equal to 0.5, in line with Pissarides & Petrongolo (2001). The prior value for the workers' bargaining power is typically 0.5 in calibration studies, satisfying the Hosios condition. We set the prior value of b to 0.71 as in Hall & Milgrom (2008), a value in the range of Shimer (2005) (0.4) and Hagedorn & Manovskii (2008) (0.95). The cost of posting a vacant job is pin down from the job creation condition and is equal to 0.27. The calibration is summarized in Table 1 and Table 2 as prior means.

For shock processes we consider a VAR representation because the two series may influence each other. We thus allow for cross correlations between the shock processes:

$$\begin{bmatrix} z_{t+1} \\ \beta_{t+1} \end{bmatrix} = \Gamma \begin{bmatrix} z_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \varepsilon^z_{t+1} \\ \varepsilon^\beta_{t+1} \end{bmatrix} \tag{12}$$

where Γ is the matrix of auto-regressive coefficients and Σ is the variance-covariance matrix of shocks. We estimate this system of two equations using structural VAR methodology. We calculate the series of productivity by dividing the real GDP by the total employment. The discount factor is calculated by differentiating the return of the SP500 with the 3-Month Treasury Bill (Secondary Market Rate) at quarterly rate. Furthermore, we assume that the discount factor shock does not have an immediate impact on the labor productivity. We impose a 0 on the variance-covariance matrix accordingly. We use an HP-filter with smoothing parameter $\lambda = 1600$ on the logarithm of the series and estimate the VAR using a 10000 bootstrap simulations. The mean of autocorrelation coefficients are reported in Table 2 as prior means[7]. We obtain a persistence coefficient of 0.78 on the lagged productivity and 0.72 on the lagged discount factor. The estimation shows that the impact of the lagged value of the discount factor has almost no impact on productivity while the opposite is true. The volatility associated to the productivity shock and the discount factor shock are equal to 0.006 and 0.022 respectively. The variance-

[6] We work with end-of-period variables in the simulations. It means that unemployment is u_t, vacancies are $\tilde{v}_t$ and the Tightness is $\tilde{v}_t/u_t$.

[7] The density is a beta-distribution but the lower bound and the upper bound are -1 and 1 respectively.

covariance matrix of innovations is given by:

$$\Sigma * 1000 = \begin{pmatrix} 0.0348 & 0.0314 \\ 0.0314 & 0.4908 \end{pmatrix} \tag{13}$$

The priors for the standard deviations of shocks follow an inverse-gamma distribution with infinite standard deviation. Finally we check at each draw of the random-walk Metropolis-Hastings algorithm if the roots associated to the shock processes remains below 1.

Variables	Symbol	Prior	Source
Discount factor	β	0.99	4% annual real interest rate
Separation rate	s	0.1	BLS, 0.032 monthly
Utility when unemployed	b	0.71	Hall & Milgrom (2008)
Worker bargaining power	ξ	0.5	Hosios condition
Elast. matching w.r.t u	γ	0.5	Pissarides & Petrongolo (2001)
Vacancy posting cost	c	0.27	Deduced
Matching efficiency	χ	0.68	Deduced
Unemployment rate	u	0.055	BLS
Vacancy rate	$v/(v+n)$	0.035	BLS
Job finding rate	f	0.63	Deduced
Job filling rate	q	0.73	Deduced

Table 1: **PRIOR PARAMETERS AND TARGET**

Table 2 reports posterior means for the estimated parameters and the associated 90% confidence intervals. The posterior mean of the workers' bargaining power remain around its prior mean despite the high standard deviation of its prior density. This result contrasts the calibration *a la* Hagedorn & Manovskii (2008) in which they assign a value of 0.052. The posterior density clearly rules out this value. It follows that firms and workers share equally the total surplus from a match according to the estimation. Furthermore, the posterior mean of the utility derived from unemployment is lower than its prior mean. It does not give support to an important workers' outside option and rejects Hagedorn & Manovskii (2008) value once again. Indeed, the posterior density assigns zero probability to $b = 0.955$.

The posterior mean for the cost of posting a vacancy moves away from its prior mean and reaches 0.37. The elasticity of the matching function with respect to unemployment is in the range of the most standard values (Pissarides & Petrongolo (2001)). The Hosios condition, $\xi = 1 - \gamma$, is likely to be satisfied on the upper range of γ. The standard deviation of the discount factor shock is three times larger than that of the productivity shock but slightly lower that the one we obtain from the VAR estimation. The volatility of the productivity shock does not move away from its prior. While the persistence coefficient of the productivity shock falls, the estimation calls for a more persistent discount factor shock. The impact of the discount factor on productivity measured by the term $\rho_{z,\beta}$ is still around zero while the estimation highlights the strong impact of z over β.

