Uncertainty in a model with credit frictions[☆]

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Abstract

Are uncertainty shocks, transmitted through financial frictions and nominal rigidities, a major source of business cycle fluctuations? This paper studies the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and to the dispersion of entrepreneurs' idiosyncratic productivity (micro uncertainty) in a financial accelerator DSGE model with sticky prices. The time series properties of macro and micro uncertainty are estimated using U.S. aggregate and firm-level data. We find that micro uncertainty shocks can account for a non-trivial share of output volatility, while macro uncertainty shocks do not. Both the degree of price stickiness and the severity of the credit friction can amplify the transmission of micro uncertainty shocks.

Keywords: Uncertainty shocks, Credit frictions, Business cycles, Micro uncertainty, Macro uncertainty, Financial accelerator.

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1 Introduction

Are uncertainty shocks, transmitted through financial frictions and nominal rigidities, a major source of business cycle fluctuations? Economists and policy makers alike have long debated this question. But recently uncertainty has been in the forefront of policy makers and economists' minds as a possible explanation for the tepid recovery in U.S. GDP and elsewhere following the collapse of Lehman Brothers.

Economists have long understood the mechanisms by which uncertainty affects key economic variables. For example, Leland (1968) and Kimball (1990) show the theoretical conditions needed for (future) uncertainty to affect consumption, later quantified empirically by Carroll and Samwick (1995) and others. Hartman (1976), Abel (1983), Bernanke (1983), Caballero (1991), and Dixit and Pindyck (1994) show the theoretical conditions needed for uncertainty to affect investment. Recently Bloom (2009) has shown that uncertainty can have sizeable effects on firms' demand for factor inputs.

Credit market imperfections can create additional channels through which fluctuations in uncertainty can affect macroeconomic outcomes. For example, when firms choose their scale before observing (uninsurable) shocks and bear the risk of a costly default, high uncertainty can lead to a reduction of factor inputs (Arellano et al., 2012); or when the relation between lender and borrower is subject to asymmetric information (leading to agency and/or moral hazard problems) an increase in uncertainty will in general raise the cost of external finance (Christiano et al., 2014, Gilchrist et al., 2014).

One important point on the notion of uncertainty is in order here. The vast majority of the recent literature has modelled uncertainty as "second-moment" shocks, i.e. changes in the variance of the shocks driving the model economy.[1] In turn, this definition of uncertainty has been used with two different notions: (i) uncertainty about *aggregate* shocks, such as the time-varying variance of the economy-wide total factor productivity; and (ii) uncertainty about *idiosyncratic* shocks, such as the cross-sectional dispersion of firm-level productivity in models with heterogeneous firms. In this paper we consider both notions of uncertainty and we refer to the former as "macro uncertainty" and to latter as "micro uncertainty".

Recent research has attempted to shed light on the role of uncertainty in driving business cycle

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[1]A notable exception is the paper by Ilut and Schneider (2014).

fluctuations using dynamic stochastic general equilibrium (DSGE) models.[2] One robust finding in the literature is that, in a closed economy, both macro and micro uncertainty shocks lead to a lack of comovement between output, consumption and investment —an important prerequisite for any shock that seeks to explain business cycle fluctuations. Basu and Bundick (2012) argue that nominal rigidities can fix this lack of comovement. When prices are sticky, in response to micro uncertainty (which depresses investment demand *via* real options effects or higher lending rates) or macro uncertainty (which depresses consumption *via* precautionary savings), prices do not fall sufficiently to keep output constant as is the case with flexible prices. Monetary policy is also responsible for generating comovement since real rates are not reduced sufficiently, thereby acting to reduce consumption and investment.

The aim of this paper is (i) to investigate the propagation of uncertainty shocks through imperfect financial markets and nominal rigidities; and (ii) to quantify the role of uncertainty in driving business cycle fluctuations. We use a general equilibrium model with sticky prices and credit frictions in the spirit of Bernanke et al. (1999). We characterize the cyclical fluctuations in macro uncertainty using aggregate total factor productivity (TFP) data for the U.S. business sector; and the cyclical fluctuations in micro uncertainty using the cross-sectional dispersion of establishment-level TFP from Census panel of manufacturing establishments. We then feed the estimated processes into the fully non-linear model solution to compute the response of the model economy to a mean-preserving shock to the variance of aggregate productivity (a "macro uncertainty shock") and to the variance of idiosyncratic productivity (a "micro uncertainty shock"). Finally, we gauge the importance of uncertainty shocks (and of the different channels through which these shocks are transmitted) for the business cycle by computing unconditional business cycle properties of some variants of the baseline model.

Our estimates of the cyclical fluctuations in micro uncertainty contribute to the debate on how to parametrize micro uncertainty processes in this class of financial accelerator models. Christiano et al. (2014) recover the time series properties of micro uncertainty (which they label a "risk shock") from macroeconomic and financial *aggregate* data through the estimation of a richer version of Bernanke et al. (1999)'s original model. Differently, in a recent paper closely related

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[2]For uncertainty about aggregate shocks see Justiniano and Primiceri (2008), Fernandez-Villaverde and Rubio-Ramirez (2010), Fernandez-Villaverde et al. (2011), Basu and Bundick (2012), Born and Pfeifer (2014), Fernandez-Villaverde et al. (2011), and Gourio (2012), and Bonciani and van Roye (2013). For uncertainty about idiosyncratic shocks see Dorofeenko et al. (2008), Bloom (2009), Gilchrist et al. (2014), Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2014). Bloom et al. (2012) and Balke et al. (2012) consider both notions of uncertainty.

to ours, Chugh (2015) use *disaggregated* plant-level data constructed by Cooper and Haltiwanger (2006) using the Longitudinal Research database to estimate the time series properties of micro uncertainty.[3] Despite the different estimation procedure, Chugh (2015)'s estimate of the volatility of micro uncertainty shocks is smaller, but of the same order of magnitude of Christiano et al. (2014)'s estimate. Using the same 'micro' approach on a different data set —that covers a larger cross-section and longer sample period— our estimates are very much in line with Chugh (2015)'s.

Impulse response analysis shows that both macro and micro uncertainty shocks generate hump-shaped responses in output, consumption and investment. Macro uncertainty is primarily transmitted through a precautionary savings channel. In the face of increased uncertainty around their future stream of income, households increase their savings and decrease their consumption. Differently, micro uncertainty shocks operate through the cost of external finance and entrepreneurial capital demand. Under asymmetric information between lender and borrower, the costly state verification problem introduces a wedge in banks' zero profit condition. In the face of increased uncertainty around entrepreneurial productivity, this wedge induces banks to raise their lending interest rates. As a result, entrepreneurs demand less capital and investment falls. Both micro and macro uncertainty shocks propagate to the rest of the economy via sticky prices, which not only are crucial for generating comovement between consumption and investment but also amplify the impact of both shocks on output. Moreover, the financial accelerator mechanism amplifies any shock that in general equilibrium affects entrepreneurial net worth.

The impact of macro and micro uncertainty shocks on economic activity, however, is strikingly different. A one standard deviation shock to micro uncertainty leads to a 0.25 percent fall in total output. This is about 25 times larger than a one standard deviation shock to macro uncertainty. Moreover, unconditional business cycle statistics obtained using simulated data from our baseline model show that micro uncertainty alone can generate about 10 percent of the total volatility of output. Differently, macro uncertainty alone do not generate any significant variation in output. Our estimates of the importance of micro uncertainty shocks fall in between the estimates from the previous studies. Using different variants of our baseline model we reconcile the available evidence on the importance of micro uncertainty shocks: the simulated business cycle statistics show that amount of output volatility generated by uncertainty shocks is increasing in the severity of credit friction and the degree of price stickiness.

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[3]Chugh (2015) builds on the analysis by Dorofeenko et al. (2008), who consider a mean preserving shock to the dispersion of firms' idiosyncratic productivity in the financial accelerator set up of Carlstrom and Fuerst (1997).

The remainder of the paper proceeds as follows: Section 2 provides some intuition for the results in the paper; Section 3 presents the model, including the sources of uncertainty; section 4 discusses the choice of parameters in the model; the estimation of the time series properties of micro and macro uncertainty; and the solution method employed; Section 5 presents the key results and Section 6 concludes. Three appendices describe the equilibrium conditions of the model; the technical details on how the impulse responses were computed; and a comparative statics exercise to gain intuition on the transmission of micro uncertainty shocks.

2 Intuition for our results

Four key ingredients drive our results. The first two are sticky prices and monetary policy which, as discussed above, are key for generating comovement between consumption, investment, hours and output in response to both macro and micro uncertainty. The next two ingredients are credit frictions and GHH preferences as in Greenwood et al. (1988), which act to amplify the effects of the shocks and do not in themselves generate comovement. On the one hand, credit frictions act to amplify the impact of both uncertainty shocks on investment since both shocks reduce the price of capital and therefore entrepreneurial net worth. On the other hand, GHH preferences prevent outward shifts in labor supply, following falls in consumption, that would act to mitigate the fall in output.

The comovement problem in DSGE models with precautionary savings is similar to the co-movement problem which arises in these models with "news shocks" models.[4] Precautionary savings, like news shocks, are equivalent to exogenous changes in consumption demand. At the heart of the comovement problem is the labor market.[5] Consider a closed economy where output, Y, can be used for consumption, C, and investment, I:

$$Y_t = C_t + I_t, \tag{1}$$

and where output is produced using predetermined capital, K, total factor productivity, A, and

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[4]In these models, agents obtain information about future shocks —news shocks. Forward-looking agents will react to these shocks but, because these shocks only materialize in the future, production can only be modified by changes to endogenous factor inputs (labour, capital services, etc). See, for example, Jaimovich and Rebelo (2009).

[5]The arguments in this section follow those in Basu and Bundick (2012), Eusepi and Preston (2009) and Wang (2012). The co-movement problem was first noted by Barro and King (1984).

labor, N:

$$Y_t = F\left(A_t, K_{t-1}, N_t\right). \tag{2}$$

These two equations show that for consumption and investment to move in the same direction, labor must do so too. Moreover, labor must move by more than the changes in consumption and investment. Given these observations, understanding the labor market is crucial for understanding the comovement problem. Equilibrium in the labor market is observed when labor demand:

$$W_t^R = F^N\left(A_t, K_{t-1}, N_t\right), \tag{3}$$

is equal to labor supply:

$$W_t^R = -\frac{U^N\left(C_t, N_t\right)}{U^C\left(C_t, N_t\right)}, \tag{4}$$

where W_t^R is the real wage, F^N the marginal product of labor, U^N the marginal (dis)utility of labor and U^C is the marginal utility of consumption. Equating the demand and supply of labor, assuming a Cobb-Douglas production function, separable preferences in consumption and labor, taking logs and ignoring constants yields:

$$a_t + \alpha k_{t-1} - \left(\alpha + \upsilon\right) n_t = \varrho c_t, \tag{5}$$

where α is the capital share; υ is the inverse of the Frisch elasticity of labor supply; ϱ is the coefficient of risk aversion; and lower case letters denote the logarithm of the variable. This equation implies that, absent an exogenous shock to TFP, consumption and hours are negatively correlated. This negative correlation is driven by the income effect of labor supply (υ) and decreasing marginal returns to labor (α) from labor demand, with the coefficient of risk aversion governing the strength of these effects *via* the substitution effect.

The presence of sticky prices introduces a wedge between labor demand and the real wage (sticky wages introduce a similar wedge but this time between labor supply and the real wage), which in logs is expressed as:

$$mc_t + a_t + \alpha k_{t-1} - \alpha n_t = w_t^R \tag{6}$$

where mc is the nominal marginal cost faced by firms. The nominal marginal cost is inversely related to firms mark-up by virtue that optimizing firms set prices as a mark-up over costs.

Denoting the price mark-up by μ_t^P and substituting the equation for labor supply we now have

$$a_t + \alpha k_{t-1} - (\alpha + \upsilon) n_t = \varrho c_t + \mu_t^P. \tag{7}$$

Just like TFP, movements in the markup are able to break the negative relationship between labor and consumption. Consider a fall in consumption brought about by an increase in precautionary savings. This leads to a reduction in firms' demand such that firms would like to lower their prices. However, due to sticky prices, firms do not decrease prices sufficiently to fully accommodate the fall in demand and markups increase. If the increase in the markup is larger than the fall in consumption, it is possible for the right hand side of (7) to be positive. As a result the labor input needs to fall. Although not shown algebraically here, monetary policy acts to amplify the effect of sticky prices. This is because with naive rules such as the Taylor rule real interest rates do not fall sufficiently to mitigate the fall in demand thereby depressing consumption and investment further. As a result markups are higher.

With GHH preferences there is no income effect in labor supply so consumption does not shift the labor supply schedule. In that case, labor market equilibrium is given by:

$$a_t + \alpha k_{t-1} - (\alpha + \upsilon) n_t = \mu_t^P. \tag{8}$$

As this equation shows, by themselves, GHH preferences do not solve the comovement problem, but mitigate it. It is the presence of sticky prices that generates the comovement. Of course, there are other mechanisms that can aid the comovement problem such as adding sticky wages or introducing additional factors of production (e.g., capital utilization). Wang (2012) provides a convenient summary of these mechanisms.

