# Optimal Monetary Policy Rule and Cyclicality of Fiscal Policy in a Developing Oil Economy

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**Abstract**

This paper constructs a dynamic stochastic general equilibrium model of joint monetary and fiscal policy for a developing oil economy to find an optimal monetary policy rule combined with pro-/countercyclical fiscal stance based on an explicitly derived welfare measure. The model captures a set of structural specifics: two monetary instruments—interest rate and foreign exchange intervention, two fiscal instruments—public consumption and public investment, two production sectors—oil and non-oil, and the two types of households—optimizers and rule-of-thumb households. It further includes a Sovereign Wealth Fund, foreign exchange reserves accumulation, the foreign debt of private sector via collateral constraint, and a world oil price shock. The welfare measure is derived as a second-order approximation of households' utility, based on which an optimal Taylor rule is examined given pro-/countercyclical fiscal policy. Neutral fiscal stance serves as a benchmark to study the composition of welfare loss. Impulse response functions produced at optimal policy combination are analyzed as well.

*Keywords:* oil economy, monetary policy, fiscal policy, welfare, oil price shock, foreign exchange reserves, SWF

*JEL Classification:* E31, E52, E62, E63, F31, F41, H54, H63, Q33, Q38

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# 1 Introduction

Most macroeconomic DSGE models are constructed for the developed world incorporating its advanced market structure and relevant policy environment. Emerging market economies have their own unique features, which can modify the existing core frameworks in several respects. First, public investment should be considered separately from public consumption as a growth inducing instrument of fiscal policy, since it is usually associated with infrastructure and human capital which developing countries often lack. Thus, fiscal stimulus in terms of raised public investment and/or public consumption may directly accumulate public debt breaking the Ricardian equivalence assumption. Second, monetary policy is typically a hybrid of inflation targeting and managed exchange rate regime; thus, interest rate and foreign exchange intervention represent the two separate instruments of monetary policy. Third, in underdeveloped domestic financial market, the investments of firms are often financed by foreign funds, so that physical capital and foreign debt can be linked through a collateral constraint. Fourth, households are heterogeneous in their income and access to financial market; a certain portion of the population may be liquidity constrained having only wages, without making savings. These four structural specifics are incorporated in the model of Algozhina (2012) calibrated for Hungary as a first economy among all emerging markets severely hit by the global financial crisis in mid-October 2008.

This paper extends Algozhina (2012) for a subset of emerging open economies which export oil, but it can be applied to any commodity. Oil exporting developing economies obviously differ from other emerging countries and need to be examined through their own DSGE framework with the following structural specifics in addition to those outlined above. The oil and non-oil production sectors should be specified separately. The economy is exposed to a volatile exogenous world oil price shock. A Sovereign Wealth Fund (SWF) is established collecting the oil taxes, saving them abroad, and partly transferring them to the government budget[1]. The foreign exchange intervention accumulates central bank reserves that may positively contribute to welfare. And motivated by Frankel and Catao (2011), hereafter as F&C, monetary policy can follow product price targeting (PPT) as an alternative to consumer price index (CPI); thus, these two monetary anchors need to be compared in a general equilibrium framework jointly with fiscal policy based on an explicitly derived welfare measure.

F&C argue that commodity exporting economies are better off targeting the

[1] The mechanism of SWF accumulation differs across countries, but since the model is calibrated for Kazakhstan, its experience is specifically captured.

output price index which includes export commodities and excludes import products; such monetary policy is automatically countercyclical against the volatile terms of trade shock. The argument is that if the world oil price increases and there is PPT, then monetary policy tightens by raising its interest rate, thus causing the exchange rate appreciation which is the objective of offsetting the initial positive terms of trade shock. And vice versa, an adverse terms of trade shock, such as a fall in oil price, can be mitigated by the exchange rate depreciation under PPT. The CPI inflation targeting, in contrast, does not respond to export prices, but to import prices. If there is an adverse terms of trade shock, such as an increase of import prices, CPI targeting brings the exchange rate appreciation exacerbating further the initial negative shock for producers. "Bottom line: a Product Price Targeter would appreciate in response to an increase in world prices of its commodity exports, not in response to an increase in world prices of its imports. CPI targeting gets this backwards." (F&C, p. 4).

The aim of this paper, therefore, is to construct a DSGE model for a developing resource-rich economy capturing its structural specifics, as defined above, and explicitly derive the utility-based welfare in order to examine an optimal monetary policy rule combined with pro-/countercyclical fiscal policy. The calibration is based on Kazakhstan as a small open oil exporting economy severely hit by the global financial crisis 2008 due to the high foreign debt of private sector. Since 2006, the IMF has added Kazakhstan to its "fuel exporters" group analyzed in the *World Economic Outlook*. The classification is made on the evidence that over past five years the average share of fuel exports in total exports exceeds 40 percent. In 2000, Kazakhstan established its SWF managed by the National Bank on behalf of the Ministry of Finance. Oil taxes directly accumulate the SWF saved abroad, but regularly on an ad hoc basis there are transfers from SWF to the government budget. Monetary policy is independently conducted by the National Bank pursuing price stability goal and managed exchange rate regime at the same time.

The utility-based welfare measure is derived according to Edge (2003), who studies welfare criterion à la Woodford (2003) in a model with endogenous capital accumulation. Yet my model extends it in several respects: there is private and public capital, the private capital of non-oil sector has its investment adjustment costs, a collateral constraint exists, public consumption and public investment are the two separate fiscal instruments, and central bank reserves are accumulated by the foreign exchange intervention. In addition, this is a small open economy framework with the households utility inseparable in consumption and hours worked.

Based on a constructed model and its derived welfare measure, my research question focuses on an optimal Taylor rule given the cyclicality of fiscal policy in order to understand which monetary anchor is a preferable option: CPI, PPT or exchange rate targeting. Output response in the Taylor rule is set to low and high value while searching for optimal inflation and exchange rate responses. Neutral fiscal policy, associated with the zero output response of public spending, is taken as a benchmark to calculate welfare loss in deviation from it; thus, pro-/countercyclical fiscal stance corresponds to positive/negative output response of public spending respectively. I also examine the impulse response functions to five exogeous shocks to identify their propagation channels: two fiscal shocks—public consumption and public investment, two monetary shocks—interest rate and foreign exchange intervention, and the world oil price shock.

In section two, I outline the model with two types of households, standard optimizers and rule-of-thumb households, non-oil firms acting in a monopolistically competitive market, oil sector owned by the foreigners and government, two monetary policy rules for each its instrument, and respective fiscal policy rules. Section three describes the calibrated values for parameters, the list of which is provided in Appendix A. Section four lays out the steps taken to derive welfare loss as a second-order approximation of households utility. Section five examines the main results followed by the sensitivity analyses of no investment adjustment costs and no collateral constraint to welfare implications.

# 2 Model

The model has several frictions: an incomplete asset market, investment adjustment costs, collateral constraint, and the Calvo price setting. The crucial underlying assumption is that the foreign world is a saver, while the domestic economy is a borrower; thus, foreign discount factor is higher than domestic discount factor as the domestic households might be relatively impatient compared to the rest of the world. This assumption implies in turn that the interest rate of an emerging economy is always higher than the foreign interest rate, which is consistent with the evidence.

There are two producers in the model: oil firms and non-oil firms. The foreigners and government own the oil firms. Capital-intensive oil production has only capital input affected by a real FDI shock. The world oil price has an exogenous shock as well. The non-oil firms are monopolistically competitive and set prices on their intermediate goods à la Calvo (1983); their profits are transferred to optimizing households. The government share of oil profits together with taxes on oil sector accumulate the SWF, while the remainder of oil profits goes to the foreigners. The interest income of SWF constitutes the oil budget revenues, which have an exogenous shock interpreted as the SWF transfers discretionary made to the government budget.

Since there are two types of households, only optimizers borrow from abroad and have a collateral constraint on non-oil physical capital. They also hold the domestic government bonds, own the non-oil firms, rent physical capital to these firms, and decide about investment. The liquidity constrained (rule-of-thumb) households consume their wages each period. The labor market is assumed to be competitive, as unions or high households bargaining power over wages might be irrelevant in the emerging market setting.

The CPI Taylor rule includes the lagged interest rate, CPI inflation, output, and exchange rate, yet there is also a rule for the foreign exchange intervention responding to the exchange rate and its change according to Sarno and Taylor (2001). The PPT Taylor rule, in contrast, involves the oil price inflation and domestic price inflation weighted by the oil and non-oil GDP shares respectively. Public consumption and public investment rules have fiscal debt, oil revenues of the government budget, and aggregate output to capture pro-/countercyclical fiscal policy. Public investment is productive in accumulating public capital, which is an additional input in the Cobb-Douglass non-oil production function beyond labor and physical capital.

The foreign world is modeled by its Phillips curve, AR (1) process for output, and the Taylor rule for interest rate. All foreign variables are denoted by an asterisk in this paper.

## 2.1 Households

The economy is populated by a continuum of households on the interval [0,1], where the fraction $\mu$ is rule-of-thumb households. They do not have access to financial markets and consume all of their disposable income each period. In other words, they act myopically without any effect of a future policy on their economic decisions. The other $(1-\mu)$ fraction of households are forward-looking optimizers who hold government bonds, borrow from abroad, invest in non-oil physical capital, rent that capital to the non-oil firms, and receive profits from those monopolistic firms. The labor market is competitive, wage is the same across all households, and both types of households work the same number of hours. The superscript $S$ indicates a variable associated with savers (optimizers), while $N$ is for non-savers (rule-of-thumb households).

The optimizing household maximizes its utility (Schmitt-Grohe & Uribe, 2003):

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^S - \phi^{-1} N_t^{\phi}]^{1-\sigma} - 1}{1-\sigma}, \qquad \phi > 1,\ \sigma > 1 \tag{1}$$

subject to the following budget constraint:

$$C_t^S + I_t + b_t + R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} + T_t^S = W_t N_t + R_t^{kno} K_{t-1}^{no} + R_{t-1} \frac{b_{t-1}}{\pi_t} + b_t^* + \Pi_t, \tag{2}$$

where $b_t = \frac{B_t}{P_t}$ is the real purchase of government bonds, $RER_t$ is a real exchange rate (the price of foreign goods basket in terms of the domestic goods baskets), $b_t^* = RER_t \frac{B_t^*}{P_t^*}$ is the real foreign borrowings expressed in domestic goods, $R_{t-1}$ and $R_{t-1}^*$ are the nominal gross domestic and foreign interest rates respectively, $T_t^S$ is the real lump-sum taxes, $W_t$ is a real wage, $R_t^{kno}$ is the real rental cost of non-oil physical capital, $\pi_t = \frac{P_t}{P_{t-1}}$ is inflation, and $\Pi_t$ is the real profit of monopolistic non-oil firms[2].

Each $i \in \{S, N\}$ type of households has the composite CES consumption preference over domestic and foreign goods with $\eta > 0$ as an elasticity of substitution between goods:

$$C_t(i) = \left[ \gamma^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}(i) + (1-\gamma)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}(i) \right]^{\frac{\eta}{\eta-1}},$$

where $\gamma$ is a home-bias parameter, while $(1-\gamma)$ is a degree of openness. The typical consumption expenditures minimization by a household delivers the following CPI index:

$$P_t^{1-\eta} = \gamma P_{h,t}^{1-\eta} + (1-\gamma) P_{f,t}^{1-\eta} \tag{3}$$

The aggregate consumption in turn is $C_t = \mu C_t^N + (1-\mu) C_t^S$.

The law of motion for non-oil capital is specified according to Berg, Portillo, Yang, and Zanna (2013) incorporating the investment adjustment costs:

$$K_t^{no} = (1-\delta) K_{t-1}^{no} + \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad \text{where } \kappa > 0 \tag{4}$$

The collateral constraint relates gross foreign liabilities to a future value of capital (Faia & Iliopulos, 2011) and always binds, assuming that foreign debt is permanently

[2] $\Pi_t = Y_t^{no}(P_t^h - MC_t)$, where $Y_t^{no}$ is non-oil output, $P_t^h$ is the relative domestic price of non-oil goods to composite consumption, and $MC_t$ is the real marginal cost of non-oil firms.

high in this economy[3]:

$$R_t^* b_t^* = E_t\{\Omega \frac{Q_{t+1}\pi_{t+1}^*}{RER_{t+1}/RER_t} K_t^{no}\}, \tag{5}$$

where $Q_t$ is a real shadow value of capital (Tobin's Q) and $\Omega$ is an upper bound of leverage ratio.

The problem of the optimizer is, therefore, to maximize (1) with respect to $C_t^S$, $I_t^S$, $K_t^{no}$, $b_t$, $b_t^*$, $N_t$ subject to (2), (4), and (5). The first-order conditions of this problem are below, where $\lambda_t$, $\lambda_t^k$, and $\lambda_t \xi_t$ are the Lagrange multipliers of the constraints (2), (4), (5) respectively.

$$U_{C_t^S} = \lambda_t = \frac{1}{\left[C_t^S - \frac{N_t^\phi}{\phi}\right]^\sigma} \tag{6}$$

$$\frac{1}{Q_t} = 1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} + \beta\kappa E_t\left\{\frac{Q_{t+1}\lambda_{t+1}}{Q_t\lambda_t}\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}, \tag{7}$$

where $Q_t = \frac{\lambda_t^k}{\lambda_t}$

$$Q_t = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left[R_{t+1}^{kno} + Q_{t+1}(1-\delta)\right] + \xi_t \Omega \frac{Q_{t+1}\pi_{t+1}^*}{RER_{t+1}/RER_t}\right\} \tag{8}$$

$$\frac{1}{R_t} = \beta E_t\left\{\frac{U_{C_{t+1}^S}}{U_{C_t^S}\pi_{t+1}}\right\} \tag{9}$$

$$\frac{1}{R_t^*} = \beta E_t\left\{\frac{U_{C_{t+1}^S}}{U_{C_t^S}}\frac{RER_{t+1}}{RER_t\pi_{t+1}^*}\right\} + \xi_t \tag{10}$$

$$W_t = N_t^{\phi-1} \tag{11}$$

By dividing (10) into (9), the following uncovered interest rate parity (UIP) condition is obtained:

$$\frac{R_t}{R_t^*} = E_t\left\{\frac{RER_{t+1}}{RER_t}\frac{\pi_{t+1}}{\pi_{t+1}^*}\right\} + \frac{\xi_t}{\beta}E_t\left\{\frac{U_{C_t^S}}{U_{C_{t+1}^S}}\pi_{t+1}\right\} + cov_t, \tag{12}$$

where $cov_t$ captures covariance terms.

The rule-of-thumb household has the same preference as the optimizer. It chooses

[3] Occasionally binding collateral constraint is ruled out because it requires global solution methods that may be infeasible to apply in this complex model.

only consumption and labor and its budget constraint is simply this:

$$C_t^N + T_t^N = W_t N_t \tag{13}$$

The first-order conditions with respect to $N_t$ and $C_t^N$ are identical to the optimizer's solutions. Thus, the rule-of-thumb household faces the same labor supply condition (11).