Variables	Symbol	Prior mean	Posterior mean
Separation rate	s	$\beta(0.10, 0.01)$	0.084 [0.072, 0.099]
Worker bargaining power	ξ	$\beta(0.50, 0.20)$	0.50 [0.31, 0.69]
Matching elasticity	γ	$\beta(0.50, 0.20)$	0.30 [0.09, 0.55]
Utility when unemployed	b	$\Gamma(0.71, 0.20)$	0.46 [0.28, 0.62]
Vacancy posting cost	κ	$\Gamma(0.27, 0.10)$	0.37 [0.17, 0.55]
Matching efficiency	χ	$\Gamma(0.68, 0.20)$	0.46 [0.29, 0.63]
Persistence z, z	$\rho_{z,z}$	$\beta(0.79, 0.20)$	0.73 [0.64, 0.83]
Persistence z, β	$\rho_{z,\beta}$	$\beta(-0.04, 0.10)$	-0.01 [-0.03, -0.00]
Persistence β, β	$\rho_{\beta,\beta}$	$\beta(0.73, 0.20)$	0.78 [0.71, 0.85]
Persistence β, z	$\rho_{\beta,z}$	$\beta(0.43, 0.10)$	0.50 [0.34, 0.65]
Productivity Std.	σ_z	$\Gamma^{-1}(0.006, \inf)$	0.006 [0.006, 0.007]
Discount Std.	σ_β	$\Gamma^{-1}(0.022, \inf)$	0.019 [0.013, 0.093]

Table 2: **POSTERIOR ESTIMATES - BENCHMARK MODEL**. For persistence parameters, the β distribution is bounded between -1 and 1. We check through a stability analysis that the roots associated to the stochastic process are below one.

3.3 Cyclical properties of the model

Table 3 describes the empirical moments for U.S. data. Unemployment and vacancies are about 9 times more volatile than output. Both are strongly correlated as suggested by the negative correlation coefficient of -0.89. The labor market tightness is about 17 times more volatile than output while the wage is less volatile than output.

We report the unconditional moments of the variables using the mode of the posterior distribution for parameters' values to simulate the model. It is shown that the canonical search and matching model generates large fluctuations in the labor market when the two shocks are considered. The relative volatility of unemployment, vacancies and the tightness are close to their empirical counterparts. The high level of the workers' bargaining power involves a volatility of the wage larger than the data. This result is of great importance because it highlights that the wage rigidity is not needed to match the strong volatility of the labor market. Of course, the model produces excessive volatility in wages. We tackle this issue in our robustness analysis. Although a little bit lower than in the data, the model is able to reproduce a strong negative correlation between unemployment and vacancies.

Variables	Data	Model
Std. Output.	1.58	1.66
Rel. Std. Unemployment	8.58	9.42
Rel. Std. Vacancies	8.90	7.79
Rel. Std. Tightness	16.71	16.14
Rel. Std. Wages	0.58	2.04
corr (u,v)	-0.89	-0.76

Table 3: **MOMENTS.** The model is simulated 10^5 times at the mode of the parameters estimate. Except for output, all standard deviations are relative to output.

Last but not least, we analyze the extent to which the estimated sequence of shocks departs from the data. Figure 3 and 4 depict the smoothed series of the productivity and the discount factor obtained by Bayesian estimation[8] against those from the data. The productivity fit is very good and follow closely the series from the data. Furthermore, it exhibits a clear decline in amplitude since the end of the 1984 recession. Although the estimated discount factor shock is not perfectly correlated with our proxy calculated using the SP500, the correlation between the two is very satisfactory. The volatility of the two are also almost the same. Regarding the simplicity of the model, we argue that the model provides an adequate transmission channel for the fit of the business cycles.

[8]For the sake of clarity, we look at z_t and β_t rather than ε_t^z and ε_t^β since the former have a direct empirical counterpart.

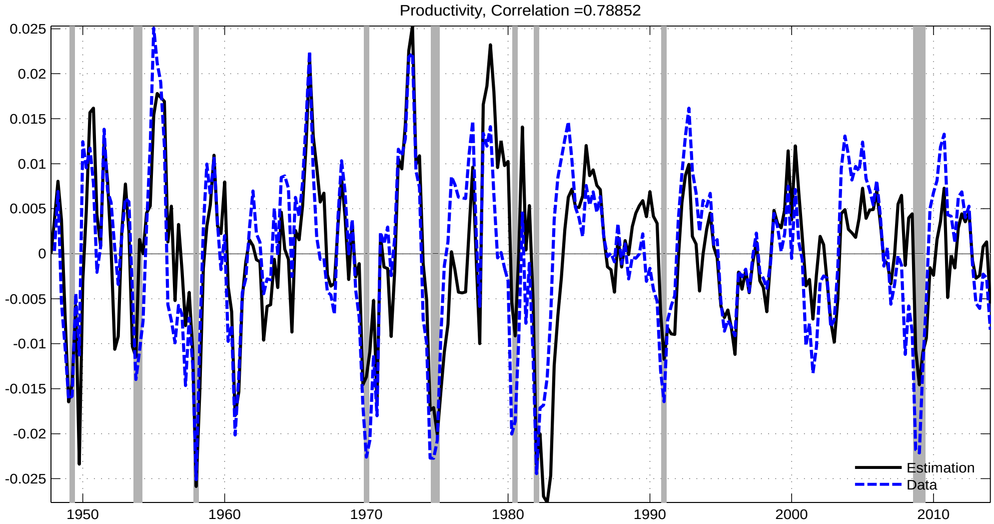

Figure 3: ***Productivity shock***.

