

# Explaining the Boom-Bust Cycle in the U.S. Housing Market: A Reverse-Engineering Approach\*

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## Abstract

We use a simple quantitative asset pricing model to “reverse-engineer” the sequences of stochastic shocks to housing demand and lending standards that are needed to exactly replicate the boom-bust patterns in U.S. household real estate value and mortgage debt over the period 1995 to 2012. Conditional on the observed paths for U.S. disposable income growth and the mortgage interest rate, we consider four different specifications of the model that vary according to the way that household expectations are formed (rational versus moving average forecast rules) and the maturity of the mortgage contract (one-period versus long-term). We find that the model with moving average forecast rules and long-term mortgage debt does best in plausibly matching the patterns observed in the data. Counterfactual simulations show that shifting lending standards (as measured by a loan-to-equity limit) were an important driver of the episode while movements in the mortgage interest rate were not. All models deliver rapid consumption growth during the boom, negative consumption growth during the Great Recession, and sluggish consumption growth during the recovery when households are deleveraging.

Keywords: *Housing bubbles, Mortgage debt, Borrowing constraints, Lending standards, Macroprudential policy.*

JEL Classification: D84, E32, E44, G12, O40, R31.

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# 1 Introduction

Starting in the mid-1990s, the U.S. economy experienced correlated booms and busts in household real estate value, household mortgage debt, and personal consumption expenditures (all measured relative to personal disposable income), as shown in Figure 1. The ratio of housing value to income peaked in 2005.Q4. The ratio of mortgage debt to income peaked 8 quarters later in 2007.Q4—coinciding with the start of the Great Recession. Throughout this period, the ratio of imputed housing rent to disposable income declined steadily.<sup>1</sup> Given that rents are a measure of the “dividend” or service flow from housing, the quiet behavior of rents during the boom lends support to non-fundamental explanations of the episode. Our aim is to develop a transparent quantitative model that can account for the patterns observed in Figure 1. In so doing, we assess the plausibility of the driving forces that are needed to make the model fit the data.

A wide variety of empirical evidence links the U.S. housing boom to relaxed lending standards.<sup>2</sup> The report of the U.S. Financial Crisis Inquiry Commission (2011) emphasizes the effects of a self-reinforcing feedback loop in which an influx of new homebuyers with access to easy mortgage credit helped fuel an excessive run-up in house prices. The run-up, in turn, encouraged lenders to ease credit further on the assumption that house prices would continue to rise. As house prices rose, the lending industry marketed a range of exotic mortgage products, e.g., loans requiring no down payment or documentation of income, monthly payments for interest-only or less, and adjustable rate mortgages with low introductory ‘teaser’ rates that reset higher over time. Within the United States, house prices rose faster in areas where subprime and exotic mortgages were more prevalent (Mian and Sufi 2009, Pavlov and Wachter 2011, Berkovec, Chang, and McManus 2012). In a given area, past house price appreciation had a significant positive influence on subsequent loan approval rates in the same area (Dell’Ariccia, Igan, and Laeven 2012, Goetzmann, Peng, and Yen 2012).

In the aftermath of the 2001 recession, the Federal Reserve reduced the federal funds rate to just 1% and held it there for over 12 months during 2003 and 2004. While some studies find evidence that low interest rates were an important contributor to the run-up in house prices (Taylor 2007, McDonald and Stokes 2011) others argue that low interest rates were not a major factor (Dokko, et al. 2011, Glaeser, Gottlieb, and Gyourko 2013). Aside from the possible effect on house prices, there is clear evidence that low mortgage interest rates during this period set off a refinancing boom, allowing consumers to tap the equity in their homes to pay for all kinds of goods and services. According to data compiled by Greenspan and Kennedy (2008), free cash generated by home equity extraction contributed an average of \$136

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<sup>1</sup>Data on household real estate value and household mortgage debt are from the Federal Reserve’s Flow of Funds Accounts. Data on personal disposable income and personal consumption expenditures are from the Federal Reserve Bank of St. Louis’ FRED data base. Data on imputed rents from owner-occupied housing are from [www.lincolinst.edu](http://www.lincolinst.edu), as documented in Davis, Lehnert, and Martin (2008).

<sup>2</sup>See, for example, Demyanyk and Van Hemert (2011), Duca, Muellbauer, and Murphy (2010, 2011), and Dokko, et al. (2011).

billion per year in personal consumption expenditures from 2001 to 2006—more than triple the average yearly contribution of \$44 billion from 1996 to 2000 (p. 131). Kermani (2012) finds that U.S. counties that experienced the largest increases in house prices from 2000 to 2006 also tended to experience the largest increases in auto sales over the same period. The same counties tended to suffer the largest declines in auto sales from 2006 to 2009 when house prices were falling.<sup>3</sup> Similarly, Mian and Sufi (2014) identify a significant effect on auto spending that operates through home equity borrowing during the period 2002 to 2006. Laibson and Mollerstrom (2010) argue that the U.S. consumption boom from 1996 to 2006 was driven mainly by bubbly movements in house prices, not lower real interest rates.

In this paper, we use four different versions of a simple quantitative asset pricing model to “reverse-engineer” the sequences of stochastic shocks that are needed to match the boom-bust patterns observed in Figure 1. The four model specifications differ according to the way that household expectations are formed (rational versus moving average forecast rules) or the maturity of the mortgage contract (one-period versus long-term). Conditional on the observed paths for U.S. disposable income growth and the mortgage interest rate, we back-out sequences for: (1) a shock to housing preferences, and (2) a shock to lending standards (as measured by a loan-to-equity limit) so as to exactly replicate the boom-bust patterns in household real estate value and mortgage debt over the period 1995.Q1 to 2012.Q4, as plotted in the top panels of Figure 1. We also examine the model predictions for the evolution of other variables, such as the rent-income ratio, the consumption-income ratio, and consumption growth during three phases of the episode, i.e., the boom, the Great Recession, and the recovery.

Under rational expectations, we show that the model requires large and persistent housing preference shocks to account for the boom-bust cycle in U.S. housing value from 1995 to 2012. According to the model, an increase in housing preference will increase the housing service flow, as measured by the imputed rent from owner-occupied housing. Consequently, the rational expectations model predicts a similar boom-bust cycle in the ratio of housing rent to income. But this did not happen in the data.

As an alternative to rational expectations, we consider a setup where households employ simple moving average forecast rules, i.e., adaptive expectations. This type of forecast rule is consistent with a wide variety of survey evidence that directly measures agents’ expectations (Coibion and Gorodnichenko 2012, Williams 2013). We show that the moving average model can match the boom-bust cycle in U.S. housing value with much smaller movements in the housing preference shock. Indeed, the standard deviation of the shock innovation in the moving average model is only one-tenth as large as the value needed in the rational expectations model. This is because the household’s forecast rule embeds a unit root which serves to magnify asset price volatility in response to shocks.<sup>4</sup> Consequently, the moving average model does a much

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<sup>3</sup>A similar pattern can be found in cross-country data on house prices and consumption. See Glick and Lansing (2010) and International Monetary Fund (2012).

<sup>4</sup>This mechanism for magnifying the volatility of house prices is also employed by Gelain, Lansing, and Mendicino (2013) and Gelain and Lansing (2014).

better job of matching the quiet behavior of the U.S. rent-income ratio plotted in the lower left panel of Figure 1. More generally, the moving average model captures the idea that much of the run-up in U.S. house prices and credit during the boom years appears to be linked to an influx of unsophisticated homebuyers. Given their inexperience, these buyers would be more likely to employ simple backward-looking forecast rules for future house prices, income, lending standards, etc. One can also make the case that many U.S. lenders behaved similarly by approving subprime and exotic mortgage loans that could only be repaid if housing values continued to trend upward.<sup>5</sup>

Mortgage debt in the model is governed by a standard collateral constraint that depends on the market value of the housing stock. With one-period mortgage contracts, the entire stock of outstanding debt is refinanced each period, causing the stock of debt to move in tandem with housing value. All else equal, the one-period debt model would therefore predict a rapid deleveraging from 2006 onwards when U.S. housing values were falling rapidly. In the data, however, the deleveraging proceeded gradually, as debt declined at a much slower pace than housing value, as shown in Figure 1. To avoid the counterfactual prediction of a rapid deleveraging, the one-period debt model requires a post-2007 relaxation of lending standards (a larger loan-to-equity limit) to simultaneously match the patterns of housing value and mortgage debt in the data. This prediction conflicts with evidence from the Federal Reserve’s Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS) which shows that banks started to tighten lending standards before the onset of the Great Recession and often continued to tighten standards even after the recession ended.

Following Kydland, Rupert, and Šustek (2012), we model long-term mortgage debt by approximating the amortization schedule of a conventional 30-year mortgage loan. Such a loan has the feature that the borrower’s early payments consist mainly of interest while later payments consist mainly of principal. With long-term mortgages, the borrowing constraint applies only to new loans, not to the entire stock of outstanding mortgage debt. In any given period, the household’s new loan cannot exceed a fraction of accumulated home equity. When the borrowing constraint is binding, a sustained period of progressively relaxed lending standards leads to an increase in the flow of new loans which, in turn, contributes to a buildup in household leverage. A rapid decline in housing value leads to a rapid decline in the flow of new loans, but the stock of outstanding mortgage debt declines slowly, as in the data. Using impulse response functions, we show that models with long-term mortgage debt exhibit the feature that housing value peaks earlier than the mortgage debt, consistent with the data plotted in Figure 1. Now when we undertake the reverse-engineering exercise, we identify a relaxation of lending standards during the boom years of 2001 to 2005 followed by a period of progressively tightening lending standards, consistent with the SLOOS data. The reverse-

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<sup>5</sup>According to the report of the U.S. Financial Crisis Inquiry Commission (2011), p. 70, new subprime mortgage originations went from \$100 billion in the year 2000 to around \$650 billion at the peak in 2006. In that year, subprime mortgages represented 23.5% of all new mortgages originated. On p. 165, the report states “Overall, by 2006, no-doc or low-doc loans made up 27% of all mortgages originated.”

engineered shifts in lending standards produce the necessary flow of new loans to allow the model to match the path of the debt-income ratio in the data.

Given the reverse-engineered paths for the stochastic shocks, all models deliver identical paths for the consumption-income ratio and consumption growth. According to the simple household budget constraint, the consumption-income ratio is driven by movements in the debt-income ratio and the mortgage interest rate which, by construction, are the same across models for the reverse-engineering exercise. We show that a smoothed version of the model consumption-income ratio roughly resembles the hump-shaped pattern observed in the U.S. data from 1995 to 2012. Consequently, all models deliver rapid consumption growth during the boom phase from 1995.Q1 to 2007.Q4, negative consumption growth during the Great Recession from 2007.Q4 to 2009.Q2, and sluggish consumption growth during the recovery from 2009.Q2 to 2012.Q4 when households are deleveraging.

A virtue of our reverse-engineering approach is that we can construct counterfactual scenarios by shutting off a particular shock sequence and then examining the evolution of model variables versus those in the U.S. data. For example, shutting off the reverse-engineered housing preference shock in the rational expectations model serves to completely eliminate the boom-bust cycle in housing value. In contrast, the moving average model continues to generate a boom-bust cycle, albeit smaller in magnitude, due to the asset price response to the other identified shocks. This result illustrates the ability of the moving average model to generate an income- and credit-fueled boom in housing value.

When we shut off the reverse-engineered lending standard shock, the models with long-term mortgage debt exhibit no significant run-up in debt, regardless of the expectation regime. This result indicates that shifting lending standards were an important driver of the boom-bust episode. Put another way, the amplitude of the boom-bust episode could have been mitigated if mortgage regulators had been more vigilant in enforcing prudent lending standards.

When we shut off the income growth shock, there is little noticeable effect in the rational expectations models. In contrast, the moving average models now exhibit a delayed boom-bust episode relative to the data. This is due to the absence of the persistently positive income growth shocks that occurred during the late 1990s. According to the moving average models, movements in income growth did play a role in the magnitude and timing of the episode.

When we shut off the mortgage interest rate shock, all of the models continue to exhibit significant boom-bust cycles in both housing value and debt. This is because the magnitude of the mortgage interest rate drop in the data is simply too small to have much impact on the trajectories of housing value and debt. All of the models imply that movements in the U.S. mortgage interest rate were not a major driver of the episode.

Overall, we find that the moving average model with long-term mortgage debt does best in plausibly matching the patterns in Figure 1. This version lends support to the view that the U.S. housing boom was a classic credit-fueled bubble involving over-optimistic projections about future housing values, relaxed lending standards, and ineffective mortgage regulation.

## 1.1 Related Literature

A common feature of all bubbles is the emergence of seemingly-plausible fundamental arguments that seek to justify the dramatic rise in asset prices. During the boom years of the U.S. housing market, many economists and policymakers argued that a housing bubble did not exist and that numerous fundamental factors were driving the run-ups in housing values and mortgage debt.<sup>6</sup> Commenting on the rapid growth in subprime mortgage lending, Fed Chairman Alan Greenspan (2005) offered the view that the lending industry had been dramatically transformed by advances in information technology: “Where once more-marginal applicants would simply have been denied credit, lenders are now able to quite efficiently judge the risk posed by individual applicants and to price that risk appropriately.” In a July 1, 2005 interview on the CNBC network, Ben Bernanke, then Chairman of the President’s Council of Economic Advisers, asserted that fundamental factors such as strong growth in jobs and incomes, low mortgage rates, demographics, and restricted supply were supporting U.S. house prices. In the same interview, Bernanke stated his view that a substantial nationwide decline in house prices was “a pretty unlikely possibility.”

Numerous recent studies have employed quantitative theoretical models to try to replicate various aspects of the boom-bust cycle in the U.S. housing market. Most of these studies preempt bubble explanations by assuming that all agents are fully rational. For example, taking the observed paths of U.S. house prices, aggregate income, and interest rates as given, Chen, Michaux, and Roussanov (2013) show that a model with rational expectations and long-term (interest-only) mortgages can approximate the observed patterns in U.S. household debt and consumption. Their quantitative exercise is similar in spirit to ours with the important exception that they do not attempt to explain movements in U.S. house prices.

Standard dynamic stochastic general-equilibrium (DSGE) models with fully-rational expectations have difficulty producing large swings in housing values that resemble the patterns observed in the U.S. and other countries. Indeed, it is common for such models to employ extremely large and persistent exogenous shocks to rational agents’ preferences for housing in an effort to bridge the gap between the model and the data.<sup>7</sup> We obtain a similar result here when we impose rational expectations. But, as noted above, large housing preference shocks are not a plausible explanation for the boom-bust episode because these shocks generate extremely large movements in the imputed housing rent, which are counterfactual. We show that households’ use of moving average forecast rules serves to shrink substantially the required magnitude of the housing preference shocks that are needed to match the data.

Justiniano, Primiceri, and Tambalotti (2015b) develop a stylized model that distinguishes between a credit supply constraint and the more conventional borrowing constraint. They argue that the U.S. housing boom is best explained as a relaxation of the credit supply con-

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<sup>6</sup>See, for example, McCarthy and Peach (2004) and Himmelberg, Mayer, and Sinai (2005).