## 2.2 Firms

Following Gali, Lopez-Salido, and Valles (2007), there are monopolistically competitive non-oil firms producing differentiated intermediate goods, and a perfectly competitive non-oil firm producing a final domestic good. The final domestic non-oil producer has a constant returns technology:

$$Y_t^{no} = \left( \int_0^1 X_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $X_t(j)$ is the input amount of intermediate good $j$ and $\varepsilon > 1$ is the elasticity of substitution between differentiated intermediate goods. It maximizes profit taking as given the domestic final goods price $P_t^h$ and intermediate goods prices $P_t^h(j)$ such that the optimal demand allocation is as follows:

$$X_t(j) = \left( \frac{P_t^h(j)}{P_t^h} \right)^{-\varepsilon} Y_t^{no} \tag{14}$$

Each intermediate goods non-oil firm has the identical Cobb-Douglass production function, which includes the private non-oil capital, labor, and public capital:

$$Y_t^{no}(j) = u_t^{no} K_{t-1}^{no}(j)^{\alpha} N_t(j)^{1-\alpha} K_{G,t-1}^{\psi}, \tag{15}$$

where the level of technology, $u_t^{no}$, and the usage of public capital are common to all firms.

Intermediate goods producers solve their problem in two stages. First, cost minimization subject to the production function (15) provides the following real marginal cost common to all non-oil firms, taking the real wage and rental cost of capital as given:

$$MC_t = \frac{W_t^{1-\alpha} (R_t^{kno})^{\alpha}}{u_t^{no} K_{G,t-1}^{\psi} (1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{16}$$

Second, intermediate non-oil producers choose the price $P_t^{hop}$ to maximize their discounted real profit:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_{t+m}^h} - MC_{t+m} \right) \right\}, \tag{17}$$

where $D_{t,t+m} = \beta^m E_t(\frac{U_{C_{t+m}^S}}{U_{C_t^S}})$ is a stochastic discount factor coming from the optimizing household's problem, subject to the demand constraint according to (14):

$$Y_{t+m}^{no}(j) = \left( \frac{P_t^{hop}}{P_{t+m}^h} \right)^{-\varepsilon} Y_{t+m}^{no}$$

In other words, a fraction $(1 - \theta)$ of non-oil firms adjusts their prices each period, while the respective fraction $\theta$ keeps their prices unchanged; thus, $\theta$ is an index of price stickiness according to Calvo (1983). The domestic price index, therefore, evolves as follows:

$$(P_t^h)^{1-\varepsilon} = \theta (P_{t-1}^h)^{1-\varepsilon} + (1-\theta)(P_t^{hop})^{1-\varepsilon}$$

The first-order condition of this price setting decision (17) is below:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_{t+m}^h} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+m} \right) \right\} = 0, \tag{18}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is a frictionless price markup.

The production function of oil firm has only capital input assuming that oil production is a capital-intensive sector and to avoid any complication originating from possible labor mobility between two sectors:

$$Y_t^o = (K_{t-1}^o)^{\alpha^o} \tag{19}$$

The oil capital is accumulated by FDI which follows an exogenous AR (1) process[4]:

$$K_t^o = (1-\delta) K_{t-1}^o + FDI_t^* \tag{20}$$

$$\widehat{FDI_t^*} = \rho_{FDI} \widehat{FDI_{t-1}^*} + \varepsilon_t^{FDI} \tag{21}$$

[4]The quarterly data of balance of payments for FDI are plotted in Appendix H in mln US dollars over 2002Q1-2012Q2. The graph shows that FDI indeed exhibits a stochastic process, rather than being constant over time.

Hats, hereafter, denote the deviation of variables from their steady state.

The world oil price has an exogenous shock to its AR(1) process as well:

$$\widehat{P_t^{o*}} = \rho_o \widehat{P_{t-1}^{o*}} + \epsilon_t^o \tag{22}$$

The oil firm receives its profit $\Pi_t^{o*}$ net of royalties levied on production quantity at a rate $\tau^o$:

$$\Pi_t^{o*} = (1 - \tau^o) P_t^{o*} Y_t^o \tag{23}$$

The oil sector is owned by the foreigners and government: the dividend share of oil profit that the government receives is denoted by $\iota^{\text{div}}$.

## 2.3 Fiscal policy

The government collects lump-sum taxes $T_t$ and oil revenues $OR_t$ as the transfers from SWF. It issues one-period bonds to finance public consumption $G_t^C$ and public investment $G_t^I$, which are assumed to have the same CPI price. The government budget constraint in real terms can be written as follows:

$$b_t + T_t + \underbrace{e^{\epsilon_t^{\Xi}} (R_t^* - 1) OF_{t-1}^* RER_t}_{OR_t} = G_t^I + G_t^C + R_{t-1} \frac{b_{t-1}}{\pi_t}, \tag{24}$$

where $T_t = (1 - \mu) T_t^S + \mu T_t^N$, $OR_t = e^{\epsilon_t^{\Xi}} (R_t^* - 1) OF_{t-1}^* RER_t$ is the oil revenues specified as the interest income of SWF transferred to the budget and multiplied by a shock to those transfers $e^{\epsilon_t^{\Xi}}$.

Public investment is productive so that the law of motion for public capital is given by:

$$K_t^G = (1 - \delta^g) K_{t-1}^G + G_t^I \tag{25}$$

Oil taxes, collected in foreign currency, consist of royalties and government share of oil sector's profit

$$T_t^{o*} = \tau^o P_t^{o*} Y_t^o + \iota^{\text{div}} \Pi_t^{o*} \tag{26}$$

and go directly to SWF accumulated according to the equation below.

$$OF_t^* = \rho_{OF} OF_{t-1}^* + T_t^{o*} \tag{27}$$

Two fiscal instruments, public investment and public consumption, have the following rules with their aggregate output response ($\vartheta_{GI}$ and $\vartheta_{GC}$) associated with fiscal cyclicality:

$$\widehat{G_t^I} = \rho_{GI}\widehat{G_{t-1}^I} + (1-\rho_{GI})[\vartheta_{GI}\widehat{Y}_t - \gamma_{GI}\widehat{b}_{t-1} + \gamma_{OF}^{GI}\widehat{OR}_t] + \epsilon_t^{GI} \tag{28}$$

$$\widehat{G_t^C} = \rho_{GC}\widehat{G_{t-1}^C} + (1-\rho_{GC})[\vartheta_{GC}\widehat{Y}_t - \gamma_{GC}\widehat{b}_{t-1} + \gamma_{OF}^{GC}\widehat{OR}_t] + \epsilon_t^{GC} \tag{29}$$

Since fiscal debt clears the government budget constraint, the lump-sum taxes require a separate equation, which includes fiscal debt and public spending similar to Gali, Lopez-Salido, and Valles (2007) minus oil revenues specific to this model:

$$\widehat{T}_t = \varphi_b\widehat{b}_{t-1} + \varphi_I\widehat{G_t^I} + \varphi_C\widehat{G_t^C} - \varphi_{OF}\widehat{OR}_t \tag{30}$$

## 2.4 Monetary policy

The nominal interest rate responds to its lagged value, CPI inflation, aggregate output, and real exchange rate according to the CPI targeting Taylor rule below:

$$CPI:\ \widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left[\phi_\pi\pi_t + \phi_y\widehat{Y}_t + \phi_e\widehat{RER}_t\right] + \epsilon_t \tag{31}$$

The PPT Taylor rule, in contrast, has the product price inflation that is a weighted average of oil price inflation in domestic goods basket $\pi_t^o = \triangle\ \widehat{P_t^{o*}} + \triangle\ \widehat{RER}_t$ and domestic price inflation $\pi_t^h$ according to Appendix C, with weights corresponding to the GDP shares of oil $s_o$ and non-oil sector $(1-s_o)$ respectively:

$$PPT:\ \widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left[\begin{array}{c}\phi_\pi\left[s_o\pi_t^o + (1-s_o)\left(\pi_t - \frac{1-\gamma}{\gamma}\triangle\ \widehat{RER}_t\right)\right] + \\ +\phi_e\widehat{RER}_t + \phi_y\widehat{Y}_t\end{array}\right] + \epsilon_t \tag{32}$$

The foreign exchange intervention, as a purchase of foreign currency by a central bank, has its separate rule responding to the exchange rate and its rate of depreciation[5] (Sarno & Taylor, 2001):

$$\widehat{Int_t^*} = \alpha_1\widehat{RER}_t + \alpha_2\triangle\ \widehat{RER}_t + \epsilon_t^{int}, \quad \alpha_1 < 0,\ \alpha_2 < 0, \tag{33}$$

where $Int_t^*$ is the real foreign exchange intervention denominated in foreign currency.

Foreign exchange reserves or net foreign assets of a central bank are accumulated by the foreign exchange intervention:

$$NFA_t^* = \rho_{nfa}NFA_{t-1}^* + Int_t^* \tag{34}$$

[5]Since exchange rate is defined as the price of foreign currency in terms of domestic currency, the higher $\triangle\ \widehat{RER}_t$ is, the more domestic currency depreciates.

## 2.5 Market clearing conditions

The goods market clearing condition is as follows:

$$P_t^h Y_t^{no} + P_t^{o*} Y_t^o RER_t = C_t + I_t + RER_t FDI_t^* + G_t^C + G_t^I + NX_t \tag{35}$$

The labor and capital markets clear according to these conditions:

$$N_t = \int_0^1 N_t(j)dj, \qquad K_t^{no} = \int_0^1 K_t^{no}(j)dj$$

The balance of payments requires that the sum of current account and financial account should be equal to a change of foreign exchange reserves. The current account includes net exports, interest income of SWF assets, as those assets are saved abroad, minus the foreign share of oil sector's profit, while the financial account constitutes the foreign borrowings of optimizers and FDI:

$$NX_t + (R_t^* - 1)OF_{t-1}^* RER_t - (1 - \iota^{\text{div}}) RER_t \Pi_t^{o*} = \left( R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} - b_t^* \right) - RER_t FDI_t^* + RER_t \bigtriangleup NFA_t^*$$

## 2.6 The rest of the world

The rest of the world is a relatively large economy governed by three exogenous equations below:

$$\widehat{Y}_t^* = \rho_{Y^*} \widehat{Y}_{t-1}^* + \epsilon_t^{Y^*} \tag{36}$$

$$\widehat{R}_t^* = \phi_\pi^* \pi_t^* + \phi_y^* \widehat{Y}_t^* + \epsilon_t^{R^*} \tag{37}$$

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \lambda^* \left( \sigma + \frac{\phi^* + \alpha^*}{1 - \alpha^*} \right) \widehat{Y}_t^* \tag{38}$$

The model includes 25 endogenous variables constituting a system of 25 equations, where the variables are represented in log-deviation from their steady state: inflation $\pi_t$, the aggregate consumption of households $\widehat{C}_t$, hours worked $\widehat{N}_t$, domestic interest rate $\widehat{R}_t$, net exports $\widehat{NX}_t$, net foreign assets of a central bank $\widehat{nfa}_t^*$, foreign exchange intervention $\widehat{int}_t^*$, foreign interest rate $\widehat{R}_t^*$, foreign inflation $\pi_t^*$, foreign output $\widehat{Y}_t^*$, foreign debt $\widehat{b}_t^*$, oil capital $\widehat{K}_t^o$, non-oil capital $\widehat{K_t^{no}}$, public capital $\widehat{K}_{G,t}$, real exchange rate $\widehat{RER}_t$, fiscal debt $\widehat{b}_t$, lump-sum taxes $\widehat{T}_t$, public consumption $\widehat{G_t^C}$, public investment $\widehat{G_t^I}$, private investment $\widehat{I}_t$, oil output $\widehat{Y_t^o}$, non-oil output $\widehat{Y_t^{no}}$, aggregate output $\widehat{Y}_t$, SWF assets $\widehat{OF}_t$, and the domestic prices $\widehat{P_t^h}$. The system of log-linear equations consists of the Taylor rule (31), foreign exchange intervention rule (33), public investment equation (28), public consumption equation (29), lump-sum taxes equation (30), three foreign world expressions (36, 37 and 38), the Phillips curve represented in Appendix C, and the other 16 equations shown in Appendix D.

# 3 Calibration

The model is calibrated using the averages of Kazakhstani data over 1995Q1-2012Q2 for the steady state values of endogenous variables derived in Appendix B. The data are available from the webpages of the National Bank, Ministry of Finance, Agency of Statistics, and the IFS. They include real GDP, private consumption, public consumption, fixed capital formation, net exports, oil output, wages, nominal T-bill rate, CPI, real US dollar-tenge exchange rate, the external debt of banks and other private sectors, FDI, petroleum UK Brent price, public debt, fiscal capital expenditures, oil and non-oil revenues of the government budget, SWF assets as a stock variable and SWF inflows. The list of calibrated parameters is provided in Appendix A excluding GDP ratios and parameters for the rest of the world which are described in this section.

All parameters can be divided into three sets: standard values borrowed from other studies because of the non-availability of relevant data, values obtained from the basic time-series regressions, and parameters specifically calibrated for this model. The first set includes the depreciation rates for private and public capital $\delta = 0.025$, $\delta^g = 0.02$ (Traum & Yang, 2011), the elasticity of substitution between differentiated intermediate goods $\varepsilon = 6$ and output response in the Taylor rule $\phi_y = 0.125$ (Gali, 2008), price stickiness $\theta = 0.9$ and inflation response in the Taylor rule $\phi_\pi = 1.37$ (Jakab & Vilagi, 2008), the inverse of intertemporal elasticity of substitution for consumption $\sigma = 2$ (Schmitt-Grohe & Uribe, 2003), exchange rate change response in the intervention rule $\alpha_2 = -0.62$ (Gartner, 1987), investment adjustment costs parameter $\kappa = 20$ (Berg, Portillo, Yang & Zanna, 2013), and the fiscal debt response of lump-sum taxes $\varphi_b = 0.4$ (Algozhina, 2012). The foreign parameters are set to their standard values: the elasticity of wages with respect to hours worked $\phi^* = 2$, discount factor $\beta^* = 0.99$, $\phi^*_\pi = 1.5$, $\phi^*_y = 0.125$ (Gali, 2008), $\theta^* = 0.75$ (Gali, Lopez-Salido & Valles, 2007), output elasticity to capital $\alpha^* = 0.32$, and output persistence $\rho_{Y*} = 0.8$.

The second set consists of significant regression parameters according to model's equations using the seasonally adjusted log of real variables. In particular, fiscal debt response in the public investment equation (28) is equal to $\gamma_{GI} = 0.54$, while the autoregressive coefficients $\rho_{GC}$ and $\rho_{GI}$ appear to be 0.3. According to the lump-sum taxes equation (30), the regression of non-oil fiscal revenues on public consumption, public investment, fiscal debt, and oil revenues of the government budget produces the significant responses to public consumption $\varphi_C = 1$ and public investment $\varphi_I = 0.2$. The autoregressive coefficients in the world oil price, FDI, foreign exchange reserves, and Taylor rule regressions suggest to be $\rho_o = 0.98$, $\rho_{FDI} = 0.65$, $\rho_{nfa} = 0.98$, and $\rho = 0.95$ respectively.

