Uncertainty in a model with credit frictions☆

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Abstract

This paper investigates the relationship between uncertainty and economic activity in a DSGE model with sticky prices and credit frictions. We analyse the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and we compare it to the effect of a mean preserving shock to the dispersion of entrepreneurs' idiosyncratic productivity (micro uncertainty). We find that micro uncertainty has a larger impact on economic activity. While macro uncertainty is transmitted through precautionary savings, micro uncertainty primarily acts through the cost of external debt and capital demand and, therefore, it is greatly magnified by the credit friction.

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1 Introduction

> "[...] the euro-area crisis has [...] create[d] a large black cloud of uncertainty hanging over not only the euro area but our economy too, and indeed the world economy as a whole. [...] The paralysing effect of uncertainty, with consumers and businesses holding back from commitments to spending, raises the question of whether any conventional macroeconomic measure could do much to stimulate private sector spending. And that has led some to question whether further monetary easing would prove effective."
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> Speech by Sir Mervyn King, at the Lord Mayor's Banquet for Bankers and Merchants of the City of London at the Mansion House, 14 June 2012.

Is uncertainty a key driver of business cycle fluctuations or are other forces more important? Economists, policy makers and financial commentators alike have long debated this question. As the above quote shows, uncertainty has been in the forefront of policy makers and economists' minds as a possible explanation for the tepid recovery in GDP in the UK and elsewhere following the collapse of Lehman Brothers.[1]

Economists have long understood the mechanisms by which uncertainty affects key economic variables. For example, Leland (1968), Kimball (1990) and show the theoretical conditions needed for (future) uncertainty to affect consumption, later quantified empirically by Carroll and Samwick (1995) and others. Hartman (1976), Abel (1983), Bernanke (1983), Caballero (1991), and Dixit and Pindyck (1994) show the theoretical conditions needed for uncertainty to affect investment. Recently Bloom (2009) has shown that uncertainty can have sizeable effects on firms' demand for factor inputs. But all of this research is undertaken within a partial equilibrium framework and as such ignores the general equilibrium effects that uncertainty can have on the economy.

Recent research has attempted to shed light on this issue using dynamic stochastic general equilibrium (DSGE) models. One important point on the notion of uncertainty is in order here. The vast majority of this recent literature has modelled uncertainty as "second-moment" shocks, i.e. changes in the dispersion of the primitive shocks driving the model economy.[2] In turn, this definition of uncertainty has been used with two different notions: (i) uncertainty about *aggregate* shocks, such as the time-varying variance of the economy-wide total factor productivity; and (ii) uncertainty about *idiosyncratic* shocks, such as the cross-sectional dispersion of firm-level productivity in models with heterogeneous firms. In this paper we consider both notions of uncertainty and we refer to the former as "macro uncertainty" and to latter as "micro

[1]Similar explanations have been given for the weakness in investment in the U.S., see for example the Federal Reserve's Monetary Policy Report of February 2012.

[2]A notable exception is the paper by Ilut and Schneider (2014).

uncertainty".[3]

Impaired credit markets have also been noted as a key contributing factor for the Great Recession as well as the subsequent slow recovery. A number of recent papers (for example Adrian and Song Shin (2010), Curdia and Woodford (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Gertler and Kiyotaki (2012), Iacoviello (2011), Khan and Thomas (2013) among others) have argued that financial frictions are important for understanding business cycle fluctuations.[4] In these models, financial frictions amplify the effects of shocks on the economy and, in some cases, shocks emanating from the financial sector are in themselves important drivers of the business cycle. Note that, in addition to that, credit market imperfections can create additional channels through which fluctuations in uncertainty can affect macroeconomic outcomes. For example, when firms choose their scale before observing (uninsurable) shocks and bear the risk of a costly default, high uncertainty can lead to a reduction of factor inputs and, therefore, of output (Arellano et al., 2012); or when the relation between lender and borrower is subject to asymmetric information (leading to agency and/or moral hazard problems) an increase in uncertainty will in general raise the cost of external finance (Gilchrist et al., 2012, Christiano et al., 2013).

The aim of this paper is to investigate the role of uncertainty for economic activity in the context of imperfect credit markets. How are uncertainty shocks transmitted to economic activity? Do credit market imperfections amplify the impact (of all types) of uncertainty? Does the nature of the transmission channel matter? We try to answer some of these questions using a general equilibrium model with sticky prices and credit frictions in the spirit of Bernanke et al. (1999). We analyse the effect of a mean preserving shock to the variance of aggregate total factor productivity (a "macro uncertainty shock") and we compare it with a mean preserving shock to the dispersion of entrepreneurs idiosyncratic productivity (a "micro uncertainty shock").

Our paper contributes to the literature along two dimensions. First, we jointly analyse the impact of the two types of uncertainty typically considered in the literature. Second, we parametrize uncertainty shocks using U.S. macro and micro data and show under which conditions uncertainty shocks are likely to significantly affect economic activity.

We find that micro uncertainty shocks appear to be more relevant than macro

[3]For uncertainty about aggregate shocks see Justiniano and Primiceri (2008), Fernandez-Villaverde and Rubio-Ramirez (2010), Fernandez-Villaverde et al. (2011), Basu and Bundick (2012), Born and Pfeifer (2013), Fernandez-Villaverde et al. (2011). For uncertainty about idiosyncratic shocks see Dorofeenko et al. (2008), Bloom (2009), Christiano et al. (2014). Bloom et al. (2012), Chugh (2013), Bachmann and Bayer (2013) consider both notions of uncertainty.

[4]The papers build on the workhorse models of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Holmstrom and Tirole (1997).

uncertainty shocks. According to our simulations, a 2 standard deviations shock to micro uncertainty depresses output by about 1.4 percent, relative to a fall of less than 0.1 percent in response to a 2 standard deviations shock to macro uncertainty. This result is driven by (i) the different nature of the uncertainty shock and its transmission mechanism and (ii) by the amplifying role of the credit friction.

Macro uncertainty is primarily transmitted through a precautionary savings channel. In the face of increased uncertainty around their future stream of income, households increase their savings and decrease their consumption. Differently, micro uncertainty shocks operate through the cost of external finance and entrepreneurial capital demand. Under asymmetric information between lender and borrower, the costly state verification problem introduces a wedge in banks' zero profit condition. In the face of increased uncertainty around entrepreneurial productivity, this wedge induces banks to raise their lending interest rates. As a result, entrepreneurs demand less capital and investment falls.

Both micro and macro uncertainty shocks propagate to the rest of the economy *via* sticky prices, which not only are crucial for generating co-movement between consumption and investment but also amplify the impact of both shocks on output. However, differently from macro uncertainty shocks, micro uncertainty shocks are greatly magnified by the credit friction. There are two reasons for why this is the case. First, the financial accelerator mechanism amplifies any shock that in general equilibrium affects entrepreneurial net worth: since the micro uncertainty shock has larger impact on net worth, it is not surprising that the micro uncertainty shock also displays more amplification. Second, and maybe more importantly, in the case of micro uncertainty the credit friction is not just an amplifying mechanism: it is also the economic rationale for why micro uncertainty shocks affect the real economy. By playing this dual role, a higher degree of credit market imperfections affects both the impact and the amplification of micro uncertainty shocks.

We do not interpret this evidence as suggesting that uncertainty affects the economy mainly through investment and only to a lesser extent through consumption. Indeed, if households were to borrow in imperfect credit markets, the same amplification mechanism observed for entrepreneurs would be at work. This result is consistent with the empirical evidence provided in a recent paper by Caldara et al. (2013), where uncertainty shocks are found to have a large impact on GDP if transmitted through the financial channel but a significantly smaller impact otherwise.

Finally note that both theoretical studies (Bachmann and Moscarini, 2011, Christiano and Ikeda, 2013) and empirical studies (Bachmann et al., 2013) assume that uncertainty endogenously reacts to developments in the economy and explore the implications of this assumption. We leave the investigation of the direction of the causality for future research.

Literature. While virtually all papers that investigate the relation between uncertainty and economic activity in models with financial frictions consider the micro notion of uncertainty, our paper considers both micro and macro uncertainty. A brief review of the literature is presented below.[5]

Dorofeenko et al. (2008) consider a mean preserving shock to the dispersion of firms' idiosyncratic productivity (a type of micro uncertainty shock) in the financial accelerator set up of Carlstrom and Fuerst (1997) and show that an increase in uncertainty leads to a fall in investment supply. Christiano et al. (2013) use a version of the Bernanke et al. (1999) model, BGG henceforth, to consider the importance of a similar micro uncertainty shock. The authors argue that this shock, which they label a "risk shock", is important for explaining the fluctuations of U.S. GDP and of other variables over the 1985-2010 period. Moreover, this shock is able to generate co-movement between output, consumption, investment and hours. This is an important prerequisite for any shock that seeks to explain business cycle fluctuations, since such co-movement is observed in the data.[6] In contrast, using the "output model" of Carlstrom and Fuerst (1998), Chugh (2013) argues that micro uncertainty explains a large share of financial variables' fluctuations but a very small share of real variables' fluctuations.[7] In a model with heterogeneous firms, costly entry and default, Arellano et al. (2012) show that uncertainty shocks tighten credit constraints and have large real effects on the economy. Gilchrist et al. (2012) use a model with heterogeneous firms, non-convex capital adjustment costs and financial distortions in the debt and equity markets to replicate the negative effect of uncertainty on the economy that they document in their empirical analysis. Finally, in a model based on Gerali et al. (2010), Bonciani and van Roye (2013) show that frictions in the banking sector amplify the effect of macro uncertainty on economic activity.

The remainder of the paper proceeds as follows: Section 2 provides some intuition for the results in the paper; Section 3 presents the model, including the sources of uncertainty; section 4 discusses the calibration of the model parameters and the solution method employed; Section 5 presents the key results and Section 6 concludes. An appendix describes the solution procedures and some technical details on how the

[5] An increasingly large literature —not reviewed here for sake of brevity— has analyzed the impact of macro uncertainty shocks in models without financial frictions. See Fernandez-Villaverde et al. (2011), Gourio (2012), Basu and Bundick (2012), and Born and Pfeifer (2013) among others.

[6] This point is eloquently made by Basu and Bundick (2012) in the context of macro uncertainty shocks. Since many of the models that consider aggregate uncertainty shocks tend to find that consumption, investment, hours and output do not move in the same direction such shocks cannot be candidates for explaining the business cycle.

[7] In an independently conducted yet unfinished paper, Balke et al. (2012) also study the impact of macro and micro uncertainty shocks in an environment with financial frictions. The main difference lies in the details of the model specification (a sticky price version of Carlstrom and Fuerst, 1997).

impulse responses were computed.

2 Intuition for our results

Four key ingredients drive our results. The first two are sticky prices and monetary policy which are key for generating co-movement between consumption, investment, hours and output in response to both macro and micro uncertainty. Christiano et al. (2013) and Basu and Bundick (2012) provide intuition: with sticky prices, output is demand determined implying that in response to micro uncertainty (which depresses investment demand *via* higher lending rates) or macro uncertainty (which depresses consumption *via* precautionary savings), prices do not fall sufficiently to keep output constant as is the case with flexible prices. The policy maker is also responsible for generating co-movement since real rates are not reduced sufficiently thereby acting to reduce consumption and investment.

The next two ingredients are credit frictions and GHH preferences as in Greenwood et al. (1988), which act to amplify the effects of the shocks and do not in themselves generate co-movement. On the one hand, credit frictions act to amplify the impact of both uncertainty shocks on investment since both shocks reduce the price of capital and therefore entrepreneurial net worth. On the other hand, GHH preferences prevent outward shifts in labour supply, following falls in consumption, that would act to mitigate the fall in output.

The co-movement problem in DSGE models with precautionary savings is similar to the co-movement problem which arises in these models with "news shocks" models.[8] Precautionary savings, like news shocks, are equivalent to exogenous changes in consumption demand. At the heart of the co-movement problem is the labour market.[9] Consider a closed economy where output, Y, can be used for consumption, C, and investment, I:

$$Y_t = C_t + I_t, \tag{1}$$

and where output is produced using predetermined capital, K, total factor productivity, A, and labour, N:

$$Y_t = F\left(A_t, K_{t-1}, N_t\right). \tag{2}$$

These two equations show that for consumption and investment to move in the same

[8]In these models, agents obtain information about future shocks —news shocks. Forward-looking agents will react to these shocks but, because these shocks only materialize in the future, production can only be modified by changes to endogenous factor inputs (labour, capital services, etc). See, for example, Jaimovich and Rebelo (2009).