<sup>7</sup>See for example, Iacoviello and Neri (2010) and Justiniano, Primiceri, and Tambalotti (2015a), among others.

straint, as this reduces mortgage interest rates and thereby can generate a sizeable increase in the steady-state house price. In their quantitative exercises, they compare sequences of steady states, where each movement in the credit supply limit “is unanticipated by the agents” (p. 25). Hence, their proposed explanation can be interpreted as departing from rational expectations, as done here. In contrast to their approach, our simulations account for the model’s out-of-steady-state transition dynamics. We find that the observed decline in the U.S. real mortgage interest was not a major contributor to the run-up in U.S. housing value—consistent with the empirical findings of Dokko, et al. (2011) and Glaeser, Gottlieb, and Gyourko (2013). In this regard, it’s worth noting that U.S. real mortgage interest rates continued to decline for several years after 2007 while housing values also continued to fall. Our model ascribes a key role to relaxed borrowing constraints, consistent with the empirical evidence on the rapid growth of subprime mortgage lending during the boom years.

Boz and Mendoza (2014) show that a model with Bayesian learning about a regime shifting loan-to-value limit can produce a pronounced run-up in credit and land prices followed by a sharp and sudden drop. The one-period debt contract in their model causes credit and the land price to move in tandem on the downside—a feature that is not consistent with the gradual deleveraging observed in the data. Nevertheless, the Bayesian updating mechanism in their model shares some of the flavor of the moving average forecast rules in our model. Using a model that abstracts from shifts in lending standards, Adam, Kuang, and Marcet (2012) show that the introduction of constant-gain learning can help account for recent cross-country patterns in house prices and current account dynamics. Constant-gain learning algorithms are similar in many respects to moving average forecast rules; both formulations assume that agents apply exponentially-declining weights to past data when constructing forecasts of future variables.

In a review of the literature on housing bubbles, Glaeser and Nathanson (2014) conclude: “It seems silly now to believe that housing price changes are orderly and driven entirely by obvious changes in fundamentals operating through a standard model” (p. 40). Moving average forecast rules depart from the “standard model” of rational expectations but nevertheless are consistent with a wide variety of survey evidence. In a study of data from the Michigan Survey of Consumers, Piazzesi and Schneider (2009) report that “starting in 2004, more and more households became optimistic after having watched house prices increase for several years” (p. 407). Along these lines, Burnside, Eichenbaum, and Rebelo (2015) develop a model where agents’ optimistic beliefs about future house prices can spread like an infectious disease. In a review of the time series evidence on housing investor expectations from 2002 to 2008, Case, Shiller, and Thompson (2012) find that “1-year expectations [of future house prices changes] are fairly well described as attenuated versions of lagged actual 1-year price changes” (p. 282). Similarly, Greenwood and Shleifer (2014) show that measures of investor expectations about future stock returns are strongly correlated with past stock returns. Jurgilas and Lansing (2013) show that the balance of households in Norway and Sweden expecting a house price

increase over the next year is strongly correlated with nominal house price growth over the preceding year. Ling, Ooi, and Te (2015) find that past real house price changes help to predict future real house price changes even after taking into account measures of buyer, builder, and lender sentiment plus every conceivable fundamental variable that the theory says should matter. Their results can be interpreted as evidence that U.S. housing market participants employ some type of backward-looking, extrapolative, or moving-average forecast rule.

Garriga, Manuelli, and Peralta-Alva (2014) develop a model of house price swings that shares some common features with ours, i.e., long-term mortgage debt and shocks to the mortgage interest rate and lending standards. Under perfect foresight, their model cannot explain the U.S. house price boom-bust episode. In contrast, a version with “shocks to expectations” does a much better job of fitting the data. Gete (2014) shows that introducing the Case-Shiller-Thompson survey expectations into a standard DSGE model can help account for movements in U.S. house prices over the period 1994 to 2012.

## 2 Model

Housing services are priced using a version of the frictionless pure exchange model of Lucas (1978). The representative household’s problem is to choose sequences of  $c_t$  and  $h_t$  to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t c_t h_t^{\theta_t}, \quad (1)$$

subject to the following equations

$$\theta_t = \theta \exp(u_t) \quad (2)$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \quad \varepsilon_{u,t} \sim N(0, \sigma_u^2), \quad (3)$$

$$c_t + p_t h_t + (r_t + \delta_t) b_t = y_t + p_t h_{t-1} + \ell_t, \quad (4)$$

$$b_{t+1} = (1 - \delta_t) b_t + \ell_t, \quad (5)$$

$$x_t \equiv \log(y_t / y_{t-1}) = \bar{x} + \rho_x (x_{t-1} - \bar{x}) + \varepsilon_{x,t} \quad \varepsilon_{x,t} \sim N(0, \sigma_x^2), \quad (6)$$

$$R_t \equiv 1 + r_t = R \exp(\tau_t), \quad (7)$$

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2), \quad (8)$$

where  $c_t$  is real household consumption expenditures,  $h_t$  is the housing service flow,  $y_t$  is real disposable income,  $\beta$  is the subjective time discount factor, and  $\theta_t \geq 0$  measures the strength of the agent’s housing preference which is subject to a persistent exogenous shock  $u_t$ . The symbol  $\widehat{E}_t$  represents the household’s subjective expectation, conditional on information available at time  $t$ , as explained more fully below. Under rational expectations,  $\widehat{E}_t$  corresponds to the mathematical expectation operator  $E_t$  evaluated using the objective distribution of shocks, which are assumed known to the rational household. The symbol  $p_t$  is the price of housing services in consumption units. The law of motion for the stock of household debt is given by

equation (5), where  $\ell_t$  is new borrowing during the period, and  $\delta_t \in (0, 1]$  is the amortization rate, i.e., the fraction of outstanding mortgage debt that is repaid during the period. Real disposable income growth  $x_t$  follows an exogenous AR(1) process given by equation (6). The gross quarterly real mortgage interest rate  $R_t \equiv 1 + r_t$  is subject to a persistent exogenous shock  $\tau_t$ .

Following Kydland, Rupert, and Šustek (2012), we model the mortgage amortization rate using the following law of motion

$$\delta_{t+1} = \left(1 - \frac{\ell_t}{b_{t+1}}\right) \delta_t^\alpha + \frac{\ell_t}{b_{t+1}} (1 - \alpha)^\kappa, \quad (9)$$

where  $\alpha \in [0, 1)$  and  $\kappa \geq 0$  are parameters and the ratio  $\ell_t/b_{t+1}$  measures the size of the new loan relative to the end-of-period stock of mortgage debt. When  $\alpha = 0$ , we have  $\delta_{t+1} = 1$  for all  $t$  from (9) and  $\ell_t = b_{t+1}$  from (4), such that we recover a one-period mortgage contract where all outstanding debt is repaid each period. When  $\alpha > 0$ , the above law of motion captures the realistic feature that the amortization rate is low during the early years of a mortgage (i.e., when  $\ell_t/b_{t+1} \simeq 1$ ) such that mortgage payments consist mainly of interest. The amortization rate rises in later years as more principal is repaid. Kydland, Rupert, and Šustek (2012) show that appropriate settings for the parameters  $\alpha$  and  $\kappa$  can approximately match the amortization schedule of a 30-year conventional mortgage.

We assume that households face the following constraint on the amount of new borrowing each period

$$\ell_t \leq m_t \left[ \widehat{E}_t p_{t+1} h_t - b_{t+1} \right], \quad (10)$$

$$m_t = m \exp(v_t), \quad (11)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t} \quad \varepsilon_{v,t} \sim N(0, \sigma_v^2), \quad (12)$$

where  $m_t$  is a lending standard variable that is subject to a persistent exogenous shock  $v_t$ . Equation (10) says that the size of the new loan  $\ell_t$  cannot exceed a fraction  $m_t$  of expected home equity, i.e., next period's expected housing value  $\widehat{E}_t p_{t+1} h_t$  minus next period's mortgage debt  $b_{t+1}$ . We interpret an increase in  $m_t$  to represent a relaxation of lending standards while a decrease in  $m_t$  is a tightening of standards.<sup>8</sup> For simplicity, we assume that the lender's subjective forecast  $\widehat{E}_t p_{t+1} h_t$  coincides with the household's subjective forecast.

The representative household's optimization problem be formulated as

$$\max_{c_t, h_t, b_{t+1}, \delta_{t+1}} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t, \quad (13)$$

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<sup>8</sup> Along these lines, Duca Muellbauer, and Murphy (2011) find that movements in the LTV ratio of U.S. first-time homebuyers help to explain movements in the ratio of U.S. house prices to rents, particularly in the years after 2000.

where the current-period Lagrangian  $\mathcal{L}_t$  is given by

$$\begin{aligned}\mathcal{L}_t = & c_t h_t^{\theta_t} + \lambda_t [y_t + p_t (h_{t-1} - h_t) + b_{t+1} - R_t b_t - c_t] \\ & + \mu_t \left[ \frac{m_t}{1+m_t} E_t p_{t+1} h_t + \frac{(1-\delta_t)}{1+m_t} b_t - b_{t+1} \right] \\ & + \eta_t \{ \delta_{t+1} b_{t+1} - b_t (1-\delta_t) [\delta_t^\alpha - (1-\alpha)^\kappa] - (1-\alpha)^\kappa b_{t+1} \},\end{aligned}\quad (14)$$

where  $\lambda_t$ ,  $\mu_t$ , and  $\eta_t$  are the Lagrange multipliers on the budget constraint (4), the borrowing constraint (10), and the law of motion for the endogenous amortization rate (9), respectively. In each constraint, we have used equation (5) to eliminate the new loan amount  $\ell_t$ .

The household's first-order conditions with respect to  $c_t$ ,  $h_t$ ,  $b_{t+1}$ , and  $\delta_{t+1}$  are given by

$$\lambda_t = h_t^{\theta_t}, \quad (15)$$

$$\lambda_t p_t = \theta_t c_t h_t^{\theta_t-1} + \mu_t \frac{m_t}{1+m_t} \widehat{E}_t p_{t+1} + \beta \widehat{E}_t \lambda_{t+1} p_{t+1}, \quad (16)$$

$$\begin{aligned}\mu_t = & \lambda_t - \beta \widehat{E}_t \lambda_{t+1} R_{t+1} + \beta (1-\delta_{t+1}) \widehat{E}_t \frac{\mu_{t+1}}{1+m_{t+1}} \\ & + \eta_t [\delta_{t+1} - (1-\alpha)^\kappa] - \beta \underbrace{(1-\delta_{t+1}) [\delta_{t+1}^\alpha - (1-\alpha)^\kappa]}_{\equiv f(\delta_{t+1})} \widehat{E}_t \eta_{t+1}\end{aligned}\quad (17)$$

$$\eta_t = \beta \underbrace{[\alpha \delta_{t+1}^{\alpha-1} (1-\delta_{t+1}) - \delta_{t+1}^\alpha + (1-\alpha)^\kappa]}_{\equiv g(\delta_{t+1})} \widehat{E}_t \eta_{t+1} + \beta \widehat{E}_t \frac{\mu_{t+1}}{1+m_{t+1}}, \quad (18)$$

where we make use of the fact that  $\delta_{t+1}$  is known at time  $t$  because  $b_{t+1}$  is known at time  $t$ . In equation (18), we have simplified things by dividing both sides by  $b_{t+1}$ . After dividing both sides of equation (16) by  $\lambda_t$ , we can see that the ‘‘dividend’’ or imputed rent from owner-occupied housing consists of two parts: (1) a utility flow that is influenced by the stochastic preference variable  $\theta_t$ , and (2) the marginal collateral value of the house in the case when the borrowing constraint is binding, i.e., when  $\mu_t > 0$ .<sup>9</sup>

Equation (17) shows that when mortgage debt extends beyond one period ( $\delta_{t+1} < 1$ ), the household takes into account the expected lending standard variable  $m_{t+1}$  when deciding how much to borrow in the current period. This is an element of shock propagation that is unique to an environment with long-term mortgage debt. With one-period debt ( $\delta_{t+1} = 1$ ,  $\alpha = 0$ ), equation (17) simplifies to  $\mu_t = \lambda_t - \beta \widehat{E}_t \lambda_{t+1} R_{t+1}$ .<sup>10</sup>

<sup>9</sup>We confirm that the borrowing constraint is binding at the ergodic mean values of the state variables. As is common in the literature, we solve the model assuming that the borrowing constraint is always binding in a neighborhood around the ergodic mean.

<sup>10</sup>Given that  $\lambda_t = 1$  for all  $t$  in equilibrium, the rational expectations model with one-period debt will exhibit a binding borrowing constraint at the ergodic mean if  $\beta R \exp(\sigma_\tau^2/2) < 1$ . This condition is satisfied in our calibration of the model parameters, as described in Section 3.

Assuming that housing exists in unit net supply, we have  $h_t = 1$  such that  $\lambda_t = 1$  for all  $t$ . Imposing  $\lambda_t = 1$  in the above equations and dividing both sides of the applicable equilibrium conditions by current period income  $y_t$  to obtain expressions in stationary variables yields:

$$\frac{p_t}{y_t} = \theta_t \frac{c_t}{y_t} + \left[ \mu_t \frac{m_t}{1+m_t} + \beta \right] \widehat{E}_t \frac{p_{t+1}}{y_t}, \quad (19)$$

$$\begin{aligned} \mu_t &= 1 - \beta \widehat{E}_t R_{t+1} + \beta (1 - \delta_{t+1}) \widehat{E}_t \frac{\mu_{t+1}}{1+m_{t+1}} \\ &\quad + \eta_t [\delta_{t+1} - (1 - \alpha)^\kappa] - \beta f(\delta_{t+1}) \widehat{E}_t \eta_{t+1}, \end{aligned} \quad (20)$$

$$\eta_t = \beta g(\delta_{t+1}) \widehat{E}_t \eta_{t+1} + \beta \widehat{E}_t \frac{\mu_{t+1}}{1+m_{t+1}}, \quad (21)$$

$$\frac{c_t}{y_t} = 1 + \frac{b_{t+1}}{y_t} - R_t \frac{b_t}{y_{t-1}} \exp(-x_t), \quad (22)$$

$$\frac{b_{t+1}}{y_t} = \frac{m_t}{1+m_t} \widehat{E}_t \frac{p_{t+1}}{y_{t+1}} \exp(x_{t+1}) + \frac{(1-\delta_t)}{1+m_t} \frac{b_t}{y_{t-1}} \exp(-x_t), \quad (23)$$

$$\delta_{t+1} = \frac{b_t/y_{t-1}}{b_{t+1}/y_t} \exp(-x_t) (1-\delta_t) [\delta_t^\alpha - (1-\alpha)^\kappa] + (1-\alpha)^\kappa, \quad (24)$$

where the last three equations are the normalized versions of the budget constraint, the borrowing constraint, and the law of motion for the amortization rate.

## 2.1 Rational Expectations

Details regarding the rational expectations solution are contained in the appendix. We transform the equilibrium conditions (19) through (24) so that the household's decision variables correspond to the three endogenous objects that the household must forecast, namely: (1) the normalized housing value  $p_{n,t} \equiv p_t/y_{t-1}$ , (2) a composite variable  $w_t \equiv \mu_t/(1+m_t)$  that depends on the borrowing constraint shadow price  $\mu_t$  and the lending standard variable  $m_t$ , and (3) the amortization rate shadow price  $\eta_t$ . There are six state variables: (1) the normalized stock of mortgage debt  $b_{n,t} \equiv b_t/y_{t-1}$ , (2) the mortgage amortization rate  $\delta_t$ , (3) the housing preference shock  $u_t$ , (4) the lending standard shock  $v_t$ , (5) the income growth rate  $x_t$ , and (6) the mortgage interest rate shock  $\tau_t$ . The state variables  $b_{n,t}$  and  $\delta_t$  are endogenous while the other four state variables are exogenous, as governed by the AR(1) laws of motion (3), (12), (6), and (8). To solve for the household decision rules, we employ a log-linear approximation of the transformed equilibrium conditions. The approximation point is the ergodic mean rather than the deterministic steady state.<sup>11</sup>

<sup>11</sup>Lansing (2010) demonstrates the accuracy of this solution method in a standard asset pricing model.