3 Model

This section outlines the baseline DSGE model that we use in our analysis. It closely resembles the BGG variant formulated by Faia and Monacelli (2007), but it is modified in two dimensions. First, we consider the role of different preferences; second, and more importantly, we consider the effect of uncertainty shocks. The model comprises optimizing households; monopolistic firms that can set prices and produce final output; capital producers that transform output into unfinished capital goods; entrepreneurs that purchase this capital, rent it to firms and are subject to a credit

friction; financial intermediaries that channel households' savings into loans for entrepreneurs; and a policy maker that sets interest rates. In what follows we consider the problems faced by each agent.

3.1   Households

There is a continuum of households, each indexed by $i \in (0, 1)$. They consume a composite final good, invest in safe bank deposits, supply labor, and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative household chooses the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and one-period nominal deposits $\{D_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{P_t, W_t, (1 + R_t^n)\}_{t=0}^{\infty}$ and the initial condition D_0 to maximize:

$$\max_{\{C_t, N_t, D_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=o}^{\infty} \beta^t U(C_t, N_t), \tag{9}$$

subject to the sequence of budget constraints:

$$P_t C_t + D_{t+1} \leq (1 + R_t^n) D_t + W_t N_t + \Pi_t, \tag{10}$$

where C_t is workers' consumption of the final good, W_t is the nominal wage, N_t is total labor hours, R_t^n is the nominal net interest rate paid on deposits, Π_t are the nominal profits that households receive from running production in the monopolistic sector.

The first order conditions of the above problem read as follows:

$$\begin{aligned} U_{c,t} &= \beta(1 + R_t^n)\mathbb{E}_t \left[U_{c,t+1} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= -\frac{U_{n,t}}{U_{c,t}}, \end{aligned} \tag{11}$$

together with $\lim_{j \longrightarrow \infty} D_{t+j}/(1 + R_t^n) = 0$ and (10) holding with equality.

3.2   Unfinished capital producers

A competitive sector of capital producers combines investment (expressed in the same composite as the final good, hence with price P_t) and existing (depreciated) capital stock to produce unfinished capital goods. This activity entails physical adjustment costs. The corresponding constant

return to scale production function is $\phi(\frac{I_t}{K_t})K_t$ where $\phi(\cdot)$ is increasing and convex. We assume the following functional form:

$$\phi(\frac{I_t}{K_t})K_t = \left[\frac{I_t}{K_t} - \frac{\phi_k}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right]K_t, \tag{12}$$

so that capital accumulation obeys to:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{13}$$

Defining Q_t as the re-sell price of the capital good, capital producers maximize profits:

$$\max_{I_t} Q_t \left[I_t - \frac{\phi_k}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t\right] - P_t I_t, \tag{14}$$

implying the following first order condition:

$$Q_t \left[1 - \phi_k\left(\frac{I_t}{K_t} - \delta\right)\right] = P_t. \tag{15}$$

3.3 Entrepreneurs

The activity of entrepreneurs is at the heart of the credit friction. These agents are risk neutral. At the end of period t, each entrepreneur j purchases unfinished capital from the capital producers at the price Q_t and transforms it into finished capital (that will be used for production in $t + 1$).

The transformation of unfinished capital into finished capital is performed with a technology that is subject to idiosyncratic productivity shocks (ω_{t+1}^j). The idiosyncratic shocks are assumed to be independently and identically distributed (*i.i.d.*) across entrepreneurs and time, and to follow a log normal distribution, namely $\omega \sim \log \mathcal{N}(1, \sigma_\omega^2)$, with cumulative distribution function denoted by $F(\omega)$. Note that, for the solution of the entrepreneurial problem, we take the variance of ω as a given parameter. However, as we shall see in section 3.8, allowing for time variation in σ_ω^2 in the solution of the model will constitute a major source of uncertainty in our economy (that we labelled micro uncertainty).[6]

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[6]Note that other papers in the earlier literature have considered a similar definition of time-varying uncertainty (or "risk") as the one used here. See, for example Christiano et al. (2003), Dorofeenko et al. (2008), Christiano et al. (2010), and Christiano et al. (2014).

To finance the purchase of unfinished capital entrepreneurs employ internal funds but also need to acquire an external loan from a financial intermediary (banks). The relationship with the lender is modelled assuming asymmetric information between entrepreneurs and banks and a costly state verification as in Townsend (1979) and Gale and Hellwig (1985). Specifically, the idiosyncratic shock to entrepreneurs is private information for the entrepreneur. To observe this, the lender must pay an auditing cost that is a fixed proportion $\mu \in [0, 1]$ of the realized gross return to capital held by the entrepreneur. The optimal loan contract will induce the entrepreneur to not misreport his earnings and will minimize the expected auditing costs incurred by the lender. Under these assumptions, the optimal contract is a standard debt contract with costly bankruptcy. If the entrepreneur does not default, the lender receives a fixed payment independent of the realization of the idiosyncratic shock; in contrast, if the entrepreneur defaults, the lender audits and seizes whatever is left. As we shall see below, for this reason the interest rate on entrepreneurial loans will be given by a spread over the risk free rate. The section below reports the derivation of the optimal contract.

3.3.1 The optimal loan contract

There are two agents, entrepreneurs and banks. At the end of period t, an entrepreneur j holds nominal net worth NW_{t+1}^j and acquires the following amount of credit to finance capital purchases:

$$B_{t+1}^j = Q_t K_{t+1}^j - NW_{t+1}^j. \tag{16}$$

Before defining entrepreneurs' problem we first need to define the expected *nominal income from holding one unit of finished capital*. Assume that, at the end of period t, an entrepreneur buys one unit of capital at price Q_t. In period $t+1$ this unit of capital is available in the rental market and the entrepreneur gets income from renting that unit to firms (Z_{t+1}) and from re–selling the undepreciated capital to capital producers at price Q_{t+1}; note moreover that, in presence of adjustment costs, the nominal income has to be adjusted for the marginal utility of holding one additional unit of capital next period. Hence, the nominal income from holding one unit of finished capital can be written as:

$$
\begin{aligned}
\mathcal{Y}_{t+1}^k &= Q_t \left(1 + R_{t+1}^k \right) = \\
&= Z_{t+1} + Q_{t+1} \left[(1 - \delta) - \frac{\phi_k}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi_k \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right]
\end{aligned} \tag{17}
$$

However, the idiosyncratic shock realizes before the beginning of period $t+1$. Entrepreneur j will repay his loans only if $\omega_{t+1}^j \mathcal{Y}_{t+1}^k K_{t+1}^j \geq B_{t+1}^j \left(1 + R_{t+1}^L\right)$ where R_{t+1}^L is the lending rate paid on loans. Therefore, the above expression defines the cut–off value of the idiosyncratic shock that separates bankrupt and non-bankrupt entrepreneurs. An entrepreneur who experiences an idiosyncratic shock equal to:

$$\omega_{t+1}^j < \bar{\omega}_{t+1}^j = \frac{B_{t+1}^j \left(1 + R_{t+1}^L\right)}{\mathcal{Y}_{t+1}^k K_{t+1}^j} \tag{18}$$

will default on his debt and the bank will seize all his remaining assets after paying the monitoring cost.

On the other hand, banks operate only if the following condition is satisfied:

$$\mathcal{Y}_{t+1}^k K_{t+1}^j \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j)\right) \geq (1 + R_t^n) B_{t+1}^j. \tag{19}$$

where $G(\bar{\omega}_{t+1}^j) = \int_0^{\bar{\omega}} \omega_{t+1}^j dF(\omega)$ and $\Gamma(\bar{\omega}_{t+1}^j) = \left[1 - \int_0^{\bar{\omega}} dF(\omega)\right] \bar{\omega}_{t+1}^j + G(\bar{\omega}_{t+1}^j)$. Note here that, as in BGG, $\Gamma(\bar{\omega}_{t+1}^j)$ is the share of finished capital going to banks. Symmetrically, $1 - \Gamma(\bar{\omega}_{t+1}^j)$ is the shared of finished capital going to entrepreneurs. Finally, $G(\bar{\omega}_{t+1}^j)$ is the average value of the idiosyncratic shock for bankrupt entrepreneurs.

The optimal contract can be derived by maximizing over $\{\bar{\omega}_{t+1}^j, B_{t+1}^j\}$ entrepreneurial profits:

$$\max_{\{\bar{\omega}_{t+1}^j, B_{t+1}^j\}} \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j)\right), \tag{20}$$

subject to the definition of borrowing (16) and the zero profit condition implied by (19) holding with equality. By equalizing the Lagrangian multipliers in the first order conditions of the above problem and using the definition of the nominal income from holding one unit of finished capital (17) we get:

$$\frac{1 + R_{t+1}^k}{1 + R_t^n} = \psi_t, \tag{21}$$

where:

$$\psi_t = \left(\frac{\left(1 - \Gamma(\bar{\omega}_{t+1}^j)\right)\left(\Gamma'(\bar{\omega}_{t+1}^j) - \mu G'(\bar{\omega}_{t+1}^j)\right)}{\Gamma'(\bar{\omega}_{t+1}^j)} + \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j)\right)\right)^{-1}, \tag{22}$$

is the *external finance premium*. As in BGG, $\psi_t = f(\bar{\omega}_{t+1}^j)$ with $f'(\bar{\omega}_{t+1}^j) > 0$. Moreover, the ratio

between the lending rate and the risk free rate gives the *risk premium*, which can be computed from the zero profit condition as:

$$\frac{1 + R_{t+1}^L}{1 + R_t^n} = \frac{\psi_t}{\bar{\omega}_{t+1}^j} \left(1 - \frac{NW_{t+1}^j}{Q_t K_{t+1}^j} \right), \tag{23}$$

where we notice that $NW_{t+1}^j / Q_t K_{t+1}^j$ is the inverse of the leverage ratio. Interestingly, equation (23) shows that, in the presence of credit market imperfections, the premium paid on the risk free interest rate for a loan depends on the entrepreneur's balance-sheet condition. Specifically, the higher the leverage, the higher is the premium charged on entrepreneurial risky loans.

Finally notice that the zero-profit condition can be written as a demand function for capital:

$$K_{t+1}^j = \left(\frac{1}{1 - \psi_t \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j) \right)} \right) \frac{NW_{t+1}^j}{Q_t}. \tag{24}$$

Demand for capital is increasing in net worth and decreasing in price.

3.3.2 Evolution of net worth

To ensure that entrepreneurs do not accumulate enough funds to finance their expenditures on capital entirely with net worth, we assume that they have a finite lifetime. In particular, we assume that each entrepreneur survives until the next period with probability γ. Entrepreneurs who "die" in period t are not allowed to purchase capital, but instead simply consume their accumulated resources and depart from the scene. Therefore, entrepreneurial consumption in each period will be:

$$C_t^e = (1 - \gamma) \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right), \tag{25}$$

where $\mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right)$ is the share of finished capital going to entrepreneurs in each period. Symmetrically, entrepreneurs who survive will accumulate net worth according to the following equation:

$$NW_{t+1}^j = \gamma \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right). \tag{26}$$

Remembering that $\mathcal{Y}_{t+1}^k = Q_t \left(1 + R_{t+1}^k \right)$, net worth is positively related to the price and the stock of capital. In contrast, as noted by Faia and Monacelli (2007), the aggregate return on finished capital R_{t+1}^k has an ambiguous impact on net worth. On the one hand, an increase in R_{t+1}^k generates a higher return for each unit of finished capital owned by entrepreneurs. On the

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other hand, an increase in R_{t+1}^k also generates an increase in the external finance premium, as showed in equation (21), which contributes to the risk premium and therefore reduces net worth.

3.4 Firms

Each domestic household owns an equal share of the intermediate-goods producing firms. Each firm assembles labor (supplied by the workers) and (finished) entrepreneurial capital to operate a constant return to scale production function for the variety i of the intermediate good:

$$Y_t = F\big(A_t, N_t(i), K_t(i)\big) \tag{27}$$

where A_t is a productivity shifter common to all firms (i.e., total factor productivity). Note that total factor productivity will be of crucial importance for the definition of our macro uncertainty shock, as discussed in section 3.8.

Each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to:

$$\frac{\omega_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_t \right)^2 \tag{28}$$

where π is the steady-state inflation rate and where the parameter ω_p measures the degree of nominal price rigidity. The higher ω_p the more sluggish is the adjustment of nominal prices. In the particular case of $\omega_p = 0$, prices are flexible.