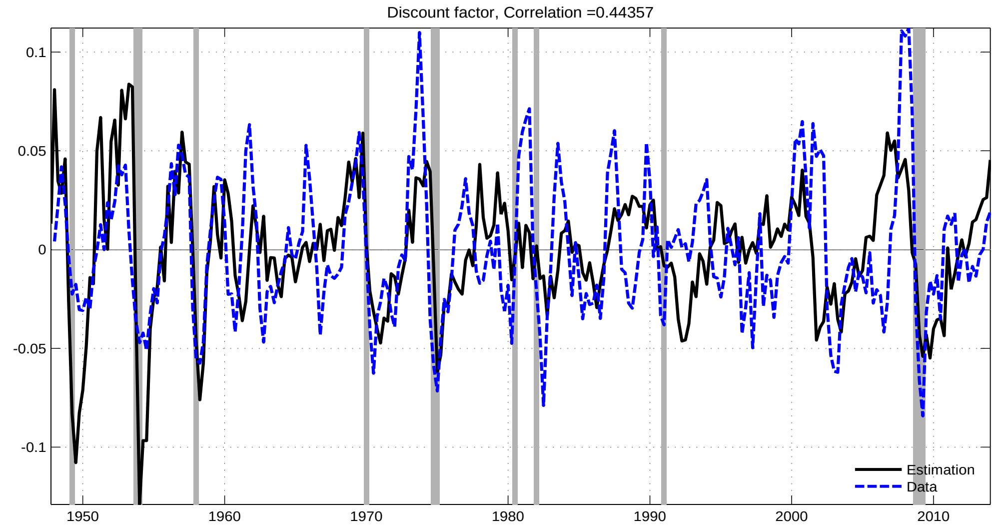

Figure 4: ***Discount factor shock***.

We now turn to the major differences between the two shock by analyzing the impulse response functions displayed in Figure 5. For the sake of comparison, we simulate the responses to a temporary shock of one percent (on ε_t^z and ε_t^β) instead of their respective standard deviation. In addition, we make a counterfactual experiment by comparing the impulse responses to a case where the interactions between the shocks are muted. This exercise helps to disentangle the role played by the cross-correlation between the two shocks in explaining the dynamic of the labor market. Here, we simply assume that $\rho_{z,\beta} = 0$ and $\rho_{\beta,z} = 0$ (the rest of the parameters being the same *i.e.* the value at the posterior mode) without re-estimating the model in order to abstract from potential changes in propagation mechanisms.

On impact the response of the variables is about two times stronger following a productivity shock (solid black line) as compared to a discount factor shock (dotted red line). In addition, the variables return more slowly to their equilibrium values after a shock on productivity. However, the jump of output is about 10 times bigger following a shock to productivity. In other words, when the adjustment of unemployment is related to the variation in output, the discount factor shock implies a relative response of the labor market which is 5 times larger. This result is of a great importance because the consistency of the matching model in dynamic issues depends on its ability to match the observed relative volatility.

When the interactions between the shock are muted (dotted line), we observe that the impulse responses resulting from the discount factor shock remains virtually unchanged. The reason comes from the weak feedback effect of β_t on z_{t+1} measured by $\rho_{z,\beta}$. According to Table 2, it is almost equal to zero. It implies no variation in productivity and then, no additional disturbance. On the other side, the strong feedback effect from z_t on β_{t+1} amplify the initial impacts of the productivity shock. By imposing $\rho_{\beta,z} = 0$ we remove the rise in β coming from the productivity shock. On impact, the response of the labor market is two times weaker and become identical to the one resulting from the discount factor shock. The path of output being weakly affected in the counterfactual scenario, the overall impact on the relative response of the labor market is lower for the productivity shock and unchanged for the discount factor shock. We conclude that the largest impact of the labor market to the productivity shock in absolute terms comes from the feedback effect of the discount factor. The productivity shock alone can not produce a relative response of a correct magnitude.

The mechanism behind the discount factor shock lies in the movements of the expected hiring costs (the last term on the RHS in Equation (10)). It directly impacts the payoff to job creation but impacts output indirectly through employment. As firms experience drastic variations in the expected returns from hiring a new worker they adjust job openings very sharply. The productivity shock also impacts firms profits but has a direct impact on output which dampens the relative volatility.