The third set includes the GDP ratios of consumption, public consumption, net exports[6], FDI, foreign debt, oil output, SWF assets, net foreign assets of a central bank, foreign exchange intervention, and fiscal capital expenditures as a proxy for public investment: $c_y = 0.61$, $g_y^C = 0.08$, $nx_y = 0.25$, $fdi_y = 0.02$, $b_y^* = 2.17$, $s_o = 0.52$, $of_y = 0.65$, $nfa_y^* = 0.52$, $int_y^* = 0.03$, and $g_y^I = 0.02$ respectively. The degree of openness is calculated as a ratio of imports to GDP, $1 - \gamma = 0.32$; thus, home-bias parameter $\gamma$ is equal to 0.68. The domestic discount factor is around 0.978 because the average T-bill rate is used as a proxy for policy interest rate, 2.3 percent per quarter[7]. The upper bound of leverage ratio $\Omega$ appears to be 0.54. Using data on wages, the elasticity of wages with respect to hours worked $\phi$ is 1.45 according to the labor supply condition (11), in which hours are obtained from the non-oil production function (15). The elasticity of output with respect to capital $\alpha$ is equal to 0.3, while with respect to public capital is $\psi = 0.16$ generated by the steady state wage equation in Appendix B. The royalties rate levied on oil production quantity $\tau^o = 0.27$ is calculated as the SWF inflows share in oil output. The dividend share of oil profit that the government receives $\iota^{\text{div}}$ is set to 0.1, while the elasticity of oil output with respect to oil capital $\alpha^o$ is technically feasible at 0.7. The persistence in SWF process $\rho_{OF}$ is fixed to 0.9, whereas the exchange rate response in the intervention rule $\alpha_1$ is set to $-0.15$.

Fiscal parameters are calibrated based on the fiscal rules at steady state and differ depending on the stance of fiscal policy: procyclical, countercyclical, and neutral. The neutral fiscal policy is a benchmark to calculate welfare loss in deviation from it. It is associated with the zero output response of public consumption and public investment in their rules ($\vartheta_{GC} = 0$ and $\vartheta_{GI} = 0$). The public investment rule (28) at neutral fiscal stance suggests the oil revenues response $\gamma_{OF}^{GI}$ of 1.66, which is set

[6] According to data, the GDP ratio to net exports is equal to 0.07. However, at this value the exchange rate and investment variations generate welfare gains instead of loss. Therefore, it is set to 0.25 in order to obtain the negative coefficient in front of quadratic real exchange rate and private investment in the welfare measure. This, in turn, affects the GDP ratios to public investment, FDI, and private investment to be equal to 0.02 for simplicity.

[7] The domestic interest rate matters exclusively for the government bonds in the model, as investments are financed by the foreign funds rather than domestic financial market.

for $\gamma_{OF}^{GC} = 1.66$ too, based on equation below:

$$\gamma_{OF}^{GI} = \frac{\ln \overline{G_I} + \gamma_{GI} \ln \overline{b} - \vartheta_{GI} \ln \overline{Y}}{\ln \overline{OR}}$$

Similarly, the public consumption rule (29) gives its public debt response $\gamma_{GC} = 0.56$ according to the following equation:

$$\gamma_{GC} = \frac{\vartheta_{GC} \ln \overline{Y} + \gamma_{OF}^{GC} \ln \overline{OR} - \ln \overline{G_C}}{\ln \overline{b}}$$

These two parameters apply to pro- and countercyclical fiscal policy as well, including the oil revenues response of lump-sum taxes $\varphi_{OF} = 0.66$ according to taxes rule (30):

$$\varphi_{OF} = \frac{\varphi_b \ln \overline{b} + \varphi_I \ln \overline{G_I} + \varphi_C \ln \overline{G_C} - \ln \overline{T}}{\ln \overline{OR}}$$

The procyclical fiscal policy corresponds to positive output response of public consumption $\vartheta_{GC} = 0.7$ and public investment rules $\vartheta_{GI} = 0.7$ to achieve the same steady state consumption, while the countercyclical fiscal policy is simulated at their negative values, $\vartheta_{GC} = -0.7$ and $\vartheta_{GI} = -0.7$.

# 4 Welfare loss derivation

The second-order approximation of the aggregate utility of two types of households is represented in Appendix E in terms of the aggregate consumption and hours worked. This welfare can be further extended by substituting consumption and hours worked with the respective endogenous variables according to the equilibrium model conditions. Such a substitution allows explicitly deriving the quadratic loss function, so that other variances may constitute welfare beyond the standard inflation and output deviations. Following Edge (2003), but not going into natural rates discussion, I express welfare loss in terms of the log-linear deviation of variables from their steady state, as they are used in the model, to find an optimal Taylor rule given fiscal policy cyclicality.

In order to substitute the aggregate welfare with its quadratic expression, two major derivation steps are taken. First, I second-order approximate the labor demand condition of non-oil firm in its cost minimization problem to replace the hours worked in utility. According to Gali (2008), the domestic price dispersion term is replaced by the second-order approximation of CPI index. Non-oil output in the labor demand can be substituted by the aggregate output via GDP supply condition, since oil output is essentially an exogenous shock and can be referred as a term independent of policy (t.i.p.).

Second, the market clearing condition is approximated to its second-order to express utility's consumption in terms of the aggregate output. The non-oil capital accumulation is used to replace the current-period capital with investment. The net exports can be obtained from the balance of payments equation, which in turn requires the approximation of foreign exchange reserves, intervention rule, and collateral constraint to replace the foreign debt. The latter involves the second-order approximation of first-order condition with respect to private investment moved one period ahead in the household's problem due to the Tobin's Q term (7).

Finally, collecting all quadratic endogenous variables and expressing them in deviation from their steady state, welfare includes the aggregate output, CPI inflation, real exchange rate, private investment, public consumption, public investment, lagged non-oil private and public capital as the state variables, lagged real exchange rate, lagged foreign exchange reserves, lagged private investment, lead private investment, and lead real exchange rate (Appendix E). The existence of quadratic lead terms and lagged private investment is due to the investment adjustment costs and collateral constraint, while lagged real exchange rate and foreign exchange reserves are driven by the balance of payments equation, intervention rule, and CPI index for obtaining inflation. The public capital and foreign exchange reserves appear to be welfare improving, while the other components contribute to loss. This reduced-form welfare, omitting an extensive part of numerous cross-product terms that can be eliminated if natural rate stance is taken according to Edge (2003), is useful by itself to study the compositional loss rather than aggregate consumption and hours worked in utility.

# 5 Results

The Taylor rule is examined by searching its loss minimizing two parameters, inflation $\phi_\pi$ and exchange rate response $\phi_e$, at fixed low $\phi_y = 0.125$ and high output response $\phi_y = 1$. Table 1 summarizes the numerical results of this search with a range set for parameters between 0 and 3 based on the derived welfare loss measure. It shows that fiscal policy cyclicality matters for an optimal monetary rule. In particular, procyclical fiscal policy does not distinguish the CPI/PPT monetary anchor, but relates to the output response of Taylor rule. The high reaction to output should be complemented with the exchange rate targeting because an interest rate affects the exchange rate via an uncovered interest rate parity. Whereas under low output reaction, no need to focus on exchange rate and inflation, since there is no inflation pressure from the procyclical fiscal stance that is offset by an active monetary policy, causing the exchange rate appreciation. In contrast, countercyclical fiscal policy should be combined with a non-zero inflation response at low output reaction to handle the investment variation, which is relatively smooth given investment adjustment costs. PPT yet assumes a significantly higher inflation response than CPI targeting due to accommodating oil price inflation which may contribute to investment variation.

Table 1. **Optimal Taylor rule**

| | Procyclical fiscal | | Countercyclical fiscal | | | | Neutral fiscal policy | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CPI and PPT | | CPI | | PPT | | CPI | | PPT | |
| $\phi_y$ | 0.125 | 1 | 0.125 | 1 | 0.125 | 1 | 0.125 | 1 | 0.125 | 1 |
| $\phi_\pi$ | 0 | 0 | 0.07 | 0 | 0.873 | 0 | 0.3 | 0 | 0.6 | 0 |
| $\phi_e$ | 0 | 2.4 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2 summarizes the contribution of welfare loss components at $\phi_y = 0.125$ and corresponding optimal Taylor rule provided by the Table 1. All entries are in percent deviation of steady state consumption from the benchmark neutral fiscal policy combined with the optimized PPT anchor, i.e., $\phi_\pi = 0.6$, $\phi_e = 0$, and $\phi_y = 0.125$. Positive values mean the percentage gain in consumption relative to the benchmark, while negative values indicate higher loss contributed by a respective component than the benchmark delivers. Column 3 represents the case of procyclical fiscal policy combined with the exchange rate targeting at $\phi_y = 1$ according to Table 1. The corresponding column of countercyclical fiscal policy is omitted, since optimal Taylor rule at $\phi_y = 0.125$ is preferred to $\phi_y = 1$ because the fiscal stance is already in control of output.

Table 2. **Contribution to welfare loss**

| | Procyclical fiscal policy | | Countercyclical fiscal policy | |
|---|---|---|---|---|
| | CPI and PPT | $\phi_y = 1$ | CPI | PPT |
| Total | 50.57 | 77.27 | 8.69 | 37.43 |
| $\widehat{Y_t^2}$ | 0.327 | 0.318 | 0.245 | 0.235 |
| $\widehat{\pi_t^2}$ | 0.01131 | 0.0113 | 0.0049 | 0.0046 |
| $\widehat{RER_t^2}$ | 3.021 | 3.022 | 1.46 | 1.25 |
| $\widehat{I_t^2}$ | 13.76 | 21.9 | 1.8 | 10.7 |
| $\widehat{G_{C,t}^2}$ | 0.00479 | 0.00481 | 0.014 | 0.012 |
| $\widehat{G_{I,t}^2}$ | 0.001146 | 0.00115 | 0.0028 | 0.0025 |
| $\widehat{K_{t-2}^{no2}} + \widehat{K_{t-1}^{no2}}$ | 0.002 | 0.0017 | 0.0005 | 0.0013 |
| $\widehat{K_{G,t-1}^2}$ | -5.9·10$^{-6}$ | -5.8·10$^{-6}$ | -5.4·10$^{-6}$ | -4.9·10$^{-6}$ |
| $\widehat{nfa_{t-1}^{*2}}$ | -0.004201 | -0.004205 | -0.0021 | -0.0017 |
| $\widehat{RER_{t-1}^2} + \widehat{RER_{t+1}^2}$ | 2.023 | 2.0234 | 0.98 | 0.83 |
| $\widehat{I_{t-1}^2} + \widehat{I_{t+1}^2} + \widehat{I_{t+2}^2}$ | 31.4 | 50 | 4.2 | 24.4 |

All entries are in percent deviation of steady state consumption from the benchmark neutral fiscal policy combined with PPT Taylor rule, i.e., $\phi_\pi = 0.6$, $\phi_e = 0$, and $\phi_y = 0.125$.

The compositional welfare loss in Table 2 suggests several findings. Procyclical fiscal policy and exchange rate targeting at high output response of monetary rule is a best policy combination in terms of stabilizing the private investment, which is very volatile compared to other macro-variables. A counteracting optimal force against procyclical fiscal stance seems to be the high output response of monetary policy to contain the investment variation that should be complemented with high exchange rate response at the same time, since an interest rate affects the exchange rate. If, however, fiscal policy is countercyclical, then investment volatility is under fiscal control as a part of output, so PPT is preferred to CPI targeting due to its higher optimal inflation response which includes the domestic price inflation $\pi_t^h$, an important indicator for investment decisions. Overall, Table 2 shows that neutral fiscal policy shouldn't be run and emerging market economies are better off by having their persistent procyclical fiscal policy in contrast to countercyclical stance traditionally observed in advanced countries. As for a preferred monetary anchor, the ranking of policy combinations is listed as follows.

**Ranking of policy combinations**

1. Procyclical fiscal policy and exchange rate targeting at high output response of monetary policy

2. Procyclical fiscal policy and low output response of monetary policy

3. Countercyclical fiscal policy and PPT at low output response of monetary

policy

4. Countercyclical fiscal policy and CPI targeting at low output response of monetary policy

5. Benchmark: neutral fiscal policy and PPT at low output response of monetary policy

The transmission channels of shocks change depending on fiscal cyclicality. Procyclical fiscal stance with the exchange rate targeting as a best policy combination (rank 1 above) produces the impulse response functions in Appendix F, where monetary policy is quite influential by responding significantly to output. An increase of interest rate appreciates the exchange rate that stimulates the foreign debt because debt's volume denominated in domestic currency decreases. Inflation falls in response to high interest rate, encouraging consumption which drives the domestic prices. However, an interest rate shock per se reduces the domestic prices that contract the aggregate output, thus non-oil output as well, including hours worked as its main production input (Figure 3 in Appendix F).

In contrast, a notion of fiscal policy dominance over monetary policy in its ultimate effect on the main macro-variables appears to hold under countercyclical fiscal stance (Appendix G). This is because public consumption, in response to output, tends to depreciate the exchange rate which contributes to inflation via a standard pass-through channel. High inflation implies high wages that stimulate hours worked and thus non-oil output as well, requiring more foreign debt to finance investment. An interest rate responds to inflation or output, discouraging consumption and making savings more attractive. Decreased consumption, which is associated with the fall in domestic absorption, means for the net exports to rise, boosting the aggregate output in turn. The direct effect of fiscal variables on the real exchange rate can be seen in the impulse response functions to public consumption and public investment shocks: the exchange rate depreciates in Figure 1 of Appendix G and appreciates in Figure 2 of Appendix G irrespective of the interest rate movement. Moreover, fiscal dominance is associated with the unconventional effect of interest rate shock on output by boosting it in Figure 3 of Appendix G, instead of typical contraction consistent with the standard monetarist doctrine. Yet this finding that high interest rate may promote the aggregate demand seems to be a result of countercyclical fiscal policy that weakens the direct effect of interest rate on domestic prices, supporting therefore the fiscal theory of price level (Leeper, 1991; 2013).

The impulse response functions to the world oil price shock, as a sudden improvement of the terms of trade, can be examined with respect to F&C argument. It appears that the monetary policy parameters matter for the interest rate dynamics, but not CPI or PPT anchor per se. The interest rate does not rise under PPT and does not fall under CPI targeting as F&C suggest, but responds according to the relative magnitude of Taylor rule's parameters. This is especially observed in the case of countercyclical fiscal stance, since the optimal monetary policy differs across CPI and PPT rule according to Table 1. In particular, the interest rate falls in response to a decline in inflation under PPT rule due to its high optimal inflation response ($\phi_\pi = 0.873$), while under CPI targeting, there is no immediate change in the interest rate because inflation and output move in opposite direction at the almost close magnitude of optimal inflation and output responses ($\phi_\pi = 0.07$ and $\phi_y = 0.125$).

In the case of procyclical fiscal policy, the world oil price shock transmits to output differently depending on the monetary policy, and the interest rate reacts to output. The exchange rate targeting at high output response (rank 1) strengthens the direct positive effect of oil price on domestic prices, while low output response without any monetary anchor (rank 2) reinforces the channel from decreased consumption to low domestic prices. Therefore, output and interest rate increase in rank 1 (Figure 5 of Appendix F), whereas they decline in rank 2 case (Figure 6 of Appendix F). Overall, it looks like that the volatile terms of trade can be offset by the appropriate monetary policy rule at given fiscal cyclicality in terms of insulating their ultimate effect on aggregate output.

# 6 Sensitivity analysis

This sensitivity analysis is performed to see whether the welfare composition changes if there are no investment adjustment costs ($\kappa = 0$) in the non-oil capital accumulation equation (4). Table 3 shows the optimal monetary policy parameters found in the same way as before, while Table 4 provides the contribution to welfare loss at $\phi_y = 0.125$ for a comparison with Table 2. All entries are in percent deviation of steady state consumption from the benchmark neutral fiscal policy combined with the following optimized Taylor rule's parameters, i.e., $\phi_\pi = 0$, $\phi_e = 0.01$, and $\phi_y = 0.125$. Positive values mean the percentage gain in consumption relative to the benchmark, while negative values indicate higher loss contributed by a respective component than the benchmark delivers.