The problem of each monopolistic firm is the one of choosing the sequence of factors of production $\{K_t(i), N_t(i)\}_{t=0}^{\infty}$ and prices $\{P_t(i)\}_{t=0}^{\infty}$ in order to maximize expected discounted real profits:

$$\max_{\{K_t(i), N_t(i), P_t(i)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=o}^{\infty} \frac{\beta^t}{P_t} \left(\begin{array}{c} P_t(i) Y_t(i) - (W_t N_t(i) + Z_t K_t(i)) - \\ \frac{\omega_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_t \right)^2 \end{array} \right), \tag{29}$$

subject to the technological constraint in (27). Let's denote by $\{mc_t\}_{t=0}^{\infty}$ the sequence of Lagrange multipliers on the above demand constraint, and by $\tilde{p}_t \equiv P_t(i)/P_t$ the relative price of variety i.

The first order conditions of the above problem read:

$$
\begin{aligned}
\frac{W_t}{P_t} &= mc_t Y_{n,t} \\
\frac{Z_t}{P_t} &= mc_t Y_{k,t} \\
0 &= Y_t \tilde{p}_t^{-\varepsilon} \left((1 - \varepsilon) + \vartheta mc_t \right) - \omega_p \left(\pi_t \frac{\tilde{p}_t}{\tilde{p}_{t-1}} - \pi \right) \frac{\pi_t}{\tilde{p}_{t-1}} + \\
&\quad + \omega_p \left(\pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} - \pi \right) \pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t^2}
\end{aligned}
\tag{30}
$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, ε is the elasticity of substitution between the $Y(i)$ goods, and where we have suppressed the superscript i, since all firms employ an identical capital to labor ratio in equilibrium. Note that the Lagrange multiplier mc_t plays the role of the real marginal cost of production. In a symmetric equilibrium it must hold that $\tilde{p}_t = 1$. This implies that $FOC(P_t)$ in (30) can be written in the form of a forward-looking Phillips curve:

$$
(\pi_t - \pi) \pi_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} (\pi_{t+1} - \pi) \pi_{t+1} \right\} + Y_t \frac{\varepsilon}{\omega_p} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right)
\tag{31}
$$

3.5 Final Good Sector

The aggregate final good Y_t is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by i, *via* the aggregate production function:

$$
Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}}.
\tag{32}
$$

Maximization of profits yields typical demand functions:

$$
Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t,
\tag{33}
$$

for all i, where $Y_t = \left(\int_0^1 P_t(i)^{-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

3.6 Monetary policy

We assume that monetary policy is conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

$$\frac{1+R_t^n}{1+R^n} = \left(\frac{1+R_{t-1}^n}{1+R^n}\right)^{\phi^r} \left(\frac{1+\pi_t^n}{1+\pi}\right)^{(1-\phi^r)\phi^\pi} \left(\frac{1+Y_t}{1+Y_{t-1}}\right)^{(1-\phi^r)\phi^y}. \tag{34}$$

The parameter $\phi^r \in [0,1)$ generates interest-rate smoothing. The parameters $\phi^\pi > 0$ and $\phi^y \geq 0$ control the responses to deviations of inflation from target π and from output growth. Given the inflation target π, the steady-state nominal interest rate R^n is determined by the equilibrium of the economy.

3.7 Market clearing

Equilibrium in the final good market requires that the production of the final good be allocated to private consumption by households and entrepreneurs, investment, and to resource costs that originate from the adjustment of prices as well as from the banks' monitoring of entrepreneurial activity:

$$Y_t = C_t + C_t^e + I_t + \frac{\omega_p}{2}(\pi_t - \pi)^2 + \mu G(\bar{\omega})\frac{\mathcal{Y}_t^k}{P_t}K_t. \tag{35}$$

3.8 Sources of uncertainty in the model

We assume that three exogenous processes drive the dynamics of our model economy. As it is standard in the literature, we assume that the *level* of total factor productivity follows an autoregressive process:

$$A_t = \rho^A A_{t-1} + e^{W_t}\sigma^A \varepsilon_t^A, \tag{36}$$

where ε_t^A follows a $\mathcal{N}(0,1)$ process and the parameter σ^A is the standard deviation of innovations to A_t (i.e., the TFP shock). The parameter σ^A is pre-multiplied by an additional process, e^{W_t}, which acts as a shifter of the variance of A_t. We refer to e^{W_t} as to the stochastic volatility of TFP. We also assume that W_t follows an autoregressive process of the type:

$$W_t = \rho^W W_{t-1} + \sigma^W \varepsilon_t^W, \tag{37}$$

where ε_t^W follows a $\mathcal{N}(0,1)$ process and the parameter σ^W is the standard deviation of innovations to W_t.

By allowing the variance of TFP shocks to rise, the probability of events that are distant from the mean increases. In the face of an increase in uncertainty, economic agents are likely to modify their behavior even though the mean outcome is unchanged (i.e., there are no first moment shocks to TFP). We define *macro uncertainty shocks* exogenous changes in the variance of TFP (i.e., movements in W_t) that do not affect its level. Figure 1 shows the difference between a TFP shock and a macro uncertainty shock.
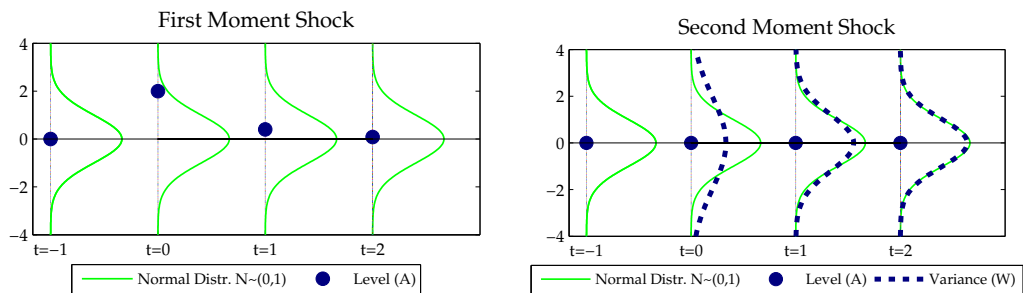
Figure 1 A MACRO UNCERTAINTY SHOCK. The charts illustrate the effect of a TFP shock (left hand side) and a macro uncertainty shock (right hand side) hitting at $t = 0$. A TFP shock (first moment shock) consists of an increase in the level of A_t, while a macro uncertainty shock (second moment shock) consists of an increase in the variance of A_t, without affecting its mean.

The last source of uncertainty in our model is the dispersion of idiosyncratic entrepreneurial productivity. As introduced by Dorofeenko et al. (2008) and Christiano et al. (2014) —and deviating from BGG— we allow the variance of the idiosyncratic shocks to vary over time. Note that, if ω is log-normally distributed with $\omega \sim \log \mathcal{N}(1, \sigma_\omega^2)$, then the log of ω is normally distributed, i.e. $\log(\omega) \sim \mathcal{N}(M, S^2)$, where M and S^2 are the mean and the variance of the underlying normal distribution. For technical purposes, it is easier to model the variance of the underlying Normal distribution, which —after fixing the mean of ω to 1— is defined as $S^2 = \log(1 + \sigma_\omega^2)$. As in Christiano et al. (2014), we model the log-deviation of S_t from its steady state value as:

$$\log\left(\frac{S_t}{\bar{S}}\right) = \rho^S \log\left(\frac{S_{t-1}}{\bar{S}}\right) + \sigma^S \varepsilon_t^S, \tag{38}$$

where ε^S follows a $\mathcal{N}(0,1)$ and σ^S is the standard deviation of innovations to S_t.

Therefore, when S_t increases, the dispersion of entrepreneurial outcomes increases too. Despite leaving the mean of the outcomes unaffected, an increase in S_t will have an impact on the

conditions in the entrepreneurial loans market. Intuitively, a higher dispersion of returns implies, *ceteris paribus*, a higher probability of entrepreneurial bankruptcy. Given the information asymmetry between banks and entrepreneurs and the costly state verification, this will affect the level of lending rates and, therefore, of capital demand. We refer to the next section for a better description of the mechanism through which micro uncertainty is transmitted to the real economy.

Figure 2 displays the effect of an increase in the variance of the idiosyncratic shock to entrepreneurs. We refer to exogenous movements in S_t as to *micro uncertainty shocks*.
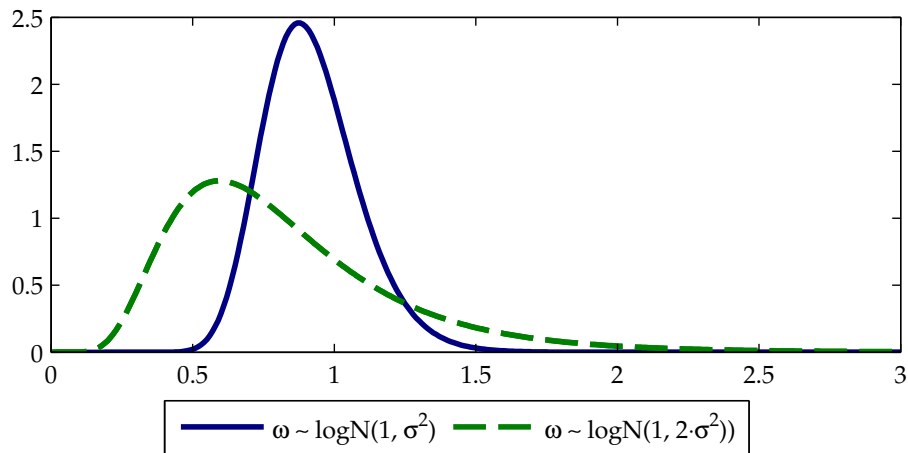
Figure 2 A MICRO UNCERTAINTY SHOCK. The chart illustrates the effect of a micro uncertainty shock. An increase in the variance of the idiosyncratic shock to entrepreneurs (ω) changes the shape of the distribution, shifting the mass of the distribution to the left tail (dashed line), without affecting the mean of the distribution.

4   Calibration and solution of the model

In this section we describe how we pin down the parameters of the model. We partition the parameter space in two sets. The first set contains the deep parameters of the model, while the second set contains parameters relating to the exogenous processes. Finally, we discuss the methodology we use to solve and simulate the model.

4.1   Parameters of the model

The time unit is a quarter. We need to make assumptions on both the standard parameters of New Keynesian DSGE models (such as economic agents' preferences, degree of price stickiness

and monopolistic competition, etc) and on the parameters relating to the credit friction. Table 1 summarizes the parameter values.

Table 1 Parameters of the model

General			
Monitoring Cost	μ	0.25	Carlstrom and Fuerst (1997)
Survivial Probability	γ	0.985	Christiano et al. (2014)
Capital Share	α	0.3	Labor share of 70%
Depreciation Rate	δ	0.025	Investment/output of 18%
Discount Factor	β	0.994	Annual real rate of 2.4%
Risk Aversion	ϱ	2	Standard
Inv. Firsh Elasticity	ν	1	Christiano et al. (2014)
GHH Scaling Factor	τ	2.5	Steady-state hours ($N = 1/3$).
Mark-up	ϵ	11	Standard
Rotemberg	θ	105	Calvo price stick. of 0.75
Investment Adj. Cost	ϕ_k	1.5	Calibrated (invest. volatility)
Steady State Inflation	π	2%	Fernandez-Villaverde et al. (2011)
Monetary policy			
Int. Rate Smoothing	ρ_r	0.25	Standard
Output	ρ_y	0.5	Standard
Inflation	ρ_π	1.5	Standard

The parameters relating the credit friction are set so as to obtain reasonable steady state values for some key financial variables, namely the external finance premium and the entrepreneurial default rate.

In order to do that, we first need to fix two parameters to pin down the solution of the entrepreneurial problem defined in section 3.3.1. The annual steady state inflation, π, is set to 2 percent; and the time discount factor, β, is set to 0.994 so as to target an annualized average real risk–free rate of interest of 2.4 percent, similar to Fernandez-Villaverde et al. (2010).

Turning now to the parameters relating to the credit friction, we set the steady state value of the quarterly survival rate of entrepreneurs γ to 0.985, the same value used by Christiano et al. (2014) and fairly similar to the value originally used by BGG; the monitoring cost μ to 0.25 as in Carlstrom and Fuerst (1997), and close to the value estimated by Christiano et al. (2014) at 0.21; and, finally, the steady state value of the standard deviation of the idiosyncratic productivity $\bar{S}$ to 0.225, slightly lower but very close to the value estimated by Christiano et al. (2014) (namely, 0.26). This parametrization yields reasonable values for our target variables. The quarterly, steady state probability of default is of about 1 percent, very close to 0.974 percent value used in

Carlstrom and Fuerst (1997) and Fisher (1999), and not far from the original 0.75 percent value used by BGG; finally, the implied steady state external finance premium is of about 188 basis points, almost identical to the value used by Carlstrom and Fuerst (1997). Moreover, the steady state value of leverage ratio implied by the above calibration is of about 2 —the same value used in BGG.

Household preferences are given by a GHH utility function (see Appendix A for a description of the functional form of GHH preferences). As is commonly done in the literature, we set the coefficient τ so that the value of hours worked is equal to $1/3$ in the steady state. Also, the coefficient of risk aversion in the utility function ϱ is fixed to 2 as in Fernandez-Villaverde et al. (2011), while the inverse of the Frisch elasticity of labor supply υ is fixed to 1 as in Christiano et al. (2014). We assume the production technology to have a Cobb-Douglas form with constant returns to scale. Without deviating from the standard values used in the literature, we set the quarterly aggregate capital depreciation rate δ to 0.025 and the capital's share α to 0.3.