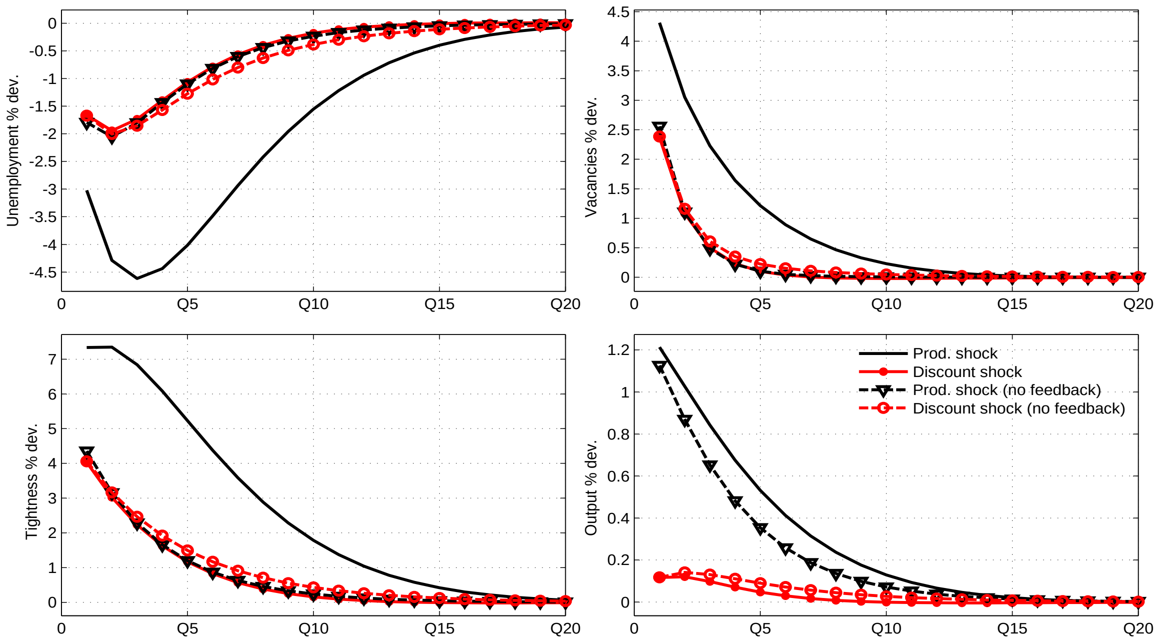

Figure 5: ***Impulse response function****. We simulate a one percent positive aggregate productivity shock and discount factor shock.*

3.4 Shock decomposition

In this section, our objective is to understand which shock is the main driver for unemployment, vacancies and output. Does the model predict a change in the source of business cycle fluctuations? An inspection of Table 4 shows that over the full sample, the bulk of variation in unemployment and vacancy is mainly governed by the disturbances pertaining to the discount factor. It explains more than 60% of labor market fluctuations. Only a small fraction of the fluctuations are generated by the productivity shock during the last 50 years which never exceed 40% on a five year moving average basis. On the other side, the productivity shock contributes to output variations up to 60%.

Table 4 split the experiment into subperiods[9]. In line with Barnichon (2010b), we consider a pre-84 and a post-84 period. In addition, we also consider a shorter and more recent episode of economic cycles: 2000Q4-2014Q1 which only includes the last two recessions as in Lubik (2013). The contribution of the discount factor is larger after 1984 for all the variables of interest. The model predicts a 20% increase in the contribution of the discount factor shock between pre-84 and post-84. The subperiod 2000-2014 highlights the important role played by the discount factor. Its contribution to output variations increases by 50% and to unemployment, vacancies and the tightness variations

[9]In this exercise we do not re-estimate the parameters. We simply cut the shock decomposition in different subperiods. By re-estimating the models' parameters over each subperiods we still find the same results.

by around 30%, 20% and 30% respectively.

Variables	Symbol	1948Q1 1984Q4	1985Q1 2014Q1	2000Q4 2014Q1	Full sample
Output	y	41	51	61	46
Unemployment	u	60	72	79	65
Vacancies	v	63	69	75	66
Tightness	θ	60	71	79	65

Table 4: **Shock decomposition.** *Contribution of the discount factor shock ε_t^{β} to the variance of the variables (in percentage). The contribution of the productivity shock is 100 minus the displayed values.*

4 Robustness of results

In this section, our objective is to analyze if our major results are robust to alternative specifications of the model: (i) the ability of the discount factor shock to produce a large response in unemployment and vacancies relative to output and (ii) the increasing contribution of the discount factor. We focus on key assumptions that we consider important for our results: the timing of events and the sensitivity of wages to shocks. Results are reported[10] in Table 5, Figure 6 and Figure 7.

4.1 Backward employment

An important implication of the contemporaneous hirings assumption is the presence of the discount factor shock in the wage equation. Workers bargain over expected hiring costs discounted at the stochastic rate β_{t+1}. The discount factor impacts the wage directly which alter the elasticity of the tightness to β. In order to mute this effect, we consider the following law of motion for employment:

$$n_t = (1-s)n_{t-1} + m_{t-1} \tag{14}$$

where $m_t = f_t j_t$ and $j_t = 1-(1-s)n_t$. An employed worker that separates with probability s in period t can find a job within the same period but will start working only in period $t+1$. Employment and unemployment are determined at the beginning of the period and wages are bargained over the surplus which includes the resources devoted to hirings the last period. The major consequence is that current hiring costs instead of future hiring costs are used as a

[10] The prior values, the posterior modes and the confident intervals of the estimated parameters under the alternative model specification are reported in a separate appendix.

threat by workers during the wage negotiations. The job creation condition is such that the average cost of filling a vacancy equals the expected future flows of profits. The wage equation[11] no longer depends directly on the realizations of the discount factor shock. The response of wages to the discount shock is a matter of importance because it governs the channel through which the model relies on rigid wage to propagate the discount factor. We label this case as "*Backward employment*".