Table 3. **Optimal Taylor rule, no investment adjustment costs**

| | Procyclical fiscal | | Countercyclical fiscal | | Neutral fiscal policy | |
|---|---|---|---|---|---|---|
| | CPI and PPT | | CPI and PPT | | CPI and PPT | |
| $\phi_y$ | 0.125 | 1 | 0.125 | 1 | 0.125 | 1 |
| $\phi_\pi$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\phi_e$ | 0.03 | 2.6 | 0.02 | 0 | 0.01 | 0 |

Table 4. **Contribution to welfare loss, no investment adjustment costs**

| | Procyclical fiscal policy | | Countercyclical fiscal policy | |
|---|---|---|---|---|
| | CPI and PPT | $\phi_y = 1$ | CPI and PPT | $\phi_y = 1$ |
| Total | 97.83 | 97.72 | 45 | 19.8 |
| $\widehat{Y_t^2}$ | 6.8 | 6.7 | 4.91 | 4.99 |
| $\widehat{\pi_t^2}$ | 0.218 | 0.2179 | 0.09 | 0.098 |
| $\widehat{RER_t^2}$ | 53.92 | 53.98 | 23.7 | 10.7 |
| $\widehat{I_t^2}$ | -0.43 | -0.4 | -0.13 | 0.5 |
| $\widehat{G_{C,t}^2}$ | 0.031 | 0.027 | 0.2 | 0.0006 |
| $\widehat{G_{I,t}^2}$ | 0.0096 | 0.0087 | 0.047 | 0.0005 |
| $\widehat{K_{t-2}^{no2}} + \widehat{K_{t-1}^{no2}}$ | 0.5 | 0.4 | 0.2 | -2.5 |
| $\widehat{K_{G,t-1}^2}$ | $-9.4 \cdot 10^{-5}$ | $-9.1 \cdot 10^{-5}$ | $-9 \cdot 10^{-5}$ | $-1.9 \cdot 10^{-5}$ |
| $\widehat{nfa_{t-1}^{*2}}$ | -0.0724 | -0.0725 | -0.03 | -0.009 |
| $\widehat{RER_{t-1}^2} + \widehat{RER_{t+1}^2}$ | 36.02 | 36.06 | 15.8 | 7 |
| $\widehat{I_{t-1}^2}$ | 0.78 | 0.72 | 0.2 | -0.9 |

All entries are in percent deviation of steady state consumption from the benchmark neutral fiscal policy combined with the following Taylor rule, i.e., $\phi_\pi = 0$, $\phi_e = 0.01$, and $\phi_y = 0.125$.

The lead terms of private investment disappear in the welfare loss if there are no investment adjustment costs, but at the expense of increased contribution made by the real exchange rate variation. An absence of adjustment costs makes investment more volatile and therefore hard to stabilize, while the exchange rate affects many variables in this model, suggesting its significant role for welfare. The columns 2 and 3 show almost the same welfare because the output response of monetary policy does not matter now to capture the investment variation, which is volatile anyway without investment adjustment costs.

Table 5 below demonstrates the case without collateral constraint under no investment adjustment costs, at the optimal Taylor rule of Table 3, as an additional sensitivity analysis in terms of reducing further the welfare components: lead exchange rate, lagged investment, and second lag of non-oil capital. Still it shows that the real exchange rate significantly contributes to welfare and procyclical fiscal policy is robustly preferred to countercyclical fiscal stance in a small open economy. The future research might focus on explaining the driving welfare forces of real exchange rate.

Table 5. **No collateral constraint, no investment adjustment costs**

| | Procyclical fiscal policy | | Countercyclical fiscal policy | |
|---|---|---|---|---|
| | CPI and PPT | $\phi_y = 1$ | CPI and PPT | $\phi_y = 1$ |
| Total | 98.7 | 98.4 | 52 | 11.8 |
| $\widehat{Y_t^2}$ | 9.86 | 9.76 | 6.6 | 9.3 |
| $\widehat{\pi_t^2}$ | 0.0562 | 0.056 | 0.0254 | 0.0252 |
| $\widehat{RER_t^2}$ | 69.59 | 69.65 | 35.3 | 3.6 |
| $\widehat{I_t^2}$ | 0.2 | -0.07 | 0.04 | -2.15 |
| $\widehat{G_{C,t}^2}$ | 0.447 | 0.444 | 0.497 | 0.02 |
| $\widehat{G_{I,t}^2}$ | 0.0946 | 0.094 | 0.1 | 0.004 |
| $\widehat{K_{t-1}^{no2}}$ | $-9.5 \cdot 10^{-7}$ | $2.3 \cdot 10^{-5}$ | $1.08 \cdot 10^{-7}$ | 0.0001 |
| $\widehat{K_{G,t-1}^2}$ | -0.000123 | -0.000122 | -0.0001 | $-7.5 \cdot 10^{-6}$ |
| $\widehat{nfa_{t-1}^{*2}}$ | -0.26007 | -0.26001 | -0.13 | -0.001 |
| $\widehat{RER_{t-1}^2}$ | 18.72 | 18.74 | 9.5 | 0.97 |

# 7 Conclusion

This paper develops the DSGE model for an emerging oil economy and derives its associated utility-based welfare to study the optimal monetary policy rule jointly with pro-/countercyclical fiscal policy. The model captures a set of structural specifics: two monetary instruments—interest rate and foreign exchange intervention, two fiscal instruments—public consumption and public investment, non-oil and oil sectors with the exogenous world oil price, SWF and foreign exchange reserves accumulation, and the foreign debt of private sector to finance investment via collateral constraint. The constructed framework combines the New Keynesian model of a small open economy with the two types of households, optimizing individuals and rule-of-thumb households, and integrates three equations of the rest of the world relaxing the assumption of Ricardian equivalence.

The utility-based quadratic welfare loss includes the variations in aggregate output, inflation, real exchange rate, private investment, public consumption, public investment, foreign exchange reserves, and non-oil private and public capital as the state variables. There are also some lagged and lead terms of these variables that are due to investment adjustment costs, collateral constraint, and the balance of payments equation. The public capital and foreign exchange reserves appear to be welfare improving, while the other components contribute to loss.

The novelty of this paper is threefold, as it reveals the following findings along the joint study of optimal monetary rule and fiscal cyclicality in a single oil exporting setting. First, the best policy combination is procyclical fiscal policy and exchange rate targeting at high output response of monetary rule that largely stabilizes the private investment without the welfare reducing fiscal policy dominance as in the case of its countercyclical stance. Second, the propagation channel of interest rate shock may work unconventional from the monetarist point of view, yet seems to be dependent on fiscal countercyclicality and generally supportive to the fiscal theory of price level. Third, the impulse response functions to the world oil price shock, as a sudden improvement of the terms of trade, show that the monetary policy parameters matter for the interest rate and output dynamics, but not CPI or PPT anchor as F&C suggest. The volatile terms of trade can be offset by an appropriate domestic policy combination in terms of insulating their ultimate effect on aggregate output in a small open economy.

# References


[1] Algozhina, A. (2012). Monetary and Fiscal Policy Interactions in an Emerging Open Economy: a Non-Ricardian DSGE Approach. *CERGE-EI Working Paper, 476*, 1-28.

[2] Allegret, J. P., & Benkhodja, M. T. (2011). External shocks and monetary policy in a small open oil exporting economy. *EconomiX Working Paper, 2011-39,* 1-43.

[3] Berg, A., Portillo, R., Yang, S., & Zanna, L-F. (2013). Public investment in resource-abundant developing countries. *IMF Economic Review, 61*(1), 92-129.

[4] Blanchard, O., Dell'Ariccia, G., & Mauro, P. (2010). Rethinking macroeconomic policy. *IMF Staff Position Note, SPN/10/03*, 1-18.

[5] Bodenstein, M., Erceg, C. J., & Guerrieri, L. (2011). Oil shocks and external adjustment. *Journal of International Economics, 83*(2), 168-184.

[6] Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics, 12*(3), 383-398.

[7] Cochrane, J. H. (2011). Understanding policy in the great recession: some unpleasant fiscal arithmetic. *European Economic Review, 55*(1), 2-30.

[8] Dagher, J., Gottschalk, J., & Portillo, R. (2010). Oil windfalls in Ghana: a DSGE approach. *IMF Working Paper, WP/10/116,* 1-36.

[9] Davig, T., & Leeper, E. M. (2011). Monetary-fiscal policy interactions and fiscal stimulus. *European Economic Review, 55*(2), 211-227.

[10] De Paoli, B. (2009). Monetary policy and welfare in a small open economy. *Journal of International Economics, 77*(1), 11-22.

[11] Dib, A. (2008). Welfare effects of commodity price and exchange rate volatilities in a multi-sector small open economy model. *Bank of Canada Working Paper, 2008-8*, 1-53.

[12] Edge, R. M. (2003). A utility-based welfare criterion in a model with endogenous capital accumulation. *Finance and Economics Discussion Series of the Federal Reserve Board, 2003-66*, 1-39.

[13] Faia, E., & Iliopulos, E. (2011). Financial openness, financial frictions and optimal monetary policy. *Journal of Economic Dynamics and Control*, *35*(11), 1976-1996.

[14] Frankel, J. A., & Catao, L. A. V. (2011). A comparison of product price targeting and other monetary anchor options for commodity exporters in Latin America. *Economia, 12*(1), 1-70.

[15] Gali, J. (2008). *Monetary policy, inflation, and the business cycle: an introduction to the New Keynesian framework.* USA: Princeton University Press, 1-203.

[16] Gali, J., Lopez-Salido, J. D., & Valles, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association, 5*(1), 227-270.

[17] Gartner, M. (1987). Intervention policy under floating exchange rates: an analysis of the Swiss case. *Economica, 54*(216), 439-453.

[18] Jakab, Z. M., & Vilagi, B. (2008). An estimated DSGE model of the Hungarian economy. *Magyar Nemzeti Bank Working Paper, 2008/9,* 3-81.

[19] Leeper, E. M. (1991). Equilibria under "active" and "passive" monetary and fiscal policies. *Journal of Monetary Economics, 27*(1), 129-147.

[20] Leeper, E. M. (2013). Fiscal limits and monetary policy. *NBER Working Paper, 18877*, 1-22.

[21] Nakov, A., & Pescatori, A. (2010). Oil and the Great Moderation. *Economic Journal, 120*(543), 131-156.

[22] Pieschacon, A. (2012). The value of fiscal discipline for oil-exporting countries. *Journal of Monetary Economics, 59*(3), 250-268.

[23] Sarno, L., & Taylor, M. P. (2001). Official intervention in the foreign exchange market: is it effective and, if so, how does it work? *Journal of Economic Literature, 39*(3), 839-868.

[24] Schmitt-Grohe, S., & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics, 61*(1), 163-185.

[25] Senbeta, S. R. (2011). How applicable are the New Keynesian DSGE models to a typical low-income economy? *University of Antwerp Research Paper, 2011-016*, 1-63.

[26] Traum, N., & Yang, S. (2011). When does government debt crowd out investment? *Society for Economic Dynamics 2011 Meeting Papers, 479*, 1-42.

[27] Walsh, C. E. (2010). *Monetary Theory and Policy.* USA: The MIT Press, 1-613.

[28] Woodford, M. (2003). *Interest and prices: foundations of a theory of monetary policy.* USA: Princeton University Press, 1-785.

# A Calibration

| Parameter | Definition |
|---|---|
| $\beta = 0.978$ | discount factor |
| $\gamma = 0.68$ | home-bias in consumption |
| $\Omega = 0.54$ | the upper bound of leverage ratio |
| $\mu = 0.5$ | the fraction of rule-of-thumb households |
| $\alpha = 0.3$ | non-oil output elasticity to private capital |
| $\psi = 0.16$ | non-oil output elasticity to public capital |
| $\alpha^o = 0.7$ | oil output elasticity to private capital |
| $\phi = 1.45$ | wage elasticity to hours worked |
| $\sigma = 2$ | the inverse of intertemporal elasticity of substitution for $C_t$ |
| $\delta = 0.025$ | the depreciation rate of private capital (oil and non-oil) |
| $\delta^g = 0.02$ | the depreciation rate of public capital |
| $\theta = 0.9$ | the index of price stickiness |
| $\varepsilon = 6$ | the elasticity of substitution b/w differentiated intermediate goods |
| $\kappa = 20$ | investment adjustment costs parameter |
| $\phi_y = 0.125$ | output response in the Taylor rule |
| $\phi_\pi = 0$ | inflation response in the Taylor rule |
| $\phi_e = 2.4$ | exchange rate response in the Taylor rule |
| $\alpha_1 = -0.15$ | exchange rate response in the intervention rule |
| $\alpha_2 = -0.62$ | exchange rate change response in the intervention rule |
| $\tau^o = 0.27$ | oil royalty rate |
| $\iota^{\text{div}} = 0.1$ | the dividend share of oil profit accrued to the government |
| $\gamma_{GC} = 0.56$ | the response of public consumption to fiscal debt |
| $\gamma_{GI} = 0.54$ | the response of public investment to fiscal debt |
| $\vartheta_{GI} = 0.7$ | the response of public investment to output |
| $\vartheta_{GC} = 0.7$ | the response of public consumption to output |
| $\gamma_{OF}^{GC} = 1.66$ | the response of public consumption to oil revenues |
| $\gamma_{OF}^{GI} = 1.66$ | the response of public investment to oil revenues |
| $\gamma_b^o = -0.35$ | the response of oil revenues shock to fiscal debt |
| $\varphi_b = 0.4$ | the response of lump-sum taxes to fiscal debt |
| $\varphi_{OF} = 0.66$ | the response of lump-sum taxes to oil revenues |
| $\varphi_C = 1$ | the response of lump-sum taxes to public consumption |
| $\varphi_I = 0.2$ | the response of lump-sum taxes to public investment |
| $\rho_{GC} = \rho_{GI} = 0.3$ | persistence in public consumption and public investment |
| $\rho_{FDI} = 0.65$ | persistence in FDI process |
| $\rho_{OF} = 0.9$ | persistence in SWF process |
| $\rho_o = 0.98$ | persistence in the world oil price process |
| $\rho_{nfa} = 0.98$ | persistence in the foreign exchange reserves of a central bank |
| $\rho = 0.95$ | persistence in the interest rate process |
| $\rho_{Y^{no}} = 0.8$ | persistence in the non-oil productivity process |

# B Steady state

The steady state values for the endogenous variables denoted by bars are presented in this Appendix. The first-order condition of optimizing household with respect to the government bonds (9) gives that $\overline{R} = \frac{1}{\beta}$, while with respect to the foreign debt (10) suggests $\overline{\xi} = \beta^* - \beta$ at steady state. Similarly, $\overline{R}^* = \frac{1}{\beta^*}$.

The first-order condition of oil producer with respect to capital equalizes the marginal factor product to its price:

$$\alpha^o(1-\tau^o)(\overline{K^o})^{\alpha^o-1} = \overline{R^{ok}} = \frac{1}{\beta} - (1-\delta),$$

from which steady state oil capital can be found.

$$\overline{K^o} = \left[\frac{1/\beta - (1-\delta)}{\alpha^o(1-\tau^o)}\right]^{\frac{1}{\alpha^o-1}}$$

Since oil capital is known, the oil output, FDI, and SWF are obtained from their respective equations (19), (21), and (23, 26, 27):

$$\overline{Y^o} = (\overline{K^o})^{\alpha^o}, \quad \overline{FDI^*} = \delta\overline{K^o}, \quad \overline{OF} = \frac{[\tau^o + \iota^{\text{div}}(1-\tau^o)](\overline{K^o})^{\alpha^o}}{1-\rho_{OF}}$$

The steady state real exchange rate, according to data, is set to 139.85. It can be found from the Taylor rule, but since there are two of them and monetary policy parameters are eventually optimized, it is rather fixed to data.