The elasticity of substitution across varieties in the CES aggregator (ε) is set to be 10, consistent with a price markup of roughly 11 percent, as in Born and Pfeifer (2014). Since the parameters associated with the adjustment costs and nominal rigidity cannot be pinned down by the deterministic steady state (because all adjustment costs are zero), we assign conventional values to these parameters following the literature. As noticed by Faia and Monacelli (2007), it is possible build a mapping between the frequency of price adjustment in the Calvo–Yun model and the degree of price stickiness ω_p in the Rotemberg setup. By log-linearizing equation (31) it is possible to derive the elasticity of inflation to the real marginal cost and compare it with empirical studies on the New-Keynesian Phillips curve, such as Gali and Gertler (1999) and Carlstrom et al. (2010). The Rotemberg price adjustment parameter, ω_p, is chosen such that, in an equivalent Calvo price-setting model, prices are fixed for 4 quarters on average. The above calibration implies the following great ratios in steady state: consumption over total output is roughly 76 percent; investment over total output is 18 percent, and entrepreneurial consumption over total output is 6 percent.

The coefficients on the Taylor rule are standard, namely 1.5 for the coefficient on inflation, ϕ^π, and 0.5 for the coefficient on output growth, ϕ^y. We set the interest-rate smoothing parameter, ϕ^r, to 0.25, similar to Fernandez-Villaverde et al. (2010) and Born and Pfeifer (2014).

4.2 Exogenous processes

We estimate from the data the persistence and standard deviation of both macro and micro uncertainty. The exogenous process for the level of TFP is instead calibrated to match persistence and standard deviation of HP filtered U.S. GDP data. Our procedure yields sensible parameter values which are in line with other studies in the literature. Table 2 summarizes the persistence and standard deviation of the exogenous processes in our model. The details of our procedure are instead reported below.

Table 2 Parameter values of the exogenous processes in the model

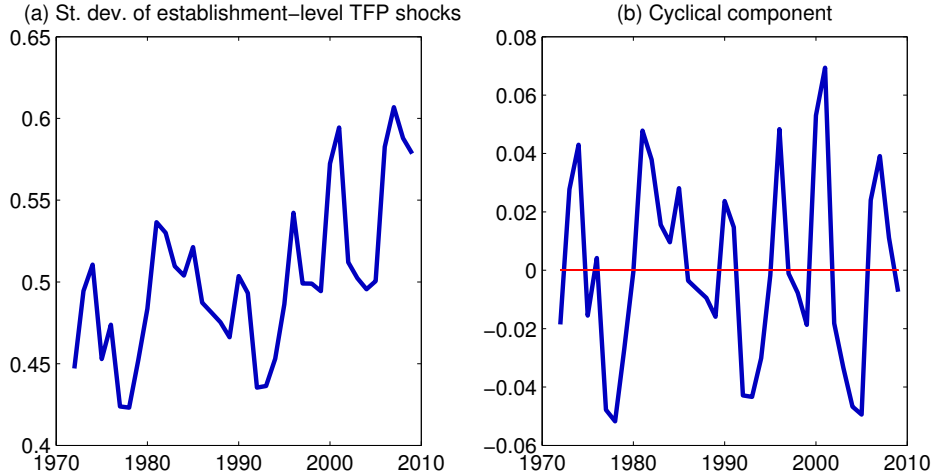| **Shocks**			
Persistence TFP	ρ^A	0.95	King and Rebelo (1999)
Std. Dev. TFP	σ^A	0.009	Calibrated (output volatility)
Persistence of Micro Uncert.	ρ^S	0.79	Data
St. Dev. of Micro Uncert.	σ^S	0.025	Data
St. Dev. of Micro Uncert. (SS)	S	0.225	Calibrated (ext. fin. premium)
Persistence of Macro Uncert.	ρ^W	0.63	Data
St. Dev. of Macro Uncert.	σ^W	0.048	Data

Let us discuss micro uncertainty first. As a proxy for the dispersion of the idiosyncratic productivity of entrepreneurs we use the cross-sectional standard deviation of establishment-level TFP shocks. This measure of uncertainty (which we label σ_t^{micro}) is made available by Bloom et al. (2012), who compute it using annual data from the Census panel of manufacturing establishments over the sample period 1972–2009.[7] We then use σ_t^{micro} to parametrize the micro uncertainty process in equation (38), closely following the careful procedure used by Chugh (2015). Relative to his paper, however, we use a different data set that covers a longer sample period (including the global financial crisis) and a larger number of cross-sectional units.[8]

Figure 3 (left panel) displays the cross-sectional standard deviation of establishment-level TFP shocks (σ_t^{micro}), which displays a modest upward trend. We consider deviations of σ_t^{micro} from an HP-filter as the model-consistent measure of micro uncertainty, i.e. as a proxy for S_t. The cyclical component of the cross-sectional standard deviation of establishment-level TFP shocks

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[7]The data is available at the following website: http://www.stanford.edu/~nbloom/index_files/Page315.htm. It includes data on over 50,000 establishments from 1972 to 2009. Bloom et al. (2012) focus on a sub-set of 15,673 establishments with 25+ years of data. To measure uncertainty, they first calculate establishment-level TFP. Then, they define TFP shocks as the residual from the regression of log(TFP) at year t+1 on its lagged value (year t), a full set of year dummies and establishment fixed effects.

[8]Chugh (2015) uses annual data of plant-level profitability constructed by Cooper and Haltiwanger (2006) from the Longitudinal Research Database (LRD). The data set covers approximately 7,000 large U.S. manufacturing plants over the period 1974-1988.

(S_t) is reported in Figure 3 (right panel). The standard deviation of S_t over the 1972–2009 sample is 3.22 percent, a number very much in line with Chugh (2015)'s estimate of 3.15 percent. We then estimate equation (38) with OLS. The point estimate of the autoregressive parameter, ρ^S, is 0.40 with a t-statistic of 2.60. Given standard deviation of 3.22 percent, the standard deviation of the (annual) micro uncertainty innovations, σ^S, can be computed to be 2.95 percent.

Figure 3 Micro Uncertainty: Cross-sectional Dispersion Of TFP Shocks. The left panel plots the cross-sectional standard deviation of establishment-level TFP shocks ($sigma_t^{micro}$) from Bloom et al. (2012). The right panel plots the deviation of $sigma_t^{micro}$ from an HP trend with smoothing parameter equal to 100, i.e. our proxy for S_t.

The annual frequency of the establishment-level TFP data poses a challenge for the calibration of the model which, instead, is at quarterly frequency. To address this mismatch, we follow the approach used by Chugh (2015). Assuming sufficient smoothness in the micro uncertainty process, we compute the quarterly persistence parameter as $\rho^S = 0.40^{0.25} = 0.79$. To set the parameter σ^S we proceed as follows. We simulate data from the quarterly model and time-aggregate the simulated data, varying σ^S so that the standard deviation of the micro uncertainty process matches its annual empirical counterpart. The procedure yields $\sigma^S = 0.025$.

Our estimates fall in between the values found by similar studies in the literature. Bloom et al. (2012) assume a two-point Markov chain process for micro uncertainty, where idiosyncratic volatility is set to a low value of 0.039 (which approximately triples in the heightened uncertainty state). Christiano et al. (2014) derive the (unanticipated component of) the standard deviation of micro uncertainty innovations directly from their DSGE model through Bayesian estimation techniques using U.S. aggregate macro-financial data: they find a value for σ^s of 0.07. Chugh (2015) finds a value of 0.037. Despite the different data and sample period, this value is remark-

ably close to what we find (especially considering that the small difference with our estimate is mainly due to the different degree of persistence of S_t in the Census panel of manufacturing establishments).

As a proxy for macro uncertainty we use the conditional heteroskedasticity of the Solow residual (as in Bloom et al. (2012)). Specifically, we estimate the conditional heteroskedasticity of the growth rate of quarterly TFP for the U.S. business sector (which we label σ_t^{macro}) with a GARCH(1,1) model over the 1972Q1-2009Q4 period.[9] The estimated series of σ_t^{macro} is reported in Figure 4 (left panel) and displays a modest downward trend during the Great Moderation period. We consider deviations of σ_t^{macro} from an HP-filter as the model-consistent measure of macro uncertainty, i.e. as a proxy for W_t. The cyclical component of the conditional heteroskedasticity of the growth rate of quarterly TFP (W_t) is reported in Figure 4 (right panel). We then estimate equation (37) with OLS. According to the AR(1) estimation, we set the persistence of macro uncertainty (ρ^W) to 0.63 and its standard deviation (σ^W) to 0.048. Our parametrization of the macro uncertainty process is robust to these alternative methodologies and it yields parameter values that are very similar to the ones used by similar studies in the literature, such as Bloom et al. (2012) and Caldara et al. (2012).[10]

Finally, we set the persistence of TFP shocks (ρ^A) to 0.95 and we use the standard deviation of TFP shocks (σ^A) as a free parameter to match the volatility of HP filtered U.S. GDP data over the 1972Q1-2009Q4 period. Specifically, we set $\sigma^A = 0.009$, which implies a standard deviation of HP filtered GDP of 1.58 percent in our model simulations relative to a value of 1.55 in the data.[11] Our values of both the persistence and the standard deviations of TFP are standard for the time series properties of the Solow residual. Indeed, our parametrization is in line with many similar papers in the literature, such as Fernandez-Villaverde et al. (2011), Bloom et al. (2012), and Christiano et al. (2014).

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[9]The data can be downloaded at the following webpage: http://www.frbsf.org/economic-research/total-factor-productivity-tfp/. See Fernald (2012), Fernald and Matoba (2009), and Kimball et al. (2006).

[10]For robustness, we proxied W_t with the conditional variance of a 20-quarter rolling window AR(1) model on the growth rate of the Solow residual; and also with a stochastic volatility AR(1) model estimated with Bayesian techniques as in (as in Primiceri, 2005).

[11]To obtain the moments implied by the model, we simulate the model economy for 2000 periods. We then use the last 152 periods (i.e., the same number of observations that we have in the data, from 1972Q1 to 2009Q4) to compute the standard deviation of the log-difference of output from its HP trend (computed with smoothing parameter 1600).
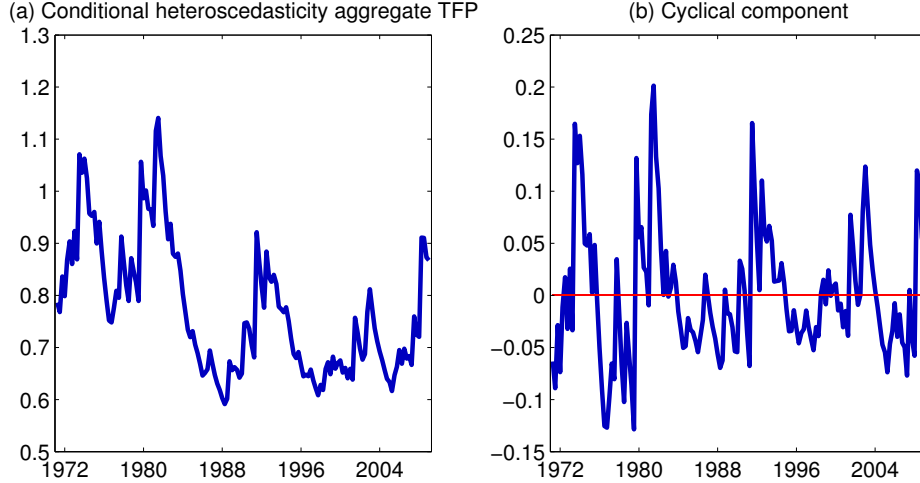
Figure 4 Macro Uncertainty: Conditional Heteroskedasticity Of Aggregate Productivity (TFP). The left panel plots the conditional heteroscedasticity of the growth rate of quarterly TFP (σ_t^{macro}) estimated using a GARCH(1,1) model using the TFP data from Kimball et al. (2006). The right panel plots the deviation of σ_t^{macro} from an HP trend with smoothing parameter equal to 1600, i.e. our proxy for W_t.

4.3 Solution method

DSGE models are normally solved by taking a linear (i.e., first-order) approximation around their non-stochastic steady state equilibrium. However, when using the traditional linear approximation, shocks to the variance of exogenous processes do not play any role, since the certainty equivalence holds and the decision rule of the representative agent is independent of shocks' second (or higher) moments.[12]

For second (or higher) moments to enter the decision rules of economic agents, higher approximation to the policy functions are needed. For example, when taking an approximation to the 2nd-order of the solution of the model, second moment shocks (as the ones defined above) would only enter as cross-products with the other state variables. This implies that a shock that affects the variance of an exogenous process can have an impact on the dynamics of the model only when also its mean is affected.