## 2.2 Moving Average Forecast Rules

The rational expectations solution is based on strong assumptions about the representative household's information set. Specifically, the rational solution assumes that households know the stochastic processes for all exogenous shocks. The survey evidence described in Section 1.1 shows that there is strong empirical support for extrapolative or moving average type forecast rules. For example, U.S. inflation expectations derived from the Survey of Professional Forecasters (SPF) systematically underpredict inflation in the sample period prior to 1979 when inflation was rising and systematically overpredict it thereafter when inflation was falling. The survey pattern is well-captured by a moving-average of past inflation rates.<sup>12</sup> More generally, a moving average forecast rule can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information.

Motivated by the empirical evidence, we postulate that the household's forecast for a given variable is an exponentially-weighted moving average of past observed values of that same variable. Constructing such a forecast requires only a minimal amount of computational and informational resources. From equations (19) through (24), we see that the household must construct four separate forecasts: (1)  $\widehat{E}_t p_{n,t+1}$  where  $p_{n,t+1} \equiv p_{t+1}/y_t$ , (2)  $\widehat{E}_t w_{t+1}$ , where  $w_{t+1} \equiv \mu_{t+1}/(1 + m_{t+1})$ , (3)  $\widehat{E}_t \eta_{t+1}$ , and (4)  $\widehat{E}_t R_{t+1}$ . The moving average forecast rule for  $\widehat{E}_t p_{n,t+1}$  is given by

$$\begin{aligned} \widehat{E}_t p_{n,t+1} &= \widehat{E}_{t-1} p_{n,t} + \lambda \left[ p_{n,t} - \widehat{E}_{t-1} p_{n,t} \right], \\ &= \lambda \left[ p_{n,t} + (1 - \lambda) p_{n,t-1} + (1 - \lambda)^2 p_{n,t-2} + \dots \right] \end{aligned} \quad (25)$$

where the parameter  $\lambda \in [0, 1]$  governs the weight assigned to the most recent observation— analogous to the gain parameter in the adaptive learning literature. When  $\lambda = 1$ , households employ a simple random walk forecast such that  $\widehat{E}_t p_{n,t+1} = p_{n,t}$ . In this case, households view all movements in  $p_{n,t}$  as permanent. In contrast, when  $\lambda = 0$ , households view all movements in  $p_{n,t}$  as temporary. The forecast rules for  $\widehat{E}_t w_{t+1}$ ,  $\widehat{E}_t \eta_{t+1}$ , and  $\widehat{E}_t R_{t+1}$  are constructed in the same way. For simplicity, we assume that the household employs the same value of  $\lambda$  for all forecasts.

Substituting the moving average forecast rules into the transformed first-order conditions yields a set of nonlinear laws of motion for the three decision variables  $p_{n,t}$ ,  $w_t$ , and  $\eta_t$ . Details are contained in the appendix.

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<sup>12</sup>This result is demonstrated by Lansing (2009) and Gelain, Lansing, and Mendicino (2013).

### 3 Parameter Values

Table 1: Model Parameter Values

Parameter	One-period Mortgage	Long-term Mortgage	Description/Target
$\alpha$	0	0.9959	Approximate 30-year mortgage schedule.
$\kappa$	0	1.0487	Approximate 30-year mortgage schedule.
$\beta$	0.9828	0.9828	House price/quarterly rent $\simeq 80$ .
$\theta$	0.0625	0.0657	Housing value/quarterly income $\simeq 6.3$ .
$m$	0.5836	0.0121	Mortgage debt/quarterly income $\simeq 2.3$ .
$\bar{x}$	0.00473	0.00473	Quarterly income growth rate = 0.473%.
$R$	1.01	1.01	Gross quarterly real mortgage rate.
$\lambda$	0.9	0.9	Forecasts for $p_{n,t+1}$ and $R_{t+1}$ in U.S. data.

Table 2: Parameters for Stochastic Shocks

Parameter	RE Model	MA Model	1995.Q1 - 2012.Q4 Target
$\rho_u$	0.95	0.95	AR(1) housing value/income.
$\sigma_u$	0.351	0.037	Std. dev. housing value/income.
$\rho_v$	0.95	0.95	AR(1) mortgage debt/income.
$\sigma_v$	0.118	0.112	Std. dev. mortgage debt/income.
$\rho_x$	-0.23	-0.23	AR(1) income growth rate.
$\sigma_x$	0.0082	0.0082	Std. dev. income growth rate.
$\rho_\tau$	0.95	0.95	AR(1) mortgage interest rate.
$\sigma_\tau$	0.00078	0.00078	Std. dev. mortgage interest rate.

Notes: RE = rational expectations. MA = moving average forecast rules.

Tables 1 and 2 show the values of the model parameters that we employ in the simulations. The parameters in Table 1 are the same for both expectation regimes but in some cases differ across mortgage specifications. From Figure 1, we see that the ratios from U.S. data are all close to their long-run means in the mid-1990s. Anticipating the reverse-engineering exercise, we choose the values of  $\beta$ ,  $\theta$ , and  $m$  simultaneously so that the ergodic-means of three model-implied ratios are close to their U.S. data counterparts at 1995.Q1. The three ratios are: (1) house price-rent, (2) housing value-income, and (3) mortgage debt-income. By construction, we also match the debt-to-value ratio at 1995.Q1. Data on U.S. house prices and imputed rents from owner-occupied housing are from the Lincoln Land Institute.<sup>13</sup> Data on U.S. residential real estate values and household mortgage debt are from the Federal Reserve Flow of Funds. Data on personal disposable income and population are from the Federal Reserve Bank of St. Louis' FRED database.

Following Kydland, Rupert, and Šustek (2012), the values of  $\alpha$  and  $\kappa$  are chosen so that the amortization schedule for the model's long-term mortgage roughly approximates the amortization schedule of a conventional 30-year mortgage. With long-term mortgage debt, we require

<sup>13</sup>See [www.lincolnlnd.edu](http://www.lincolnlnd.edu). For prices, we use the data series that includes the Case-Shiller-Weiss measure from the year 2000 onwards, as documented in Davis, Lehnert, and Martin (2008).

a lending standard parameter of  $m = 0.0124$  to match the ratios in the data whereas the one-period debt model requires  $m = 0.5951$ . In the models with long-term mortgage debt, the loan-to-value ratio differs from the debt-to-value ratio whereas these two ratios coincide in the models with one-period debt.

Starting from the normalized collateral constraint (23), it is straightforward to show that the steady state debt-to-value ratio is given by

$$\frac{b}{p} = \frac{m}{[1 + m - (1 - \tilde{\delta}) \exp(-\bar{x})]}. \quad (26)$$

where  $\tilde{\delta}$  is the mean amortization rate. Our calibration procedure yields  $\tilde{\delta} = 0.0162$  with long-term mortgage debt versus  $\tilde{\delta} = 1$  with one-period debt. Equation (26) shows changes in the value of  $\tilde{\delta}$  must be accompanied by changes in the value of  $m$  so that the model continues to match the target debt-to-value ratio implied by the data.

The parameter  $\lambda$  in the moving average model governs the forecast weight assigned to the most recent data observation. We use the same value of  $\lambda$  for each of the four conditional forecasts that appear in the households's first order conditions (19) through (24). Of the four variables that the agent must forecast, two are observable in U.S. data, namely  $p_{n,t+1}$  and  $R_{t+1}$ . The variable  $p_{n,t+1} \equiv p_{t+1}/y_t$  is constructed using data on the housing value-income ratio  $p_t h_t / y_t$ , as plotted in the top left panel of Figure 1. The variable  $R_{t+1}$  is the gross quarterly real mortgage interest rate which we construct from the data for the period 1971.Q2 to 2012.Q4.<sup>14</sup>

To get a sense of a reasonable value for  $\lambda$ , Figure 2 plots the root mean squared percentage forecast error (RMSPFE) for one-quarter ahead forecasts of  $p_{n,t+1}$  and  $R_{t+1}$  in the data using a moving average forecast rule of the form (25). For  $p_{n,t+1}$ , forecast performance is best (lowest RMSPFE) when  $\lambda \simeq 1.4$  for the boom-bust period and  $\lambda \simeq 1$  for the pre-boom period. For  $R_{t+1}$ , forecast performance is best when  $\lambda \simeq 1$  during both periods. Recall that  $\lambda = 1$  corresponds to a random walk forecast. When  $\lambda > 1$ , a positive forecast error in the prior period leads to an upward adjustment in the forecasted growth of the variable in the next period.<sup>15</sup> In the model, values of  $\lambda$  that approach or exceed unity give rise to explosive dynamics. For the simulations, we employ  $\lambda = 0.9$  which yields stable dynamics. Figure 2 shows that  $\lambda = 0.9$  does not sacrifice much in forecast performance relative to higher values of  $\lambda$ . Hence, our calibration implies that households employ a “near-optimal” value of  $\lambda$  for the simulations that exactly replicate the U.S. data.

Table 2 shows the parameter values that govern the persistence and volatility of the four stochastic shocks. The parameter values for the housing preference shock  $u_t$  and the lending

<sup>14</sup>We start with data on the nominal 30-year conventional mortgage interest rate from the Federal Reserve Bank of St. Louis' FRED database. We then convert the annualized nominal mortgage interest rate into a quarterly real rate using 4-quarter-ahead expected inflation (converted to a quarterly expected inflation rate) from the Survey of Professional Forecasters.

<sup>15</sup>To see this, equation (25) can be rearranged as follows:  $\hat{E}_t (p_{n,t+1} - p_{n,t}) = (\lambda - 1) [p_{n,t} - \hat{E}_{t-1} p_{n,t}]$ .

standard shock  $v_t$  depend on the expectation regime. We calibrate these shocks so that the rational expectations (RE) model and the moving average (MA) model can both match the standard deviations of the U.S. real estate value and mortgage debt ratios over the period 1995.Q1 to 2012.Q4. Analytical moment formulas derived from the log-linear solutions of both models are used in the calibration procedure. The calibration is done for the case of long-term mortgage debt, but we use the same set of shock parameters in the case of one-period debt. From Table 2, we see that the RE model requires a highly volatile housing preference shock with  $\sigma_u = 0.351$  versus  $\sigma_u = 0.037$  in the MA model. For the lending standard shock, the RE model requires  $\sigma_v = 0.118$  versus  $\sigma_v = 0.112$  in the MA model.

The stochastic process for income growth  $x_t$  is estimated using data on the quarterly growth rate of U.S. real per capita disposable income. The parameter values for the mortgage interest rate shock  $\tau_t$  are estimated using data on the 30-year conventional mortgage interest rate after conversion to a quarterly real rate as described above. The sample period for estimating the shock parameters is 1995.Q1 to 2012.Q4.

## 4 Quantitative Results

### 4.1 Simulations with Model-Specific Shocks

Figure 3 (RE model) and Figure 4 (MA model) show simulation results using the parameter values in Tables 1 and 2. The four panels in each figure are the model-generated versions of the corresponding U.S. data ratios plotted earlier in Figure 1.

Our calibration procedure ensures that the RE model and the MA model both exhibit realistic volatilities for the housing value-income ratio  $p_t h_t / y_t$  and the mortgage debt-income ratio  $b_t / y_t$ . Consequently, the top panels of Figure 3 look similar to the top panels of Figure 4. A crucial distinction between the two models can be seen by comparing the bottom left panels of Figures 3 and 4. The RE model predicts a substantially more volatile rent-income ratio than the MA model. This is because the RE model's housing preference shock has  $\sigma_u = 0.351$  which is nearly ten times larger than the corresponding value  $\sigma_u = 0.037$  in the MA model. The volatility of the housing preference shock directly influences the volatility of the rent-income ratio which is given by

$$\frac{Rent_t}{y_t} = \theta_t \frac{c_t}{y_t} + \mu_t \frac{m_t}{1 + m_t} \widehat{E}_t \frac{p_{t+1}}{y_t}, \quad (27)$$

where  $\theta_t = \theta \exp(u_t)$  is the stochastic housing preference variable. The first term on the right side of (27) is the housing service flow while the second term is the marginal collateral value of the house. In the simulations, the volatility of the rent-income ratio is determined mainly by movements in the housing service flow.

With long-term mortgage debt, the coefficient of variation for the rent-income ratio in the RE model is 0.66 versus 0.11 in the MA model. For comparison, the coefficient of variation for the rent-income ratio in U.S. data is 0.10 over the period 1960.Q1 to 2012.Q4. For the

more-recent period of 1995:Q1 to 2012:Q4, the coefficient of variation is even lower at 0.02. The extremely low volatility of the rent-income ratio in the data argues against fundamental demand shocks as a key driver of the boom-bust episode. A virtue of the MA model is its ability to generate realistic volatility in the housing value-income ratio without the need for large housing demand shocks.

The right-side panels in Figures 3 and 4 show that the long-term mortgage specification delivers smoother behavior in the debt-income ratio  $b_t/y_t$  and the consumption-income ratio  $c_t/y_t$  relative to the one-period mortgage version of the same model. With long-term mortgage debt, shocks can have a large impact on the size of the new loan but the impact on the stock of outstanding debt is much smaller. This is because the new loan represents only a small fraction of the end-of-period stock of debt. In contrast, the new loan and the end-of-period stock of debt are equal with one-period mortgage debt, causing the debt-income ratio to be more responsive to shocks.<sup>16</sup>

The normalized version of the household's budget constraint (22) shows that movements in the consumption-income ratio are linked to movements in the debt-income ratio. The smoother behavior of the debt-income ratio in the models with long-term mortgage debt translates into smoother behavior for the consumption-income ratio.

## 4.2 Impulse Response Functions with Common Shocks

Figures 5 and 6 illustrate how a common stochastic shock propagates differently in the four different model specifications. Figure 5 plots the model responses to a one standard deviation innovation of the housing preference shock  $u_t$ . Figure 6 plots the model responses to a one standard deviation innovation of the lending standard shock  $v_t$ . The vertical axes measure the percentage deviation of the variable from the no-shock value. All model specifications now employ  $\sigma_u = 0.037$  (Figure 5) or  $\sigma_v = 0.112$  (Figure 6). These are the original calibrated values from the MA model, as shown in Table 2.

Both figures show that, regardless of the mortgage specification, the MA model exhibits substantially more volatility in housing value than the RE model. In other words, the MA model exhibits excess volatility in the asset price in response to fundamental shocks. This result is not surprising given that the moving average forecast rule (25) embeds a unit root assumption. This is most obvious when  $\lambda = 1$  but is also true when  $0 < \lambda < 1$  because the weights on lagged variables sum to unity. Due to the self-referential nature of the equilibrium conditions, the households' subjective forecast influences the dynamics of the object that is being forecasted.<sup>17</sup>

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<sup>16</sup>In the context of a monetary DSGE model, Gelain, Lansing, and Natvik (2015) show that a tightening of monetary policy reduces the debt-income ratio with one-period mortgage debt but increases the debt-income ratio with long-term mortgage debt.