According to Table 5, the relative volatilities are still broadly consistent with the data. The standard deviation of wages is more closer to its empirical counterpart but still too high. However, the backward timing breaks the negative correlation between unemployment and vacancies. The reason is that unemployment adjusts with a one-lag period while vacancies adjust instantaneously. In the benchmark model, the strong negative correlation between unemployment and vacancies hinges on the contemporaneous hiring assumption. Unemployment adjusts more rapidly to shocks because n_t depends on the current level of available jobs. This lead to closer co-movements with vacancies.

The transmission channel of the shocks is affected in several way. The response of unemployment is larger and more persistent, especially when the economy is hit by a discount factor shock. On impact, the response of output is stronger and oscillates around its equilibrium value. However, the relative response of the variables remains way more larger following a shock to the discount factor than to the productivity. Looking at the variance decompositions, it is shown that the contribution of the discount factor is almost similar to the benchmark average. It also increases as time goes by but at a rate of lower growth.

4.2 Wage rigidity

Shimer (2005) shows that the DMP model implies a high sensitivity of wages which offsets the impact of productivity shocks. Two key parameters govern the response of wages: ξ and b. For the sake of clarity, we derive the steady state elasticities of the tightness to productivity:

$$\varepsilon_{\theta,z} = \frac{1}{\gamma}\frac{z-\varepsilon_{w,z}w}{z-w} \tag{15}$$

$$\varepsilon_{w,z}w = \xi(z+\beta(1-s)\kappa\theta\varepsilon_{\theta,z}) \tag{16}$$

and to the discount factor:

$$\varepsilon_{\theta,\beta} = \frac{1}{\gamma}\left[\frac{\beta(1-s)}{1-(1-s)\beta} - \frac{(1-(1-s)\beta)\varepsilon_{w,\beta}w}{z-w}\right] \tag{17}$$

$$\varepsilon_{w,\beta}w = \xi\beta\kappa(1-s)\theta\left(1+\varepsilon_{\theta,\beta}\right) \tag{18}$$

[11] Details are provided in appendix B.

Interestingly, they are both higher when the elasticity of wage is low. This means that rigid wages may foster the propagation of the discount factor shock as well. All other things being equal, a small surplus ($z - w$ low) is likely to increase the elasticity of the tightness to productivity but reduces the elasticity of the tightness to the discount factor. The major difference between the two shocks hinges on the level of wages which depends on the cost of posting vacancies, the tightness and the utility when unemployed.

We consider two forms of wage rigidities. Shimer (2005) proposes a simple way to solve the unemployment volatility Puzzle in a productivity-driven fluctuations setup. He assumed that the wage is given by:

$$w_t = \lambda w_{t-1}^n + (1 - \lambda) w_t^n \tag{19}$$

with w_t^n being the wage bargained from Equation 11. λ stands for the degree of wage rigidity. For this experiment we consider that the prior of λ is 0.8, a value at the upper range of the ones we can find in the literature. This case is labeled as "*Real wage rigidity*". Another way to prevent wages from adjusting rapidly is to consider a specific calibration as in Hagedorn & Manovskii (2008). They assume a very low workers' bargaining power $\xi = 0.052$ combined with a large utility when unemployed $b = 0.95$. This parametrization is known as "*Small surplus calibration*", circumvents the ad-hoc assumption proposed by Shimer. However, as shown by the estimation, the calibration adopted by the authors is clearly rejected in our estimation.

Finally, Hall & Milgrom (2008) built a DMP model with micro-funded wages bargaining. Their idea is that the Nash bargaining abstracts from the dynamic of the negotiations. Threat points are usually the outside option of the two players. When a firm and a worker meet, no one has interests to break the match, which implies that the threat points cannot be their respective outside option. The negotiation between the two is delayed over several periods. One at a time proposes its counteroffer. As in Hall & Milgrom (2008), we suppose that the firm makes the first offer w_t. The worker may accept the offer or refuse. If the offer is refused, the worker receives b and has a probability δ that the match ends during the next period and a probability $1 - \delta$ to propose a counteroffer w'_{t+1}. Since we assume that hirings are contemporaneous, we modify the Bellman equations describing the workers and employers' surplus accordingly[12]. We label this model by "*Credible bargaining*".

It is shown that the two types of wage rigidities produce excessive volatility in vacancies and in the tightness. The volatility of unemployment remains virtually unchanged but the volatility of wages become closer to its empirical counterpart. Furthermore, the two types of wage rigidities reproduce perfectly the negative correlation between unemployment and vacancies. Finally, the shock decomposition shows that the contribution of the discount factor

[12]Details are provided in appendix B.

shock depict an upward shift over time and is on average bigger than in the benchmark.

Variables	Data	Benchmark model	Real wage rigidity	Credible bargaining	Backward employment
Std. Output.	1.58	1.66	1.49	1.57	1.47
Rel. Std. Unemployment	8.58	9.42	9.37	9.61	9.72
Rel. Std. Vacancies	8.90	7.79	26.79	17.41	9.99
Rel. Std. Tightness	16.71	16.14	35.33	26.31	15.16
Rel. Std. Wages	0.58	2.04	1.64	1.20	1.45
corr (u,v)	-0.89	-0.76	-0.88	-0.89	-0.18

Table 5: **Moments.** The model is simulated 10^5 times at the mode of the parameters estimate. Except for output, all standard deviations are relative to output.