[9]The arguments in this section follow those in Basu and Bundick (2012), Eusepi and Preston (2009) and Wang (2012). The co-movement problem was first noted by Barro and King (1984).

direction, labour must do so too. Moreover, labour must move by more than the changes in consumption and investment. Given these observations, understanding the labour market is crucial for understanding the co-movement problem. Equilibrium in the labour market is observed when labour demand:

$$W_t^R = F^N \left(A_t, K_{t-1}, N_t\right), \tag{3}$$

is equal to labour supply:

$$W_t^R = -\frac{U^N \left(C_t, N_t\right)}{U^C \left(C_t, N_t\right)}, \tag{4}$$

where W_t^R is the real wage, F^N the marginal product of labour, U^N the marginal (dis)utility of labour and U^C is the marginal utility of consumption. Equating the demand and supply of labour, assuming a Cobb-Douglas production function, separable preferences in consumption and labour, taking logs and ignoring constants yields:

$$a_t + \alpha k_{t-1} - (\alpha + \upsilon) n_t = \varrho c_t, \tag{5}$$

where α is the capital share; υ is the inverse of the Frisch elasticity of labour supply; ϱ is the coefficient of risk aversion; and lower case letters denote the logarithm of the variable. This equation implies that, absent an exogenous shock to TFP, consumption and hours are negatively correlated. This negative correlation is driven by the income effect of labour supply (υ) and decreasing marginal returns to labor (α) from labour demand, with the coefficient of risk aversion governing the strength of these effects *via* the substitution effect. The presence of sticky prices introduces a wedge between labour demand and the real wage (sticky wages introduce a similar wedge but this time between labour supply and the real wage), which in logs is expressed as:

$$mc_t + a_t + \alpha k_{t-1} - \alpha n_t = w_t^R \tag{6}$$

where mc is the nominal marginal cost faced by firms. The nominal marginal cost is inversely related to firms mark-up by virtue that optimizing firms set prices as a mark-up over costs. Denoting the price mark-up by μ_t^P and substituting the equation for labour supply we now have

$$a_t + \alpha k_{t-1} - (\alpha + \upsilon) n_t = \varrho c_t + \mu_t^P. \tag{7}$$

Just like TFP, movements in the markup are able to break the negative relationship between labour and consumption. Consider a fall in consumption brought about by an increase in precautionary savings. This leads to a reduction in firms' demand such that firms would like to lower their prices. However, due to sticky prices, firms do not decrease prices sufficiently to fully accommodate the fall in demand and markups increase. If the increase in the markup is larger than the fall in consumption, it is possible for the right hand side of (7) to be positive. As a result the labour input needs to fall. Although not shown algebraically here, monetary policy acts to amplify the effect of sticky prices. This is because with naive rules such as the Taylor rule real interest rates do not fall sufficiently to mitigate the fall in demand thereby depressing consumption and investment further. As a result markups are higher.

With GHH preferences there is no income effect in labour supply so consumption does not shift the labour supply schedule. In that case, labour market equilibrium is given by:

$$a_t + \alpha k_{t-1} - (\alpha + \upsilon)\, n_t = \mu_t^P. \tag{8}$$

As this equation shows, by themselves, GHH preferences do not solve the co-movement problem, but mitigate it. It is the presence of sticky prices that generates the co-movement. Of course, there are other mechanisms that can aid the co-movement problem such as adding sticky wages or introducing additional factors of production (e.g., capital utilization). Wang (2012) provides a convenient summary of these mechanisms.

3 Model

This section outlines the baseline DSGE model that we use in our analysis. It closely resembles the BGG variant formulated by Faia and Monacelli (2007), but it is modified in two dimensions. First, we consider the role of different preferences; second, and more importantly, we consider the effect of uncertainty shocks. The model comprises optimizing households; monopolistic firms that can set prices and produce final output; capital producers that transform output into unfinished capital goods; entrepreneurs that purchase this capital, rent it to firms and are subject to a credit friction; financial intermediaries that channel households' savings into loans for entrepreneurs; and a policy maker that sets interest rates. In what follows we consider the problems faced by each agent.

3.1 Households

There is a continuum of households, each indexed by $i \in (0, 1)$. They consume a composite final good, invest in safe bank deposits, supply labour, and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative household chooses the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and one-period nominal deposits $\{D_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{P_t, W_t, (1 + R_t^n)\}_{t=0}^{\infty}$

and the initial condition D_0 to maximize:

$$\max_{\{C_t, N_t, D_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=o}^{\infty} \beta^t U(C_t, N_t), \tag{9}$$

subject to the sequence of budget constraints:

$$P_t C_t + D_{t+1} \leq (1 + R_t^n) D_t + W_t N_t + \Pi_t, \tag{10}$$

where C_t is workers' consumption of the final good, W_t is the nominal wage, N_t is total labour hours, R_t^n is the nominal net interest rate paid on deposits, Π_t are the nominal profits that households receive from running production in the monopolistic sector.

The first order conditions of the above problem read as follows:

$$\begin{aligned} U_{c,t} &= \beta(1 + R_t^n)\mathbb{E}_t \left[U_{c,t+1} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= -\frac{U_{n,t}}{U_{c,t}}, \end{aligned} \tag{11}$$

together with $\lim_{j \longrightarrow \infty} D_{t+j}/(1 + R_t^n) = 0$ and (10) holding with equality.

3.2 Unfinished capital producers

A competitive sector of capital producers combines investment (expressed in the same composite as the final good, hence with price P_t) and existing (depreciated) capital stock to produce unfinished capital goods. This activity entails physical adjustment costs. The corresponding constant return to scale production function is $\phi(\frac{I_t}{K_t})K_t$ where $\phi(\cdot)$ is increasing and convex. We assume the following functional form:

$$\phi(\frac{I_t}{K_t})K_t = \left[\frac{I_t}{K_t} - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 \right] K_t, \tag{12}$$

so that capital accumulation obeys to:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \tag{13}$$

Defining Q_t as the re-sell price of the capital good, capital producers maximize profits:

$$\max_{I_t} Q_t \left[I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \right] - P_t I_t, \tag{14}$$

implying the following first order condition:

$$Q_t \left[1 - \phi_k \left(\frac{I_t}{K_t} - \delta \right) \right] = P_t. \tag{15}$$

3.3 Entrepreneurs

The activity of entrepreneurs is at the heart of the credit friction. These agents are risk neutral. At the end of period t, each entrepreneur j purchases unfinished capital from the capital producers at the price Q_t and transforms it into finished capital (that will be used for production in $t + 1$).

The transformation of unfinished capital into finished capital is performed with a technology that is subject to idiosyncratic productivity shocks (ω_{t+1}^j). The idiosyncratic shocks are assumed to be independently and identically distributed ($i.i.d.$) across entrepreneurs and time, and to follow a log normal distribution, namely $\omega \sim \log \mathcal{N}(1, \sigma_\omega^2)$, with cumulative distribution function denoted by $F(\omega)$. Note that, for the solution of the entrepreneurial problem, we take the variance of ω as a given parameter. However, as we shall see in section 3.8, allowing for time variation in σ_ω^2 in the solution of the model will constitute a major source of uncertainty in our economy (that we labelled micro uncertainty).[10]

To finance the purchase of unfinished capital entrepreneurs employ internal funds but also need to acquire an external loan from a financial intermediary (banks). The relationship with the lender is modelled assuming asymmetric information between entrepreneurs and banks and a costly state verification as in Townsend (1979) and Gale and Hellwig (1985). Specifically, the idiosyncratic shock to entrepreneurs is private information for the entrepreneur. To observe this, the lender must pay an auditing cost that is a fixed proportion $\mu \in [0, 1]$ of the realized gross return to capital held by the entrepreneur. The optimal loan contract will induce the entrepreneur to not misreport his earnings and will minimize the expected auditing costs incurred by the lender. Under these assumptions, the optimal contract is a standard debt contract with costly bankruptcy. If the entrepreneur does not default, the lender receives a fixed payment independent of the realization of the idiosyncratic shock; in contrast, if the entrepreneur defaults, the lender audits and seizes whatever is left. As we shall see below, for this reason the interest rate on entrepreneurial loans will be given by a spread over the risk free rate. The section below reports the derivation of the optimal contract.

[10] Note that other papers in the earlier literature have considered a similar definition of time-varying uncertainty (or "risk") as the one used here. See, for example Christiano et al. (2003), Dorofeenko et al. (2008), Christiano et al. (2010), and Christiano et al. (2013).

3.3.1 The optimal loan contract

There are two agents, entrepreneurs and banks. At the end of period t, an entrepreneur j holds nominal net worth NW_{t+1}^{j} and acquires the following amount of credit to finance capital purchases:

$$B_{t+1}^{j}=Q_tK_{t+1}^{j}-NW_{t+1}^{j}. \tag{16}$$

Before defining entrepreneurs' problem we first need to define the expected *nominal income from holding one unit of finished capital.* Assume that, at the end of period t, an entrepreneur buys one unit of capital at price Q_t. In period $t+1$ this unit of capital is available in the rental market and the entrepreneur gets income from renting that unit to firms (Z_{t+1}) and from re–selling the undepreciated capital to capital producers at price Q_{t+1}; note moreover that, in presence of adjustment costs, the nominal income has to be adjusted for the marginal utility of holding one additional unit of capital next period. Hence, the nominal income from holding one unit of finished capital can be written as:

$$\begin{aligned}\mathcal{Y}_{t+1}^{k} &= Q_t\left(1+R_{t+1}^{k}\right)= \\ &= Z_{t+1}+Q_{t+1}\left[(1-\delta)-\frac{\phi_k}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2+\phi_k\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}}\right]\end{aligned} \tag{17}$$

However, the idiosyncratic shock realizes before the beginning of period $t+1$. Entrepreneur j will repay his loans only if $\omega_{t+1}^{j}\mathcal{Y}_{t+1}^{k}K_{t+1}^{j}\geq B_{t+1}^{j}\left(1+R_{t+1}^{L}\right)$ where R_{t+1}^{L} is the lending rate paid on loans. Therefore, the above expression defines the cut–off value of the idiosyncratic shock that separates bankrupt and non-bankrupt entrepreneurs. An entrepreneur who experiences an idiosyncratic shock equal to:

$$\omega_{t+1}^{j}<\bar{\omega}_{t+1}^{j}=\frac{B_{t+1}^{j}\left(1+R_{t+1}^{L}\right)}{\mathcal{Y}_{t+1}^{k}K_{t+1}^{j}} \tag{18}$$

will default on his debt and the bank will seize all his remaining assets after paying the monitoring cost.

On the other hand, banks operate only if the following condition is satisfied:

$$\mathcal{Y}_{t+1}^{k}K_{t+1}^{j}\left(\Gamma(\bar{\omega}_{t+1}^{j})-\mu G(\bar{\omega}_{t+1}^{j})\right)\geq(1+R_t^{n})B_{t+1}^{j}. \tag{19}$$

where $G(\bar{\omega}_{t+1}^{j})=\int_0^{\bar{\omega}}\omega_{t+1}^{j}dF(\omega)$ and $\Gamma(\bar{\omega}_{t+1}^{j})=\left[1-\int_0^{\bar{\omega}}dF(\omega)\right]\bar{\omega}_{t+1}^{j}+G(\bar{\omega}_{t+1}^{j})$. Note here that, as in BGG, $\Gamma(\bar{\omega}_{t+1}^{j})$ is the share of finished capital going to banks. Symmetrically, $1-\Gamma(\bar{\omega}^j_{t+1})$ is the shared of finished capital going to entrepreneurs. Finally, $G(\bar{\omega}^j_{t+1})$ is the average value of the idiosyncratic shock for bankrupt entrepreneurs.