<sup>17</sup>A simple example with  $\lambda = 1$  illustrates the point. Suppose that  $p_t = d_t + \beta \widehat{E}_t p_{t+1}$ , where  $d_t$  follows an AR(1) process with persistence  $\gamma$ . Under rational expectations, we have  $Var(p_t)/Var(d_t) = 1/(1 - \gamma\beta)^2$ . When  $\widehat{E}_t p_{t+1} = p_t$ , we have  $Var(p_t)/Var(d_t) = 1/(1 - \beta)^2$  which implies excess volatility in the model asset

Given that all shocks are governed by AR(1) laws of motion, a hump-shaped impulse response is indicative of an endogenous propagation mechanism in the model. The MA model with long-term mortgage debt is the only specification to exhibit a hump-shaped response in both housing value and mortgage debt. The effects of the shocks are temporary but highly persistent—lasting in excess of 100 quarters (25 years). The RE model with long-term mortgage debt can produce a hump-shaped response in debt but not housing value. Notice that the RE model with one-period mortgage debt does not produce a hump-shaped response in either housing value or debt. Hence, the dynamics of model variables in this version are driven entirely by the exogenous AR(1) shocks.

Another notable feature of the impulse response functions is the timing of the peaks in housing value versus mortgage debt. With one-period debt, both peaks occur at the same time. With long-term mortgage debt, the peak in housing value occurs well before the peak in debt. This is qualitatively similar to the pattern observed in Figure 1 for the U.S. data.

### 4.3 Reverse-Engineering the Shocks to Match the Data

We now undertake the main part of our quantitative analysis: reverse-engineering the sequences of stochastic shocks that are needed to exactly replicate the boom-bust patterns in U.S. household real estate value and mortgage debt over the period 1995.Q1 to 2012.Q4. All of the model’s state variables are set equal to their ergodic means at 1995.Q1. For each version of the model, we use the log-linearized versions of the decision rules and laws of motion in first-difference form to back out sequences for the change in the housing preference shock  $\Delta u_t$  and the change in the lending standard shock  $\Delta v_t$  to match the change in the U.S. housing value-income ratio and the change in the U.S. mortgage debt-income ratio. For each period of the exercise, we have a linear system of two equations and unknowns, namely,  $\Delta u_t$  and  $\Delta v_t$ . Given the sequences for  $\Delta u_t$  and  $\Delta v_t$ , we construct sequences for  $u_t$  and  $v_t$  using the initial conditions  $u_t = v_t = 0$  at 1995.Q1. We use the first-difference forms of the log-linear decision rules and laws of motion to eliminate constant terms in the model which, for some variables, may not coincide with the corresponding U.S. values in 1995.Q1.<sup>18</sup>

As inputs to the reverse-engineering exercise, we use U.S. data for the period 1995.Q1 to 2012.Q4 to identify sequences for the change in disposable income growth  $\Delta x_t$  and the change in the mortgage interest rate shock  $\Delta \tau_t$ . The data we use to identify  $\Delta x_t$  and  $\Delta \tau_t$  are plotted in Figure 7, where the trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1600. We use the trends to identify  $\Delta x_t$  and  $\Delta \tau_t$  in order to screen out high frequency movements in the data that would show up as noise in the reverse-engineered shocks, thus obscuring their economic interpretation. Given the identified sequences for  $\Delta x_t$

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price whenever  $|\gamma| < 1$ .

<sup>18</sup>For example, the ergodic mean value of  $c_t/y_t$  in the model does not coincide with the U.S. consumption-income ratio in 1995.Q1. Nevertheless, given a model-implied sequence for  $\Delta(c_t/y_t)$ , we can construct a comparable model-implied sequence for  $c_t/y_t$  using the 1995.Q1 value in the data as the initial condition.

and  $\Delta\tau_t$ , we construct sequences for the state variables  $x_t - \bar{x}$  and  $\tau_t$  using the initial conditions  $x_t = \bar{x} = 0.00473$  and  $\tau_t = 0$  at 1995.Q1. The time patterns of these state variables mimic the trends in Figure 7.

Figure 8 plots the results of the reverse-engineering exercise. The left panels show the reverse-engineered shocks in the RE models while the right panels show the corresponding shocks in the MA models.

Analogous to the model simulations, the RE model requires large movements in the reverse-engineered housing preference shock to match the data. The time pattern of the housing preference shock mimics the path of the U.S. housing value-income ratio in Figure 1. This is true for both mortgage specifications. Hence, the RE model “explains” the boom-bust cycle in U.S. housing value as a wholly exogenous phenomenon. In contrast, the top right panel of Figure 8 shows that the MA model requires much smaller movements in the housing preference shock to match the same data. Again this is true for both mortgage specifications.

Table 3 compares the properties of the reverse-engineered shocks across the four different model specifications. With long-term mortgage debt, the mean of the housing preference shock is 0.97 in the RE model versus 0.04 in the MA model. The standard deviation of the housing preference shock is 1.04 in the RE model versus 0.25 in the MA model.

The bottom panels of Figure 8 show that the reverse-engineered lending standard shock is highly dependent on the mortgage specification, but is not sensitive to the expectation regime. With one-period mortgage debt, both the RE and MA models imply a near-zero lending standard shock during the boom years prior to 2007. This is because the one-period debt specification requires the stock of debt to move in tandem with housing value. Since housing value is driven upwards by the other shocks, a lending standard shock is not needed to explain the run-up in mortgage debt. Things are different, however, during the bust years. To avoid the counterfactual prediction of a rapid deleveraging as U.S. housing values fell rapidly, the one-period debt models require a post-2007 relaxation of lending standards (i.e., a persistently positive value for the lending standard shock  $v_t$ ) to simultaneously match the patterns of housing value and mortgage debt in the data.

With long-term mortgage debt, the new loan size moves in tandem with housing value but the stock of mortgage debt adjusts more slowly than housing value. In order to match the run-up in U.S. mortgage debt during the boom years, the long-term debt versions of the RE and MA models both require a substantial relaxation of lending standards during the boom years from 2001 to 2005. This pattern is consistent with the empirical evidence cited in the introduction. Period-by-period fluctuations in stock of mortgage debt in the data translate into the need for much larger fluctuations in the flow of new loans in the models, thus accounting for the volatility of the reverse-engineered lending standard shocks. The magnitude of the lending standard shock  $v_t$  starts declining well before the peak in mortgage debt that occurs at 2007.Q4. A declining value of  $v_t$  implies a tightening of lending standards. After 2007.Q4, both models require a persistently negative value for the lending standard shock

which is indicative of even further tightening of lending standards during the Great Recession and beyond.

Figure 9 plots two indicators of lending standard tightness from the Federal Reserve’s Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS). The indicators are the net percentage of U.S. domestic banks that are tightening lending standards for either residential mortgage loans or credit card loans.<sup>19</sup> Both series show that banks started to tighten lending standards before the onset of the Great Recession in 2007.Q4. Moreover, a substantial percentage of banks continued to tighten standards even after the recession ended in 2009.Q2. Overall, the SLOOS data confirms the plausibility of the reverse-engineered lending standard shocks in the models with long-term mortgage debt.

Table 3: Properties of Reverse-Engineered Shocks

Shock	1995.Q1-2012.Q4 Statistic	One-period Mortgage		Long-term Mortgage	
		RE Model	MA Model	RE Model	MA Model
Housing	Mean	0.92	0.02	0.97	0.04
Preference	Std. dev.	1.13	0.31	1.04	0.25
Lending	Mean	0.18	0.16	0.24	0.23
Standard	Std. dev.	0.27	0.27	0.58	0.57

Notes: RE = rational expectations. MA = moving average forecast rules.

Figures 10 and 11 plot the model-implied paths for two other variables of interest, namely, the housing rent-income ratio given by equation (27) and the consumption-income ratio given by equation (22). For the rent-income ratio, we first construct a log-linearized law of motion for the change in the ratio in terms of the change in the model state variables. We then substitute in the identified sequences for  $\Delta x_t$  and  $\Delta \tau_t$  and the reverse-engineered sequences for  $\Delta u_t$  and  $\Delta v_t$ . As before, the endogenous state variables evolve according to their log-linearized laws of motion in first-difference form. For the consumption-income ratio, we simply use the exact nonlinear law of motion (22) in first-difference form.

Figure 10 shows that both versions of the RE model predict a counterfactual boom-bust cycle in the rent-income ratio that is driven by the large movements in the reverse-engineered housing preference shock. In contrast, the predicted paths for the rent-income ratio in the MA model are much closer to the data. The endogenous bubble-like dynamics in housing value generated by the moving average forecast rules allows the MA model to be much less reliant on exogenous housing preference shocks to match the data. This helps to avoid the prediction of large movements in housing rents which are not present in the data.

<sup>19</sup>The data are available from [www.federalreserve.gov/boarddocs/SnLoanSurvey/](http://www.federalreserve.gov/boarddocs/SnLoanSurvey/). Prior to 2007.Q2, the survey data do not distinguish between prime and subprime mortgages. From 2007.Q2 onwards, we plot the survey responses for prime mortgages.

In Figure 11, all model versions deliver identical paths for the consumption-income ratio. The normalized household budget constraint (22) shows that movements in the consumption-income ratio are linked mechanically to movements in the debt-income ratio and the mortgage interest rate which, by construction, are the same across models for this exercise. A smoothed version of the model-implied path (constructed using the Hodrick-Prescott filter) exhibits a hump-shaped pattern that roughly resembles the hump-shaped pattern in the data from 1995 to 2012. Hence, abstracting from high-frequency fluctuations, all models predict a boom-bust cycle in consumption that is positively correlated with the boom-bust cycles in housing value and mortgage debt.

Table 4: Per Capita Consumption Growth Rates

Time Period	U.S. Data	All Models
Boom: 1995.Q1 to 2007.Q4	3.58	2.32
Bust: 2007.Q4 to 2009.Q2	-1.83	-4.27
Recovery: 2009.Q2 to 2012.Q4	2.03	0.74

Note: Annualized compound growth rate over the period in %.

Table 4 compares annualized compound growth rates of per capita consumption from U.S. data to the predicted growth rates from the models for three phases of the simulation, i.e., the boom from 1995.Q1 to 2007.Q4, the bust coinciding with the Great Recession from 2007.Q4 to 2009.Q2, and the recovery starting in 2009.Q2 and going until the end of our data sample in 2012.Q4. Since the paths for  $c_t/y_t$  and income growth are the same across models, so too is consumption growth. In both the data and the models, consumption growth is fastest during the boom phase when income growth shocks were persistently positive and debt was rising faster than income. The bust delivers negative consumption growth in all models. Finally, the recovery is very sluggish, with model-implied consumption growth rates that are less than 1% . As in the data, the sluggish recovery coincides with a period of household deleveraging. Relative to the U.S. data, the consumption bust in the model is more severe and the consumption recovery is much weaker. But of course the model is missing the numerous automatic stabilizers and policy responses that helped to support U.S. consumer spending as these events transpired.

Recall that the shocks are not designed to match the path of  $c_t/y_t$  in the U.S. data. Nevertheless, conditional on matching the observed time paths of U.S. housing value and mortgage debt, all of the models can account for the broad patterns of U.S. consumption growth during the boom, bust, and recovery phases. It is important to recognize, however, that each model tells a different story regarding the shocks that presumably caused these events. We think the story told by the MA model with long-term mortgage debt is the most plausible.

## 5 Counterfactual Scenarios

Our final quantitative exercise examines four counterfactual scenarios that are plotted in Figures 12 through 15. In each case, we turn off one of the four shock sequences that was constructed in the reverse-engineering exercise. Turning off one shock at a time allows us to see which shocks are the most important drivers of the boom-bust episode, as interpreted through the lens of the model.

Counterfactual scenario 1 (Figure 12) shuts off the housing preference shock such that  $u_t = 0$  for all  $t$ . The RE models now exhibit no significant run-up in housing value. This result confirms the importance of the housing preference shock in “explaining” the run-up under rational expectations. In contrast, the MA models still exhibit a sizeable run-up in housing value, particularly in the version with long term mortgage debt which continues to be hit by positive income growth shocks and loosening lending standards. The run-up in housing value now starts earlier than in the data, however. This pattern is driven by the series of persistently positive income growth shocks during the late 1990s (Figure 7). From the top right panel of Figure 8, we see that the housing preference shocks in the MA models are slightly negative from 1995 to 2000—the period of “irrational exuberance” in the NASDAQ stock market index. One interpretation of these results is that the positive income growth shocks of the late 1990s helped fuel a run-up in stock prices rather than house prices. Since there is only one asset price in our model, the only way to delay the rise in housing values until after 2000 is to postulate the existence of small negative shocks to housing demand during the late 1990s—a period when investors were paying more attention to the stock market than the housing market.

Counterfactual scenario 2 (Figure 13) shuts off the lending standard shock such that  $v_t = 0$  for all  $t$ . The models with one-period mortgage debt now imply a rapid deleveraging that coincides with the rapid decline in housing value. Recall that the models with one-period debt require an implausible sequence of looser lending standard shocks after 2007 to match the gradual deleveraging in the data. Once this shock sequence is turned off, the stock of debt moves down in tandem with housing value. In contrast, the models with long-term mortgage debt exhibit much smaller run-ups in mortgage debt when the lending standard shock is turned off. According to these models, shifting lending standards were an important driver of the boom-bust episode. The MA model with long-term mortgage debt is the only one to show smaller boom-bust cycles in both housing value and mortgage debt when the lending standard shock is turned off.

Counterfactual scenario 2 can be interpreted as a macroprudential policy experiment; it shows what would have happened if mortgage regulators had enforced prudent lending standards during the boom years. According to our preferred model (MA with long-term mortgage debt), such action by regulators would have helped to restrain the boom on the upside such that the subsequent bust and the associated economic damage would have been less severe.

Counterfactual scenario 3 (Figure 14) shuts off the income growth shock such that  $x_t = \bar{x}$  for all  $t$ . There is little noticeable effect in the RE models showing that the remaining shocks are doing all the work of fitting the data. In contrast, the MA models now exhibit a brief and mild decline in housing value until the year 2000. Again, this pattern can be traced to the slightly negative housing preference shocks during the early part of the simulation. These are needed when all four shocks are present, as explained above in counterfactual scenario 1. Otherwise, the positive income growth shocks of the late 1990s would push up housing value too soon relative to the data (since there is no other asset price in the model). According to the MA models, movements in income growth did play a role in the magnitude and timing of the episode. Notice that the MA model with one-period debt now exhibits a much larger boom-bust cycle in debt. The (implausible) post-2007 loosening of lending standards in this version of the model accounts for the larger debt movements when we turn off the post-2007 negative income growth shocks. The MA model with long-term mortgage debt exhibits smaller boom-bust cycle in debt when the income growth shock is turned off.

Counterfactual scenario 4 (Figure 15) shuts off the mortgage interest rate shock such that  $\tau_t = 0$  for all  $t$ . All model versions continue to exhibit significant boom-bust cycles in both housing value and debt. This is because the magnitude of the U.S. mortgage interest rate drop is simply too small to have much impact. Figure 7 shows that the trend value of the quarterly real mortgage interest rate declined by only 30 basis points from 1995.Q1 until 2005.Q4. After 2005.Q4, the interest rate continued to decline by about 45 basis points. These interest rate moves are not sufficient to appreciably alter the trajectories of housing value and debt in the presence of the other three shocks. According to the models, the decline in the U.S. mortgage interest rate was not a major driver of the boom-bust episode.<sup>20</sup>

Our results regarding the mortgage interest rate conflict with the findings of Taylor (2007) and McDonald and Stokes (2011) who argue that the Fed's decision to keep interest rates artificially low during the boom years helped fuel the housing bubble.<sup>21</sup> But other studies find that low interest rates were not a major contributor to the U.S. housing boom. Dokko, et al. (2011) present evidence that movements in U.S. house prices were much larger than can be explained by the historical relationship between house prices and interest rates. An empirical study by Glaeser, Gottlieb, and Gyourko (2013) finds that lower real interest rates can explain only about 20% of the rise in U.S. house prices from 1996 to 2006. One way to reconcile these various findings is to postulate that the low interest rate environment at the time fostered excessive risk-taking behavior by lenders.<sup>22</sup> This explanation is consistent with widespread use of imprudent lending practices during the boom years of the U.S. housing market.