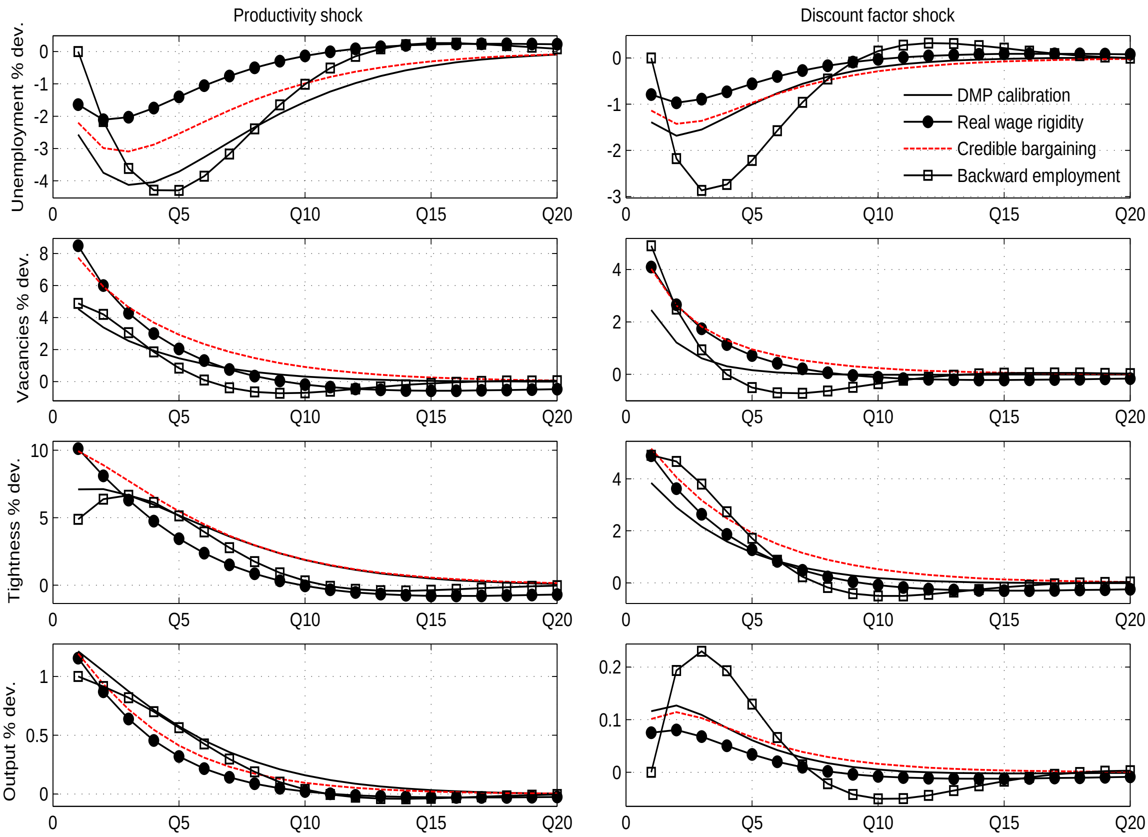

Figure 6: ***Impulse response function.*** *We simulate a one percent positive aggregate productivity shock and discount factor shock.*