The optimal contract can be derived by maximizing over $\{\bar{\omega}^j_{t+1}, B^j_{t+1}\}$ entrepreneurial profits:

$$\max_{\{\bar{\omega}^j_{t+1}, B^j_{t+1}\}} \mathcal{Y}^k_{t+1} K^j_{t+1} \left(1 - \Gamma(\bar{\omega}^j_{t+1})\right), \tag{20}$$

subject to the definition of borrowing (16) and the zero profit condition implied by (19) holding with equality. By equalizing the Lagrangian multipliers in the first order conditions of the above problem and using the definition of the nominal income from holding one unit of finished capital (17) we get:

$$\frac{1 + R^k_{t+1}}{1 + R^n_t} = \psi_t, \tag{21}$$

where:

$$\psi_t = \left(\frac{\left(1 - \Gamma(\bar{\omega}^j_{t+1})\right)\left(\Gamma'(\bar{\omega}^j_{t+1}) - \mu G'(\bar{\omega}^j_{t+1})\right)}{\Gamma'(\bar{\omega}^j_{t+1})} + \left(\Gamma(\bar{\omega}^j_{t+1}) - \mu G(\bar{\omega}^j_{t+1})\right) \right)^{-1}, \tag{22}$$

is the *external finance premium*. As in BGG, $\psi_t = f(\bar{\omega}^j_{t+1})$ with $f'(\bar{\omega}^j_{t+1}) > 0$. Moreover, the ratio between the lending rate and the risk free rate gives the *risk premium*, which can be computed from the zero profit condition as:

$$\frac{1 + R^L_{t+1}}{1 + R^n_t} = \frac{\psi_t}{\bar{\omega}^j_{t+1}} \left(1 - \frac{NW^j_{t+1}}{Q_t K^j_{t+1}}\right), \tag{23}$$

where we notice that $NW^j_{t+1}/Q_t K^j_{t+1}$ is the inverse of the leverage ratio. Interestingly, equation (23) shows that, in the presence of credit market imperfections, the premium paid on the risk free interest rate for a loan depends on the entrepreneur's balance-sheet condition. Specifically, the higher the leverage, the higher is the premium charged on entrepreneurial risky loans.

Finally notice that the zero-profit condition can be written as a demand function for capital:

$$K^j_{t+1} = \left(\frac{1}{1 - \psi_t \left(\Gamma(\bar{\omega}^j_{t+1}) - \mu G(\bar{\omega}^j_{t+1})\right)} \right) \frac{NW^j_{t+1}}{Q_t}. \tag{24}$$

Demand for capital is increasing in net worth and decreasing in price.

3.3.2 Evolution of net worth

To ensure that entrepreneurs do not accumulate enough funds to finance their expenditures on capital entirely with net worth, we assume that they have a finite lifetime. In particular, we assume that each entrepreneur survives until the next period with probability γ. Entrepreneurs who "die" in period t are not allowed to purchase capital, but instead simply consume their accumulated resources and depart from the scene. Therefore, entrepreneurial consumption in each period will be:

$$C_t^e = (1-\gamma)\mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j)\right), \tag{25}$$

where $\mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j)\right)$ is the share of finished capital going to entrepreneurs in each period. Symmetrically, entrepreneurs who survive will accumulate net worth according to the following equation:

$$NW_{t+1}^j = \gamma\mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j)\right). \tag{26}$$

Remembering that $\mathcal{Y}_{t+1}^k = Q_t\left(1 + R_{t+1}^k\right)$, net worth is positively related to the price and the stock of capital. In contrast, as noted by Faia and Monacelli (2007), the aggregate return on finished capital R_{t+1}^k has an ambiguous impact on net worth. On the one hand, an increase in R_{t+1}^k generates a higher return for each unit of finished capital owned by entrepreneurs. On the other hand, an increase in R_{t+1}^k also generates an increase in the external finance premium, as showed in equation (21), which contributes to the risk premium and therefore reduces net worth.

3.4 Firms

Each domestic household owns an equal share of the intermediate-goods producing firms. Each firm assembles labour (supplied by the workers) and (finished) entrepreneurial capital to operate a constant return to scale production function for the variety i of the intermediate good:

$$Y_t = F\big(A_t, N_t(i), K_t(i)\big) \tag{27}$$

where A_t is a productivity shifter common to all firms (i.e., total factor productivity). Note that total factor productivity will be of crucial importance for the definition of our macro uncertainty shock, as discussed in section 3.8.

Each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to:

$$\frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-\pi_t\right)^2 \tag{28}$$

where π is the steady-state inflation rate and where the parameter ω_p measures the degree of nominal price rigidity. The higher ω_p the more sluggish is the adjustment of nominal prices. In the particular case of $\omega_p = 0$, prices are flexible.

The problem of each monopolistic firm is the one of choosing the sequence of factors of production $\{K_t(i), N_t(i)\}_{t=0}^{\infty}$ and prices $\{P_t(i)\}_{t=0}^{\infty}$ in order to maximize expected discounted real profits:

$$\max_{\{K_t(i),N_t(i),P_t(i)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=o}^{\infty} \frac{\beta^t}{P_t} \left(\begin{array}{c} P_t(i)Y_t(i) - (W_t N_t(i) + Z_t K_t(i)) - \\ \frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-\pi_t\right)^2 \end{array} \right), \tag{29}$$

subject to the technological constraint in (27). Let's denote by $\{mc_t\}_{t=0}^{\infty}$ the sequence of Lagrange multipliers on the above demand constraint, and by $\tilde{p}_t \equiv P_t(i)/P_t$ the relative price of variety i. The first order conditions of the above problem read:

$$\begin{aligned} \frac{W_t}{P_t} &= mc_t Y_{n,t} \\ \frac{Z_t}{P_t} &= mc_t Y_{k,t} \\ 0 &= Y_t \tilde{p}_t^{-\varepsilon}\big((1-\varepsilon)+\vartheta mc_t\big) - \omega_p\left(\pi_t \frac{\tilde{p}_t}{\tilde{p}_{t-1}} - \pi\right)\frac{\pi_t}{\tilde{p}_{t-1}} + \\ &\quad + \omega_p \left(\pi_{t+1}\frac{\tilde{p}_{t+1}}{\tilde{p}_t} - \pi\right)\pi_{t+1}\frac{\tilde{p}_{t+1}}{\tilde{p}_t^2} \end{aligned} \tag{30}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, ε is the elasticity of substitution between the $Y(i)$ goods, and where we have suppressed the superscript i, since all firms employ an identical capital to labour ratio in equilibrium. Note that the Lagrange multiplier mc_t plays the role of the real marginal cost of production. In a symmetric equilibrium it must hold that $\tilde{p}_t = 1$. This implies that $FOC(P_t)$ in (30) can be written in the form of a forward-looking Phillips curve:

$$(\pi_t - \pi)\,\pi_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} (\pi_{t+1}-\pi)\,\pi_{t+1} \right\} + Y_t \frac{\varepsilon}{\omega_p}\left(mc_t - \frac{\varepsilon-1}{\varepsilon}\right) \tag{31}$$

3.5 Final Good Sector

The aggregate final good Y_t is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by i, *via* the aggregate production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{32}$$

Maximization of profits yields typical demand functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{33}$$

for all i, where $Y_t = \left(\int_0^1 P_t(i)^{-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

3.6 Monetary policy

We assume that monetary policy is conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

$$\frac{1+R_t^n}{1+R^n} = \left(\frac{1+R_{t-1}^n}{1+R^n} \right)^{\phi^r} \left(\frac{1+\pi_t^n}{1+\pi} \right)^{(1-\phi^r)\phi^\pi} \left(\frac{1+Y_t}{1+Y_{t-1}} \right)^{(1-\phi^r)\phi^y}. \tag{34}$$

The parameter $\phi^r \in [0,1)$ generates interest-rate smoothing. The parameters $\phi^\pi > 0$ and $\phi^y \geq 0$ control the responses to deviations of inflation from target π and from output growth. Given the inflation target π, the steady-state nominal interest rate R^n is determined by the equilibrium of the economy.

3.7 Market clearing

Equilibrium in the final good market requires that the production of the final good be allocated to private consumption by households and entrepreneurs, investment, and to resource costs that originate from the adjustment of prices as well as from the banks' monitoring of entrepreneurial activity:

$$Y_t = C_t + C_t^e + I_t + \frac{\omega_p}{2} (\pi_t - \pi)^2 + \mu G(\bar{\omega}) \frac{\mathcal{Y}_t^k}{P_t} K_t. \tag{35}$$

3.8 Sources of uncertainty in the model

We assume that three exogenous processes drive the dynamics of our model economy. As it is standard in the literature, we assume that the *level* of total factor productivity follows an autoregressive process:

$$A_t = \rho^A A_{t-1} + e^{W_t} \sigma^A \varepsilon_t^A, \tag{36}$$

where ε_t^A follows a $\mathcal{N}(0,1)$ process and the parameter σ^A is the standard deviation of innovations to A_t (i.e., the TFP shock). The parameter σ^A is pre-multiplied by an additional process, e^{W_t}, which acts as a shifter of the variance of A_t. We refer to e^{W_t} as to the stochastic volatility of TFP. We also assume that W_t follows an autoregressive process of the type:

$$W_t = \rho^W W_{t-1} + \sigma^W \varepsilon_t^W, \tag{37}$$

where ε_t^W follows a $\mathcal{N}(0,1)$ process and the parameter σ^W is the standard deviation of innovations to W_t.

By allowing the variance of TFP shocks to rise, the probability of events that are distant from the mean increases. In the face of an increase in uncertainty, economic agents are likely to modify their behaviour even though the mean outcome is unchanged (i.e., there are no first moment shocks to TFP). We define *macro uncertainty shocks* exogenous changes in the variance of TFP (i.e., movements in W_t) that do not affect its level. Figure 1 shows the difference between a TFP shock and a macro uncertainty shock.

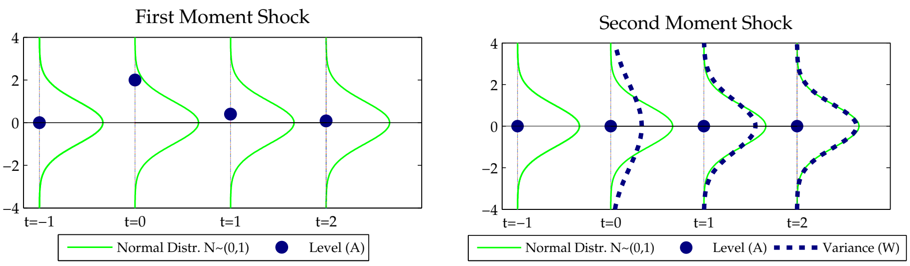

Figure 1 The charts illustrate the effect of a TFP shock (left hand side) and a macro uncertainty shock (right hand side) hitting at $t = 0$. A TFP shock (first moment shock) consists of an increase in the level of A_t, while a macro uncertainty shock (second moment shock) consists of an increase in the variance of A_t, without affecting its mean.

The last source of uncertainty in our model is the dispersion of idiosyncratic entrepreneurial productivity. As introduced by Dorofeenko et al. (2008) and Christiano et al. (2013) —and deviating from BGG— we allow the variance of the idiosyncratic shocks to vary over time. Note that, if ω is log-normally distributed with $\omega \sim \log\mathcal{N}(1,\sigma_\omega^2)$, then the log of ω is normally distributed, i.e. $\log(\omega) \sim \mathcal{N}(M,S^2)$, where M and S^2 are the mean and the variance of the underlying normal distribution. For technical purposes, it is easier to model the variance of the underlying Normal distribution, which —after fixing the mean of ω to 1— is defined as $S^2 = \log(1+\sigma_\omega^2)$. As in Christiano et al. (2013), we model the log-deviation of S_t from its steady state value as:

$$\log\left(\frac{S_t}{\bar{S}}\right) = \rho^S \log\left(\frac{S_{t-1}}{\bar{S}}\right) + \sigma^S \varepsilon_t^S, \tag{38}$$

where ε^S follows a $\mathcal{N}(0,1)$ and σ^S is the standard deviation of innovations to S_t.

Therefore, when S_t increases, the dispersion of entrepreneurial outcomes increases too. Despite leaving the mean of the outcomes unaffected, an increase in S_t will have an impact on the conditions in the entrepreneurial loans market. Intuitively, a higher dispersion of returns implies, *ceteris paribus*, a higher probability of entrepreneurial bankruptcy. Given the information asymmetry between banks and entrepreneurs and the costly state verification, this will affect the level of lending rates and, therefore, of capital demand. We refer to the next section for a better description of the mechanism through which micro uncertainty is transmitted to the real economy.

Figure 2 displays the effect of an increase in the variance of the idiosyncratic shock to entrepreneurs. We refer to exogenous movements in S_t as to *micro uncertainty shocks*.