The foreign exchange intervention, according to its rule (33), is equal to:

$$\overline{int^*} = \overline{RER}^{\alpha_1}$$

The steady state net foreign assets of a central bank, according to their equation (34), result in:

$$\overline{nfa^*} = \frac{\overline{int^*}}{1-\rho_{nfa}}$$

The oil revenues are from the government budget constraint (24) as follows:

$$\overline{OR} = (\overline{R^*} - 1)\overline{OF}\overline{RER}$$

The fiscal debt can be represented in terms of public capital and output using the public investment equation (28) and the expression of $\overline{K_G} = \frac{G_I}{\delta^g}$ from the public capital accumulation equation (25):

$$\overline{b} = \left(\frac{\overline{Y}^{\vartheta_{GI}}[(\overline{R^*}-1)\overline{OF}\overline{RER}]^{\gamma_{OF}^{GI}}}{\delta^g\overline{K_G}}\right)^{\frac{1}{\gamma_{GI}}}$$

The steady state public consumption is as follows based on its rule (29), in which fiscal debt can be plugged in from the previous equation:

$$\overline{G_C} = \overline{Y}^{\vartheta_{GC}} \overline{b}^{-\gamma_{GC}} [(\overline{R^*} - 1)\overline{OFRER}]^{\gamma_{OF}^{GC}}$$

The lump-sum taxes can be found from the government budget constraint (24) in terms of output and public capital after all previous expressions for the fiscal debt, oil revenues, public consumption, and public investment:

$$\overline{T} = \overline{K_G}\delta^g + \overline{G_C} + (\overline{R} - 1)\overline{b} - \overline{OR}$$

The government budget constraint with two unknown variables, output and public capital, constitutes a first equation of the system. The second equation comes from the market clearing condition shown gradually below.

The first-order condition with respect to non-oil capital (8) yields the following rental cost of non-oil capital:

$$\overline{R^{kno}} = \frac{1}{\beta} - (1 - \delta) - \frac{\overline{\xi}\Omega}{\beta}$$

The price setting problem of non-oil firm suggests that the real marginal cost (16) equates with the inverse of price frictionless mark-up $\frac{\varepsilon}{\varepsilon-1}$ at steady state; thus, wage can be found as:

$$\overline{W} = (1 - \alpha) \left[ \frac{\overline{K_G}^{\psi} \alpha^{\alpha} (\varepsilon - 1)}{(\overline{R}^{kno})^{\alpha} \varepsilon} \right]^{\frac{1}{1-\alpha}}$$

The labor supply condition (11) gives $\overline{N} = \overline{W}^{\frac{1}{\phi-1}}$.

According to Gali (2008), the CPI index equation (3) can be alternatively written as $P_t = P_{h,t}^{\gamma} P_{f,t}^{1-\gamma}$ if $\eta = 1$. Assuming that the law of one price holds, this can be expressed in terms of the real exchange rate $1 = (P_t^h)^{\gamma} RER_t^{1-\gamma}$, where $P_t^h$ is the real domestic prices. Therefore, the steady state domestic prices are as follows:

$$\overline{P^h} = \left( \frac{1}{\overline{RER}^{1-\gamma}} \right)^{1/\gamma}$$

As aggregate output is a sum of non-oil and oil output $\overline{Y} = \overline{P^h}\overline{Y_{no}} + \overline{RERY_o} = \overline{P^h}\overline{N}^{1-\alpha}\overline{K_G}^{\psi}\overline{K_{no}}^{\alpha} + \overline{RERY_o}$, the non-oil capital is obtained in terms of public capital and output:

$$\overline{K_{no}} = \left( \frac{\overline{Y} - \overline{RERY_o}}{\overline{P^h}\overline{N}^{1-\alpha}\overline{K_G}^{\psi}} \right)^{\frac{1}{\alpha}}$$

The law of motion for capital (4) relates investment with non-oil capital: $\overline{I} = \delta\overline{K_{no}}$.

The collateral constraint (5) allows finding the foreign debt:

$$\overline{b^*} = \frac{\Omega\overline{K_{no}}}{\overline{R}^*}$$

The steady state taxes of rule-of-thumb households are equal to $\overline{T^N} = \frac{\overline{T}-(1-\mu)\overline{T^S}}{\mu}$ given that $T_t = \mu T_t^N + (1-\mu)T_t^S$, while the taxes of optimizers can be derived from their budget constraint (2) given that both types of households have equal consumption at steady state:

$$\overline{T^S} = \mu\left[(\overline{R^{kno}} - \delta)\overline{K^{no}} + \overline{b}(\overline{R} - 1) + \overline{b^*}(1 - \overline{R^*}) + (\overline{P^h} - \frac{\varepsilon - 1}{\varepsilon})\overline{Y^{no}}\right] + \overline{T}$$

Therefore, the taxes of rule-of-thumb households are as follows:

$$\overline{T^N} = \overline{T} - (1-\mu)\left[(\overline{R^{kno}} - \delta)\overline{K^{no}} + \overline{b}(\overline{R} - 1) + \overline{b^*}(1 - \overline{R^*}) + (\overline{P^h} - \frac{\varepsilon - 1}{\varepsilon})\overline{Y^{no}}\right]$$

The budget constraint of rule-of-thumb households (13) provides their consumption $\overline{C^N} = \overline{WN} - \overline{T^N}$, which is assumed to be equal to the consumption of optimizers, thus to aggregate consumption, given that it sums up the consumption of two types households: $\overline{C} = \mu\overline{C^N} + (1-\mu)\overline{C^S}$.

The balance of payments equation provides the net exports:

$$\begin{aligned}\overline{NX} \quad &= \quad \frac{\Omega\overline{K_{no}}}{\overline{R}^*}\left(\overline{R}^* - 1\right) - \overline{RERFDI^*} - (\overline{R^*} - 1)\overline{OFRER} + \\ &\quad +(1 - \iota^{\text{div}})\overline{RER}(1 - \tau^o)\overline{Y^o}\end{aligned}$$

The market clearing condition (35) is utilized as a second equation in the system to find output and public capital:

$$\overline{Y} = \overline{C} + \delta\overline{K_{no}} + \overline{RERFDI^*} + \overline{K_G}\delta^g + \overline{G_C} + \overline{NX}$$

Once these two variables are known, all other steady state variables are extracted from our earlier expressions.

# C The Phillips curve

The Phillips curve for CPI inflation in a small open economy has been derived according to Gali (2008).

The log-linearized optimal price setting condition (18) delivers a typical equation for domestic inflation $\pi_t^h$:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t,$$

where $\widehat{mc_t}$ is the log deviation of the economy's average real marginal cost from its steady state and $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$.

The CPI inflation includes the domestic inflation $\pi_t^h$ and the terms of trade, which can be alternatively represented by the real exchange rate $RER_t$:

$$\pi_t = \pi_t^h + \frac{1-\gamma}{\gamma} \vartriangle \widehat{RER_t}$$

The Phillips curve then is as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t + \frac{1-\gamma}{\gamma} \vartriangle \widehat{RER}_t - \beta\frac{1-\gamma}{\gamma} E_t \vartriangle \widehat{RER}_{t+1},$$

where $\widehat{mc}_t = \widehat{W}_t - (\widehat{Y_t^{no}} - \widehat{N}_t) + \frac{1-\gamma}{\gamma}\widehat{RER}_t$. Wages can be substituted with the log-linearized labor supply condition (11), so that the Phillips curve used in the model is this:

$$\begin{aligned}\pi_t \;&=\; \beta E_t \pi_{t+1} + \frac{1-\gamma}{\gamma}\left(\lambda\frac{1-\alpha}{1-\alpha+\alpha\varepsilon} + \beta + 1\right)\widehat{RER}_t - \frac{1-\gamma}{\gamma}\widehat{RER}_{t-1} \\ &\quad -\beta\frac{1-\gamma}{\gamma}E_t\widehat{RER}_{t+1} + \lambda\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\phi\widehat{N}_t - \widehat{Y_t^{no}}\right)\end{aligned} \quad (39)$$

# D Log-linearized equations

The final 16 log-linearized equations are listed below.

The balance of payments equation results in the following:

$$\begin{aligned}\widehat{NX}_t \;=\;& \frac{\overline{R^*}b_y^*}{nx_y}\widehat{R}_{t-1}^* - \frac{of_y}{nx_y}\widehat{R}_t^* + [\frac{\overline{R^*}b_y^*}{nx_y} + \frac{(1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y} - \frac{(\overline{R}^*-1)of_y}{nx_y} \\ &-\frac{fdi_y}{nx_y}]\widehat{RER}_t - \frac{b_y^*}{nx_y}\widehat{b}_t^* + \frac{\overline{R^*}b_y^*}{nx_y}\widehat{b}_{t-1}^* - \frac{\overline{R^*}b_y^*}{nx_y}\pi_t^* - \frac{\overline{R^*}b_y^*}{nx_y}\widehat{RER}_{t-1} + \\ &\frac{(1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y}\widehat{Y}_t^o - \frac{(\overline{R}^*-1)of_y}{nx_y}\widehat{OF}_{t-1} + \frac{nfa_y^*}{nx_y}(\widehat{NFA_t^*} - \widehat{NFA_{t-1}^*}) \\ &+\frac{(1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y}\widehat{P_t^{o*}} - \frac{fdi_y}{nx_y}\widehat{FDI_t^*}\end{aligned} \quad (40)$$

The net foreign assets of a central bank (34) in log-linearized form are below:

$$\widehat{NFA_t^*} = \rho_{nfa}\widehat{NFA_{t-1}^*} + \frac{int_y^*}{nfa_y^*}\widehat{Int_t^*} \quad (41)$$

The collateral constraint (5) combined with the first-order condition with respect to investment (7) yields:

$$\widehat{b}_t^* = E_t\pi_{t+1}^* - \widehat{R}_t^* + \widehat{K_t^{no}} + \widehat{RER}_t - E_t\widehat{RER}_{t+1} + \kappa(1+\beta)E_t\widehat{I_{t+1}} - \kappa\beta E_t\widehat{I_{t+2}} - \kappa\widehat{I}_t \quad (42)$$

The law of motion for non-oil capital (4) is as follows:

$$\widehat{K_t^{no}} = (1-\delta)\widehat{K_{t-1}^{no}} + \delta\widehat{I}_t \quad (43)$$

Similarly, the public capital accumulation (25) in its log-linearized form is this:

$$\widehat{K_t^G} = (1-\delta^g)\widehat{K_{t-1}^G} + \delta^g\widehat{G_t^I} \quad (44)$$

The oil capital is accumulated by exogenous FDI according to its equation (20):

$$\widehat{K_t^o} = (1-\delta)\widehat{K_{t-1}^o} + \delta\widehat{FDI_t^*}, \qquad \widehat{FDI_t^*} = \rho_{FDI}\widehat{FDI_{t-1}^*} + \varepsilon_t^{FDI} \tag{45}$$

The market clearing condition (35) is represented in terms of investment:

$$\widehat{I}_t = \frac{1}{1-\widetilde{\gamma}}\left[\widehat{Y}_t - c_y\widehat{C}_t - g_y^C\widehat{G_t^C} - g_y^I\widehat{G_t^I} - nx_y\widehat{NX}_t - fdi_y(\widehat{RER_t} + \widehat{FDI_t^*})\right] \tag{46}$$

where $(1-\widetilde{\gamma}) = 1 - c_y - fdi_y - g_y^C - g_y^I - nx_y$.

The UIP condition (12) after some tedious algebra corresponds to:

$$E_t\widehat{RER}_{t+1} = \widehat{R}_t + \widehat{RER}_t - \frac{\beta^*}{\beta}\widehat{R}_t^* + \pi_{t+1}^* - \pi_{t+1} - \left(\frac{\beta^*}{\beta} - 1\right)\widehat{\xi}_t, \tag{47}$$

where $\widehat{\xi}_t$ is an UIP shock.

The non-oil and oil production functions (15 and 19) give respectively:

$$\widehat{Y_t^{no}} = \alpha\widehat{K_{t-1}^{no}} + (1-\alpha)\widehat{N}_t + \psi\widehat{K}_{G,t-1} + \widehat{u}_t^{no}, \qquad \widehat{u}_t^{no} = \rho_{Y^{no}}\widehat{u}_{t-1}^{no} + \epsilon_t^{no} \tag{48}$$

$$\widehat{Y_t^o} = \alpha\widehat{K_{t-1}^o} \tag{49}$$

The aggregate output is as follows:

$$\widehat{Y}_t = (1-s_o)\widehat{Y_t^{no}} + (1-s_o)\widehat{P_t^h} + s_o\widehat{Y_t^o} + s_o\widehat{RER}_t + s_o\widehat{P_t^{o*}}, \qquad \widehat{P_t^{o*}} = \rho_o\widehat{P_{t-1}^{o*}} + \epsilon_t^o \tag{50}$$

The combination of first-order condition with respect to non-oil capital (8) and investment (7) given that $\widehat{R_t^{kno}} = \phi\widehat{N}_t - \widehat{K_{t-1}^{no}}$ delivers the following:

$$\begin{aligned}
\kappa(1+\beta)\widehat{I}_t &= \left(\beta(1-\delta) + \Omega\overline{\xi}\right)\left[\kappa(1+\beta)E_t\widehat{I_{t+1}} - \kappa\beta E_t\widehat{I_{t+2}} - \kappa\widehat{I}_t\right] + \kappa\beta E_t\widehat{I_{t+1}} \qquad (51)\\
&\quad + (1-\beta(1-\delta) - \Omega\overline{\xi})E_t\left[\phi\widehat{N_{t+1}} - \widehat{K_t^{no}}\right] - \frac{\overline{C} - \overline{N}^{\phi}/\phi}{\sigma\overline{C}}\left(\widehat{R}_t - E_t\pi_{t+1}\right)\\
&\quad + \kappa\widehat{I_{t-1}} + \Omega\overline{\xi}(E_t\pi_{t+1}^* + \widehat{RER}_t - E_t\widehat{RER}_{t+1} + \widehat{\xi}_t)
\end{aligned}$$

The budget constraint of optimizer (2) in a log-linearized form:

$$\begin{aligned}
\widehat{C}_t &= \left(\frac{\phi\overline{N}^{\phi} + (1-\mu)\overline{K^{no}}\overline{R^{kno}}\phi}{\overline{C}} - \frac{1-\mu}{c_y}(1-s_o)\frac{\varepsilon-1}{\varepsilon}\phi\right)\widehat{N}_t + \frac{(1-\mu)}{c_y}b_y^*\overline{R^*}\pi_t^* \qquad (52)\\
&\quad + \frac{(1-\mu)\overline{b}\overline{R}}{\overline{C}}(\widehat{R}_{t-1} + \widehat{b_{t-1}}) + \frac{(1-\mu)}{c_y}b_y^*\widehat{b}_t^* - \frac{(1-\mu)}{\overline{C}}\left[\delta\overline{K^{no}}\widehat{I}_t + \overline{b}\widehat{b}_t\right] +\\
&\quad + \frac{(1-\mu)}{c_y}\overline{P^h}(1-s_o)(\widehat{Y_t^{no}} + \widehat{P_t^h}) - \frac{(1-\mu)b_y^*\overline{R^*}}{c_y}\left(\widehat{R_{t-1}^*} + \widehat{b_{t-1}^*} - \widehat{RER}_{t-1}\right) -\\
&\quad \left[\frac{(1-\mu)b_y^*\overline{R^*}}{c_y} + \frac{(1-\mu)}{c_y}(1-s_o)\frac{(\varepsilon-1)(1-\gamma)}{\varepsilon\gamma}\right]\widehat{RER}_t - \frac{\overline{T}}{\overline{C}}\widehat{T}_t - \frac{(1-\mu)}{\overline{C}}\overline{b}\overline{R}\pi_t
\end{aligned}$$

The combination of oil taxes equation (26), SWF accumulation (27), and the profit of oil producer (23) corresponds to:

$$\widehat{OF}_t = \rho_{OF}\widehat{OF}_{t-1} + \frac{[\tau^o + \iota^{\text{div}}(1-\tau^o)]\overline{Y^o}}{\overline{OF}}\widehat{Y}_t^o + \frac{[\tau^o + \iota^{\text{div}}(1-\tau^o)]\overline{Y^o}}{\overline{OF}}\widehat{P_t^{o*}} \tag{53}$$

The government budget constraint (24) in terms of fiscal debt results in:

$$\widehat{b}_t = \overline{bR}\Psi(\widehat{b}_{t-1} + \widehat{R}_{t-1} - \pi_t) + \overline{G^I}\Psi\widehat{G_t^I} + \overline{G^C}\Psi\widehat{G_t^C} - \overline{T}\Psi\widehat{T}_t - \overline{OR}\Psi\widehat{OR}_t, \tag{54}$$

where $\Psi = \frac{1}{\overline{bR}+\overline{G^I}+\overline{G^C}-\overline{T}-\overline{OR}}$ and oil revenues with their shock are as follows $\widehat{OR}_t = (\widehat{R_t^* - 1}) + \widehat{OF}_{t-1} + \widehat{RER}_t + \epsilon_t^{\Xi}$.