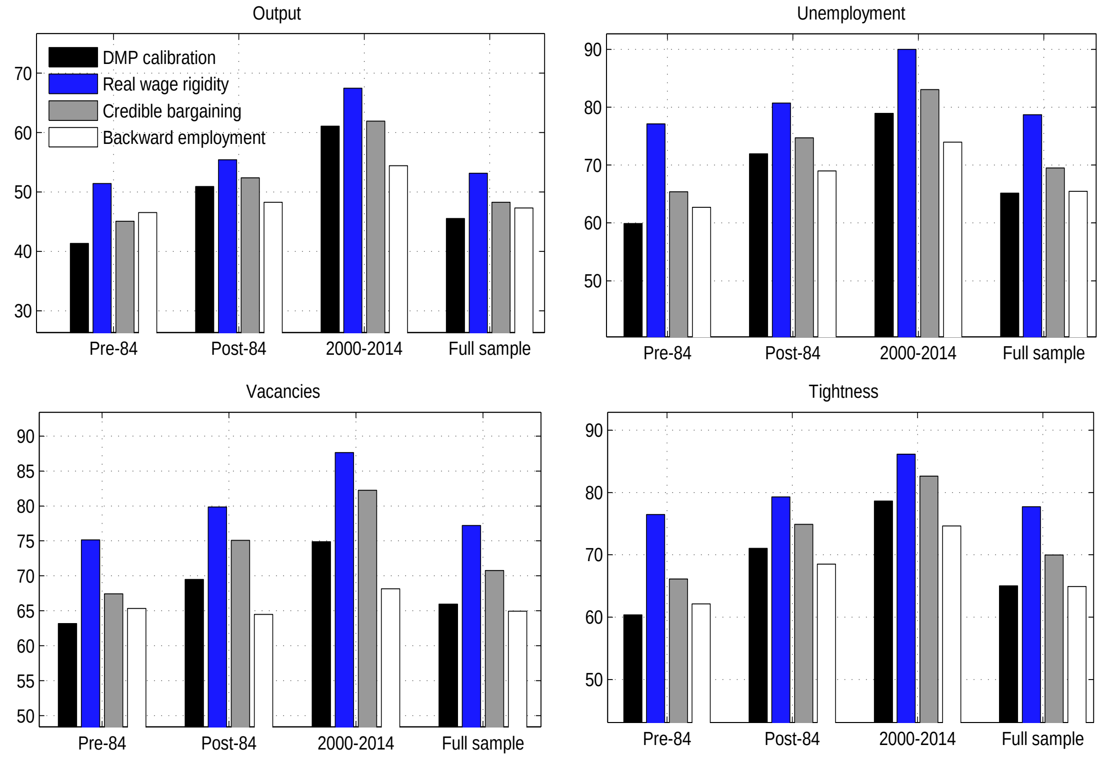

Figure 7: ***Shock decomposition***. *Contribution of the discount factor shock ε_t^β to the variance of the variables (in percentage). The contribution of the productivity shock is 100 minus the displayed values.*

5 Conclusion

The literature on the cyclical behavior of the labor market has assumed that variations in labor productivity are the main, and sometimes only, source of business cycle fluctuations. The secular use of the productivity shock is inherited from the RBC theory. It is arguably a rigorous way to assess how marginal departures from the benchmark model (which is based on this TFP shock) would lead to different outcomes. However, the changes in business cycle stylized facts observed in the US make the use of TFP shocks not convincing anymore. In this paper, we argue that the canonical search and matching model is able to generate enough volatility in unemployment and vacancies if the fluctuations are not solely driven by the productivity shock. Our starting point is that hiring can be viewed as an asset characterized by some initial costs and some expected returns. Employers and employees simply compare the payoff from an employment relationship to an alternative asset whose return is r_t. Movements in the discount factor make perfect economic sense to interpret the disturbances on the labor market as they impact the expected profit streams.

The estimation of the structural model reveals that the discount factor shock is more likely to explain labor market fluctuations than the productivity shock alone. Our study also documents the relation between productivity and the discount factor shock which are important for the dynamics. We found that the bulk of variations in unemployment and vacancies is mainly explained by disturbances pertaining to the discount factor rather than to productivity. The model predicts a significant change in business cycles sources since the eighties. While a full setup with nominal rigidities is need to explain the vanishing pro-cyclically of labor productivity, our results point toward a change in the contribution of shocks rather than a change in the propagation of shocks. Our general conclusion is that alternative sources of uncertainty like the one coming from the discount factor should be considered for future research.

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A Backward employment law of motion

The representative household maximizes aggregate consumption by solving the following problem

$$\max_{\Omega_t^H} E_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) c_t + (1 - n_t) b \tag{20}$$

subject to (i) the budget constraint:

$$c_t = w_t n_t + \Pi_t \tag{21}$$

(ii) the job seekers $j_t = 1 - (1-s)n_t$ definition and (iii) the law of motion of employment:

$$n_t = (1-s)n_{t-1} + f_{t-1} j_{t-1} \tag{22}$$

The optimality conditions of the household's problem defines the marginal value of employment for a worker:

$$\varphi_t = w_t - b + E_t \beta_{t+1} (1-s)(1-f_t) \varphi_{t+1} \tag{23}$$

The optimization problem of the firm consists of choosing the set of processes $\Omega_t^F = \{v_t, n_t\}_{t=0}^{\infty}$ taking as given the set of processes $\{w_t, q_t\}_{t=0}^{\infty}$ so as to maximize the following intertemporal profit function:

$$\max_{\Omega_t^F} E_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) (y_t - w_t n_t - \kappa v_t) \tag{24}$$

subject to the production function and the law of motion for employment:

$$
\begin{aligned}
y_t &= z_t n_t && (25)\\
n_t &= (1-s)n_{t-1} + q_{t-1}v_{t-1} && (26)
\end{aligned}
$$

The optimality conditions of the above problem gives

$$\mu_t = \frac{\kappa}{q_t} = E_t\beta_{t+1}\left[z_{t+1} - w_{t+1} + (1-s)\frac{\kappa}{q_{t+1}}\right] \tag{27}$$

Finally, wages are the outcome of a bilateral bargaining game where the wage can be derived using the FOC $\xi\mu_t = (1-\xi)\varphi_t$. It leads to

$$w_t = \xi\left(z_t + (1-s)\kappa\theta_t\right) + (1-\xi)b \tag{28}$$

Since the equations are almost the same at the steady state the Prior for parameters are very close to those from the benchmark model estimation.