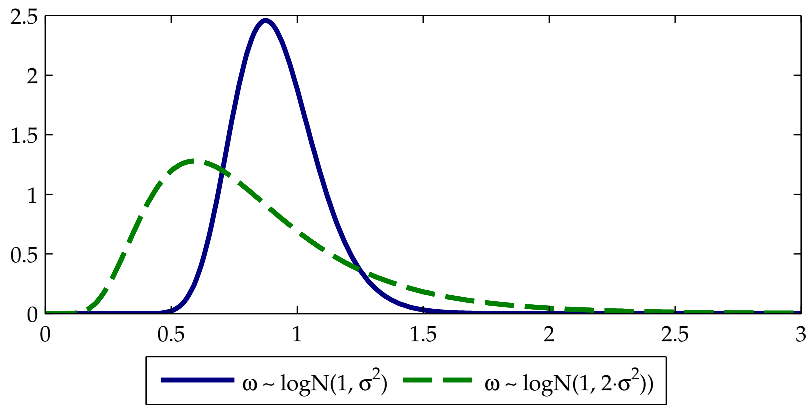

Figure 2 The chart illustrates the effect of a micro uncertainty shock. An increase in the variance of the idiosyncratic shock to entrepreneurs (ω) changes the shape of the distribution, shifting the mass of the distribution to the left tail (dashed line), without affecting the mean of the distribution.

4 Calibration and solution of the model

In this section we describe how we pin down the model's parameters. We partition the model's parameter space in two sets. The first set contains the deep parameters of the model, while the second set contains parameters relating to the exogenous processes. Finally, we discuss the methodology we use to solve and simulate the model.

4.1 Parameters of the model

The time unit is a quarter. We need to make assumptions on both the standard parameters of New Keynesian DSGE models (such as economic agents' preferences, degree of price stickiness and monopolistic competition, etc) and on the parameters relating to the credit friction. Table 1 summarizes our parameter values.

Table 1 Value of deep parameters of the model

General			
Monitoring Cost	μ	0.25	Carlstrom and Fuerst (1997)
Survivial Probability	γ	0.985	Christiano et al. (2013)
Capital Share	α	0.3	Labour share of 70%
Depreciation Rate	δ	0.025	Investment/output of 18%
Discount Factor	β	0.994	Annual real rate of 2.4%
Risk Aversion	ϱ	2	Standard
Inv. Firsh Elasticity	ν	1	Christiano et al. (2013)
GHH Scaling Factor	τ	2.5	Steady-state hours ($N = 1/3$).
Mark-up	ϵ	10	Born and Pfeifer (2013)
Rotemberg	θ	105	Calvo price stick. of 0.75
Investment Adj. Cost	ϕ_k	20	Carlstrom and Fuerst (1997)
Steady State Inflation	π	2%	Fernandez-Villaverde et al. (2011)
Std. Dev. of Micro Uncert. (SS)	S	0.225	Ext. fin. premium
Monetary policy			
Int. Rate Smoothing	ρ_r	0.5	Standard
Output	ρ_y	0.5	Standard
Inflation	ρ_π	1.5	Standard

The parameters relating the credit friction are set so as to obtain reasonable steady state values for some key financial variables, namely the external finance premium and the entrepreneurial default rate.

In order to do that, we first need to fix two parameters to pin down the solution of the entrepreneurial problem defined in section 3.3.1. The annual steady state inflation, π, is set to 2 percent; and the time discount factor, β, is set to 0.994 so as to target an annualized average real risk–free rate of interest of 2.4 percent, similar to Fernandez-Villaverde et al. (2010).

Turning now to the parameters relating to the credit friction, we set the steady state value of the quarterly survival rate of entrepreneurs γ to 0.985, the same value used by Christiano et al. (2013) and fairly similar to the value originally used by BGG; the monitoring cost μ to 0.25 as in Carlstrom and Fuerst (1997), and close to the value estimated by Christiano et al. (2013) at 0.21; and, finally, the steady state value of the standard deviation of the idiosyncratic productivity $\bar{S}$ to 0.225, a bit lower but very close to the value estimated by Christiano et al. (2013) (namely, 0.26).

This parametrization yields reasonable values for our target variables. The quarterly, steady state probability of default is of about 1 percent, very close to 0.974 percent value used in Carlstrom and Fuerst (1997) and Fisher (1999), and not far from the original 0.75 percent value used by BGG; finally, the implied steady state external finance premium is of about 188 basis points, almost identical to the value used by Carlstrom and Fuerst (1997). Moreover, the steady state value of leverage ratio implied by the above calibration is of about 2 —the same value used in BGG.

Household preferences are given by a GHH utility function (see the appendix for a description of the functional form of GHH preferences). As is commonly done in the literature, we set the coefficient τ so that the value of hours worked is equal to 1/3 in the steady state. Also, the coefficient of risk aversion in the utility function ϱ is fixed to 2 as in Fernandez-Villaverde et al. (2011), while the inverse of the Frisch elasticity of labour supply υ is fixed to 1 as in Christiano et al. (2013). We assume the production technology to have a Cobb-Douglas form with constant returns to scale. Without deviating from the standard values used in the literature, we set the quarterly aggregate capital depreciation rate δ to 0.025 and the capital's share α to 0.3.

The elasticity of substitution across varieties in the CES aggregator (ε) is set to be 10, consistent with a price markup of roughly 11 percent, as in Born and Pfeifer (2013). Since the parameters associated with the adjustment costs and nominal rigidity cannot be pinned down by the deterministic steady state (because all adjustment costs are zero), we assign conventional values to these parameters following the literature. As noticed by Faia and Monacelli (2007), it is possible build a mapping between the frequency of price adjustment in the Calvo–Yun model and the degree of price stickiness ω_p in the Rotemberg setup. By log-linearising equation (31) it is possible to derive the elasticity of inflation to the real marginal cost and compare it with empirical studies on the New-Keynesian Phillips curve, such as Gali and Gertler (1999) and Carlstrom et al. (2010). The Rotemberg price adjustment parameter, ω_p, is chosen such that, in an equivalent Calvo price-setting model, prices are fixed for 4 quarters on average. The above calibration implies the following great ratios in steady state: consumption over total output is roughly 76 percent; investment over total output is 18 percent, and entrepreneurial consumption over total output is 6 percent.

The coefficients on the Taylor rule are standard, namely 1.5 for the coefficient on inflation, ϕ^{π}, and 0.5 for the coefficient on output growth, ϕ^{y}. We set the interest-rate smoothing parameter, ϕ^{r}, to 0.5.

4.2 Exogenous processes

Since the main focus of the paper is on the impact of uncertainty shocks on economic activity, we estimate from the data the persistence and standard deviation of both macro and micro uncertainty. The exogenous process for the level of TFP is instead calibrated to match persistence and standard deviation of HP filtered U.S. GDP data. Our procedure yields sensible parameter values which are in line with other studies in the literature. Table 2 summarizes the persistence and standard deviation of the exogenous processes in our model.

Table 2 Parameter values of the exogenous processes in the model

Shocks			
Persistence TFP	ρ^A	0.95	Standard
Std. Dev. TFP	σ^A	0.008	Calibrated (Output St.Dev.)
Persistence of Micro Uncert.	ρ^S	0.78	Data
St. Dev. of Micro Uncert.	σ^S	0.043	Data
Persistence of Macro Uncert.	ρ^W	0.90	Data
Std. Dev. of Macro Uncert.	σ^W	0.070	Data

Let us discuss micro uncertainty first. As a proxy for the dispersion of idiosyncratic productivity of entrepreneurs (S_t), we use the interquartile range of industrial production growth for manufacturing industries, computed by Bloom et al. (2012) using data from the Federal Reserve Board's industrial production database.[11] To be consistent with the fact that the steady state value of the dispersion of idiosyncratic productivity of entrepreneurs ($\bar{S}$) has been fixed —so as to target the steady state value of some key financial variables as discussed above— we re-scale the series so as to have mean equal to 0.225. We then fit an AR(1) model as the one in (38) on the re-scaled series over the 1972Q1-2009Q4. According to the estimated AR(1) model, we set the persistence of micro uncertainty (ρ^s) to 0.78 and its standard deviation (σ^s) to 0.043. These estimates are robust to different measures of cross-sectional dispersion that are frequently used in the literature (such as, for example, the dispersion of output growth, sales growth, and stock market growth across industries).

Our estimates fall somewhat in between the values found by similar studies in the literature. Specifically, our parametrization is almost identical to the one used by Bloom et al. (2012), not surprisingly since we use their data for the estimation. Christiano et al. (2013) derive the (unanticipated component of) the standard deviation of micro uncertainty directly from their DSGE model (very similar to the one used in this paper) through Bayesian estimation techniques using U.S. macro-financial data:

[11] The interquartile range of industrial production growth for U.S. manufacturing industries computed by Bloom et al. (2012) is available at the following website: http://www.stanford.edu/~nbloom/index_files/Page315.htm.

they find a value for σ^s of 0.07. Note however that, using annual data of plant-level profitability constructed by Cooper and Haltiwanger (2006), Chugh (2013) estimates a much smaller value, namely $\sigma^s = 0.003$. These differences most likely arise because of the data used, in particular because of the different nature of the chosen proxy for micro uncertainty and of the frequency of the data set.

For the macro uncertainty shock, we follow again Bloom et al. (2012) and we use the conditional heteroscedasticity of the Solow residual as a proxy for the stochastic volatility of TFP. Specifically, we estimate the heteroscedasticity of the growth rate of quarterly TFP for the U.S. business sector with a GARCH(1,1) model over the 1972Q1-2009Q4 period.[12] Once we obtained our estimated proxy for the stochastic volatility of TFP (e^{W_t}), we re-scale the series so as to have mean equal to 1, in order to be consistent with (37). We then fit an AR(1) model on the log of the estimated heteroscedasticity over the 1972Q1-2009Q4. According to the AR(1) estimation, we set the persistence of macro uncertainty (ρ^W) to 0.90 and its standard deviation (σ^W) to 0.07. As an alternative, we also proxied e^{W_t} with the conditional variance of a 20-quarter rolling window AR(1) model on the growth rate of the Solow residual; and also with a stochastic volatility AR(1) model estimated with Bayesian techniques (as in Primiceri, 2005). Our parametrization of the macro uncertainty process is robust to these alternative methodologies and it yields parameter values that are very similar to the ones used by similar studies in the literature, such as Bloom et al. (2012) and Caldara et al. (2012).

Finally, we set the persistence of TFP shocks (ρ^A) to 0.95 and we use the standard deviation of TFP shocks (σ^A) as a free parameter to match the volatility of HP filtered U.S. GDP data over the 1972Q1-2009Q4 period. Specifically, we set $\sigma^A = 0.008$, which implies a standard deviation of HP filtered GDP of 1.58 percent in our model simulations relative to a value of 1.59 in the data.[13] As noted by Caldara et al. (2012), our values of both the persistence and the standard deviations of TFP are standard for the time series properties of the Solow residual. Indeed, our parametrization is in line with many similar papers in the literature, such as Fernandez-Villaverde et al. (2011), Bloom et al. (2012), and Christiano et al. (2013).

[12] The quarterly TFP series for the U.S. business sector can be downloaded at the following webpage: http://www.frbsf.org/economic-research/total-factor-productivity-tfp/. See Fernald (2012), Fernald and Matoba (2009), and Kimball et al. (2006).

[13] To obtain the moments implied by the model, we simulate the model economy for 2000 periods. We then use the last 1000 periods to compute the standard deviation of the log-difference of output from its HP trend.

4.3 Solution method

DSGE models are normally solved by taking a linear (i.e., first-order) approximation around their non-stochastic steady state equilibrium. However, when using the traditional linear approximation, shocks to the variance of exogenous processes do not play any role, since the certainty equivalence holds and the decision rule of the representative agent is independent of shocks' second (or higher) moments.[14]

For second (or higher) moments to enter the decision rules of economic agents, higher approximation to the policy functions are needed. For example, when taking an approximation to the 2^{nd}-order of the solution of the model, second moment shocks (as the ones defined above) would only enter as cross-products with the other state variables. This implies that a shock that affects the variance of an exogenous process can have an impact on the dynamics of the model only when also its mean is affected.