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<sup>20</sup>In experiments with the models, we find that doubling the magnitude of the total mortgage interest rate drop to about 150 basis points is necessary to obtain a significant effect on the trajectories of housing value and mortgage debt.

<sup>21</sup>In a long-run historical study of many countries, Jordà, Schularick, and Taylor (2014b) find that loose monetary conditions typically contribute to booms in house prices and real estate lending.

<sup>22</sup>Adrian and Shin (2010) and Jiménez, et al. (2014) present evidence that low interest rate environments contribute to increased risk-taking by lenders.

## 6 Conclusion

Episodes of explosive, bubble-like growth in house prices have occurred in many OECD countries over the past four decades (Engsted, Hviid, and Pedersen 2014). A recent cross-country empirical study by Jordà, Schularick, and Taylor (2014b) concludes that “Mortgage booms are an important source of financial instability in the post-WWII era” (p. 40). Our simple quantitative asset pricing model helps to shed light on the underlying causes of the recent boom-bust cycle in the U.S. housing market. A clear understanding of these causes is important because it can help in the design of policy actions to avoid future crises.

The official report of the U.S. Financial Crisis Inquiry Commission (2011) states: “We conclude this financial crisis was avoidable. . . Despite the expressed view of many on Wall Street and in Washington that the crisis could not have been foreseen or avoided, there were warning signs. The tragedy was that they were ignored or discounted” (p. xvii). The report lists such red flags as “an explosion in risky subprime lending and securitization, an unsustainable rise in housing prices, widespread reports of egregious and predatory lending practices, (and) dramatic increases in household mortgage debt.”

Our preferred model of the boom-bust cycle includes the following elements: (1) households who employ simple moving-average forecast rules that give rise to excess volatility in asset prices, (2) long-term mortgage contracts that cause the stock of outstanding household debt to adjust more slowly than the flow of new loans, and (3) relaxed lending standards during the run-up that created the conditions for a credit-fueled housing bubble. Our results further suggest that policy actions by regulators to control the flow of mortgage credit by enforcing prudent lending standards can limit a housing boom on the upside, such that the subsequent bust and the resulting economic fallout may be less severe.

## A Appendix: Equilibrium Conditions in Stationary Variables

Starting from the equilibrium conditions (19) through (24), we define the following stationary variables:  $p_{n,t} \equiv p_t/y_{t-1}$ ,  $w_t \equiv \mu_t/(1+m_t)$ ,  $b_{n,t} \equiv b_t/y_{t-1}$ , and  $c_{n,t} \equiv c_t/y_t$ . After substituting in these definitions, the transformed equilibrium conditions are

$$p_{n,t} = \theta \exp(u_t + x_t) c_{n,t} + [m \exp(v_t) w_t + \beta] \exp(x_t) \widehat{E}_t p_{n,t+1}, \quad (\text{A.1})$$

$$w_t = [1 + m \exp(v_t)]^{-1} \left\{ 1 - \beta R \widehat{E}_t \exp(\tau_{t+1}) + \beta (1 - \delta_{t+1}) \widehat{E}_t w_{t+1} + \eta_t [\delta_{t+1} - (1 - \alpha)^\kappa] - \beta f(\delta_{t+1}) \widehat{E}_t \eta_{t+1} \right\} \quad (\text{A.2})$$

$$\eta_t = \beta g(\delta_{t+1}) \widehat{E}_t \eta_{t+1} + \beta \widehat{E}_t w_{t+1} \quad (\text{A.3})$$

$$c_{n,t} = 1 + b_{n,t+1} - R \exp(\tau_t - x_t) b_{n,t}, \quad (\text{A.4})$$

$$b_{n,t+1} = [1 + m \exp(v_t)]^{-1} \left[ m \exp(v_t) \widehat{E}_t p_{n,t+1} + (1 - \delta_t) \exp(-x_t) b_{n,t} \right], \quad (\text{A.5})$$

$$\delta_{t+1} = \frac{b_{n,t}}{b_{n,t+1}} \exp(-x_t) (1 - \delta_t) [\delta_t^\alpha - (1 - \alpha)^\kappa] + (1 - \alpha)^\kappa, \quad (\text{A.6})$$

where  $f(\delta_{t+1}) \equiv (1 - \delta_{t+1}) [\delta_{t+1}^\alpha - (1 - \alpha)^\kappa]$ ,  $g(\delta_{t+1}) \equiv \alpha \delta_{t+1}^{\alpha-1} (1 - \delta_{t+1}) - \delta_{t+1}^\alpha + (1 - \alpha)^\kappa$ , and we have substituted in  $\theta_t = \theta \exp(u_t)$ ,  $R_t = R \exp(\tau_t)$ , and  $m_t = m \exp(v_t)$  for all  $t$ . The decision variables are  $p_{n,t}$ ,  $w_t$ , and  $\eta_t$ . The six state variables are  $b_{n,t}$ ,  $\delta_t$ ,  $u_t$ ,  $v_t$ ,  $x_t$ , and  $\tau_t$ .

## B Appendix: Solution with Rational Expectations

An approximate solution to the transformed first-order conditions (A.1) through (A.6) under rational expectations takes the form of the following (log-linear) decision rules

$$\frac{p_{n,t}}{\widetilde{p}_n} \simeq \left[ \frac{b_{n,t}}{\widetilde{b}_n} \right]^{s_1} \left[ \frac{\delta_t}{\widetilde{\delta}} \right]^{s_2} \exp [s_3 u_t + s_4 v_t + s_5 (x_t - \bar{x}) + s_6 \tau_t], \quad (\text{B.1})$$

$$\frac{w_t}{\widetilde{w}} \simeq \left[ \frac{b_{n,t}}{\widetilde{b}_n} \right]^{h_1} \left[ \frac{\delta_t}{\widetilde{\delta}} \right]^{h_2} \exp [h_3 u_t + h_4 v_t + h_5 (x_t - \bar{x}) + h_6 \tau_t], \quad (\text{B.2})$$

$$\frac{\eta_t}{\widetilde{\eta}} \simeq \left[ \frac{b_{n,t}}{\widetilde{b}_n} \right]^{f_1} \left[ \frac{\delta_t}{\widetilde{\delta}} \right]^{f_2} \exp [f_3 u_t + f_4 v_t + f_5 (x_t - \bar{x}) + f_6 \tau_t], \quad (\text{B.3})$$

where  $s_i$ ,  $h_i$ , and  $f_i$  for  $i = 1$  through 6 are solution coefficients. The ergodic mean approximation points are  $\widetilde{p}_n \equiv \exp [E \log (p_{n,t})]$ ,  $\widetilde{w} \equiv \exp [E \log (w_t)]$ ,  $\widetilde{\eta} \equiv \exp [E \log (\eta_t)]$ ,  $\widetilde{b}_n \equiv \exp [E \log (b_{n,t})]$ , and  $\widetilde{\delta} \equiv \exp [E \log (\delta_t)]$ .

We first use (A.4) through (A.6) to eliminate  $c_{n,t}$ ,  $b_{n,t+1}$  and  $\delta_{t+1}$  from (A.1) through (A.3). We take logarithms of (A.1) through (A.6) and apply a first-order Taylor series approximation to each equation. The Taylor-series coefficients are functions of the ergodic-mean approximation points  $\tilde{p}_n$ ,  $\tilde{w}$ ,  $\tilde{\eta}$ ,  $\tilde{b}_n$ , and  $\tilde{\delta}$ . The conjectured forms for the solution (B.1) through (B.3) are iterated ahead one period. From these, we eliminate  $b_{n,t+1}$  and  $\delta_{t+1}$  using the log-linearized versions of (A.5) and (A.6) and then eliminate  $u_{t+1}$ ,  $v_{t+1}$ ,  $x_{t+1}$ , and  $\tau_{t+1}$  using the AR(1) laws of motion (3), (12), (6), and (8), respectively. After collecting terms, we analytically compute the conditional rational forecasts  $E_t p_{n,t+1}$ ,  $E_t w_{t+1}$ ,  $E_t \eta_{t+1}$ , and  $E_t \exp(\tau_{t+1})$ . In each case, the forecast computation introduces a new constant term that depends on the innovation variances  $\sigma_u^2$ ,  $\sigma_v^2$ ,  $\sigma_x^2$ , or  $\sigma_\tau^2$ . We substitute the conditional rational forecasts into the log-linearized versions of (A.1) through (A.3). After collecting terms, these equations are now in the form of the conjectured solution (B.1) through (B.3). The mapping from the actual solution to the conjectured solution yields a system of 18 equations in the 18 unknown solution coefficients  $s_i$ ,  $h_i$ , and  $f_i$  for  $i = 1$  through 6.

Finally, we evaluate the original nonlinear equilibrium conditions (A.1), (A.2), (A.3), (A.5) and (A.6) at the ergodic-mean approximation points to obtain expressions for the constant terms in the corresponding Taylor-series approximations of the same equations. These relationships are substituted into the mapping from the actual solution to the conjectured solution. This mapping (which includes the new constant terms from computation of the rational forecasts) yields 5 equations that pin down the 5 approximation points  $\tilde{p}_n$ ,  $\tilde{w}$ ,  $\tilde{\eta}$ ,  $\tilde{b}_n$ , and  $\tilde{\delta}$ .

## C Appendix: Solution with Moving Average Forecast Rules

We first use (A.4) through (A.6) to eliminate  $c_{n,t}$ ,  $b_{n,t+1}$  and  $\delta_{t+1}$  from (A.1) through (A.3). The moving average forecast rules take the form  $\hat{E}_t q_{t+1} = \lambda q_t + (1 - \lambda) \hat{E}_{t-1} q_t$ , where  $q_{t+1} \in \{p_{n,t+1}, w_{t+1}, \eta_{t+1}, R_{t+1}\}$  are the four variables to be forecasted. Substituting the forecast rules into (A.1) through (A.3) and then solving for  $p_{n,t}$ ,  $w_t$ , and  $\eta_t$  yields a set of nonlinear laws of motion for the three decision variables. The lagged forecasts now appear in these laws of motion as state variables. For example, the law of motion for  $p_{n,t}$  is given by

$$p_{n,t} = \frac{\theta_t \exp(x_t) \left\{ 1 - \left[ \frac{1-\delta_t}{1+m_t} - R_t \right] \exp(-x_t) b_{n,t} \right\}}{1 - \lambda \left[ m_t w_t + \frac{\theta_t m_t}{1+m_t} + \beta \right] \exp(x_t)} + \frac{(1 - \lambda) \left[ m_t w_t + \frac{\theta_t m_t}{1+m_t} + \beta \right] \exp(x_t)}{1 - \lambda \left[ m_t w_t + \frac{\theta_t m_t}{1+m_t} + \beta \right] \exp(x_t)} \hat{E}_{t-1} p_{n,t} \quad (\text{C.1})$$

which depends in a nonlinear way on  $w_t$ . Straightforward but tedious algebra yields explicit (albeit complicated) expressions for  $p_{n,t}$ ,  $w_t$ , and  $\eta_t$  in terms of the following ten state variables:  $b_{n,t}$ ,  $\delta_t$ ,  $u_t$ ,  $v_t$ ,  $x_t$ ,  $\tau_t$ ,  $\hat{E}_{t-1} p_{n,t}$ ,  $\hat{E}_{t-1} w_t$ ,  $\hat{E}_{t-1} \eta_t$ , and  $\hat{E}_{t-1} R_t$ .

For use in the reverse-engineering exercise, we also compute log-linear approximations of the decision rules and laws of motion in the moving average model. For example, the log-linear decision rule for  $p_{n,t}$  takes the form

$$\frac{p_{n,t}}{\bar{p}_n} \simeq \left[ \frac{b_{n,t}}{\bar{b}_n} \right]^{a_1} \left[ \frac{\delta_t}{\bar{\delta}} \right]^{a_2} \exp [a_3 u_t + a_4 v_t + a_5 (x_t - \bar{x}) + a_6 \tau_t] \\ + \left[ \frac{\hat{E}_{t-1} p_{n,t}}{\bar{p}_n} \right]^{a_7} \left[ \frac{\hat{E}_{t-1} w_t}{\bar{w}} \right]^{a_8} \left[ \frac{\hat{E}_{t-1} \eta_t}{\bar{\eta}} \right]^{a_9} \left[ \frac{\hat{E}_{t-1} R_t}{R} \right]^{a_{10}},$$

where  $a_i$  for  $i = 1$  through 10 are Taylor series coefficients. The approximation points are the deterministic steady-state values  $\bar{b}_n$ ,  $\bar{\delta}$ ,  $\bar{p}_n$ ,  $\bar{w}$ ,  $\bar{\eta}$ , and  $R$ . Unlike the rational expectations solution, computation of the conditional forecasts in the moving average model does not introduce any new constant terms involving  $\sigma_u^2$ ,  $\sigma_v^2$ ,  $\sigma_x^2$ , or  $\sigma_\tau^2$ . Hence, it is not necessary to shift the approximation points away from the deterministic steady-state values in the moving average model.

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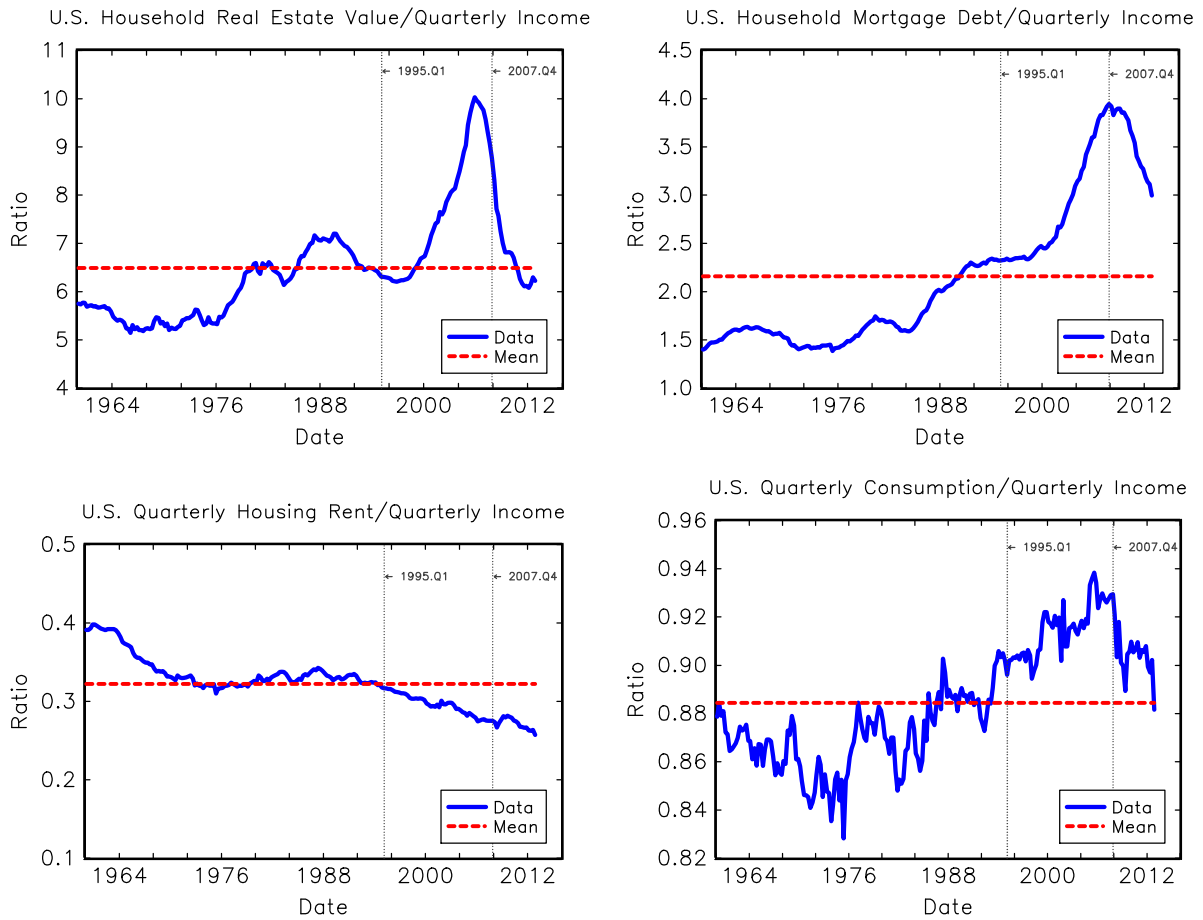


Figure 1: Starting in the mid-1990s, the U.S. economy experienced correlated booms and busts in household real estate value, mortgage debt, and personal consumption expenditures, all measured relative to personal disposable income. In contrast, the housing rent-income ratio declined steadily over the same period. The housing value-income ratio peaked in 2005.Q4. The mortgage debt-income ratio peaked 8 quarters later in 2007.Q4—coinciding with the start of the Great Recession.