In contrast, a 3rd-order Taylor expansion of the solution of the model, allows for second moment to play an independent role in the approximated policy function. Therefore, 3rd-order Taylor approximation shall allow us to simulate and evaluate the effect an uncertainty shock, while holding constant its level. Fernandez-Villaverde et al. (2010) provide a detailed discussion.

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[12]Micro uncertainty shocks (as the ones considered in this paper) and the Knightian uncertainty shocks (as in Ilut and Schneider, 2014) represent an exception and their impact can be studied with standard linear methods.

To solve the baseline model, we use Dynare 4.3. Dynare computes 3rd-order Taylor series approximation around the non-stochastic steady state of the model. As Fernandez-Villaverde et al. (2011) show, 3rd-order approximation to the policy functions is sufficient to capture the dynamics of the model, with little gain to using an approximation higher than 3rd-order.

Impulse responses functions (IRFs) are defined as the reaction of the variables of a dynamic system to an exogenous impulse of a given size. Generally, we compute them using the equilibrium of the dynamic system (i.e., steady state) as an initial condition. This is because, in linear models, IRFs do not depend on the state of the economy when the shock occurs, nor on the sign and size of the shock. However, when using a higher order approximation to the solution of the model, this is not the case anymore and impulse responses computed from the steady state are just one of the many IRFs of the non–linear model.

Moreover, an additional complication arises since —in a model approximated to the 3rd order featuring uncertainty shocks— the mean of the ergodic distributions of our endogenous variables will in general be different from their deterministic steady-state values. Fernandez-Villaverde et al. (2011) show it through model simulations and propose to compute impulse responses using the ergodic mean of the data generated by the model as an initial condition. We follow their approach and we refer the reader to Appendix B for details on how we construct our impulse responses.

5 Results

In this section we analyze the effect of both macro and micro uncertainty shocks. We shall focus on two amplification mechanisms that can affect the transmission of both shocks, namely sticky prices and the credit friction. On the one hand, sticky prices help generating comovement between consumption and investment and, by doing so, also amplify the effect of the shocks. On the other hand the presence of the financial accelerator implies that impact of any shock that in general equilibrium affects entrepreneurial net worth will get amplified.

To get insights on these amplification mechanisms we compare the impulse responses from our baseline with two alternative sets of impulse responses that we obtain by varying the degree of price stickiness and the severity of the financial friction in our model economy, respectively.

We shall do so in two separate subsections, first for macro uncertainty shocks and then for micro uncertainty shocks.

Finally, we also conduct a numerical experiment to evaluate the quantitative importance of uncertainty shocks for the business cycle.
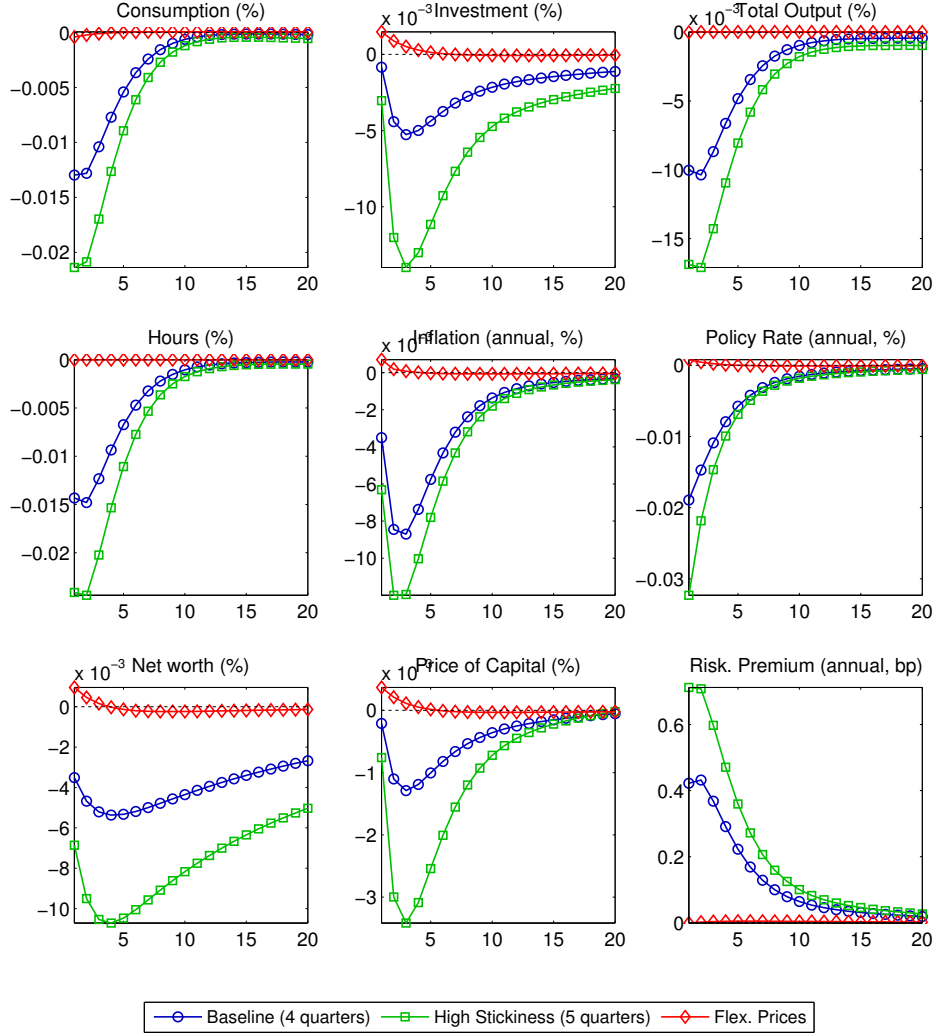
5.1 A macro uncertainty shock

We analyze the impulse responses to a 1 standard deviation increase in the macro uncertainty innovation (ε_t^W) in our model. This is equivalent to an increase in macro uncertainty of 4.8 percent.

As highlighted in the theoretical analysis in Section 2, sticky prices are crucial for generating comovement between consumption and investment, as well as amplifying the response of output in response to the macro uncertainty shock. To better understand the transmission of the shock and the role of price stickiness, in Figure 5 we consider our baseline calibration (circles); a flexible price version of the model (diamonds), obtained by setting the Rotemberg parameter $\omega_p \simeq 0$; and a version of the model with ω_p calibrated as to obtain an average probability of changing prices of 5 quarters (squares), instead of 4 quarters as in the baseline.

We focus first on the impulse responses obtained under flexible prices (diamonds). The shock acts to reduce consumption *via* precautionary saving. Since capital is predetermined output can only change in response to movements in labor. However, under flexible prices and constant markups, the labor demand schedule is unchanged. Likewise, the labor supply schedule is fixed under GHH preferences: in our model —and differently from Basu and Bundick (2012)— the macro uncertainty shock does not generate an impact increase in "precautionary labor supply", since consumption does not enter the labor supply schedule. Therefore, since both hours and wages do not move in the first period, we also do not observe any movement in output in response to the shock. As a result, since output is unchanged on impact, the lower level of consumption —i.e., households' additional saving in response to the shock— is channelled toward higher investment.

These results are clearly inconsistent with business cycle facts where both consumption and investment tend to move in the same direction as total output. Also, they are inconsistent with the empirical evidence on uncertainty shocks, where an increase in uncertainty is generally found

Figure 5 MACRO UNCERTAINTY SHOCK - THE ROLE OF PRICE STICKINESS. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (the level of TFP is held constant). The IRFs display the impact of a macro uncertainty shock under different degrees of price stickiness and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

to negatively affect both consumption and investment. Moreover, under this parametrization, note that the financial accelerator mechanism amplifies the positive response of investment and makes the comovement problem even worse. This is because the additional investment acts to boost the price of capital which in turn boosts entrepreneurs' net worth and thereby increases demand for capital.

We now describe the impulse responses obtained under sticky prices in our baseline calibration (circles). As under flexible prices, the increase in uncertainty reduces consumption through a precautionary savings channel. However, since prices are sticky in the short run, the shock puts downward pressure on the marginal cost faced by firms which implies an increase in firms'

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markups and a reduction in labor demand (see Basu and Bundick (2012)). Moreover, with GHH preferences labor supply is fixed and the fall in labor demand necessarily leads to a fall in hours and in the real wage. In other words, since under price stickiness output is demand determined (i.e., firms must satisfy whatever output is demanded at a given price), the reduction in consumption from the precautionary saving motive acts to reduce aggregate demand. Hence, demand for both labor and capital falls, and investment falls too.

Sticky prices act as a powerful amplifying mechanism: the higher price stickiness, the higher the increase in markups and the fall in labor demand. As a result, hours worked fall and consumption falls even further, therefore amplifying the effect of the shock. In Figure 5 we also report impulse responses using a version of the model calibrated as to obtain an average probability of changing prices of 5 quarters, instead of 4 quarters as in the baseline. Under high stickiness (squares), the effect of the macro uncertainty shock on output is almost twice as big as in the baseline (circles). Note, however, that in addition to sticky prices three ingredients drive the dynamics of the responses to the macro uncertainty shock. First, in contrast with the flex-price case, the financial accelerator now helps the comovement: in response to the lower price of capital, entrepreneurs' net worth falls and the risk premium increases, thereby depressing investment even further. Second, GHH preferences also act to depress consumption further: hours worked now enter the Euler equation for consumption such that a fall in the growth rate of hours acts to reduce consumption growth (see the appendix for a comparison between different functional forms for consumers' preferences). Third, in response to weaker demand, with falling prices and depressed output, monetary policy accommodates the shock by reducing the policy rate, thus supporting consumption.[13]

The resulting impact on output is, however, rather small: in our baseline, output falls by 0.01 percent and the impact on financial variables is small, too. The risk premium increases by about 0.4 annualized basis points, while the price of capital displays a maximum fall of about 0.001 percent over the 20 quarters considered. This is certainly not consistent with the behavior of macro-financial variables as we observed in the post-Lehman period.

What is the role of the credit friction in the transmission of the macro uncertainty shock? As suggested above, the financial accelerator mechanism amplifies the impact of any aggregate shock

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[13]As noted by Leduc and Liu (2012), this shows that macro uncertainty shocks largely resemble to aggregate demand shocks.

that in general equilibrium affects the net worth of entrepreneurs. Therefore, in periods where financial market distortions are more severe, macro uncertainty shocks could have a larger impact on economic activity and generate dynamics of risk premia and asset prices that are more in line with what we observed during the recent crisis. To investigate this mechanism further, Figure 6 compares our baseline results (circles) against a case where credit frictions are sensibly reduced (diamonds) and a case where frictions are more pronounced (squares). These alternative cases are computed by modifying the value of the monitoring cost parameter (μ), where remember that the higher the monitoring cost the more severe is the credit friction in the economy.
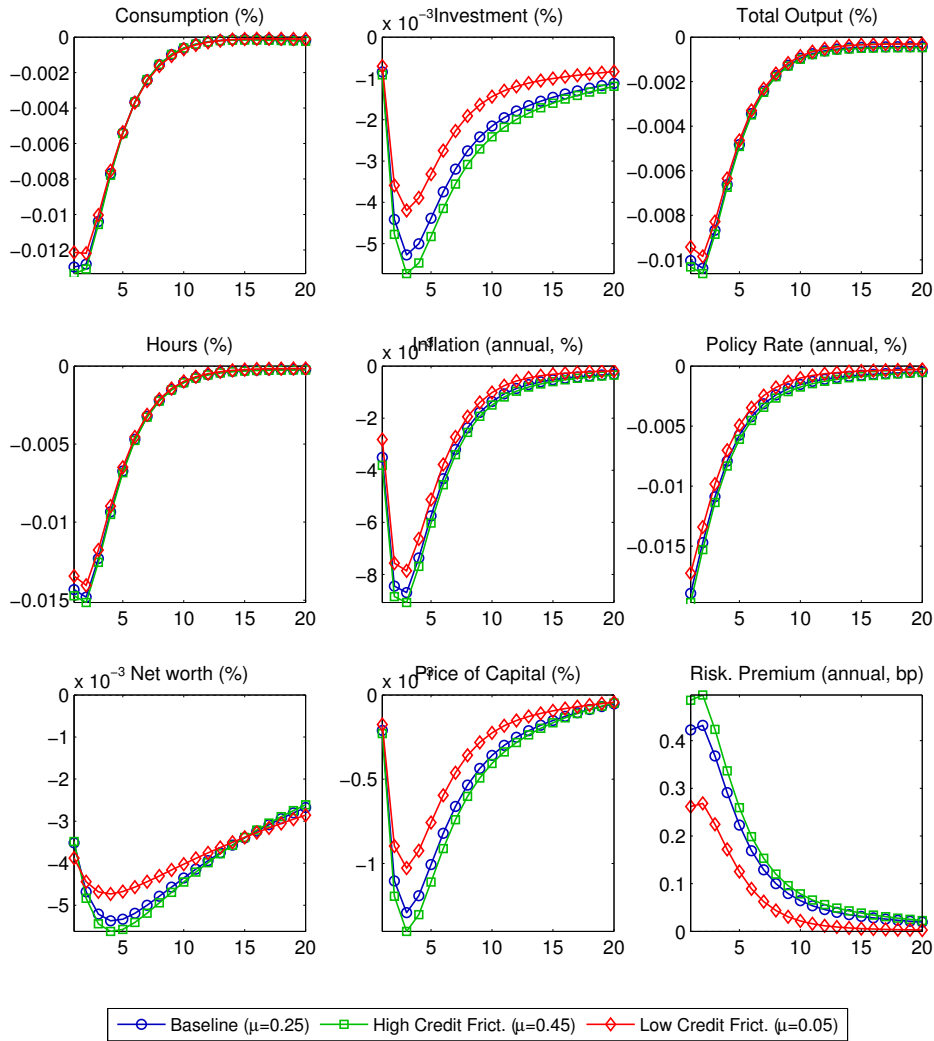
Figure 6 MACRO UNCERTAINTY SHOCK - THE ROLE OF CREDIT FRICTIONS. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (the level of TFP is held constant). The IRFs display the impact of a macro uncertainty shock under different degrees of tightness of the credit friction and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