The aggregate consumption equation is derived according to Gali, Lopez-Salido, and Valles (2007) by combining the Euler equation (9), budget constraint of the rule-of-thumb households (13), and the relationship $C_t = \mu C_t^N + (1-\mu)C_t^S$:

$$\widehat{C}_t = E_t\widehat{C}_{t+1} + \Theta_n(\widehat{N}_t - E_t\widehat{N}_{t+1}) - \Theta_i(\widehat{R}_t - E_t\pi_{t+1}) + \mu\overline{TC}^{-1}(\widehat{T_{t+1}} - \widehat{T}_t), \tag{55}$$

where $\Theta_n = \left[\mu\overline{N}^{\phi}\phi + (1-\mu)\overline{N}^{\phi}\right]\overline{C}^{-1}$ and $\Theta_i = (\sigma\overline{C})^{-1}(1-\mu)(\overline{C} - \phi^{-1}\overline{N}^{\phi})$.

# E Welfare

$$W = E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{U_t - U}{U_C C}\right)$$

$$\frac{U_t - U}{U_C C} = \widehat{C}_t + \frac{1}{2}\widehat{C_t^2} - \frac{1}{2}\frac{\sigma\overline{C}}{\overline{C} - \phi^{-1}\overline{N}^{\phi}}\widehat{C_t^2} - \frac{\overline{N}^{\phi}}{\overline{C}}\left[\begin{array}{c}\widehat{N}_t + \frac{1}{2}\widehat{N_t^2} + \\ + \left(\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}} + \phi - 1\right)\frac{1}{2}\widehat{N_t^2}\end{array}\right] + \\ + \frac{\sigma\overline{N}^{\phi}}{\overline{C} - \phi^{-1}\overline{N}^{\phi}}\widehat{C}_t\widehat{N}_t$$

**Reduced-form quadratic welfare loss**

$$\frac{U_t - U}{U_C C} = \{\tfrac{1}{2c_y}\left(1 - \tfrac{1}{c_y} - \tfrac{\sigma\overline{C}}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})c_y}\right) + \tfrac{\sigma\overline{N}^{\phi}}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})(1-\alpha)(1-s_o)}\left(\tfrac{1}{c_y} - \tfrac{\overline{N}^{\phi}}{2\overline{C}(1-\alpha)(1-s_o)}\right) - \\ - \tfrac{\overline{N}^{\phi}}{2\overline{C}(1-\alpha)(1-s_o)}\left(1 - \tfrac{1}{1-s_o} + \tfrac{\phi-2}{(1-\alpha)(1-s_o)}\right)\}\widehat{Y_t^2} + \{\tfrac{\sigma\overline{N}^{\phi}s_o fdi_y}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})(1-\alpha)(1-s_o)c_y} - \tfrac{\overline{R^*}b_y^*}{c_y} + \\ + \tfrac{(1-\iota^{\text{div}})(1-\tau^o)s_o - b_y^* - fdi_y - (\overline{R}^* - 1)of_y + int_y(\alpha_1+\alpha_2)}{c_y} \times \\ \left[\tfrac{\sigma\overline{N}^{\phi}s_o}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})(1-\alpha)(1-s_o)} - \tfrac{fdi_y}{c_y}\left(1 + \tfrac{\sigma\overline{C}fdi_y nx_y}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})c_y^2}\right)\right] + \tfrac{\overline{R}^* b_y^* - fdi_y - (\overline{R}^* - 1)of_y + (1-\iota^{\text{div}})(1-\tau^o)s_o}{2c_y} \\ \times\left[\begin{array}{c}\left(1 - \tfrac{nx_y}{c_y} - \tfrac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)\tfrac{\overline{R}^* b_y^* - fdi_y - (\overline{R}^* - 1)of_y + (1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y} - 1 - \\ -2\tfrac{b_y^*}{nx_y}(1+\overline{R}^*)\left(1 - \tfrac{nx_y}{c_y} - \tfrac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)\end{array}\right] +$$

$$
\begin{aligned}
&+\frac{b_y^{*2}}{2nx_yc_y}\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)(1+\overline{R}^{*})^2+\frac{int_y^2}{nfa_yc_y}(\alpha_1+\alpha_2)^2- \\
&-\frac{int_y}{2c_y}[\alpha_1+\alpha_2-(\alpha_1+\alpha_2)^2]+\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)\frac{int_y}{c_y}(\alpha_1+\alpha_2)\times \\
&[\frac{int_y}{2nx_y}(\alpha_1+\alpha_2)+\frac{(1-\iota^{\text{div}})(1-\tau^o)s_o-b_y^*-fdi_y-(\overline{R}^*-1)of_y}{c_y}]-\left[1+\frac{fdi_y}{c_y}\left(1+\frac{\sigma\overline{C}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}\right)\right]\times \\
&\frac{fdi_y}{2c_y}-\frac{\overline{N}^{\phi}}{\overline{C}}\begin{pmatrix}\frac{1}{2(1-\alpha)^2}\left[(\frac{s_o}{(1-s_o)})^2+\frac{1}{\lambda}(\frac{1-\gamma}{\gamma})^2\right]\left(\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}+\phi-2\right)- \\ -\frac{s_o}{2(1-\alpha)(1-s_o)^2}+\frac{\varepsilon(1-\alpha+\alpha\varepsilon)}{2(1-\alpha)^2\lambda}\left(\frac{1-\gamma}{\gamma}\right)^2\end{pmatrix}\}\widehat{RER_t^2}- \\
&-\frac{\overline{N}^{\phi}}{\overline{C}}\left[\frac{\varepsilon(1-\alpha+\alpha\varepsilon)}{2(1-\alpha)^2\lambda}+\frac{1}{2(1-\alpha)^2\lambda}\left(\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}+\phi-2\right)\right]\pi_t^2+\{\frac{b_y^*}{c_y}[\delta(\tfrac{1}{2}-\delta)- \\
&\kappa(\tfrac{1}{2}+\kappa)+\delta\kappa]-\frac{1-\widetilde{\gamma}}{2c_y}\left[1+\frac{1-\widetilde{\gamma}}{c_y}\left(1+\frac{\sigma\overline{C}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}\right)\right]-\frac{(1-\widetilde{\gamma})}{c_y^2}\left(1+\frac{\sigma\overline{C}(1-\widetilde{\gamma})nx_y}{(\overline{C}-\phi^{-1}\overline{N}^{\phi})c_y^2}\right)\times \\
&\times\left[\overline{R}^*b_y^*\kappa(1+\beta)-b_y^*(\delta-\kappa)\right]-\frac{\overline{R}^*b_y^*}{c_y}\kappa(1+\beta)[\tfrac{1}{2}-\kappa(1+\beta)]+\frac{b_y^{*2}}{2c_ynx_y}\times \\
&\times\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)[\overline{R}^*\kappa(1+\beta)-(\delta-\kappa)]^2\}\widehat{I_t^2}-\frac{g_y^C}{2c_y}\times \\
&\times\left[1+\frac{g_y^C}{c_y}\left(1+\frac{\sigma\overline{C}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}\right)\right]\widehat{G_{C,t}^2}-\frac{g_y^I}{2c_y}\left[1+\frac{g_y^I}{c_y}\left(1+\frac{\sigma\overline{C}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}\right)\right]\widehat{G_{I,t}^2}+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{b_y^*\overline{R}^*(1-\delta)}{c_y}\left[\frac{b_y^*\overline{R}^*(1-\delta)}{2nx_y}\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)-\delta+\frac{1}{2}\right]\widehat{K_{t-2}^{no2}}+ \\
&+\{\frac{b_y^*(1-\delta)}{c_y}[\frac{b_y^*(1-\delta)}{2nx_y}\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)-\frac{\alpha\sigma\overline{N}^{\phi}}{(1-\alpha)(\overline{C}-\phi^{-1}\overline{N}^{\phi})}+(\delta-\tfrac{1}{2})]- \\
&\frac{\overline{N}^{\phi}}{\overline{C}}\left[\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^2\left(\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}+\phi\right)-\frac{\alpha}{2(1-\alpha)}\right]\}\widehat{K_{t-1}^{no2}}-\frac{\overline{N}^{\phi}}{\overline{C}}[\frac{1}{2}\left(\frac{\psi}{1-\alpha}\right)^2\times \\
&\left(\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}+\phi-2\right)-\frac{\psi}{2(1-\alpha)}]\widehat{K_{G,t-1}^2}+\frac{b_y^*\overline{R}^*}{c_y}[\frac{b_y^*\overline{R}^*}{2nx_y}(\kappa-\delta)^2\begin{pmatrix}1-\frac{nx_y}{c_y}- \\ -\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\end{pmatrix}- \\
&(\delta(\tfrac{1}{2}-\delta)-\kappa(\tfrac{1}{2}+\kappa)+\delta\kappa)]\widehat{I_{t-1}^2}+\{\frac{b_y^*\overline{R}^*}{2c_y}+\frac{(int_y\alpha_2)^2}{2c_ynx_y}\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)+ \\
&\frac{int_y}{2c_y}(\alpha_2+\alpha_2^2)+\frac{(int_y\alpha_2)^2}{nfa_yc_y}-\frac{\overline{N}^{\phi}}{\overline{C}}\begin{pmatrix}\varepsilon(1-\alpha+\alpha\varepsilon)+ \\ +\frac{\sigma\overline{N}^{\phi}}{\overline{C}-\phi^{-1}\overline{N}^{\phi}}+\phi-2\end{pmatrix}\frac{1}{2(1-\alpha)^2\lambda}\left(\frac{1-\gamma}{\gamma}\right)^2\}\widehat{RER_{t-1}^2} \\
&+\{\frac{nfa_y}{2c_y}[1-\rho_{nfa}(1-\rho_{nfa})]+\frac{nfa_y^2}{c_ynx_y}[\frac{\rho_{nfa}^2+1}{2}-\rho_{nfa}]\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)\}\widehat{nfa_{t-1}^{*2}}
\end{aligned}
$$

$$
\begin{aligned}
&+\{\frac{\overline{R}^*b_y^*}{c_y}\beta\kappa(\tfrac{1}{2}+\beta\kappa)+\frac{b_y^*}{c_y}\kappa(1+\beta)[\tfrac{1}{2}-\kappa(1+\beta)]+\frac{b_y^{*2}}{2nx_yc_y}\left[\overline{R}^*\beta\kappa+\kappa(1+\beta)\right]^2 \\
&\times\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)\}\widehat{I_{t+1}^2}+\frac{b_y^*}{c_y}[\frac{b_y^*}{2nx_y}\left(1-\frac{nx_y}{c_y}-\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\right)-1]\widehat{RER_{t+1}^2} \\
&+\{\frac{[b_y^*\beta\kappa]^2}{2nx_yc_y}\begin{pmatrix}1-\frac{nx_y}{c_y}- \\ -\frac{nx_y\sigma\overline{C}}{c_y(\overline{C}-\phi^{-1}\overline{N}^{\phi})}\end{pmatrix}-\frac{b_y^*}{c_y}\beta\kappa(\tfrac{1}{2}+\beta\kappa)\}\widehat{I_{t+2}^2}+ \\
&+\text{linear terms}+\text{cross-products}+\text{t.i.p.}+O(\parallel\xi\parallel^3)
\end{aligned}
$$

# F Procyclical fiscal policy and exchange rate targeting monetary rule

The standard error of interest rate shocks is 0.05, the world oil price shock has its standard error of 0.1, and foreign output shock has 0.01, while the remaining shocks increase by 1 percent in all figures. However, tables in the main text assume that all shocks increase by 1 percent (standard error of 1).