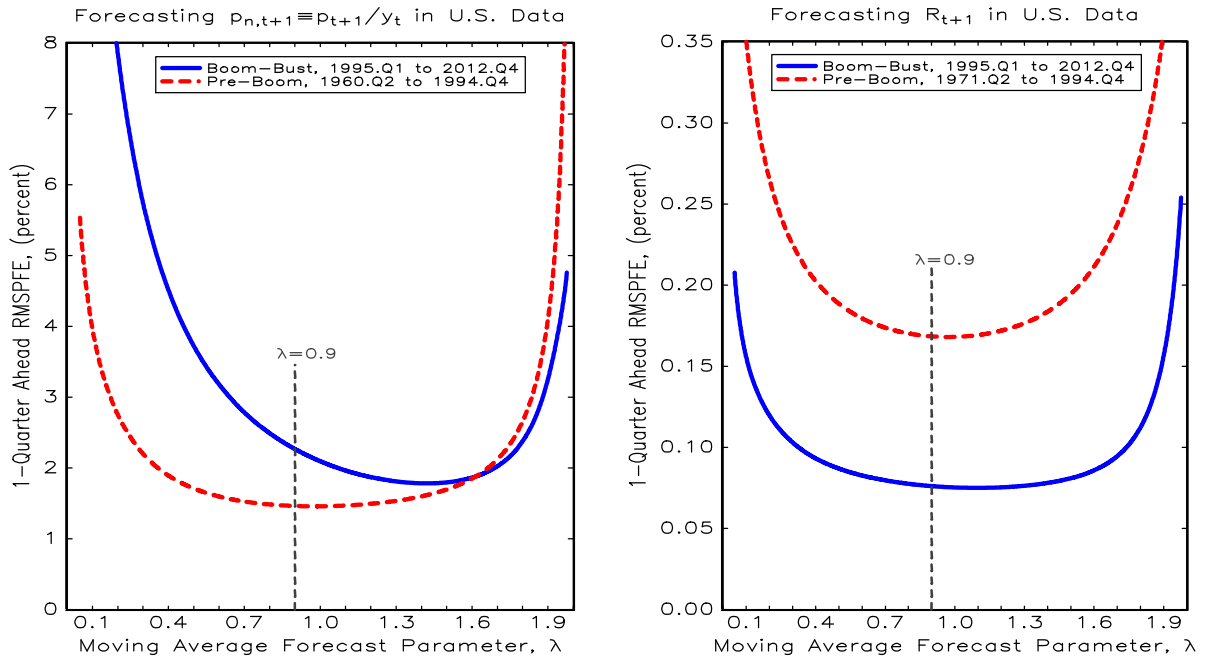


Figure 2: The figure plots the root mean squared percentage forecast error (RMSPFPE) for one-quarter ahead forecasts of each variable using a moving average forecast rule of the form (25). For the simulations, we employ  $\lambda = 0.9$  which yields stable dynamics and does not sacrifice much in forecast performance relative to higher values of  $\lambda$ .

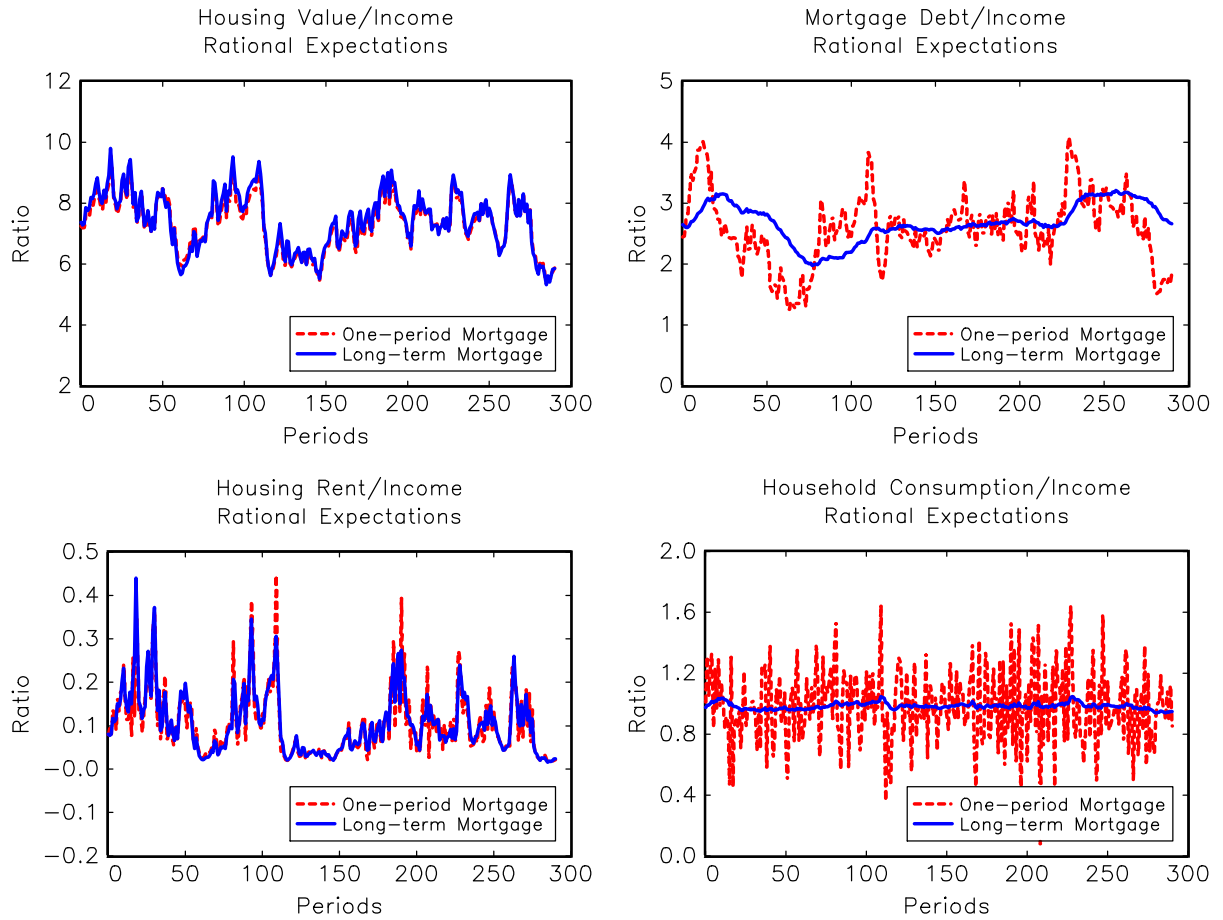


Figure 3: Model simulations with rational expectations. A volatile housing preference shock is needed to match the standard deviation of the housing value-income ratio in the data, This results in a highly volatile time series for the housing rent-income ratio, which is counterfactual. Movements in debt and consumption are much smoother with long-term mortgage debt.

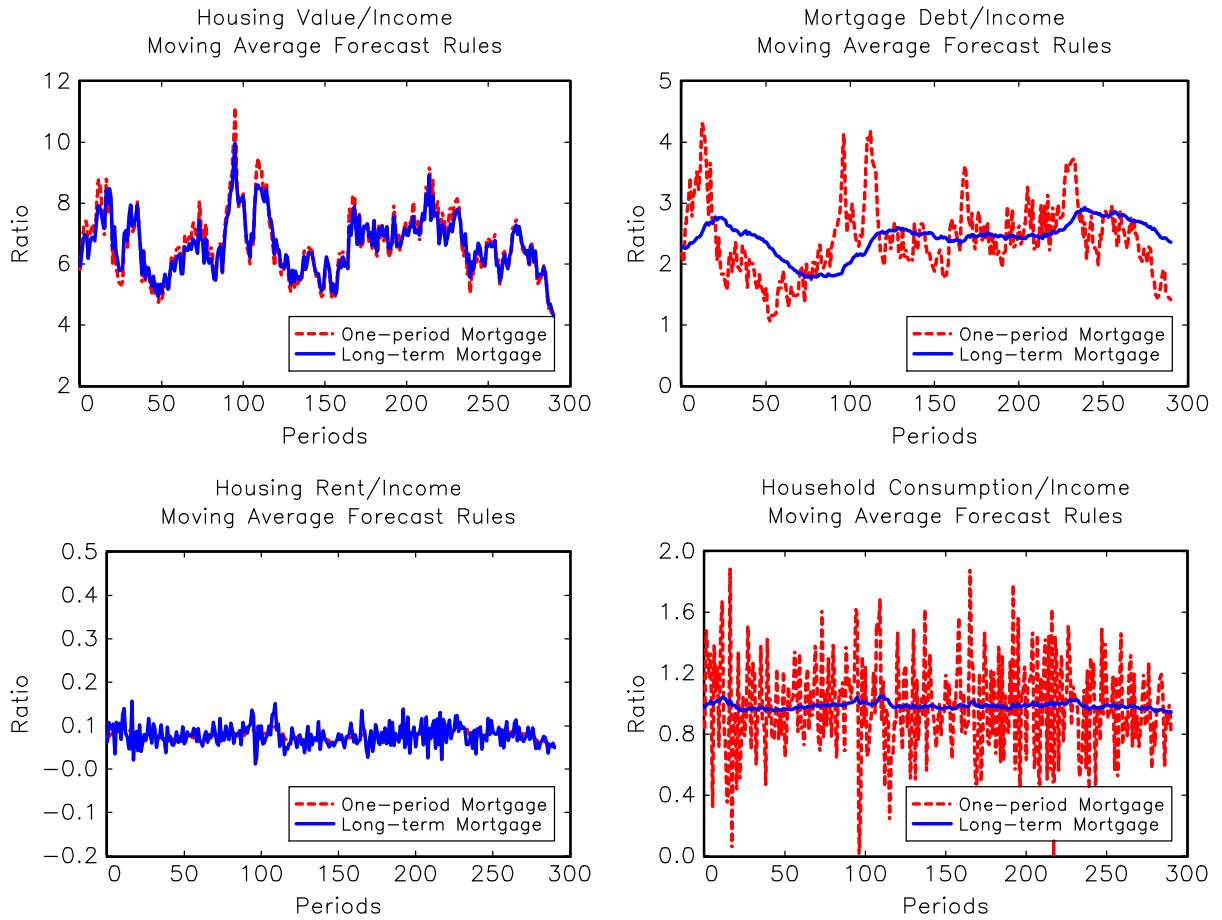


Figure 4: Model simulations with moving average forecast rules. Only a small housing preference shock is needed to match the standard deviation of the housing value-income ratio in the data. The rent-income ratio exhibits low volatility, consistent with the data. Movements in debt and consumption are much smoother with long-term mortgage debt.

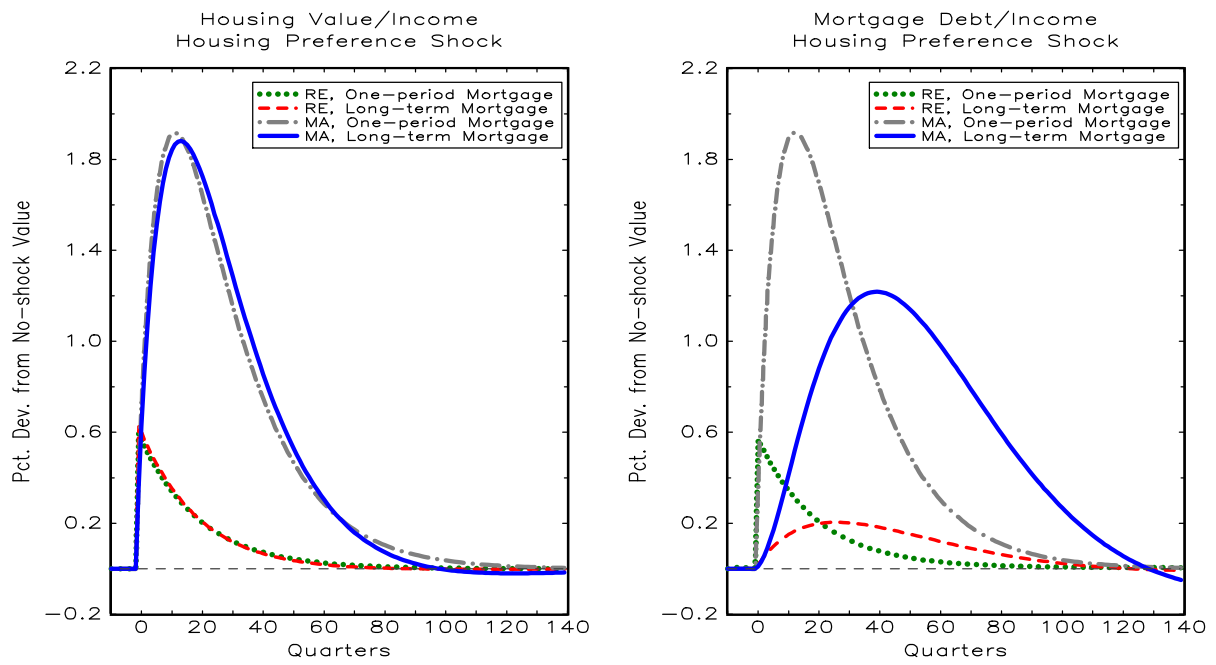


Figure 5: When all models are subjected to the same housing preference shock, the moving average model exhibits substantially more volatility in housing value. All else equal, the models with long-term mortgage debt exhibit more persistent debt dynamics than the models with one-period debt. With long-term mortgage debt, housing value peaks before debt, as in the data.