In contrast, a 3^{rd}-order Taylor expansion of the solution of the model, allows for second moment to play an independent role in the approximated policy function. Therefore, 3^{rd}-order Taylor approximation shall allow us to simulate and evaluate the effect an uncertainty shock, while holding constant its level. Fernandez-Villaverde et al. (2010) provide a detailed discussion.

To solve the baseline model, we use Dynare 4.3. Dynare computes 3^{rd}-order Taylor series approximation around the non-stochastic steady state of the model. As Fernandez-Villaverde et al. (2011) show, 3^{rd}-order approximation to the policy functions is sufficient to capture the dynamics of the model, with little gain to using an approximation higher than 3^{rd}-order.

Impulse responses functions (IRFs) are defined as the reaction of the variables of a dynamic system to an exogenous impulse of a given size. Generally, we compute them using the equilibrium of the dynamic system (i.e., steady state) as an initial condition. This is because, in linear models, IRFs do not depend on the state of the economy when the shock occurs, nor on the sign and size of the shock. However, when using a higher order approximation to the solution of the model, this is not the case anymore and impulse responses computed from the steady state are just one of the many IRFs of the non–linear model.

Moreover, an additional complication arises since —in a model approximated to the 3^{rd} order featuring uncertainty shocks— the mean of the ergodic distributions of our endogenous variables will in general be different from their deterministic steady-state values. Fernandez-Villaverde et al. (2011) show it through model simulations and

[14] Micro uncertainty shocks (as the ones considered in this paper) and the Knightian uncertainty shocks (as in Ilut and Schneider, 2014) represent an exception and their impact can be studied with standard linear methods.

propose to compute impulse responses using the ergodic mean of the data generated by the model as an initial condition. We follow their approach and we refer the reader to Appendix B for details on how we construct our impulse responses.

5 The effect of micro and uncertainty on the model economy

In this section we analyse the effect of both macro and micro uncertainty shocks, with an emphasis on their different nature and transmission mechanisms. Note that our model features two amplification mechanisms that can potentially affect both shocks, namely sticky prices and the credit friction. On the one hand, sticky prices help generating co-movement between consumption and investment and, by doing so, also amplify the effect of the shocks. On the other hand the presence of the financial accelerator implies that impact of any shock that in general equilibrium affects entrepreneurial net worth will get amplified.

In the next sections we compare the impulse responses from our baseline with two alternative sets of impulse responses that we obtain by varying the degree of price stickiness and the severity of the financial friction in our model economy. We shall do so in two separate subsections, first for macro uncertainty shocks and then for micro uncertainty shocks. Finally, for micro uncertainty, we shall also conduct an additional exercise to get better insight of the transmission mechanism of the shock.

5.1 A macro uncertainty shock

We analyse the impulse responses to a 2 standard deviation increase in the macro uncertainty innovation (ε_t^W) in our model. This is equivalent to an increase in macro uncertainty of 14 percent.

As highlighted in the theoretical analysis in Section 2 and explained in the analysis below, sticky prices are crucial for generating co-movement between consumption and investment, as well as amplifying the response of output in response to the macro uncertainty shock. To better understand the transmission of the shock and the role of price stickiness, in Figure 3 we consider our baseline calibration (circles); a flexible price version of the model (diamonds), obtained by setting the Rotemberg parameter $\omega_p \simeq 0$; and a version of the model with ω_p calibrated as to obtain an average probability of changing prices of 5 quarters (squares), instead of 4 quarters as in the baseline.

We focus first on the impulse responses obtained under flexible prices (diamonds).

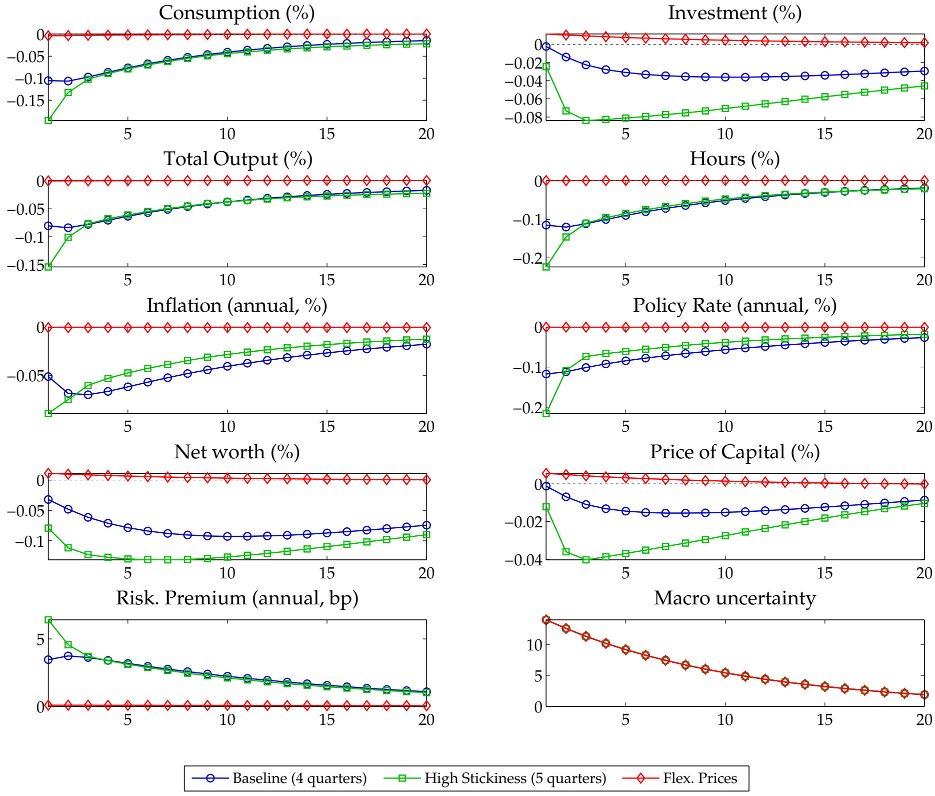

Figure 3 Impulse response functions (IRFs) to a 2 standard deviation increase in macro uncertainty (the level of TFP is held constant). The IRFs display the impact of a macro uncertainty shock under different degrees of price stickiness and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

The shock acts to reduce consumption *via* precautionary saving. Since capital is predetermined output can only change in response to movements in labour. However, under flexible prices and constant mark-ups, the labour demand schedule is unchanged. Likewise, the labour supply schedule is fixed under GHH preferences: in our model —and differently from Basu and Bundick (2012)— the macro uncertainty shock does not generate an impact increase in “precautionary labour supply”, since consumption does not enter the labour supply schedule. Therefore, since both hours and wages do not move in the first period, we also do not observe any movement in output in response to the shock. As a result, since output is unchanged on impact, the lower level of consumption —i.e., households’ additional saving in response to the shock— is channelled toward higher investment.

These results are clearly inconsistent with business cycle facts where both consumption and investment tend to move in the same direction as total output. Also, they are inconsistent with the empirical evidence on uncertainty shocks, where an increase in uncertainty is generally found to negatively affect both consumption and investment. Moreover, under this parametrization, note that the financial accelerator mechanism amplifies the positive response of investment and makes the co-movement problem even worse. This is because the additional investment acts to boost the price of capital which in turn boosts entrepreneurs' net worth and thereby increases demand for capital.

Under sticky prices (circles), however, the impulse responses are strikingly different. As already suggested, the increase in uncertainty reduces consumption through a precautionary savings channel. As explained by Basu and Bundick (2012), this puts downward pressure on the marginal cost faced by firms which —since prices are sticky in the short run— implies an increase in firms' markups and a reduction in labour demand. Note that, with GHH preferences and differently from Basu and Bundick (2012), here labour supply is fixed and the fall in labour demand necessarily leads to a fall in hours and in the real wage. In other words, since under price stickiness output is demand determined (i.e., firms must satisfy whatever output is demanded at a given price), the reduction in consumption from precautionary saving motives acts to reduce aggregate demand. Hence, demand for both labour and capital falls and, therefore, investment falls too.

Sticky prices act as a powerful amplifying mechanism: the higher price stickiness, the higher the increase in markups and the fall in labour demand. As a result, hours worked fall and consumption falls even further, therefore amplifying the effect of the shock. In Figure 3 we also compute impulse responses using a version of the model calibrated as to obtain an average probability of changing prices of 5 quarters, instead of 4 quarters as in the baseline. Under high stickiness (squares), the effect of the macro uncertainty shock on output is almost twice as big as in the baseline (circles). Note, however, that in addition to sticky prices three ingredients drive the dynamics of the responses to the macro uncertainty shock. First, in contrast with the flex-price case, the financial accelerator now helps the co-movement: in response to the lower price of capital, entrepreneurs' net worth falls and the risk premium increases, thereby depressing investment even further. Second, GHH preferences also act to depress consumption further: hours worked now enter the Euler equation for consumption such that a fall in the growth rate of hours acts to reduce consumption growth (see the appendix for a comparison between different functional forms for consumers' preferences). Third, in response to weaker demand, with falling prices and depressed output, monetary policy accommodates the shock by reducing the policy rate, thus supporting consumption. As noted by Leduc and Liu (2012), this shows that macro uncertainty shocks largely resemble to aggregate demand shocks.[15]

[15] Another amplification mechanism has been pointed out by Fernandez-Villaverde et al. (2011), who show that higher demand elasticity leads to greater convexity of firms' marginal profit function: this, in turn, implies higher mark-ups in response to uncertainty shocks and therefore, higher amplification.

The resulting impact on output is, however, rather small: in our baseline, output falls by 0.1 percent and the impact on financial variables is small, too. The risk premium increases by about 4 annualized basis points, while the price of capital displays a maximum fall of about 0.2 percent over the 20 quarters considered. This is certainly not consistent with the behavior of macro-financial variables as we observed in the post-Lehman period.

What is the role of the credit friction in the transmission of the macro uncertainty shock? As suggested above, the financial accelerator mechanism amplifies the impact of any aggregate shock that in general equilibrium affects the net worth of entrepreneurs. Therefore, in periods where financial market distortions are more severe, macro uncertainty shocks could have a larger impact on economic activity and generate dynamics of risk premia and asset prices that are more in line with what we observed during the recent crisis. To investigate this mechanism further, Figure 4 compares our baseline results (circles) against a case where credit frictions are sensibly reduced (diamonds) and a case where frictions are more pronounced (squares). These alternative cases are computed by modifying the value of the monitoring cost parameter (μ), where remember that the higher the monitoring cost the more severe is the credit friction in the economy.

When credit frictions are more pronounced (squares), uncertainty shocks tends to have a larger impact on investment and on financial variables (i.e., net worth, the price of capital and the risk premium). However, and somewhat surprisingly, the effect on total output does not seem responsive to changes in the severity of the credit friction. This result can be accounted for by the transmission mechanism of the macro uncertainty shock and the nature of the financial accelerator. As already noted above, in the face of an exogenous increase in uncertainty consumers increase their precautionary savings and consumption falls. Since there are no credit frictions directly affecting households, however, the fall of consumption is very similar in the three cases that we consider, i.e. is irrespective of the severity of the credit friction. Differently, investment —which, as noted above, co-moves with consumption because of price rigidities— gets amplified by the financial accelerator mechanism. The shock in fact reduces the price of capital and entrepreneurial net worth and increases the risk premium. However, since the impact on investment is small relative to the impact on consumption, the amplifying role of the credit friction is almost indiscernible. Moreover, and somehow puzzling, the negative impact of the macro uncertainty shock on total output is slightly larger when credit frictions are less severe. The minor difference between the impulse responses in Figure 4 are driven by the response of monetary policy: when credit frictions are more severe, in fact, the real interest is lower relative to the case

In their paper they set the elasticity of demand to $\varepsilon = 21$. As in Born and Pfeifer (2013), we take a relatively conservative stance and set $\varepsilon = 10$.