When credit frictions are more pronounced (squares), uncertainty shocks tends to have a

larger impact on investment and on financial variables (i.e., net worth, the price of capital and the risk premium). However, and somewhat surprisingly, the effect on total output does not seem responsive to changes in the severity of the credit friction. This result can be accounted for by the transmission mechanism of the macro uncertainty shock and the nature of the financial accelerator. As already noted above, in the face of an exogenous increase in macro uncertainty consumers increase their precautionary savings and consumption falls. Since there are no credit frictions directly affecting households, however, the fall of consumption is very similar in the three cases that we consider, i.e. is irrespective of the severity of the credit friction. Differently, investment —which, as noted above, co-moves with consumption because of price rigidities— gets amplified by the financial accelerator mechanism. The shock in fact reduces the price of capital and entrepreneurial net worth and increases the risk premium. However, since the impact on investment is small relative to the impact on consumption, the amplifying role of the credit friction is almost indiscernible.

These results suggest that in our model (i) macro uncertainty shocks do not seem to have a large impact on economic activity; and (ii) credit frictions *per se* do not amplify the effect of macro uncertainty shocks on total output. Since the macro uncertainty shock is primarily transmitted through consumption (*via* precautionary savings), it has little impact on entrepreneurial net worth and, therefore, there is little amplification. As a result, the profiles of output, consumption, hours worked and inflation do not display substantial differences to changes in the tightness of the credit friction.

5.2   A micro uncertainty shock

We turn now to the analysis of micro uncertainty shocks. We consider a 1 standard deviation increase in the micro uncertainty innovation (ε^S), which is equal to an increase in micro uncertainty of 2.5 percent.

As for the macro uncertainty shock, Figure 7 displays the impulse responses under our baseline calibration (circles); a flexible price version of the model (diamonds), obtained by setting the Rotemberg parameter $\omega_p \simeq 0$; and a version of the model with ω_p calibrated as to obtain an average probability of changing prices of 5 quarters (squares), instead of 4 quarters as in the baseline.
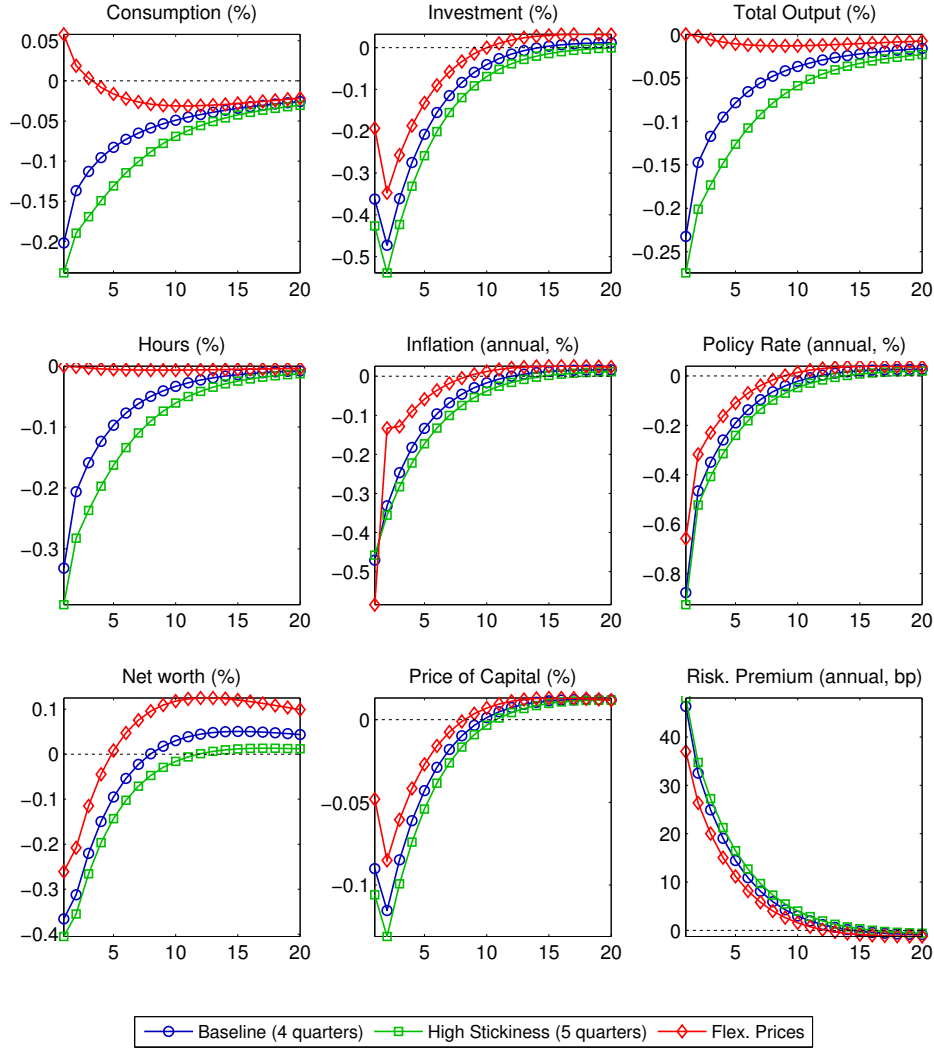
Figure 7 Micro Uncertainty Shock - The Role Of Price Stickiness. Impulse response functions (IRFs) to a 1 standard deviation increase in micro uncertainty. The IRFs display the impact of a micro uncertainty shock under different degrees of price stickiness and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

The impulse responses in Figure 7 show that the transmission of micro uncertainty shocks to the real economy is noticeably different from macro uncertainty shocks. Whilst the macro uncertainty shock operates through precautionary savings and propagates to the rest of the economy as a demand shock *via* sticky prices, the micro uncertainty shock operates through the cost of external debt and entrepreneurial capital demand. Then, similarly to the macro shock, the micro uncertainty shock propagates to the rest of the economy *via* sticky prices.

Specifically, higher dispersion of the idiosyncratic shock implies larger returns for some entrepreneurs and larger losses for other entrepreneurs. All else equal, this implies higher bankruptcy rate. With no credit frictions this would have no impact on the model economy, since banks'

expected return has not changed and both entrepreneurs and banks are risk neutral. Under asymmetric information, however, the costly state verification problem introduces a wedge (the monitoring cost) in banks' zero profit condition: a higher default rate (due to those entrepreneurs experiencing lager negative shocks) increases the expected costs for banks which as a result will charge higher lending rates. This in turn generates a fall in capital demand and hence in investment. A more detailed description of these mechanisms is explained in Appendix C with a simple comparative statics exercise.

The difference between micro and macro uncertainty shocks becomes clear when looking at the flex price economy (diamonds). Differently from the macro shock, which affects consumption, the micro uncertainty shock depresses investment. Increased micro uncertainty generates an increase in the cost of external finance since the expected cost associated with bankruptcies is now larger. Higher lending rates imply lower capital demand, therefore generating a sharp fall in investment and in the price of capital. However, symmetrically to what we observed for the macro shock, under flexible prices lower investment leads to higher consumption.

As for the macro uncertainty shock, price stickiness helps generating the comovement between consumption and investment. In our baseline (circles), weaker capital demand now acts to reduce output and, therefore, also consumption. As for macro uncertainty shocks, moreover, sticky prices act as a powerful amplifying mechanism: as Figure 7 shows, the higher high the degree of price stickiness (squares) the larger the impact on output.

As Christiano et al. (2014) note, the shock resembles an increase in the tax rate on the return on investment which should act to discourage saving (and hence investment) and boost consumption or leisure. But, in our model, there are three factors that discourage consumption. First, the response of monetary policy is such that the real rate does not fall sufficiently to encourage households to consume. Second, the fall in output leads to a fall in hours, which with GHH preferences act to decrease marginal utility on impact relative to the future. To mitigate such a fall in marginal utility, consumption falls. This effect is greater in the case of GHH preferences compared to the case of non-separable preferences.[14] Third, credit frictions act as an amplifier: lower entrepreneurial net worth and a higher external finance premium imply even weaker capital demand and lower investment.

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[14]The results with different specifications of households' preferences are not reported here for reasons of space but are available from the authors upon request.

Finally, in addition to the different transmission mechanism, an important difference between micro and macro uncertainty shocks lies in the magnitude of the response of total output to the shock, which is now much larger. As displayed in Figure 7, in our baseline a 1 standard deviation shock to micro uncertainty leads to a fall of about 0.25 percent of total output, an impact which is 25 times larger than the macro uncertainty shock. This larger impact does not apply only to output: the risk premium now increases by about 50 basis points, net worth falls by 0.35 percent and the price of capital falls by 0.1 percent.

Can credit market imperfections account for such a large difference? Or, in other words, what is the role of the credit friction for the transmission of micro uncertainty shocks? Figure 8 compares our baseline results (circles) against the case where credit frictions are sensibly reduced (diamonds) and against the case where frictions are more pronounced (squares).

The role of credit frictions in the transmission of micro uncertainty shocks can be easily understood with the simple partial equilibrium example used above. Since the monitoring cost introduces a wedge in banks' zero profit condition, a mean preserving shock to the variance of idiosyncratic productivity increases banks' expected costs and induces them to raise the spread they charge on lending interest rates. It follows that when credit frictions are less severe —i.e., when the monitoring cost is lower— the effect of micro uncertainty shock on total output should be lower. And, in the limit case where the monitoring cost is equal to zero —e.g., assuming that the both the entrepreneur and the bank could be costlessly observe idiosyncratic shocks— the impact of a micro uncertainty shock would also tend to zero.

Figure 8 shows that this is indeed the case: unlike macro uncertainty, credit frictions greatly magnify micro uncertainty shocks. Specifically, when reducing the severity of the credit friction (i.e., the monitoring cost is reduced from 0.25 to 0.05), the impact of a micro uncertainty shock on the risk premium and on investment reduces substantially. When the degree of credit frictions is low (diamonds), investment falls by one half relative to baseline, while the risk premium increases by only 20 basis points (relative to an increase of 45 basis points in the baseline). The impact on net worth and on the price of capital reduces substantially, too. Total output —which falls by about 0.25 percent in our baseline— falls by less than 0.15 percent when the severity of the credit friction is reduced.
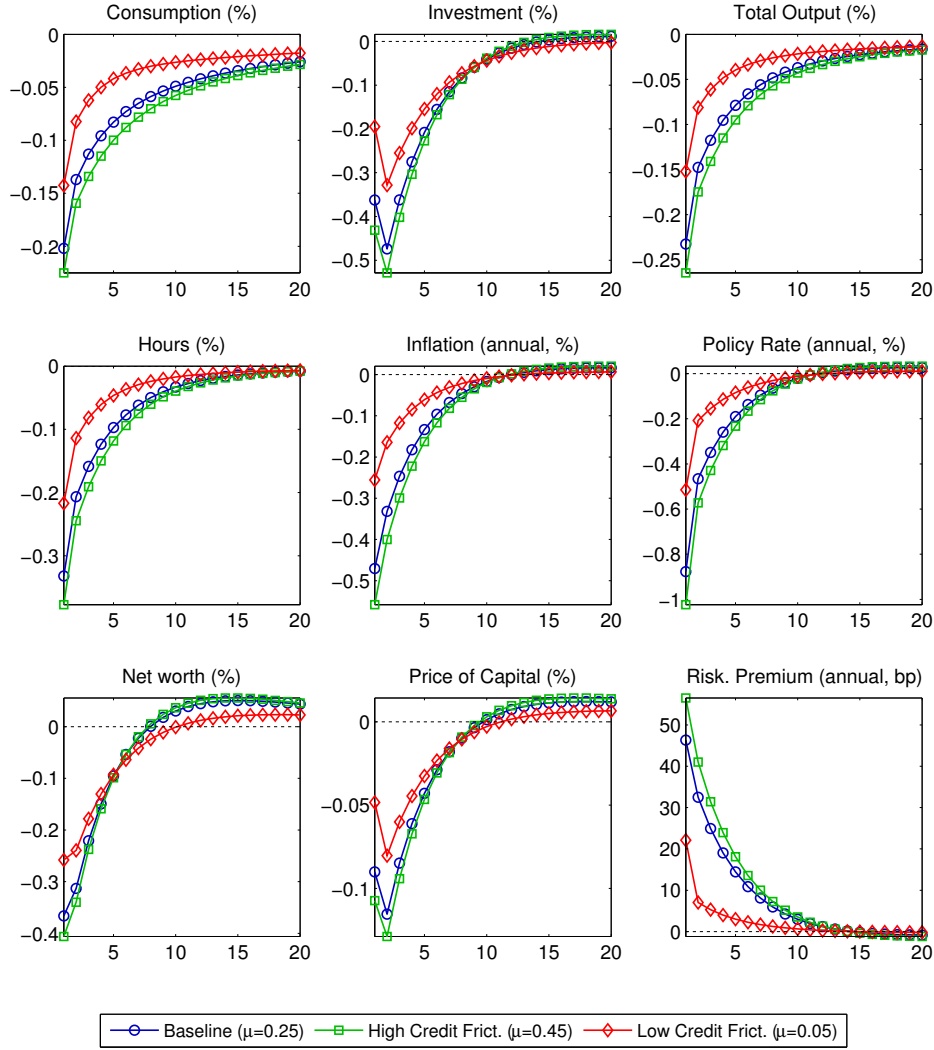
Figure 8 MICRO UNCERTAINTY SHOCK - THE ROLE OF CREDIT FRICTIONS. Impulse response functions (IRFs) to a 1 standard deviation increase in micro uncertainty. The IRFs display the impact of a micro uncertainty shock under different degrees of tightness of the credit friction and are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points.