B Credible bargaining

The wage offered by the firm must satisfy the following indifference condition for the worker:

$$J^w_{N_t} = \delta J_{U_t} + (1-\delta)\left(b + \mathbb{E}_t J^{w'}_{N_{t+1}}\right) \tag{29}$$

$J^w_{N_{t+1}}$ is the marginal value for the worker if he accepts the firm's offer w_t and $J^{w'}_{N_{t+1}}$ is the worker's threat point. J_{U_t} is the worker's value when unemployed. $J^w_{N_{t+1}}$ and J_{U_t} are given by:

$$
\begin{aligned}
J^w_{N_t} &= w_t + E_t\beta_{t+1}\left(J^w_{N_{t+1}}(1-s) + s(1-f_{t+1})J_{U_{t+1}} + sf_{t+1}J^w_{N_{t+1}}\right) && (30)\\
J_{U_t} &= b + E_t\beta_{t+1}\left(J_{U_{t+1}}(1-f_{t+1}) + f_{t+1}J_{N^w_{t+1}}\right) && (31)
\end{aligned}
$$

The difference between the two gives the marginal value of employment for the worker: $\varphi_t = J^w_{N_t} - J_{U_t}$. Replacing $J^w_{N_t}$ in equation (30) by equation (29) and rearranging with respect to w_t gives us the wage proposed by the firm:

$$w_t = b + \mathbb{E}_t\beta_{t+1}\left[(1-\delta)\varphi^{w'}_{t+1} - (1 - s(1-f_{t+1}) - \delta f_{t+1})\varphi^w_{t+1}\right] \tag{32}$$

We can rewrite the flow of profits for the firms in a recursive form:

$$\Pi_t^w = z_t n_t + w_t n_t - \kappa v_t + \mathbb{E}_t \beta_{t+1} \Pi_{t+1}^w \tag{33}$$

By replacing w_t by w_t' in Equation (33) and differentiating by n_t we obtain the expected surplus from a filled job for the firm when it accepts the wage offer by the worker:

$$\mu_t^{w'} \quad = \quad z_t - w_t' + \mathbb{E}_t \beta_{t+1} (1-s) \mu_{t+1}^{w'} \tag{34}$$

If the employer refuses the worker's offer w_t', the negotiation fails, incurring a cost of delaying for the firm equal to X. The firm's flow of profits from filled job is μ_{t+1}^w and the indifference condition for firms writes:

$$\mu_t^{w'} = \delta \times 0 + (1-\delta) \left[-X + \mathbb{E}_t \beta_{t+1} \mu_{t+1}^w \right] \tag{35}$$

Combining equations (34) and (35) gives:

$$z_t - w_t' + \mathbb{E}_t (1-s) \beta_{t+1} \mu_{t+1}^{w'} = \delta \times 0 + (1-\delta) \left[-X + \mathbb{E}_t \beta_{t+1} \mu_{t+1}^w \right] \tag{36}$$

Finally, combining (35) and (8) and by rearranging the previous equation we obtain the alternative wage offered by the worker:

$$w_t' \quad = \quad z_t - (1-\delta) \left[\mathbb{E}_t \left[\beta_{t+1} \frac{\kappa}{q_{t+1}} - X \right] - (1-s) \mathbb{E}_t \beta_{t+1} \left[\beta_{t+2} \frac{\kappa}{q_{t+2}} - X \right] \right] \tag{37}$$

For this model we follow closely the calibration provided by Hall & Milgrom (2008) to set the priors. We set the prior mean of $b = 0.71$ and $\kappa = 0.43$. δ is set to 0.1 as in their calibration albeit our model works at quarterly frequencies. Finally we deduce $X = 0.05$ from the steady state equations.

C Data

Variables	Type	Source	Code
Unemployment rate	Rate, s.a, 16 years and over	Bureau of Labor Statistics (BLS)	LNS14000000
Employment	Level, Civilian, s.a 16 years and over Quarterly	Federal Reserve Bank of St. Louis (BLS)	LNS12000000
Output	Quantities, s.a, Index numbers, 2005=100	Bureau of Economic Analysis (BEA)	Table 1.1.3
Unemployment	level, s.a, 16 years and over	Bureau of Labor Statistics (BLS)	LNS13000000
Vacancies	level, s.a, Job openings Total nonfarm and Help-wanted index.	Bureau of Labor statistics and Barnichon (2010a)	JTS00000000JOL
Real wages	Average Weekly Earnings in \$, s.a, Private, divided GDP Deflator, s.a, 2009=100	FRED BEA	$\frac{\text{CES0500000030}}{\text{GDPDEF}}$
Risky rate	S&P 500	Shiller	SP500
Risk free rate	3-month Treasury Bill Secondary Market Rate	FRED	TB3MS

Table 6: ***Data source and definitions.***

Data used to compute the moments cover the periods 1948Q1-2014Q1. We use the cyclical component of real GDP and unemployment over 1951Q1-2013Q2 for the estimation. All data are taken or built at quarterly frequencies using average over months if necessary. Vacancies come from Barnichon (2010a) series which is specified as a base 1998=100. The tightness is simply equal to vacancies in level divided by unemployment in level. The discount factor is calculated in the following manner:

$$
\begin{aligned}
R_t &= \left(\frac{1 + risky\ \ rate}{1 + Risk\ \ free\ \ rate}\right)^{\frac{1}{4}} \\
\beta_t &= \frac{1}{R_t}
\end{aligned}
$$

The return corresponds to our measure of the risky rate. It is calculated at annual change (quarter i of year j / quarter i of year $j+1$) using the following definition:

$$
risky\ \ rate_t = \frac{P_t + D_t}{P_{t-1}} - 1 \tag{38}
$$

where P_t stands for the price of the stock market index and D_t is the dividend.