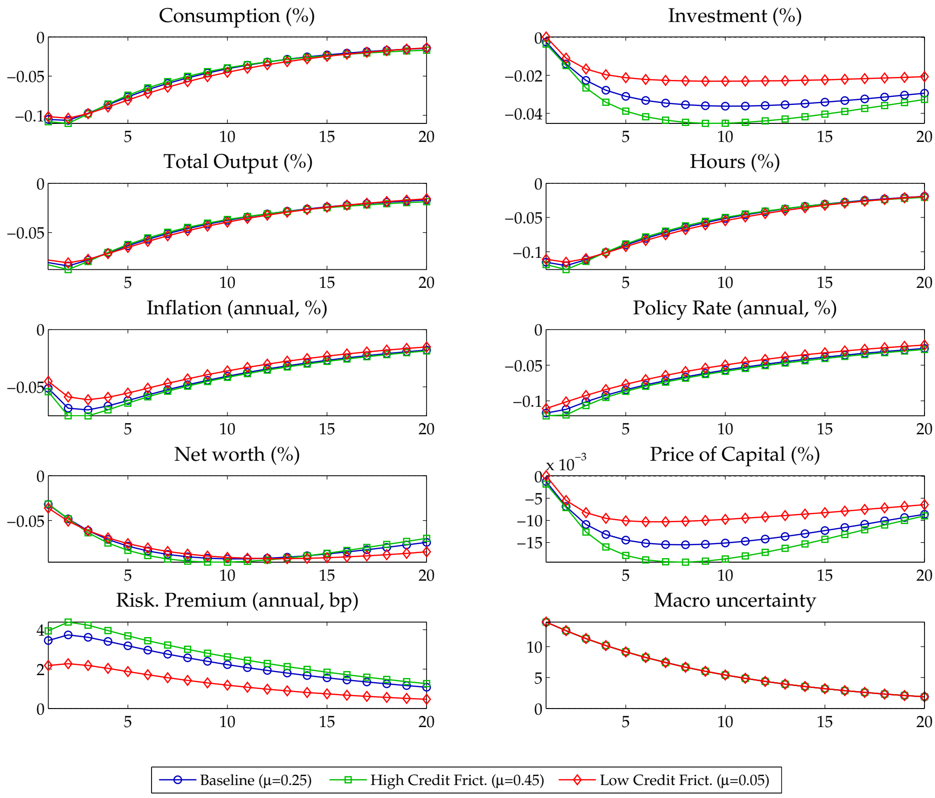

Figure 4 Impulse response functions (IRFs) to a 2 standard deviation increase in macro uncertainty (the level of TFP is held constant). The IRFs display the impact of a macro uncertainty shock under different degrees of tightness of the credit friction and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

in which credit frictions are less severe, therefore affecting the profile of consumption and output.

These results suggest that in our model (i) macro uncertainty shocks do not seem to have a large impact on economic activity; and (ii) credit frictions *per se* do not amplify the effect of macro uncertainty shocks on total output. Since the macro uncertainty shock is primarily transmitted through consumption (*via* precautionary savings), it has little impact on entrepreneurial net worth and, therefore, there is little amplification. As a result, the profiles of output, consumption, hours worked and inflation do not display substantial differences to changes in the tightness of the credit friction.

5.2 A micro uncertainty shock

We turn now to the analysis of micro uncertainty shocks. We consider a 2 standard deviation increase in the micro uncertainty innovation (ε^S), which is equal to an increase in micro uncertainty of 8.6 percent.

As for the macro uncertainty shock, Figure 5 displays the impulse responses under our baseline calibration (circles); a flexible price version of the model (diamonds), obtained by setting the Rotemberg parameter $\omega_p \simeq 0$; and a version of the model with ω_p calibrated as to obtain an average probability of changing prices of 5 quarters (squares), instead of 4 quarters as in the baseline.

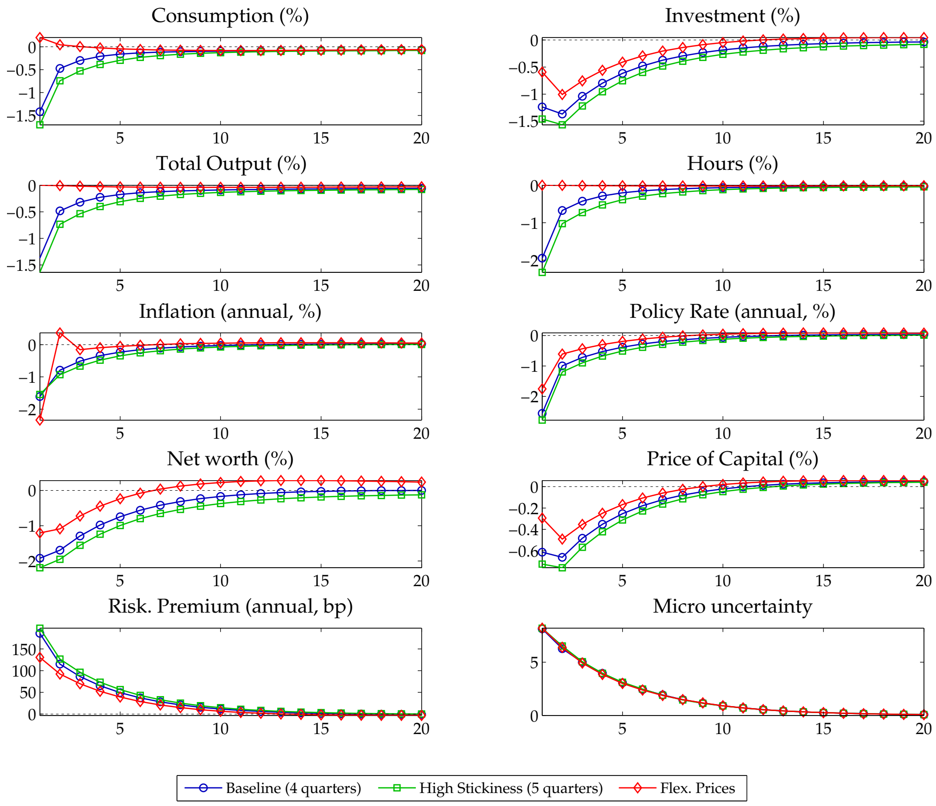

Figure 5 Impulse response functions (IRFs) to a 2 standard deviation increase in micro uncertainty. The IRFs display the impact of a micro uncertainty shock under different degrees of price stickiness and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

At first sight, the impulse responses to the micro uncertainty shock look similar to those obtained for the macro uncertainty shock. However, the way micro uncertainty gets transmitted to the real economy is noticeably different. Whilst the macro uncertainty shock operates through precautionary savings and propagates to the rest of the economy as a demand shock *via* sticky prices, the micro uncertainty shock operates through the cost of external debt and entrepreneurial capital demand. Then, similarly to the macro shock, the micro uncertainty shock propagates to the rest of the economy *via* sticky prices.

Specifically, higher dispersion of the idiosyncratic shock implies larger returns for some entrepreneurs and larger losses for other entrepreneurs. All else equal, this implies higher bankruptcy rate. With no credit frictions this would have no impact on the model economy, since the expected return has not changed and both entrepreneurs and banks are risk neutral. Under asymmetric information, however, the costly state verification problem introduces a wedge (the monitoring cost) in banks' zero profit condition: a higher default rate (due to those entrepreneurs experiencing lager negative shocks) increases the expected costs for banks which as a result will charge higher lending rates. This in turn generates a fall in capital demand and hence in investment.

The difference between micro and macro uncertainty shocks becomes clear when looking at the flex price economy (diamonds). Differently from the macro shock, which affects consumption, the micro uncertainty shock depresses investment. Increased micro uncertainty generates an increase in the cost of external finance (i.e., risk premium) since the expected cost associated with bankruptcies is now larger. Higher lending rates imply lower capital demand, therefore generating a sharp fall in investment and in the price of capital. However, symmetrically to what we observed for the macro shock, under flexible prices lower investment leads to higher consumption.

As for the macro uncertainty shock, price stickiness helps generating the co-movement between consumption and investment. In our baseline (circles), weaker capital demand now acts to reduce output and, therefore, also consumption. As for macro uncertainty shocks, moreover, sticky prices act as a powerful amplifying mechanism: as Figure 5 shows, the higher high the degree of price stickiness (squares) the larger the impact on output.

As Christiano et al. (2013) note, the shock resembles an increase in the tax rate on the return on investment which should act to discourage saving (and hence investment) and boost consumption or leisure. But there are three factors that discourage consumption. First, the response of monetary policy is such that the real rate does not fall sufficiently to encourage households to consume. Second, the fall in output leads to a fall in hours, which with GHH preferences act to decrease marginal utility on impact relative to the future. To mitigate such a fall in marginal utility, consumption falls. This effect is greater in the case of GHH preferences compared to the case of non-separable preferences.[16] Third, credit frictions act as an amplifier: lower entrepreneurial net worth and a higher external finance premium imply even weaker capital demand and lower investment.

Note that the model simulations accord well with empirical facts along a number of dimensions. An increase in uncertainty causes corporate bond prices to fall and credit spreads to widen immediately as investors demand greater protection against the increased downside risk. The rise in private yields pushes up the effective cost of

[16]The results with different specifications of households' preferences are not reported here for reasons of space but are available from the authors upon request.

capital, because the firms cannot costlessly replace debt with new equity to finance their investment projects.

Finally, in addition to the different transmission mechanism, an important difference between micro and macro uncertainty shocks lies in the magnitude of the response of total output to the shock, which is now much larger. As displayed in Figure 5, in our baseline a 2 standard deviation shock to micro uncertainty leads to a fall of about 1.4 percent of total output, an impact which is almost 20 times larger than the macro uncertainty shock. This larger impact does not apply only to output: the risk premium now increases by about 190 basis points, net worth falls by 2 percent and the price of capital falls by 0.6 percent. This is somewhat surprising, particularly if we take into account that the micro shock has a smaller size than the macro shock.

Can credit market imperfections account for such a large difference? Or, in other words, what is the role of the credit friction for the transmission of micro uncertainty shocks? Figure 6 compares our baseline results (circles) against the case where credit frictions are sensibly reduced (diamonds) and against the case where frictions are more pronounced (squares).

The role of credit frictions in the transmission of micro uncertainty shocks can be easily understood with the simple partial equilibrium example used above. Since the monitoring cost introduces a wedge in banks' zero profit condition, a mean preserving shock to the variance of idiosyncratic productivity increases banks' expected costs and induces them to raise the spread they charge on lending interest rates. It follows that when credit frictions are less severe —i.e., when the monitoring cost is lower— the effect of micro uncertainty shock on total output should be lower.

Figure 6 shows that this is indeed the case: unlike macro uncertainty, credit frictions greatly magnify micro uncertainty shocks. Specifically, when reducing the severity of the credit friction (i.e., the monitoring cost is reduced from 0.25 to 0.05), the impact of a micro uncertainty shock on the risk premium and on investment reduces substantially. When the degree of credit frictions is low (diamonds), investment falls by one half relative to baseline, while the risk premium increases by only 90 basis points, relative to an increase of 190 basis points in the baseline.

In addition, the impact on net worth and on the price of capital reduces substantially, too. Relative to the baseline, the fall in both entrepreneurial net worth and in the price of capital almost halves. Accordingly, total output —which falls by 1.4 percent in our baseline— falls by less than 1 percent when the severity of the credit friction is reduced.