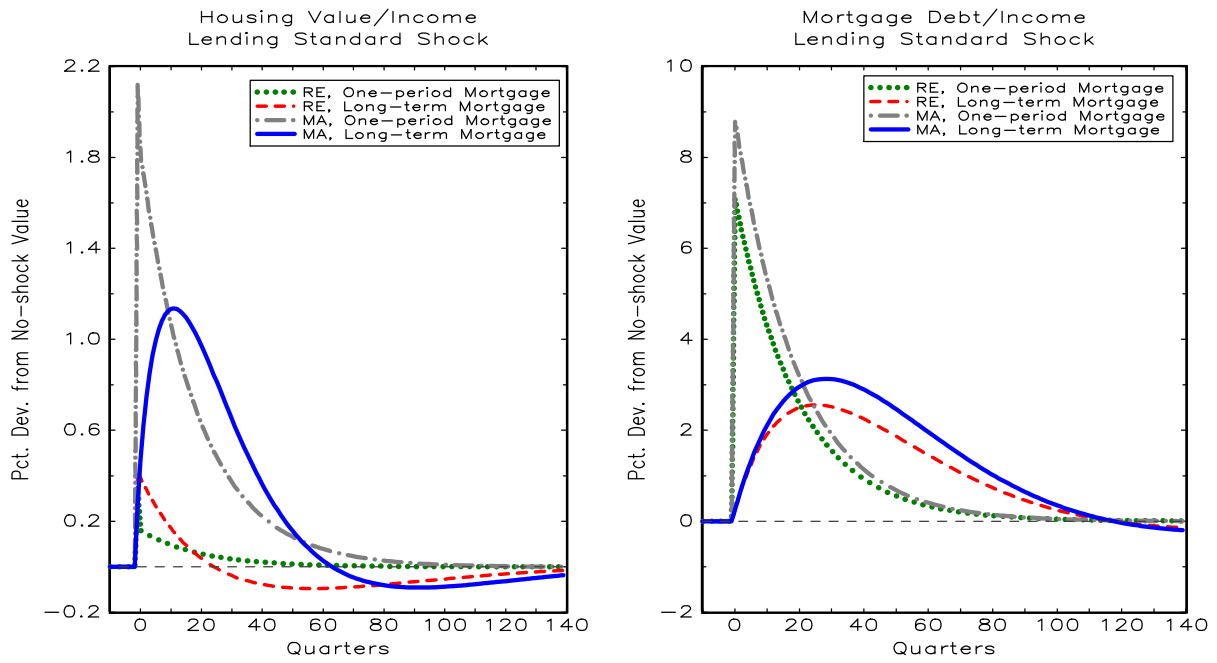


Figure 6: When all models are subjected to the same lending standard shock, the moving average model exhibits substantially more volatility in housing value. All else equal, the models with long-term mortgage debt exhibit more persistent debt dynamics than the models with one-period debt. With long-term mortgage debt, housing value peaks before debt, as in the data.

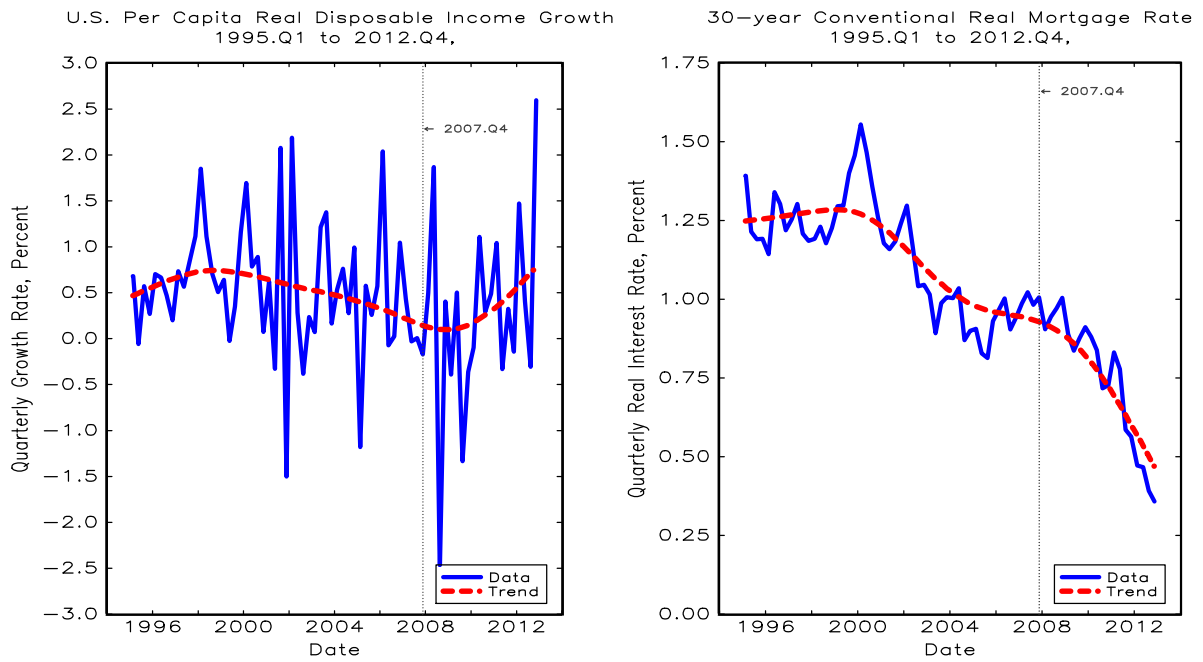


Figure 7: Inputs to the reverse-engineering exercise. Smoothed versions of the U.S. quarterly growth rate of per capita real disposable income and the U.S. quarterly real mortgage interest rate are used to identify the sequences for  $x_t - \bar{x}$  and  $\tau_t$  that appear in the household decision rules as state variables.

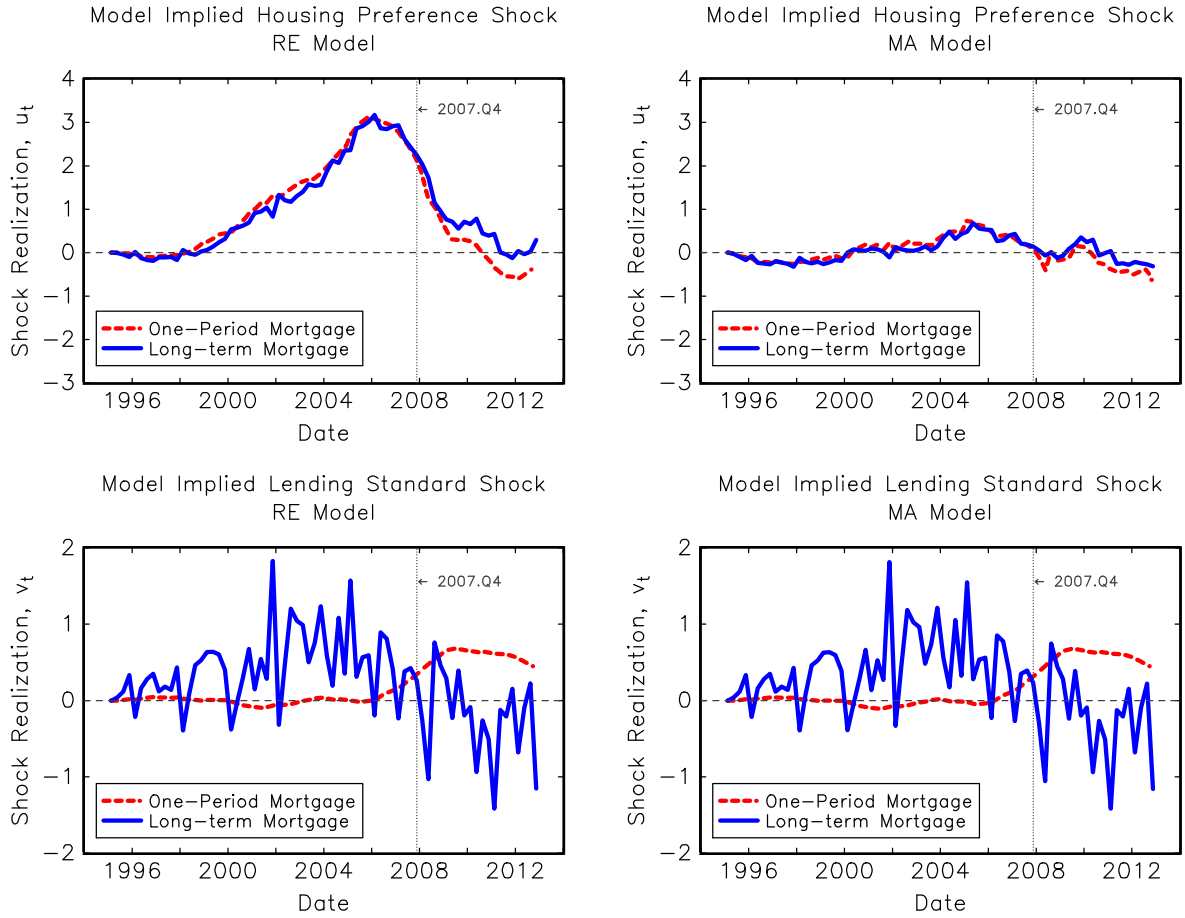
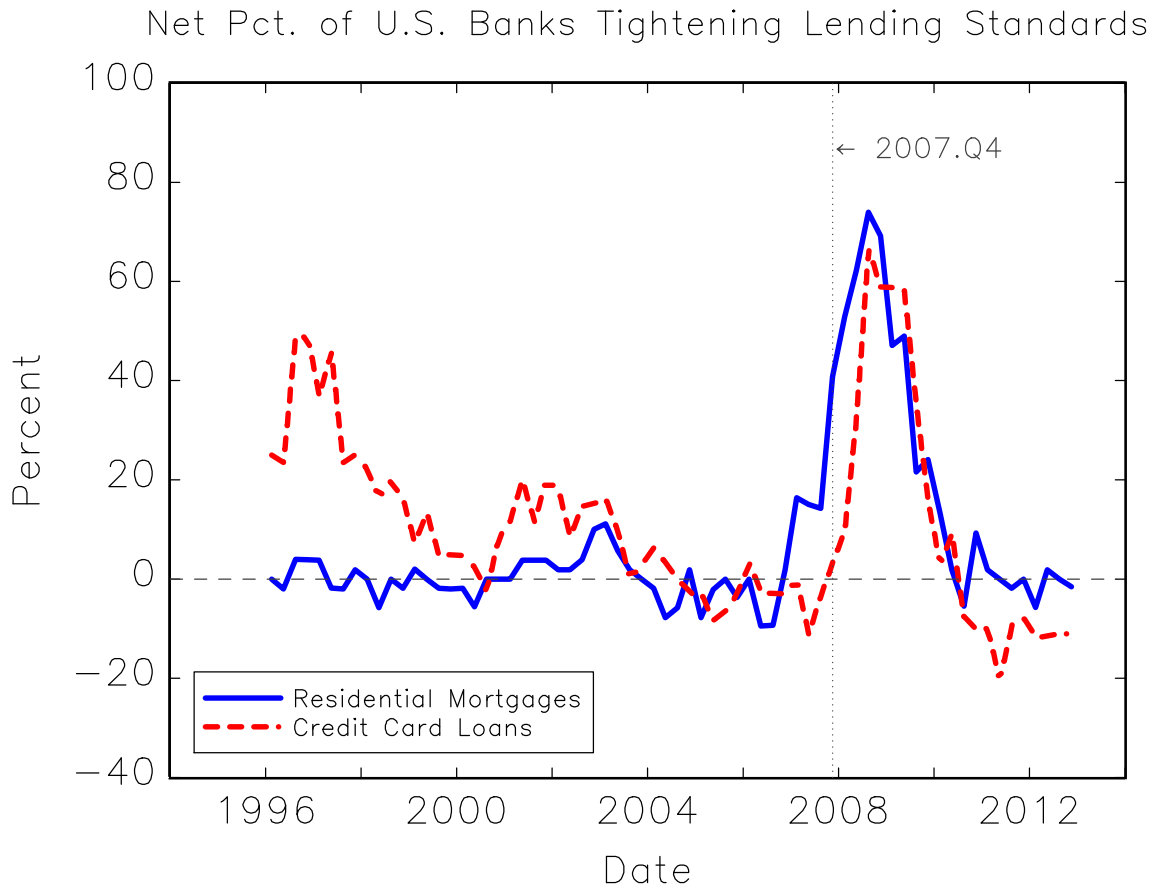
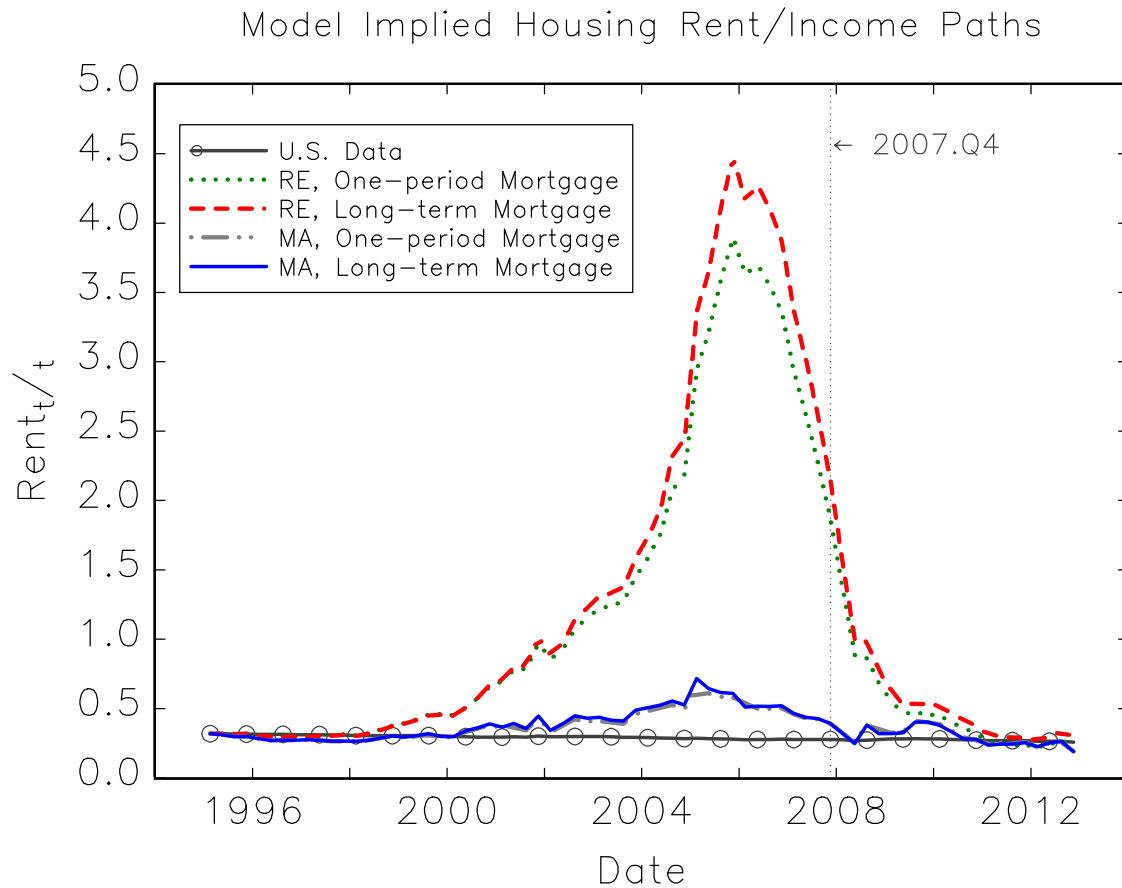


Figure 8: The left panels show the reverse-engineered shocks in the RE model. The right panels show the reverse-engineered shocks in the MA model. The MA model with long-term mortgage debt can match the boom-bust patterns in the data with smaller housing preference shocks and plausible lending standard shocks.