Figure 1. Impulse responses to a public consumption shock

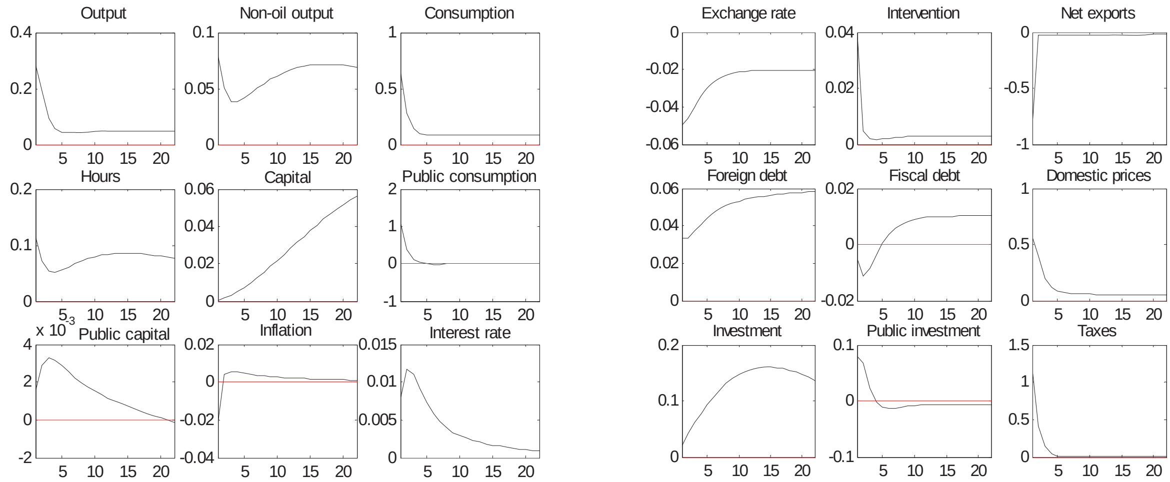



Figure 2. Impulse responses to a public investment shock

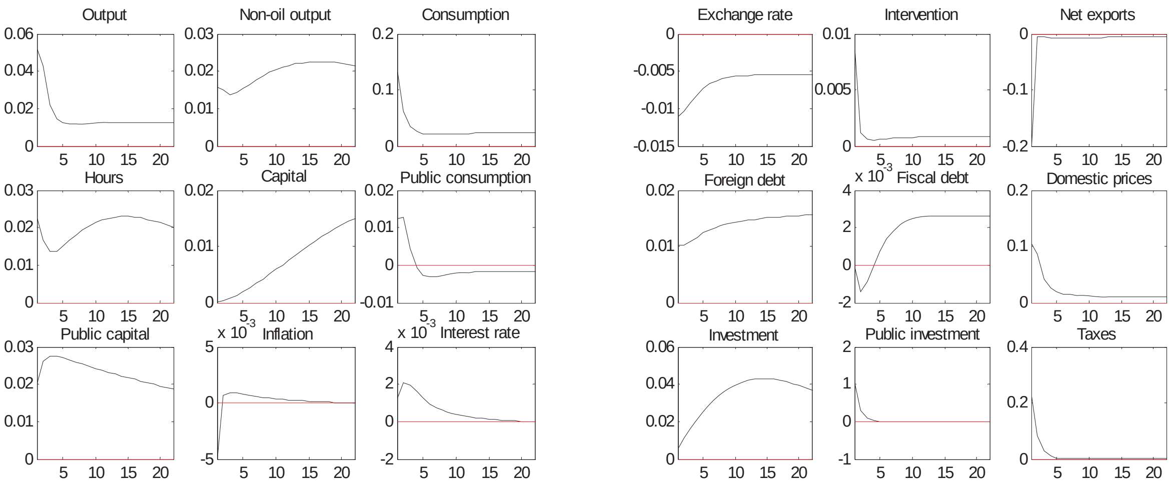


Figure 3. Impulse responses to an interest rate shock

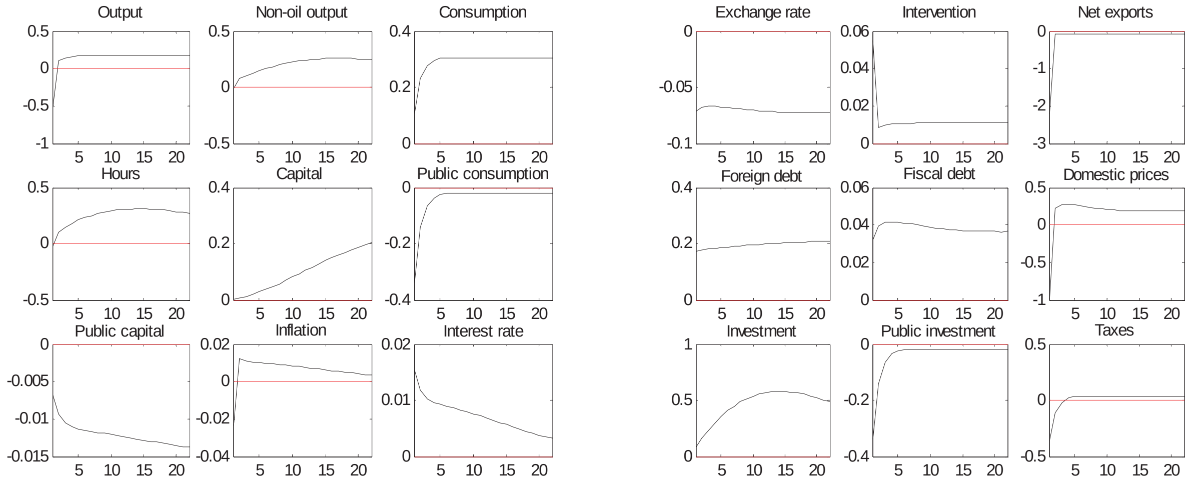



Figure 4. Impulse responses to a foreign exchange intervention shock

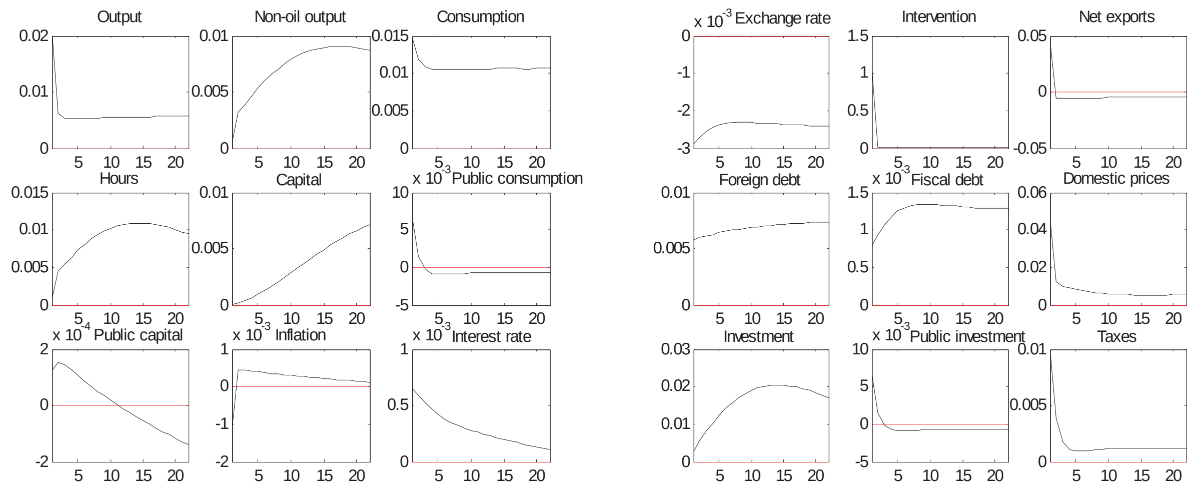


Figure 5. Impulse responses to the world oil price shock

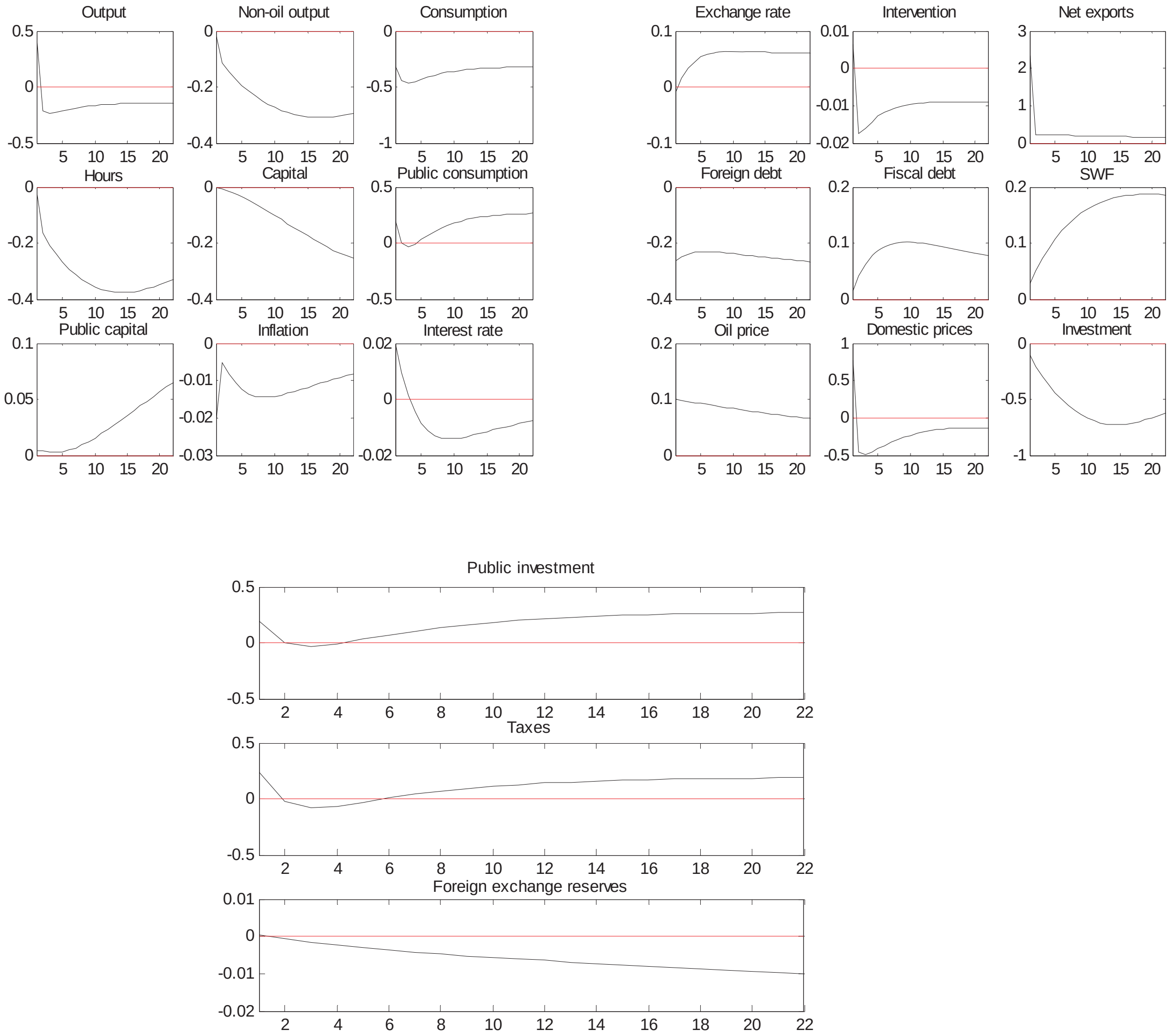
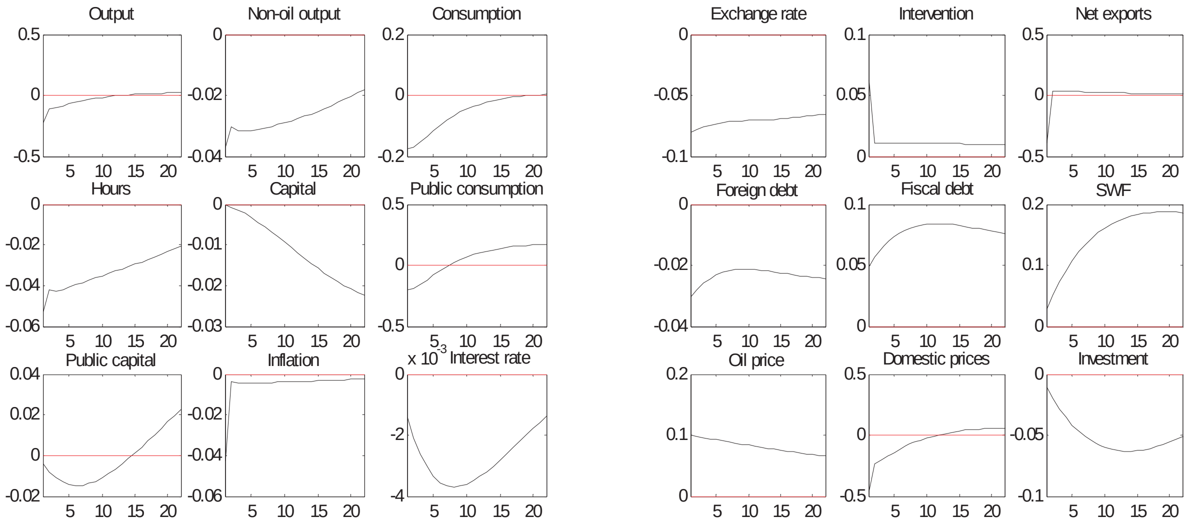
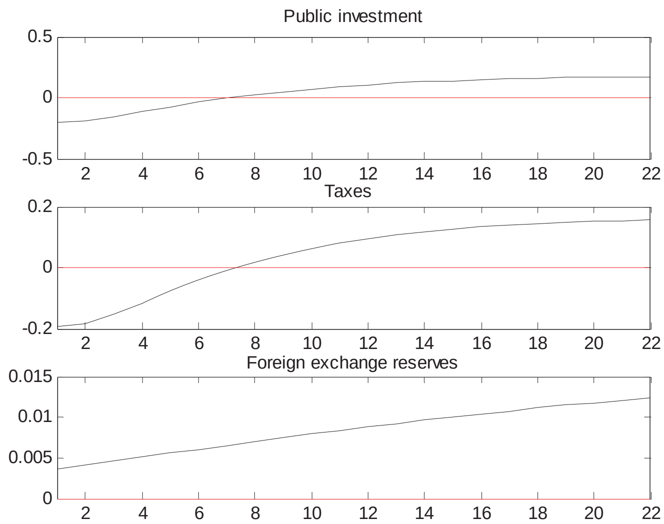



Figure 6. Impulse responses to the world oil price shock under procyclical fiscal