The impulse responses reported in Figure 6 show that tighter credit frictions imply a larger impact of micro uncertainty shocks on economic activity, while this is not the

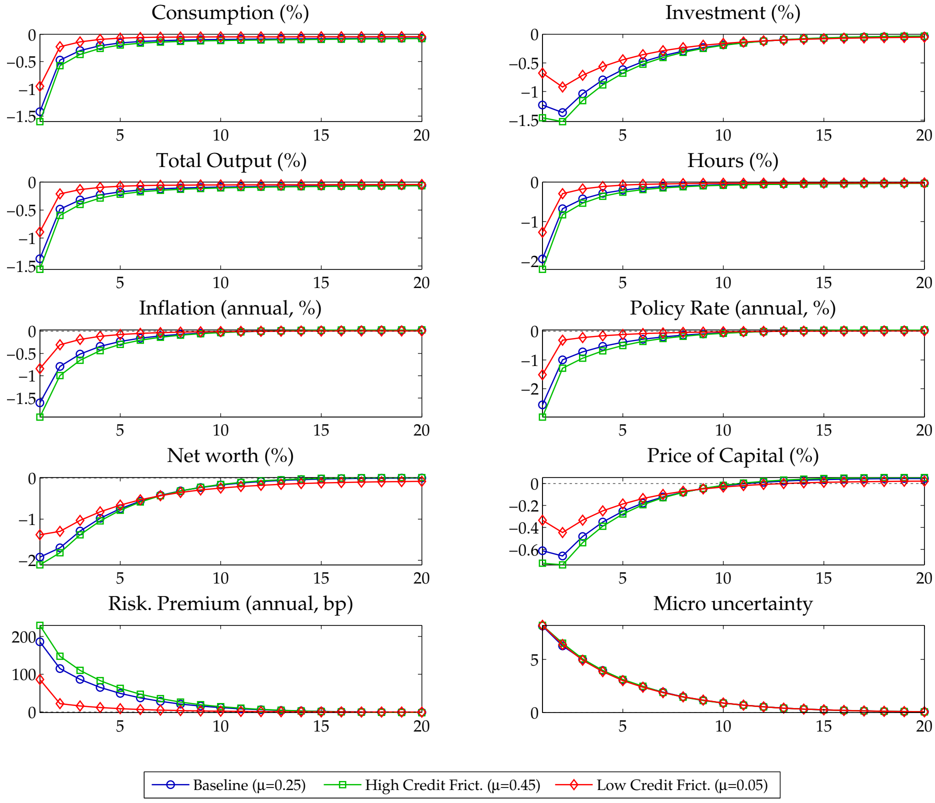

Figure 6 Impulse response functions (IRFs) to a 2 standard deviation increase in micro uncertainty. The IRFs display the impact of a micro uncertainty shock under different degrees of tightness of the credit friction and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

case for macro uncertainty shocks (see Figure 4). The main reason for why this is the case is as follows. The financial accelerator mechanism amplifies any shock that in general equilibrium affects entrepreneurial net worth. This is true for both macro and micro uncertainty shocks. Differently, in the case of micro uncertainty the credit friction is not just an amplifying mechanism, it also crucially affects the *impact* of the shock. To see this let's consider the limit case where the monitoring cost is equal to zero —e.g., let's assume that the both the entrepreneur and the bank could be costlessly observe idiosyncratic shocks. In this case the impact of a micro uncertainty shock would also tend to zero. Since both entrepreneurs and banks are risk neutral, without credit market imperfections micro uncertainty would not play any role for the dynamics of the model economy. By playing this dual role, an increase in the tightness of credit frictions in the economy implies both a larger impact of micro uncertainty shocks (through higher risk premia) and a greater amplification (through higher impact on net worth). In contrast, in the case of macro uncertainty, financial frictions are only an amplification mechanism: as shown in Figure 4 the impact effect of the shock does not depend on the severity of the credit friction. Our results are consistent with the empirical evidence provided by Caldara et al. (2013), where uncertainty shocks are found to have a large impact on GDP if transmitted through a credit channel but a significantly smaller impact otherwise.

5.2.1 Inspecting the mechanism – A comparative static exercise

In this section we perform a comparative statics exercise to get a deeper insight of the mechanism through which micro uncertainty affects the real economy. Specifically, we analyze the effect of changes in the steady state value of the standard deviation of entrepreneurial idiosyncratic productivity ($\bar{S}$) on the steady state value of other variables in our model economy. Such a simple static exercise is not only useful for a better understanding of the impulse responses in the previous section, but is also informative on the effect of micro uncertainty on the level of the economy.

We consider a wide range of steady-state standard deviations of idiosyncratic productivity, namely $\bar{S} = [0.01, 0.70]$. Then, we solve the microeconomic problem together with the steady state of our model with the algorithm described in the Appendix. In this way we pin down the steady state leverage ratio (L), the threshold value for the idiosyncratic shock ($\bar{\omega}$), and entrepreneurial real income from owning one unit of capital (y^k). Once the steady state level of these variables are determined, we can solve for the steady state value of all other variables in our model, given our baseline calibration in Table 1.

Figure 7 displays how the steady-state level of some key variables in our model varies to changes in the steady state value of S. Note that all variables expressed in levels are re-scaled to be equal to 100 for our baseline calibration ($\bar{S} = 0.225$); interest rates are in annualized percent, while the leverage ratio is not rescaled.

As described in the previous section, an increase in $\bar{S}$ is associated with an increase in the frequency of entrepreneurial default (F) which also increases banks' expected costs associated with bankruptcies. As a result, banks charge a higher spread on the risk free rate and lending rates (R^L) increase, the aggregate level of borrowing in the economy (B) falls and so do entrepreneurs' purchases of unfinished capital (K): with a lower level of capital in the economy its rental rate of return is higher (R^k).

Note that, intuitively, entrepreneurs should try to leverage up to benefit from the higher rental rate of capital. However, as already noted above, as $\bar{S}$ rises entrepreneurs face increasing interest rates (R^L), which would induce entrepreneurs to reduce borrowing and, consequently, also leverage. In a on line appendix to their paper, Christiano et al. (2013) put forth this same issue and analyse it with a similar exercise. They

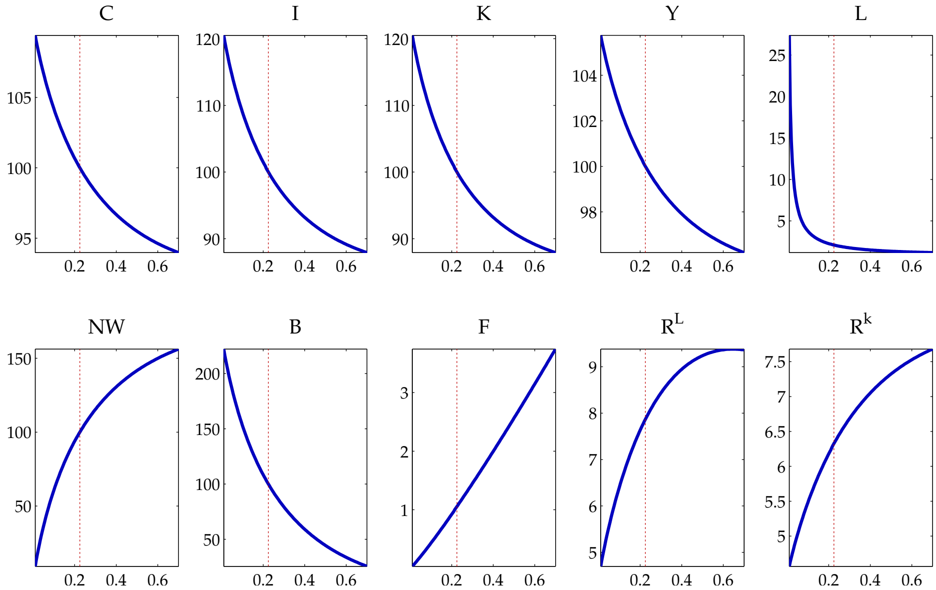

Figure 7 The charts display the effect of micro uncertainty on the steady state level of the model economy. On the horizontal axis is the steady state value of entrepreneurial idiosyncratic productivity ($\bar{S}$). On the vertical axis is the steady state value of consumption (C), investment (I), capital (K), total output (Y), leverage (L), net worth (NW), total borrowing (B), default probability (F), lending rates (R^L) and rental rate of capital (R^k).

first characterize the equilibrium in the loans market analytically in the *Risk spread - Leverage* space. Then, holding the aggregate return on capital (R^k) fixed, they show that entrepreneurs facing an exogenous increase in $\bar{S}$ would optimally choose a loan contract with a higher interest rate and lower leverage. However, they also suggest that the result of this partial equilibrium exercise could be muted by the increase in the aggregate return on capital, that would instead push entrepreneurs to increase their leverage. In an additional partial equilibrium exercise they show that this is indeed the case: holding $\bar{S}$ fixed, an exogenous increase of R^k relative to the risk free interest rate leads to an increase in leverage, therefore muting the negative impact on leverage of a jump in $\bar{S}$. In their numerical experiments, however, they find that the first effects always dominates.

In our exercise we let the rental rate of capital to be determined jointly with all other variables in our model and, consistently with Christiano et al. (2013)'s conjecture, we find that the effect of lending rates predominates on the effect of rental rate of capital. In fact, Figure 7 shows that in the face of increasing $\bar{S}$ —and therefore of increasing interest rates but also of increasing aggregate returns on capital— entrepreneurs optimally choose loans contracts with lower leverage (L).

Note that when $\bar{S}$ approaches zero, leverage is very sensitive to changes in $\bar{S}$.

Intuitively, when the variance of the idiosyncratic shock approaches zero entrepreneurs try to leverage up to infinity since their profits are unbounded and the credit friction is not binding. Analytically, this can be easily understood by recalling that leverage is defined as the ratio between capital and net worth and by observing that entrepreneurs optimally reduce their net worth (NW) as $\bar{S}$ approaches zero.

Finally, and not surprisingly, all relevant macroeconomic aggregates are decreasing in $\bar{S}$. Specifically, consumption, investment, and total output are lower for larger values of the standard deviation of idiosyncratic productivity.

To give a numerical flavour to these simple comparative statics, consider the following counterfactual exercise. Specifically, we ask what would be the impact on the level of the variables in our model economy of a permanent increase in the variance of idiosyncratic productivity? We calibrate the permanent increase of idiosyncratic productivity using the interquartile range of industrial production growth for manufacturing industries (IP IQR), computed by Bloom et al. (2012) using data from the Federal Reserve Board's industrial production database (i.e., the same series that we used to pin down the parameters governing the process for S in section 4). Specifically, we divide the sample in "high uncertainty" and "low uncertainty" regimes. The low uncertainty regime is set, somewhat arbitrarily to the Great Moderation period, namely from 1984Q1-2006Q4. In the low uncertainty regime the average level of IP IQR is 0.19; in contrast, in the high uncertainty regime (i.e., the combined 1972Q1-1983Q4 and 2007Q1-2009Q4 period) the average level of IP IQR is 0.28.

According to our calculations in Figure 7, total output, consumption and investment are 1.5, 2.2, and 4.6 percent lower in the high uncertainty regime than in the low uncertainty regime, respectively. Financial variables display even a larger difference: total lending is lower by 24 percent, leverage by 22 percent and the probability of default increases from 0.87 to 1.34 percent.

6 Conclusions

This paper investigates the relationship between uncertainty and economic activity in the context of imperfect financial markets. We use an otherwise standard general equilibrium model with sticky prices and credit frictions (in the spirit of Bernanke et al., 1999) to analyse the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and to compare it with a mean preserving shock to the dispersion of firms idiosyncratic productivity (micro uncertainty).

We find that micro uncertainty shocks appear to be more relevant than macro uncertainty shocks. According to our simulations, a 2 standard deviations shock to micro uncertainty depresses output by about 1.4 percent, relative to a fall of less than 0.1 percent in response to a 2 standard deviations shock to macro uncertainty. This result is driven by (i) the different nature of the uncertainty shock and its transmission mechanism and (ii) by the amplifying role of the credit friction.

Macro uncertainty is primarily transmitted through a precautionary savings channel. In the face of increased uncertainty around their future stream of income, households increase their savings and decrease their consumption. Differently, micro uncertainty shocks operate through the cost of external finance and entrepreneurial capital demand. Under asymmetric information between lender and borrower, the costly state verification problem introduces a wedge in banks' zero profit condition. In the face of increased uncertainty around entrepreneurial productivity, this wedge induces banks to raise their lending interest rates. As a result, entrepreneurs demand less capital and investment falls.

Both micro and macro uncertainty shocks propagate to the rest of the economy *via* sticky prices, which not only are crucial for generating co-movement between consumption and investment but also amplify the impact of both shocks on output. However, differently from macro uncertainty shocks, micro uncertainty shocks are greatly magnified by the credit friction. There are two reasons for why this is the case. First, the financial accelerator mechanism amplifies any shock that in general equilibrium affects entrepreneurial net worth: since the micro uncertainty shock has larger impact on net worth, is not surprising that it is also displays more amplification. Second, and maybe more importantly, in the case of micro uncertainty the credit friction is not just an amplifying mechanism: it is also the economic rationale for why micro uncertainty shocks affect the real economy. By playing this dual role, a higher degree of credit market imperfections affects both the impact and the amplification of micro uncertainty shocks.

Our results have one important implication. While sticky prices are certainly a powerful amplifier, financial frictions seem to be key in the transmission of uncertainty shocks. We do not interpret this evidence as suggesting that uncertainty affects the economy mainly through investment and only to a lesser extent through consumption. Indeed, if households were to borrow in imperfect credit markets, the same amplification mechanism observed for entrepreneurs would be at work.