5.3 A numerical experiment

The conditional impulse responses reported in the previous section, however, are not sufficient to gauge the importance of uncertainty shocks for the business cycle. In this section we report the results from a simple numerical experiment where we compute unconditional business cycle properties of some variants of the baseline model. Specifically, as in Bachmann and Bayer (2013), we compare the baseline model —that features both micro and macro uncertainty shocks, alongside standard aggregate productivity shocks— with variants of the model where we introduce the

shocks one at the time.[15]

Table 3 reports the results. Column (1) displays the standard deviation, persistence and correlation of output, consumption and investment in U.S. data.[16] Column (2) reports the same statistics computed from simulations of the baseline model, i.e. including standard aggregate productivity shocks alongside micro and macro uncertainty shocks. Despite the relative simplicity of the model and and the small number of shocks considered, the model does a good job at matching some key features of the data.

Table 3 Business Cycle Statistics – Data And Model Simulations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Baseline	Macro	Micro	Micro: large shock	Micro: high fin. frictions	Micro: high stickiness
Volatility							
Output	1.55	1.58	0.00	0.11	0.30	0.15	0.15
Volatility of aggregate variables relative to output volatility							
Consumption	0.82	0.81	2.36	0.78	0.77	0.77	0.84
Investment	2.92	2.35	8.66	4.56	4.85	4.05	3.63
First-order autocorrelation							
Output	0.88	0.92	0.99	0.64	0.64	0.64	0.71
Consumption	0.88	0.91	1.00	0.71	0.70	0.71	0.79
Investment	0.91	0.79	1.00	0.53	0.46	0.52	0.54
Contemporaneous correlation with aggregate consumption							
Consumption	0.88	0.94	0.95	0.98	0.98	0.98	0.98
Investment	0.95	0.83	0.91	0.94	0.79	0.93	0.89

Note. Column (1) reports business cycle statistics for U.S. data over the 1972Q1-2012Q4 sample period from OECD Main Economic Indicators. Column (2) refer to simulations from our baseline model, with both micro and macro uncertainty shocks, alongside standard aggregate productivity shocks. Columns (3) and (4) refer to simulations where there is only one shock driving the model economy, i.e. the macro uncertainty and micro uncertainty shock, respectively. Columns (5) to (7) refer to simulations where the only shock is the micro uncertainty shock. They differ in the size of the shock, which is set in (5) to $\sigma^S = 0.07$ as in Christiano et al. (2014); in the degree of severity of the financial friction, which is set in (6) to $\mu^S = 0.5$; and in the degree of price stickiness, which is set in (7) so as to obtain an average probability of changing prices of 5 quarters. All series, from data and model simulations, have been logged and HP-filtered with a smoothing parameter 1600.

We then consider versions of the model where uncertainty shocks are the sole exogenous processes in the model. First, we consider macro uncertainty only. The simulations, reported in column (4), show that the volatility of output in the simulated data is virtually zero. Differently,

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[15]Using firm-level German data, Bachmann and Bayer (2013) find that micro uncertainty shocks generate roughly 15 percent of the observed time series standard deviation of output. In their model, however, the shock is transmitted primarily *via* physical capital adjustment frictions.

[16]We use U.S. real GDP, private final consumption expenditure, and gross fixed capital formation over the sample period 1972Q1–2012Q4. The source is OECD Main Economic Indicators.

micro uncertainty shocks alone can drive a non-trivial share of output volatility. The unconditional moments in column (5) show that micro uncertainty alone can generate about 10% of the output volatility in the data.

Our estimates of the importance of micro uncertainty shocks fall in between the estimates from the previous studies. Using firm-level data Chugh (2015) finds that micro uncertainty shocks drive about 5 percent of GDP volatility; Christiano et al. (2014), instead, estimate a much important role of micro uncertainty shocks using macro-financial data (i.e., about 20 percent of GDP volatility). What is driving our results? We explore the role of three factors: non-linearities associated with the size of the shock, severity of the financial friction and degree of price stickiness.

We therefore consider three additional versions of the model with micro uncertainty only: (i) one where the size of the micro uncertainty shock is set to the largest of the available estimates from other studies, i.e. $\sigma^S = 0.07$ as in Christiano et al. (2014); (ii) one where financial frictions are more severe than in the baseline, i.e. doubling the value of the monitoring cost parameter to $\mu = 0.5$; and (iii) one where the degree of price stickiness is larger than in the baseline, i.e. setting the Rotemberg parameter ω_p so as to obtain an average probability of changing prices of 5 quarters. The unconditional moments reported in columns (5) show that the size of the shock matters but non-linearities do not seem to play a major role (remember that simulations from the fully non-linear model are used to compute the impulse responses). Finally, columns (6) and (7) show that both the severity of credit friction and the degree of price stickiness are key for transmission of uncertainty shocks.

6   Conclusions

Shocks to micro uncertainty transmitted through financial frictions and nominal rigidities can drive a small, but not insignificant, share of business cycle fluctuations. Differently, shocks to macro uncertainty do not drive any significant variation in output. We arrive at this conclusion by studying the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and to the dispersion of entrepreneurs' idiosyncratic productivity (micro uncertainty) in a financial accelerator DSGE model with sticky prices. The model is disciplined using aggregate data on total factor productivity for the U.S. business sector and disaggregated data on total factor productivity at the establishment-level. The estimated time series properties

of micro uncertainty are in line with previous estimates based on disaggregated micro data, but smaller than the estimates based on aggregated macroeconomic data: understanding what drives the gap between the two approaches should be the focus of further research in this area.

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A   Appendix: Model

A.1   Equilibrium

Let's define $q_t \equiv Q_t/P_t$, $nw_t \equiv NW_t/P_t$, $z_t \equiv Z_t/P_t$. For a given path for the exogenous processes, a recursive (imperfectly) competitive equilibrium of the model is a sequence of allocations for the endogenous variables that solves the following system of equations.
Euler equation of households:

$$U_{c,t} = \beta(1 + R_t^n)\mathbb{E}_t \left[\frac{U_{c,t+1}}{\pi_{t+1}} \right]. \tag{A.1}$$

Labour supply:

$$mc_t Y_{n,t} = -\frac{U_{n,t}}{U_{c,t}}. \tag{A.2}$$

Marginal product of capital:

$$mc_t Y_{k,t} = z_t. \tag{A.3}$$

Price of capital:

$$q_t = \left[1 - \phi_k \left(\frac{I_t}{K_t} - \delta \right) \right]^{-1}. \tag{A.4}$$

Zero profit condition:

$$y_{t+1}^k K_{t+1} \Big(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \Big) = (1 + R_t^n)(q_t K_{t+1} - nw_{t+1}). \tag{A.5}$$

NK Phillips curve:

$$(\pi_t - \pi) \pi_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} (\pi_{t+1} - \pi) \pi_{t+1} \right\} + Y_t \frac{\varepsilon}{\omega_p} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{A.6}$$

Net worth law of motion:

$$nw_{t+1} = \gamma y_{t+1}^k K_{t+1} \left(1 - \Gamma(\bar{\omega}_{t+1}) \right). \tag{A.7}$$

Entrepreneurs real consumption:

$$C_t^e = (1 - \gamma) \left(1 - \Gamma(\bar{\omega}_{t+1}) \right) y_t^k K_t. \tag{A.8}$$

Aggregate resource constraint:

$$A_t F(K_t, N_t) = C_t + C_t^e + I_t + \frac{\omega_p}{2} (\pi_t - \pi)^2 + \mu G(\bar{\omega}) y_t^k K_t. \tag{A.9}$$

Accumulation of aggregate capital:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \tag{A.10}$$

Monetary policy:

$$\frac{1 + R_t^n}{1 + R^n} = \left(\frac{1 + R_{t-1}^n}{1 + R^n} \right)^{\phi^r} \left(\frac{1 + \pi_t^n}{1 + \pi} \right)^{(1-\phi^r)\phi^\pi} \left(\frac{1 + Y_t}{1 + Y_{t-1}} \right)^{(1-\phi^r)\phi^y}. \tag{A.11}$$

Definition of real income from holding one unit of finished capital:

$$y_t^k = z_t + q_t \left[1 - \delta - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 + \phi_k \left(\frac{I_t}{K_t} - \delta \right) \frac{I_t}{K_t} \right]. \tag{A.12}$$

$$y_{t+1}^k = \frac{(1 + R_{t+1}^k)q_t}{\pi_{t+1}}. \tag{A.13}$$

Optimal contract:

$$\frac{1 + R_{t+1}^k}{1 + R_t^n} = \psi_t. \tag{A.14}$$

where:

$$\psi_t = \left(\frac{\left(1 - \Gamma(\bar\omega_{t+1}^j)\right) \left(\Gamma'(\bar\omega_{t+1}^j) - \mu G'(\bar\omega_{t+1}^j)\right)}{\Gamma'(\bar\omega_{t+1}^j)} + \left(\Gamma(\bar\omega_{t+1}^j) - \mu G(\bar\omega_{t+1}^j)\right) \right)^{-1}. \tag{A.15}$$

A.2 Households' Preferences

In the paper we compare different functional forms for households' preferences, namely standard separable preferences, log-separable preferences of the King et al. (1988) type, and GHH preferences of the Greenwood et al. (1988) type. Below, we describe the functional form of those preferences and we show how they affect the households' key equations, namely the Euler equation for consumption and labour supply.

Separable Preferences

Agents' utility is additively separable in consumption and labour:

$$\frac{C_t^{1-\varrho}}{1 - \varrho} - \tau \frac{N_t^{1+\upsilon}}{1 + \upsilon}. \tag{A.16}$$

Note that this functional form is separable in that the utility (loss) from working does not directly affect the utility (gain or loss) from consumption, i.e. the cross-derivative of utility with respect to consumption and labour is zero. In fact:

$$\begin{aligned} U_{c,t} &= C_t^{-\varrho}, \\ U_{n,t} &= -\tau N_t^{\upsilon}. \end{aligned} \tag{A.17}$$

The Euler equation and labour supply conditions are:

$$\begin{aligned} C_t^{-\varrho} &= \beta(1 + R_t^n)\mathbb{E}_t \left[C_{t+1}^{-\varrho} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= \tau \frac{N_t^{\upsilon}}{C_t^{-\varrho}}. \end{aligned} \tag{A.18}$$

The Euler equation shows that expected consumption growth is a function of the real interest rate only whilst labour supply is a positive function of the real wage and of consumption.

Non–Separable Preferences

Agents' utility is log-separable in consumption and labour:

$$\frac{\left(C_t(1 - N_t)^{\tau^{NS}}\right)^{(1-\varrho)}}{1 - \varrho}. \tag{A.19}$$

Then:

$$\begin{aligned} U_{c,t} &= C_t^{-\varrho}(1 - N_t)^{\tau^{NS}(1-\varrho)}, \\ U_{n,t} &= -\tau^{NS} C_t^{1-\varrho}(1 - N_t)^{\tau^{NS}(1-\varrho)-1}. \end{aligned} \tag{A.20}$$

This implies that the Euler equation and labour supply conditions are:

$$C_t^{-\varrho}(1 - N_t)^{\tau^{NS}(1-\varrho)} = \beta(1 + R_t^n)\mathbb{E}_t\left[C_{t+1}^{-\varrho}(1 - N_{t+1})^{\tau^{NS}(1-\varrho)}\frac{P_t}{P_{t+1}}\right], \quad \text{(A.21)}$$

$$\frac{W_t}{P_t} = \tau^{NS}\frac{C_t}{(1 - N_t)}.$$

In this case the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour whereas labour supply is similar to the non-separable case.