policy and low output response of monetary rule



# G Countercyclical fiscal policy and PPT monetary rule

Figure 1. Impulse responses to a public consumption shock

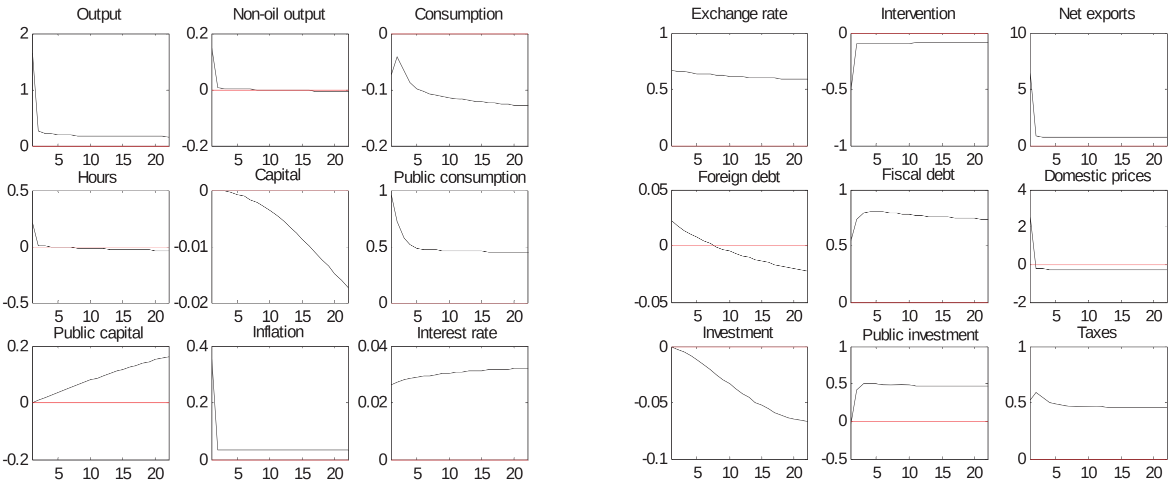



Figure 2. Impulse responses to a public investment shock

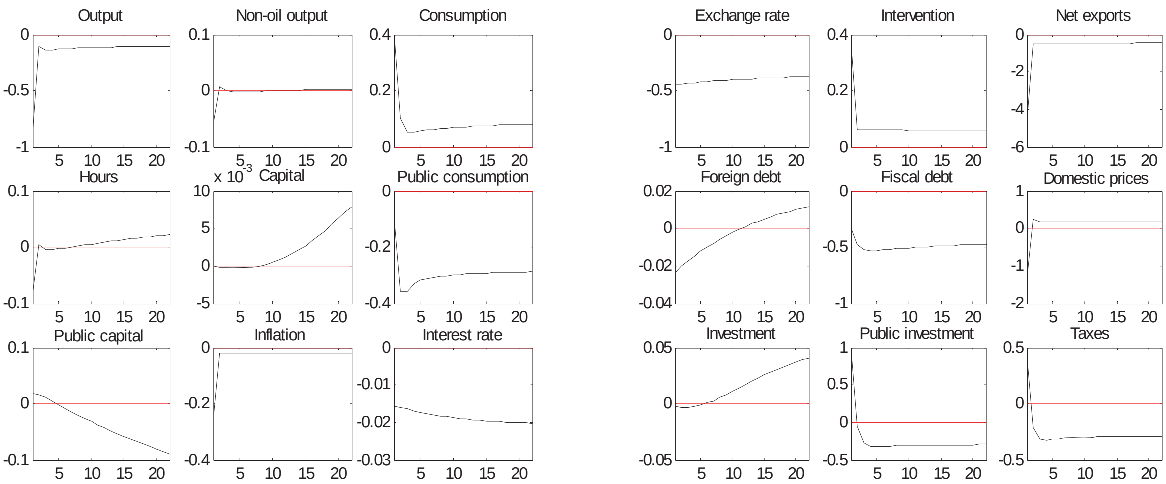


Figure 3. Impulse responses to an interest rate shock

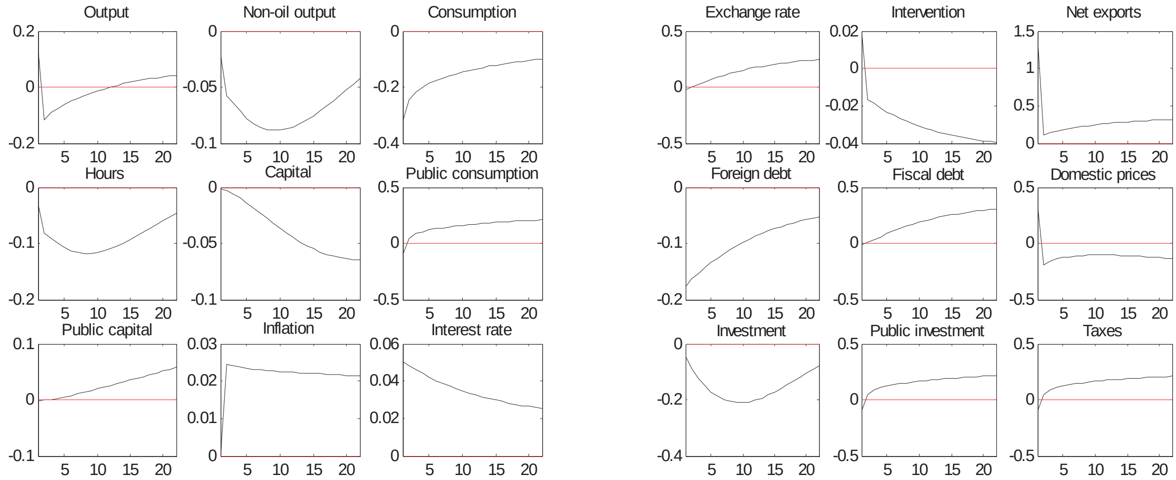



Figure 4. Impulse responses to a foreign exchange intervention shock

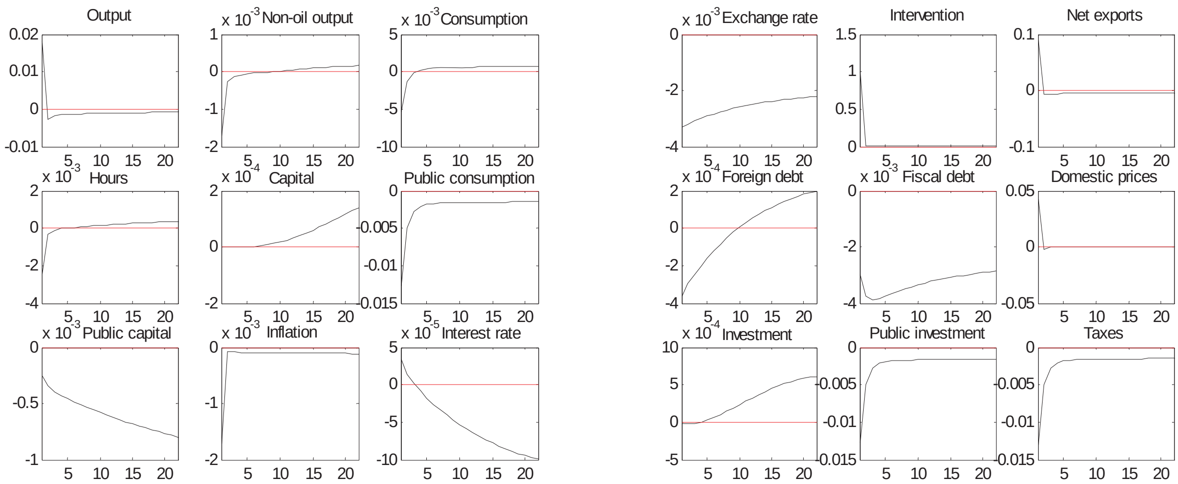


Figure 5. Impulse responses to the world oil price shock

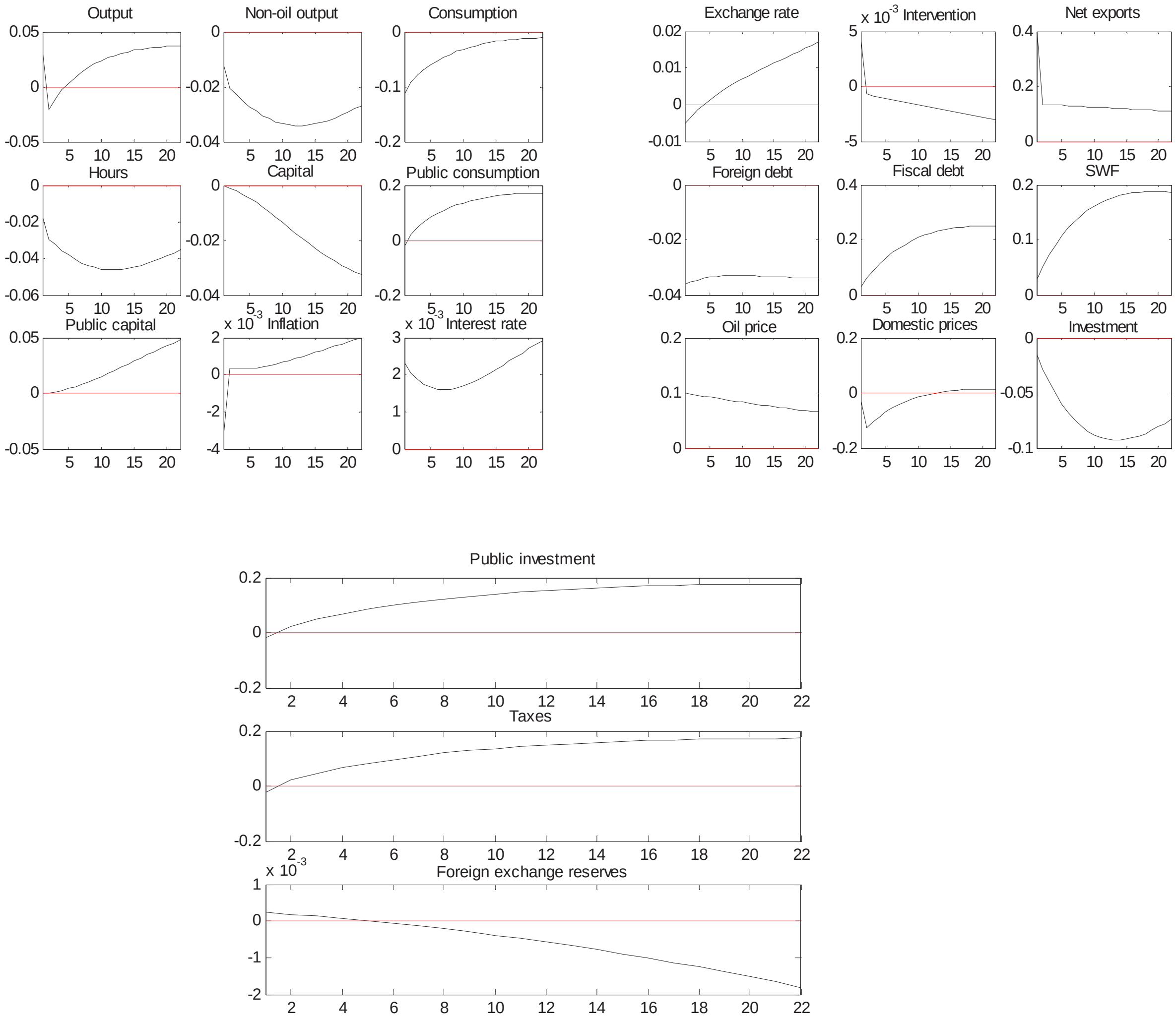
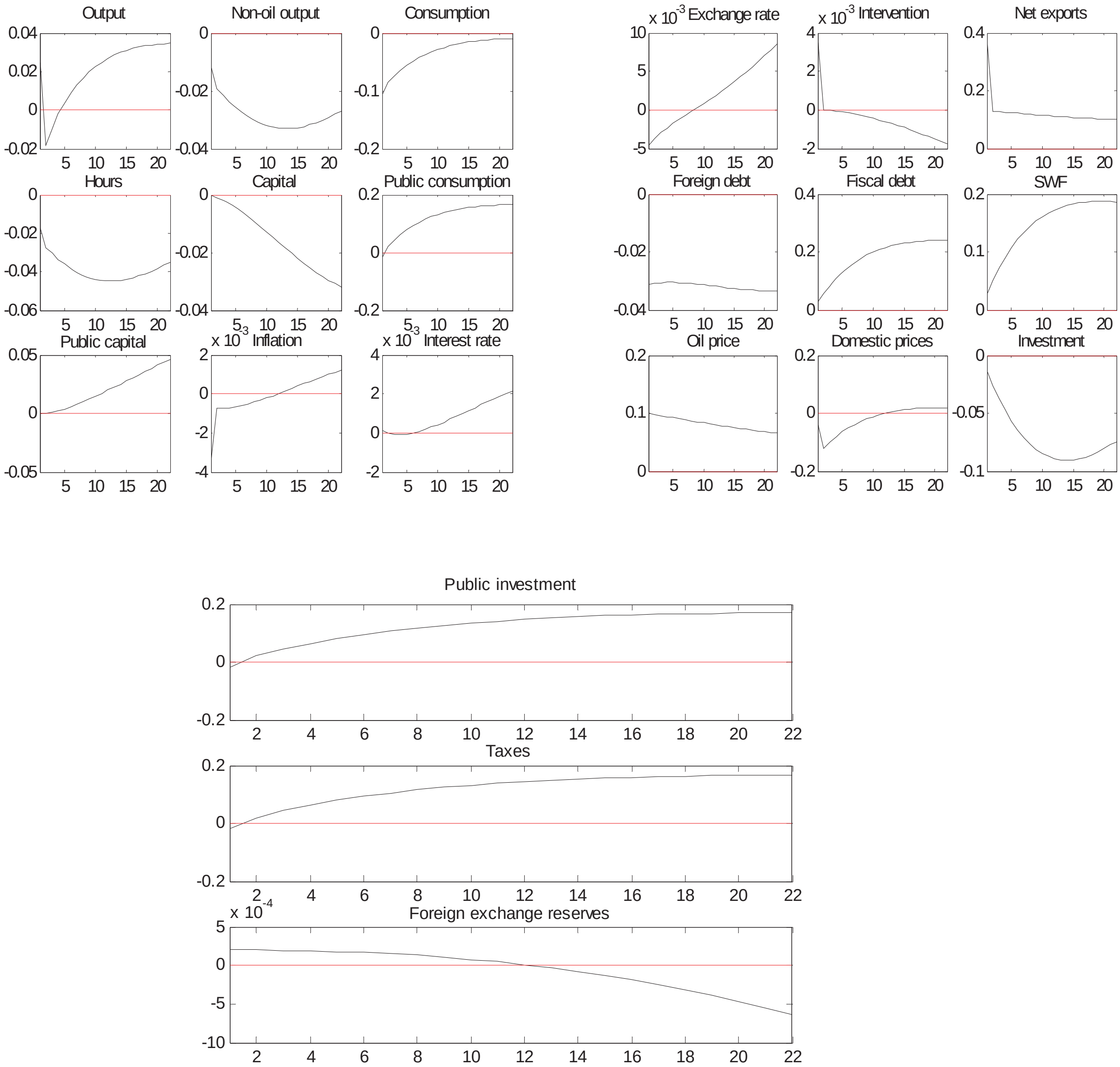
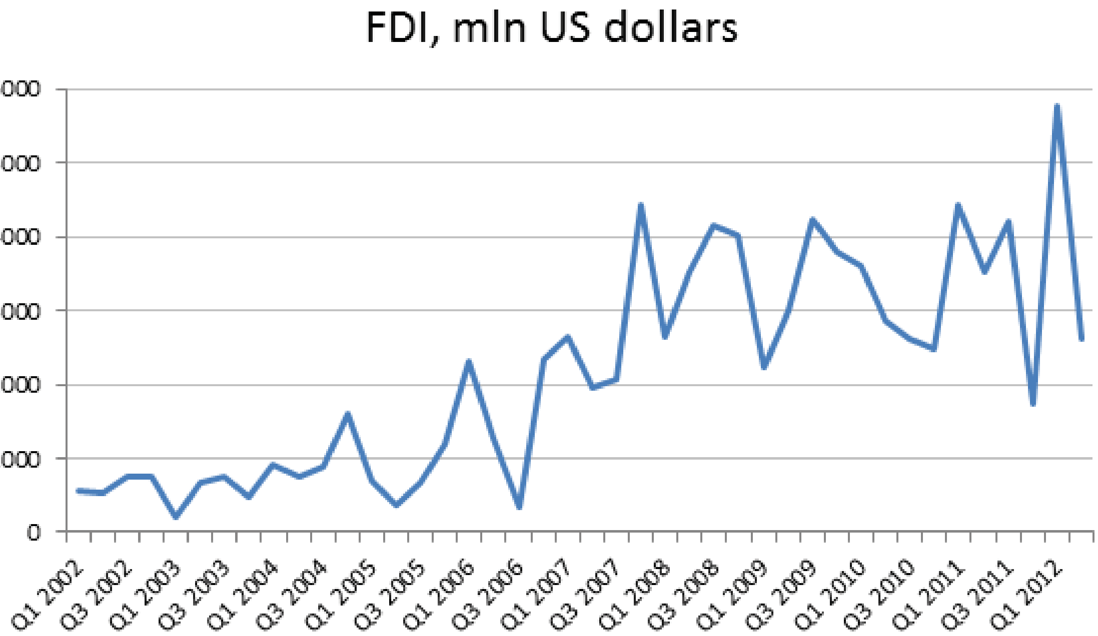


Figure 6. Impulse responses to the world oil price shock under countercyclical fiscal policy and CPI targeting monetary rule



## H FDI plot


FDI, mln US dollars
6000
5000
4000
3000
2000
1000
0
Q1 2002
Q3 2002
Q1 2003
Q3 2003
Q1 2004
Q3 2004
Q1 2005
Q3 2005
Q1 2006
Q3 2006
Q1 2007
Q3 2007
Q1 2008
Q3 2008
Q1 2009
Q3 2009
Q1 2010
Q3 2010
Q1 2011
Q3 2011
Q1 2012