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A Appendix: Model

A.1 Equilibrium

Let's define $q_t \equiv Q_t/P_t$, $nw_t \equiv NW_t/P_t$, $z_t \equiv Z_t/P_t$. For a given path for the exogenous processes, a recursive (imperfectly) competitive equilibrium of the model is a sequence of allocations for the endogenous variables that solves the following system of equations.

Euler equation of households:

$$U_{c,t} = \beta(1+R_t^n)\mathbb{E}_t\left[\frac{U_{c,t+1}}{\pi_{t+1}}\right]. \tag{A.1}$$

Labour supply:

$$mc_t Y_{n,t} = -\frac{U_{n,t}}{U_{c,t}}. \tag{A.2}$$

Marginal product of capital:

$$mc_t Y_{k,t} = z_t. \tag{A.3}$$

Price of capital:

$$q_t = \left[1 - \phi_k\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1}. \tag{A.4}$$

Zero profit condition:

$$y_{t+1}^k K_{t+1}\Big(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\Big) = (1+R_t^n)(q_t K_{t+1} - nw_{t+1}). \tag{A.5}$$

NK Phillips curve:

$$(\pi_t - \pi)\,\pi_t = \beta\mathbb{E}_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\left(\pi_{t+1} - \pi\right)\pi_{t+1}\right\} + Y_t\frac{\varepsilon}{\omega_p}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right) \tag{A.6}$$

Net worth law of motion:

$$nw_{t+1} = \gamma y_{t+1}^k K_{t+1}\left(1 - \Gamma(\bar{\omega}_{t+1})\right). \tag{A.7}$$

Entrepreneurs real consumption:

$$C_t^e = (1-\gamma)\left(1 - \Gamma(\bar{\omega}_{t+1})\right) y_t^k K_t. \tag{A.8}$$

Aggregate resource constraint:

$$A_t F(K_t, N_t) = C_t + C_t^e + I_t + \frac{\omega_p}{2}\left(\pi_t - \pi\right)^2 + \mu G(\bar{\omega}) y_t^k K_t. \tag{A.9}$$

Accumulation of aggregate capital:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi_k}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{A.10}$$

Monetary policy:

$$\frac{1+R_t^n}{1+R^n} = \left(\frac{1+R_{t-1}^n}{1+R^n}\right)^{\phi^r} \left(\frac{1+\pi_t^n}{1+\pi}\right)^{(1-\phi^r)\phi^\pi} \left(\frac{1+Y_t}{1+Y_{t-1}}\right)^{(1-\phi^r)\phi^y}. \tag{A.11}$$

Definition of real income from holding one unit of finished capital:

$$y_t^k = z_t + q_t\left[1-\delta-\frac{\phi_k}{2}\left(\frac{I_t}{K_t}-\delta\right)^2 + \phi_k\left(\frac{I_t}{K_t}-\delta\right)\frac{I_t}{K_t}\right]. \tag{A.12}$$

$$y_{t+1}^k = \frac{(1+R_{t+1}^k)q_t}{\pi_{t+1}}. \tag{A.13}$$

Optimal contract:

$$\frac{1+R_{t+1}^k}{1+R_t^n} = \psi_t. \tag{A.14}$$

where:

$$\psi_t = \left(\frac{\left(1-\Gamma(\bar{\omega}_{t+1}^j)\right)\left(\Gamma'(\bar{\omega}_{t+1}^j)-\mu G'(\bar{\omega}_{t+1}^j)\right)}{\Gamma'(\bar{\omega}_{t+1}^j)} + \left(\Gamma(\bar{\omega}_{t+1}^j)-\mu G(\bar{\omega}_{t+1}^j)\right)\right)^{-1}. \tag{A.15}$$

A.2 Households' Preferences

In the paper we compare different functional forms for households' preferences, namely standard separable preferences, log-separable preferences of the King et al. (1988) type, and GHH preferences of the Greenwood et al. (1988) type. Below, we describe the functional form of those preferences and we show how they affect the households' key equations, namely the Euler equation for consumption and labour supply.

Separable Preferences

Agents' utility is additively separable in consumption and labour:

$$\frac{C_t^{1-\varrho}}{1-\varrho} - \tau\frac{N_t^{1+\upsilon}}{1+\upsilon}. \tag{A.16}$$

Note that this functional form is separable in that the utility (loss) from working does not directly affect the utility (gain or loss) from consumption, i.e. the cross-derivative of utility with respect to consumption and labour is zero. In fact:

$$\begin{aligned} U_{c,t} &= C_t^{-\varrho}, \\ U_{n,t} &= -\tau N_t^{\upsilon}. \end{aligned} \tag{A.17}$$

The Euler equation and labour supply conditions are:

$$\begin{aligned} C_t^{-\varrho} &= \beta(1+R_t^n)\mathbb{E}_t\left[C_{t+1}^{-\varrho}\frac{P_t}{P_{t+1}}\right], \\ \frac{W_t}{P_t} &= \tau\frac{N_t^{\upsilon}}{C_t^{-\varrho}}. \end{aligned} \tag{A.18}$$

The Euler equation shows that expected consumption growth is a function of the real interest rate only whilst labour supply is a positive function of the real wage and of consumption.

Non–Separable Preferences

Agents' utility is log-separable in consumption and labour:

$$\frac{\left(C_t(1-N_t)^{\tau^{NS}}\right)^{(1-\varrho)}}{1-\varrho}. \tag{A.19}$$

Then:

$$\begin{aligned} U_{c,t} &= C_t^{-\varrho}(1-N_t)^{\tau^{NS}(1-\varrho)}, \\ U_{n,t} &= -\tau^{NS}C_t^{1-\varrho}(1-N_t)^{\tau^{NS}(1-\varrho)-1}. \end{aligned} \tag{A.20}$$

This implies that the Euler equation and labour supply conditions are:

$$\begin{aligned} C_t^{-\varrho}(1-N_t)^{\tau^{NS}(1-\varrho)} &= \beta(1+R_t^n)\mathbb{E}_t\left[C_{t+1}^{-\varrho}(1-N_{t+1})^{\tau^{NS}(1-\varrho)}\frac{P_t}{P_{t+1}}\right], \\ \frac{W_t}{P_t} &= \tau^{NS}\frac{C_t}{(1-N_t)}. \end{aligned} \tag{A.21}$$

In this case the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour whereas labour supply is similar to the non-separable case.

GHH Preferences

These preferences are as in Greenwood et al. (1988). With this utility function, the amount of hours worked by households will actually affect the amount of utility received from consumption, i.e. the cross-derivative of utility with respect to consumption and labour is unequal to zero.

$$\frac{1}{1-\varrho}\left(C_t-\tau^{GHH}N_t^{1+\upsilon}\right)^{1-\varrho}. \tag{A.22}$$

Then:

$$\begin{aligned} U_{c,t} &= \left(C_t-\tau^{GHH}N_t^{1+\upsilon}\right)^{-\varrho}, \\ U_{n,t} &= -\tau^{GHH}\left(1+\upsilon\right)N_t^{\upsilon}\left(C_t-\tau^{GHH}N_t^{1+\upsilon}\right)^{-\varrho}. \end{aligned} \tag{A.23}$$

This implies that the Euler equation and labour supply conditions are:

$$\begin{aligned}\left(C_t - \tau^{GHH} N_t^{1+\upsilon}\right)^{-\varrho} &= \beta(1+R_t^n)\mathbb{E}_t\left[\left(C_{t+1} - \tau^{GHH} N_{t+1}^{1+\upsilon}\right)^{-\varrho} \frac{P_t}{P_{t+1}}\right], \quad \text{(A.24)}\\ \frac{W_t}{P_t} &= \tau^{GHH}\left(1+\upsilon\right) N_t^{\upsilon}.\end{aligned}$$

In this case, like the non-separable case, the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour. But unlike the non-separable case, labour supply is a positive function of the real wage only. Therefore, as the marginal rate of substitution is independent of consumption and only depends on the real wage, there is no wealth effect on the labour supply.

A.3 Steady State

To compute the steady state of the model, we take an approach similar to Faia and Monacelli (2007). First, notice that some value steady state values can be pinned down simply by the calibrated parameters. For example, from the Euler equation of consumption notice that:

$$1 + R^n = \pi/\beta.$$

From the New Keynesian Phillips curve:

$$mc = \frac{\varepsilon - 1}{\varepsilon}.$$

From the price of capital equation:

$$q = 1.$$

Second, the entrepreneurial problem has to be solved to compute the cut-off value of the idiosyncratic productivity. In order to do that, notice that it is possible to compute the net worth to capital ratio from both the zero profit condition of banks in (19):

$$\mathcal{NK}_1 = \frac{nw}{K} = 1 - \frac{y^k}{1+R^n}\Big(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\Big)$$

and from the law of motion of net worth in (26):

$$\mathcal{NK}_2 = \frac{nw}{K} = \gamma y^k \left(1 - \Gamma(\bar{\omega})\right),$$

where remember that $y^k = (1 + R^k)/\pi$. By guessing an initial value for $\bar{\omega}$ we can compute R^k from the efficiency conditions associated with the optimal contract (21) and (22). With a simple algorithm in MatLab, it is then possible to modify $\bar{\omega}$ until the following condition $\mathcal{NK}_1 = \mathcal{NK}_2$ is satisfied. Once the steady state level of $\bar{\omega}$ is determined, R^k, y^k, ψ, $\Gamma(\bar{\omega})$, and $G(\bar{\omega})$ are also determined.

To compute the steady state value of the remaining variables, notice that from the definition of the nominal income from holding one unit of capital in equation (17):

$$z = y^k - 1 + \delta.$$

Then, we can compute the following ratios from the production function:

$$\begin{aligned}\frac{Y}{K} &= \frac{z}{\alpha \cdot mc},\\ \frac{K}{N} &= \left(\frac{Y}{K}\right)^{\frac{1}{\alpha-1}},\end{aligned}$$

from the law of motion of capital:

$$\frac{I}{K} = \delta,$$

and from the aggregate resource constraint:

$$\frac{C}{K} = \frac{Y}{K} - \frac{I}{K} - \mu G(\bar{\omega}) y^k.$$

Finally, by fixing the steady state level of hours $N = 1/3$ it is possible to solve the above equations and easily compute the remaining endogenous variables of the model.

B Appendix: Impulse response calculation

To compute the impulse responses reported in the paper we use a two steps procedure. As noted by Fernandez-Villaverde et al. (2011), the higher order approximation makes the simulated paths of states and controls in the model move away from their steady-state values. This is actually one of the results of Schmitt-Grohe and Uribe (2004): in a first-order approximation of the model, the expected value of any variable coincides with its value in the non-stochastic steady state, while in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value only by a constant.

In a third order approximation, the expected value of the variable will also depend on the variance of the shocks in the economy. Therefore, it is more informative to compute impulse responses as percentage deviations from their mean, rather than their steady state.

However, a well-known flaw of higher-order perturbations is that when the approximated decision rules are used to produce simulated time series from the model, the simulated data often display an explosive behaviour. We address the problem of explosive paths of simulated data by applying the pruning procedure by Kim et al. (2008).[17]

In the first step we simulate the model and compute the mean of the state and control variables. In particular we:

1. Draw a series of random shocks $\varepsilon_t = (\varepsilon_t^A, \varepsilon_t^W, \varepsilon_t^S)$ for T periods ($T = 4000$)
2. Starting from the steady state, perform simulation of the model using ε_t and get Y_t (i.e., the simulated data)

[17] We thank Martin Andreasen for sharing the codes for the pruning of DSGE models approximated to the 3^{rd}-order. See Andreasen et al. (2013) for details.

3. Discard the first half of observations as a burn in, and compute the ergodic mean of Y over the last $0.5 \cdot T$ periods:

$$Y_0 = \frac{\sum\limits_{0.5T+1}^{T} Y_t}{0.5T}$$

In the second step, we compute impulse responses. For example, for the macro uncertainty shock (ε_t^W) we:

1. Draw a series of random shocks ε_t^W for N periods ($N = 40$)
2. Perform simulation Y_t^1 starting from initial conditions Y_0 and using ε_t^W
3. Add one standard deviation to ε_t^W in period 1 and get $\tilde{\varepsilon}_t^W$
4. Perform simulation Y_t^2 starting from initial conditions Y_0 and using $\tilde{\varepsilon}_t^W$
5. IRF is equal to $Y_t^2 - Y_t^1$
6. Perform $R = 50$ replications of steps 1) to 5) and report the average IRF