GHH Preferences

These preferences are as in Greenwood et al. (1988). With this utility function, the amount of hours worked by households will actually affect the amount of utility received from consumption, i.e. the cross-derivative of utility with respect to consumption and labour is unequal to zero.

$$\frac{1}{1 - \varrho}\left(C_t - \tau^{GHH}N_t^{1+\upsilon}\right)^{1-\varrho}. \quad \text{(A.22)}$$

Then:

$$U_{c,t} = \left(C_t - \tau^{GHH}N_t^{1+\upsilon}\right)^{-\varrho}, \quad \text{(A.23)}$$

$$U_{n,t} = -\tau^{GHH}(1 + \upsilon)N_t^{\upsilon}\left(C_t - \tau^{GHH}N_t^{1+\upsilon}\right)^{-\varrho}.$$

This implies that the Euler equation and labour supply conditions are:

$$\left(C_t - \tau^{GHH}N_t^{1+\upsilon}\right)^{-\varrho} = \beta(1 + R_t^n)\mathbb{E}_t\left[\left(C_{t+1} - \tau^{GHH}N_{t+1}^{1+\upsilon}\right)^{-\varrho}\frac{P_t}{P_{t+1}}\right], \quad \text{(A.24)}$$

$$\frac{W_t}{P_t} = \tau^{GHH}(1 + \upsilon)N_t^{\upsilon}.$$

In this case, like the non-separable case, the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour. But unlike the non-separable case, labour supply is a positive function of the real wage only. Therefore, as the marginal rate of substitution is independent of consumption and only depends on the real wage, there is no wealth effect on the labour supply.

A.3 Steady State

To compute the steady state of the model, we take an approach similar to Faia and Monacelli (2007). First, notice that some value steady state values can be pinned down simply by the calibrated parameters. For example, from the Euler equation of consumption notice that:

$$1 + R^n = \pi/\beta.$$

From the New Keynesian Phillips curve:

$$mc = \frac{\varepsilon - 1}{\varepsilon}.$$

From the price of capital equation:

$$q = 1.$$

Second, the entrepreneurial problem has to be solved to compute the cut-off value of the idiosyncratic productivity. In order to do that, notice that it is possible to compute the net worth to

capital ratio from both the zero profit condition of banks in (19):

$$\mathcal{NK}_1 = \frac{nw}{K} = 1 - \frac{y^k}{1+R^n}\left(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\right)$$

and from the law of motion of net worth in (26):

$$\mathcal{NK}_2 = \frac{nw}{K} = \gamma y^k \left(1 - \Gamma(\bar{\omega})\right),$$

where remember that $y^k = (1+R^k)/\pi$. By guessing an initial value for $\bar{\omega}$ we can compute R^k from the efficiency conditions associated with the optimal contract (21) and (22). With a simple algorithm in MatLab, it is then possible to modify $\bar{\omega}$ until the following condition $\mathcal{NK}_1 = \mathcal{NK}_2$ is satisfied. Once the steady state level of $\bar{\omega}$ is determined, R^k, y^k, ψ, $\Gamma(\bar{\omega})$, and $G(\bar{\omega})$ are also determined.

To compute the steady state value of the remaining variables, notice that from the definition of the nominal income from holding one unit of capital in equation (17):

$$z = y^k - 1 + \delta.$$

Then, we can compute the following ratios from the production function:

$$\begin{aligned}
\frac{Y}{K} &= \frac{z}{\alpha \cdot mc}, \\
\frac{K}{N} &= \left(\frac{Y}{K}\right)^{\frac{1}{\alpha-1}},
\end{aligned}$$

from the law of motion of capital:

$$\frac{I}{K} = \delta,$$

and from the aggregate resource constraint:

$$\frac{C}{K} = \frac{Y}{K} - \frac{I}{K} - \mu G(\bar{\omega})y^k.$$

Finally, by fixing the steady state level of hours $N = 1/3$ it is possible to solve the above equations and easily compute the remaining endogenous variables of the model.

B   Appendix: Impulse response calculation

To compute the impulse responses reported in the paper we use a two steps procedure. As noted by Fernandez-Villaverde et al. (2011), the higher order approximation makes the simulated paths of states and controls in the model move away from their steady-state values. This is actually one of the results of Schmitt-Grohe and Uribe (2004): in a first-order approximation of the model, the expected value of any variable coincides with its value in the non-stochastic steady state, while in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value only by a constant.

In a third order approximation, the expected value of the variable will also depend on the variance of the shocks in the economy. Therefore, it is more informative to compute impulse responses as percentage deviations from their mean, rather than their steady state.

However, a well-known flaw of higher-order perturbations is that when the approximated decision rules are used to produce simulated time series from the model, the simulated data often display an explosive behaviour. We address the problem of explosive paths of simulated data by

applying the pruning procedure by Kim et al. (2008).[17]

In the first step we simulate the model and compute the mean of the state and control variables. In particular we:

1. Draw a series of random shocks $\varepsilon_t = (\varepsilon_t^A, \varepsilon_t^W, \varepsilon_t^S)$ for T periods ($T = 4000$)

2. Starting from the steady state, perform simulation of the model using ε_t and get Y_t (i.e., the simulated data)

3. Discard the first half of observations as a burn in, and compute the ergodic mean of Y over the last $0.5 \cdot T$ periods:
$$Y_0 = \frac{\sum\limits_{0.5T+1}^{T} Y_t}{0.5T}$$

In the second step, we compute impulse responses. For example, for the macro uncertainty shock (ε_t^W) we:

1. Draw a series of random shocks ε_t^W for N periods ($N = 40$)

2. Perform simulation Y_t^1 starting from initial conditions Y_0 and using ε_t^W

3. Add one standard deviation to ε_t^W in period 1 and get $\tilde{\varepsilon}_t^W$

4. Perform simulation Y_t^2 starting from initial conditions Y_0 and using $\tilde{\varepsilon}_t^W$

5. IRF is equal to $Y_t^2 - Y_t^1$

6. Perform $R = 50$ replications of steps 1) to 5) and report the average IRF

## C   Appendix: Micro uncertainty - A comparative static exercise

This appendix presents a comparative statics exercise to get a deeper insight of the mechanism through which micro uncertainty affects the real economy. Specifically, we analyze the effect of changes in the steady state value of the standard deviation of entrepreneurial idiosyncratic productivity ($\bar{S}$) on the steady state value of other variables in our model economy. Such a simple static exercise is useful for a better understanding of the impulse responses reported in Figures 7 and 8 in the main tetx.

We consider a wide range of steady-state standard deviations of idiosyncratic productivity, namely $\bar{S} = [0.01, 0.70]$. Then, we solve the microeconomic problem together with the steady state of our model with the algorithm described in the Appendix. In this way we pin down the steady state leverage ratio (L), the threshold value for the idiosyncratic shock ($\bar{\omega}$), and entrepreneurial real income from owning one unit of capital (y^k). Once the steady state level of these variables are determined, we can solve for the steady state value of all other variables in our model, given our baseline calibration in Table 1.

Figure C.1 displays how the steady-state level of some key variables in our model varies to changes in the steady state value of S. Note that all variables expressed in levels are re-scaled to be equal to 100 for our baseline calibration ($\bar{S} = 0.225$); interest rates are in annualized percent, while the leverage ratio is not rescaled.

As described in the main text, an increase in $\bar{S}$ is associated with an increase in the frequency of entrepreneurial default (F) which also increases banks' expected costs associated with bankruptcies. As a result, banks charge a higher spread on the risk free rate and lending rates

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[17]We thank Martin Andreasen for sharing the codes for the pruning of DSGE models approximated to the 3rd-order. See Andreasen et al. (2013) for details.
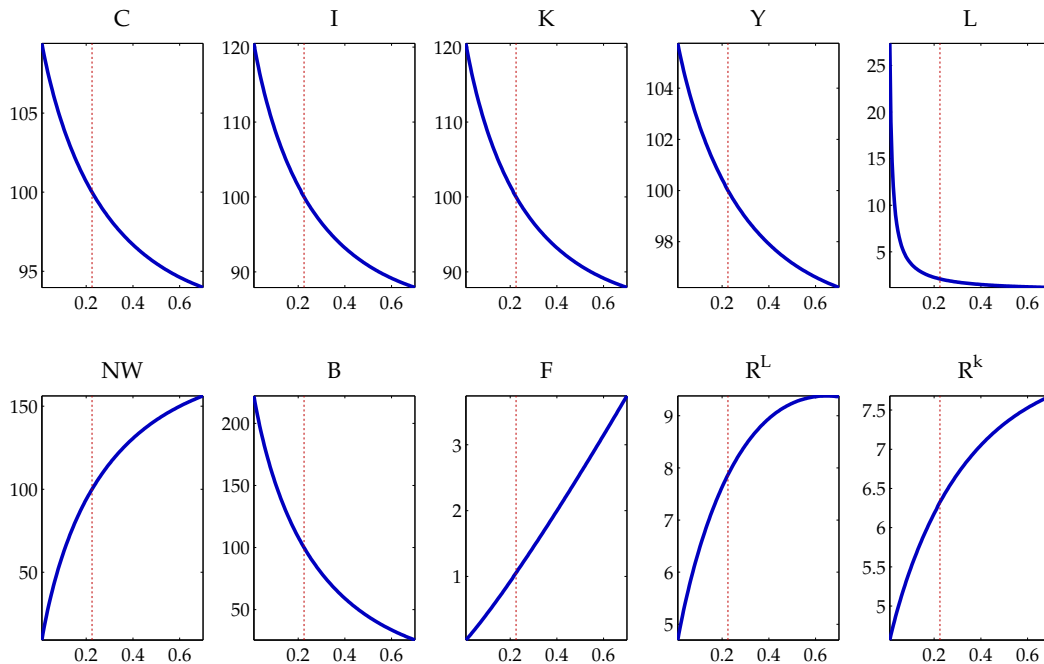
Figure C.1 The charts display the effect of micro uncertainty on the steady state level of the model economy. On the horizontal axis is the steady state value of entrepreneurial idiosyncratic productivity ($\bar{S}$). On the vertical axis is the steady state value of consumption (C), investment (I), capital (K), total output (Y), leverage (L), net worth (NW), total borrowing (B), default probability (F), lending rates (R^L) and rental rate of capital (R^k).

(R^L) increase, the aggregate level of borrowing in the economy (B) falls and so do entrepreneurs' purchases of unfinished capital (K): with a lower level of capital in the economy its rental rate of return is higher (R^k).

Note that, intuitively, entrepreneurs should try to leverage up to benefit from the higher rental rate of capital. However, as already noted above, as $\bar{S}$ rises entrepreneurs face increasing interest rates (R^L), which would induce entrepreneurs to reduce borrowing and, consequently, also leverage. In a on line appendix to their paper, Christiano et al. (2014) put forth this same issue and analyse it with a similar exercise. They first characterize the equilibrium in the loans market analytically in the *Risk spread - Leverage* space. Then, holding the aggregate return on capital (R^k) fixed, they show that entrepreneurs facing an exogenous increase in $\bar{S}$ would optimally choose a loan contract with a higher interest rate and lower leverage. However, they also suggest that the result of this partial equilibrium exercise could be muted by the increase in the aggregate return on capital, that would instead push entrepreneurs to increase their leverage. In an additional partial equilibrium exercise they show that this is indeed the case: holding $\bar{S}$ fixed, an exogenous increase of R^k relative to the risk free interest rate leads to an increase in leverage, therefore muting the negative impact on leverage of a jump in $\bar{S}$. In their numerical experiments, however, they find that the first effects always dominates.

In our exercise we let the rental rate of capital to be determined jointly with all other variables in our model and, consistently with Christiano et al. (2014)'s conjecture, we find that the effect of lending rates predominates on the effect of rental rate of capital. In fact, Figure C.1 shows that in the face of increasing $\bar{S}$ —and therefore of increasing interest rates but also of increasing aggregate returns on capital— entrepreneurs optimally choose loans contracts with lower leverage (L).

Note that when $\bar{S}$ approaches zero, leverage is very sensitive to changes in $\bar{S}$. Intuitively, when the variance of the idiosyncratic shock approaches zero entrepreneurs try to leverage up to infinity since their profits are unbounded and the credit friction is not binding. Analytically,

this can be easily understood by recalling that leverage is defined as the ratio between capital and net worth and by observing that entrepreneurs optimally reduce their net worth (NW) as $\bar{S}$ approaches zero.

Finally, and not surprisingly, all relevant macroeconomic aggregates are decreasing in $\bar{S}$. Specifically, consumption, investment, and total output are lower for larger values of the standard deviation of idiosyncratic productivity.