**Figure 9:** Two indicators of lending standard tightness from the Federal Reserve’s Senior Loan Officer Opinion Survey (SLOOS). Both series show that banks started to tighten lending standards before the onset of the Great Recession in 2007.Q4. Moreover, a substantial percentage of banks continued to tighten standards even after the recession ended in 2009.Q2.



**Figure 10:** The large reverse-engineered housing preference shocks in the RE models generate a counterfactual boom-bust cycle in the rent-income ratio. The much smaller reverse-engineered housing preference shocks in the MA models generate less movement in the rent-income ratio, which is closer to the pattern observed in the data.

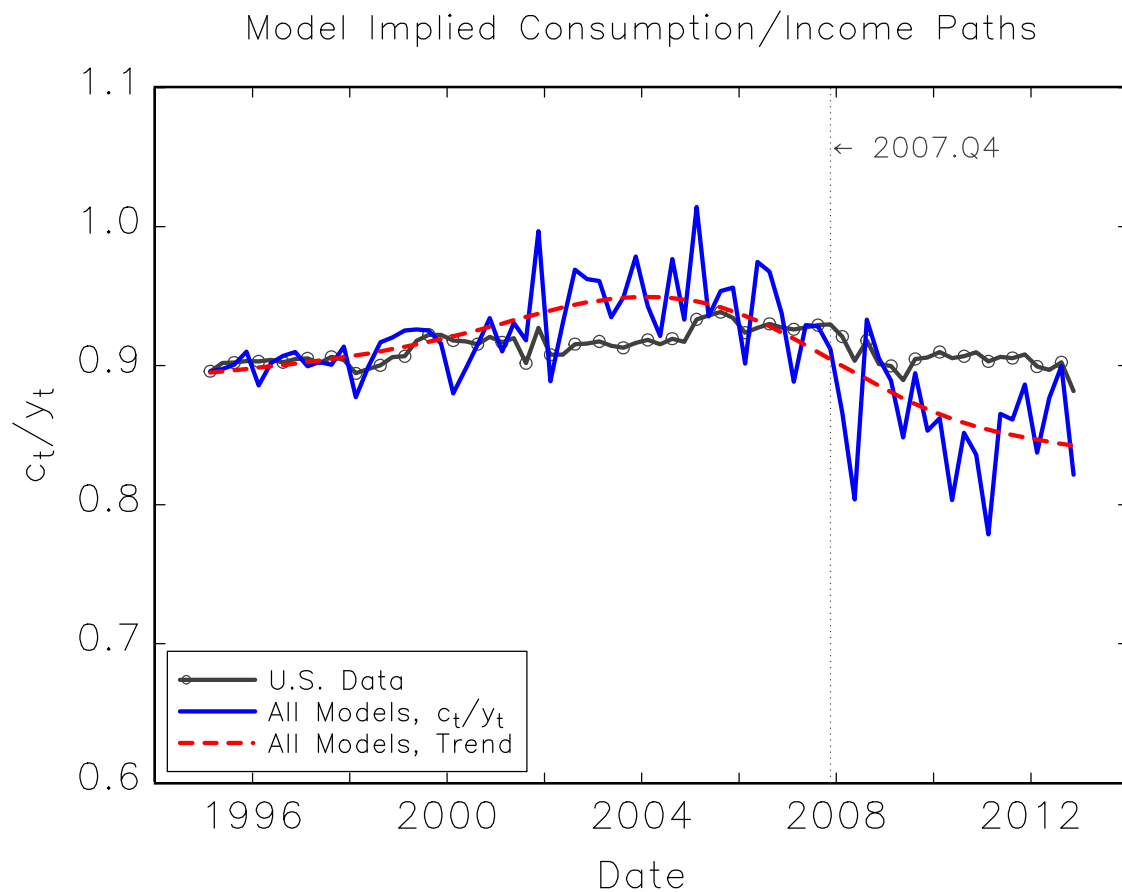


Figure 11: By construction of the reverse-engineered shocks, all models imply identical hump-shaped paths for the consumption-income ratio. A smoothed version of the model-implied path roughly resembles the hump-shaped pattern observed in the data.

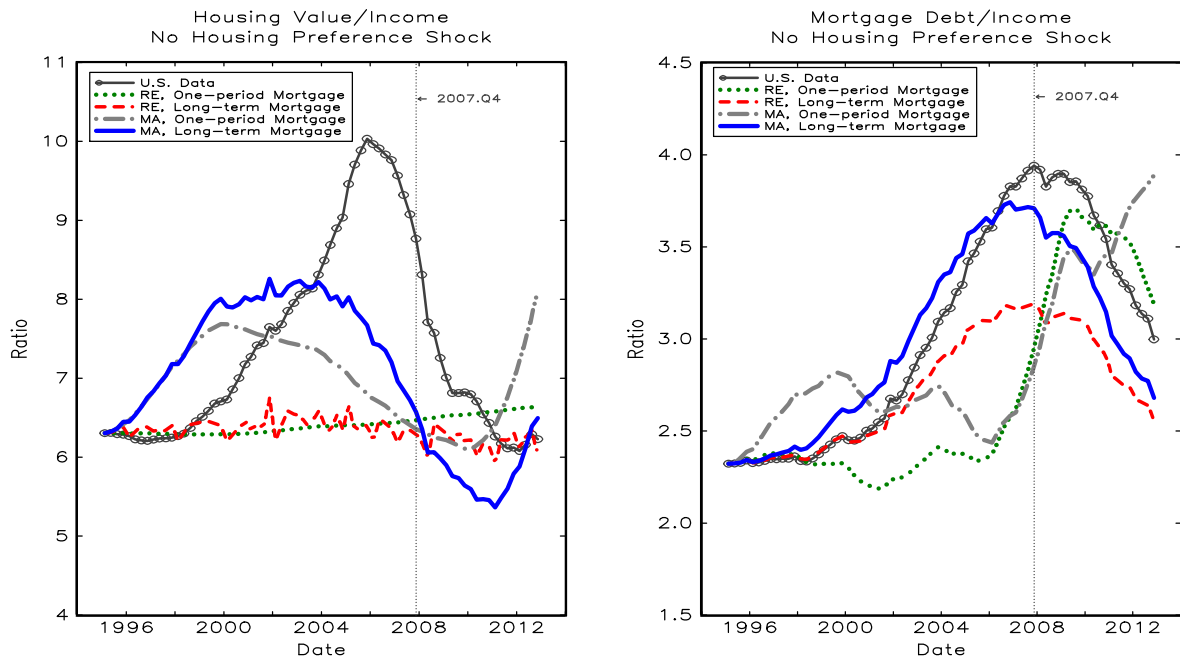


Figure 12: Counterfactual scenario 1: No housing preference shock. The RE models now exhibit no significant run-up in housing value. The MA models still exhibits a sizeable run-up in housing value, particularly in the version with long-term mortgage debt which continues to be hit by positive income growth shocks and loosening lending standards during the run-up. This result illustrates the ability of our preferred model to generate an income- and credit-fueled boom in housing value.

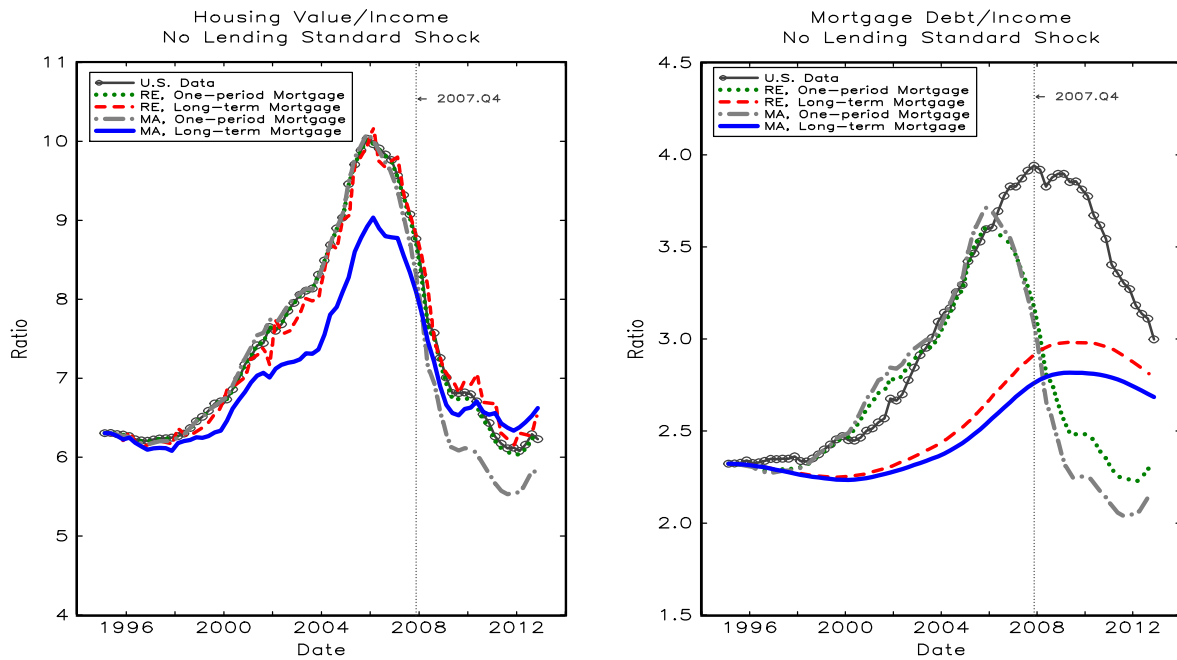


Figure 13: Counterfactual scenario 2: No lending standard shock. Models with one-period mortgage debt now imply a rapid deleveraging that coincides with the rapid decline in housing value. Models with long-term mortgage debt now exhibit much smaller run-ups in debt, suggesting that shifting lending standards were an important driver of the episode. The MA model with long-term mortgage debt is the only one to show smaller boom-bust cycles in both housing value and debt when the lending standard shock is turned off.

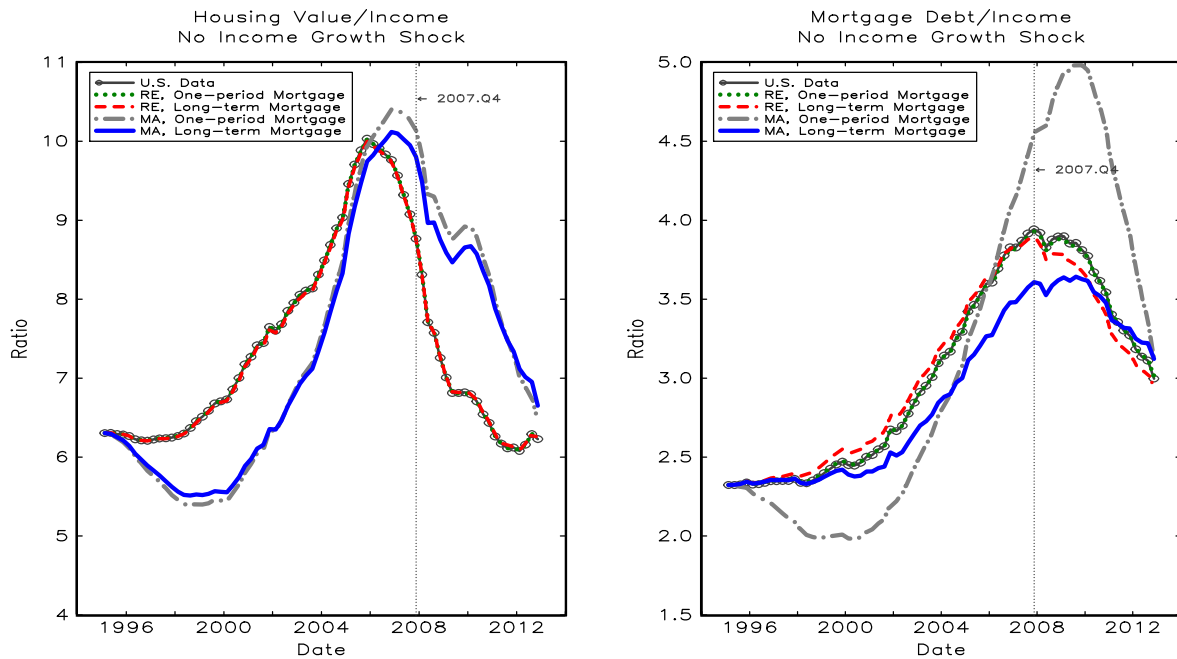


Figure 14: Counterfactual scenario 3: No income growth shock. There is little noticeable effect in the RE models. In contrast, turning off the positive income growth shocks of the late 1990s causes the MA models to exhibit a somewhat delayed boom-bust cycle in housing value relative to the data. This result implies that movements in income growth did contribute to the magnitude and timing of episode. The MA model with one-period mortgage debt exhibits a larger boom-bust cycle in debt in response to the implausible post-2007 lending standard shocks implied by this version of the model.

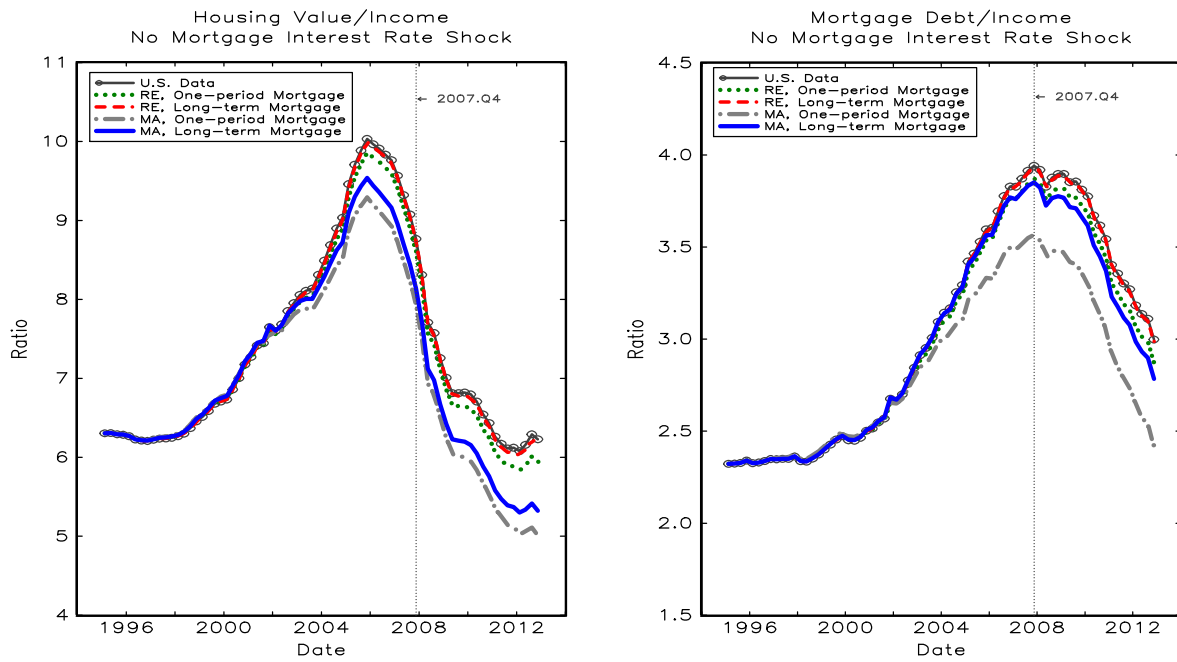


Figure 15: Counterfactual scenario 4: No mortgage interest rate shock. All models continue to exhibit significant boom-bust cycles in both housing value and debt, suggesting that the decline in the U.S. mortgage interest rate was not a major driver of the episode.