Optimal monetary policy in a New Keynesian model with heterogeneous expectations*

Giovanni Di Bartolomeo
Sapienza University of Rome
giovanni.dibartolomeo@uniroma1.it

Marco Di Pietro
Sapienza University of Rome
marco.dipietro@uniroma1.it

Bianca Giannini
Sapienza University of Rome
bianca.giannini@uniroma1.it

April, 2015

Abstract

In a world where expectations are heterogeneous, what is the design of the optimal policy? Are canonical policies robust if heterogeneous expectations are considered or would they be associated with large welfare losses? We aim to answer these questions in a stylized simple New Keynesian model where agents' beliefs are not homogeneous. Assuming that a fraction of agents can form their expectations by some adaptive or extrapolative schemes, we focus on an optimal monetary policy by second-order approximation of the policy objective from the consumers' utility functions. We find that the introduction of bounded rationality in

*The authors are grateful to Jordi Galì, Luisa Lambertini, Salvatore Nisticò, Carolina Serpieri, and Patrizio Tirelli for useful comments on earlier drafts. The authors also acknowledge financial support by Sapienza University of Rome.

the New Keynesian framework matters. The presence of heterogeneous agents adds a new dimension to the central bank's optimization problem—consumption inequality. Optimal policies must be designed to stabilize the cross-variability of heterogeneous expectations. In fact, as long as different individual consumption plans depend on different expectation paths, a central bank aimed to reduce consumption inequality should minimize the cross-sectional variability of expectations. Moreover, the traditional trade-off between the price dispersion and aggregate consumption variability is also quantitatively affected by heterogeneity.

Jel codes: E52, E58, J51, E24.

Keywords: monetary policy, bounded rationality, heterogeneous expectations.

1 Introduction

The New Keynesian approach has undoubtedly become the workhouse for academic and practical discussions about monetary policy. Optimal monetary policies are usually designed on the rational expectations paradigm, although heterogeneity in the expectations formation mechanism is well documented in both survey data and laboratory experiments.[1] Our paper aims to investigate the impact on optimal monetary policy in a New Keynesian framework where not all agents are rational in forming their expectations. We are interested in determining the degree to which the presence of heterogeneous expectations affect the canonical prescriptions with respect to the conduct of monetary policy and, therefore, their degree of robustness. In our investigation, we focus on optimal monetary policies aimed at maximizing a welfare-based criterion derived from agent utility functions.

Several approaches have been proposed that include, in a standard model, some modifications that take into account empirical evidence showing that inflation and output forecasts are not rational, at least for some agents. For instance, Krusell and Smith (1996), Mankiw (2000), Amato and Laubach (2003) and Galí *et al.* (2004, 2007) introduce in macroeconomic models a fraction of agents who are not completely rational as they make their decisions according to some "rule of thumb," i.e., consuming all their income.[2] Mankiw and Reis (2002, 2007) propose an approach based on sticky information, while Evans and Honkapohja (2001, 2003) focus on learning models. Other approaches borrow some ideas from behavioral economics or develop

[1] An alterative approach is that of agent-based models (see Cirillo *et al.*, 2011).

[2] An alternative interpretation of the assumption is to see it a short cut to model limited asset market participation (see e.g. Bilbiie, 2008).

some concepts of near-rationality.[3] By deviating somehow from the full rational paradigm, all of these studies have challenged important economic puzzles, such as inflation intrinsic persistence, data-consistent disinflation paths, and fiscal and monetary policy effects.

Along the above lines, a notable number of studies explicitly consider non-homogeneous expectations and address the issue of how this heterogeneity potentially affects aggregate economic dynamics. These studies include Brock and Hommes (1997), Preston (2006), Branch and Evans (2006), Branch and McGough (2009) and Massaro (2013). In particular, by using two alternative approaches to model the heterogeneity of expectations, Branch and McGough (2009) and Massaro (2013) develop parsimonious micro-founded sticky price models that are consistent with boundedly rational individuals.

In Branch and McGough (2009) and Massaro (2013), expectations operators may differ across groups of agents, who are of two kinds—a fraction of them are rational and the remainder are not—and agents' optimal choices are modeled to be consistent with their specific forecasts. By aggregating individual decision rules, Branch and McGough (2009) and Massaro (2013) derive aggregate demand and supply equations of New Keynesian kind embedding bounded rationality. The resulting reduced form model is analytically tractable and encompasses the representative rational agent canonical benchmark as a special case.

Formally, consistent with Preston (2006), Massaro (2013) focuses on long horizon forecasts and assumes that agents with subjective expectations choose optimal plans while considering forecasts of macroeconomic conditions over an infinite horizon. By contrast, Branch and McGough (2009) assume that individuals with subjective beliefs choose optimal plans that satisfy their individual Euler equations. Using an axiomatic approach to heterogeneous expectations, they provide restrictions on the admissible forms of non-fully rational beliefs sufficient to ensure the laws of motion as the aggregate variables are analytically tractable and easily comparable to those obtained under complete rationality.[4] As a result, in Massaro (2013), the predicted aggregate dynamics hinge on long horizon forecasts, while in Branch and McGough (2009), the aggregate dynamics depend on one-period ahead subjective heterogeneous forecasts.

Both Branch and McGough (2009) and Massaro (2013) show that heterogeneous expectations can undermine some standard results regarding equilibrium determinacy. However, they do not consider optimal policies; rather, they assume that the central bank sets the interest rate

[3] Among others see, e.g., Roberts (1997), Akerlof *et al.* (2000), Gaffeo *et al.*, (2010), De Grauwe (2011), and De Grauwe and Kaltwasser (2012).

[4] In particular, by their restrictions, they are able to decouple the aggregate dynamics from the dynamics of the wealth distribution

according to a Taylor-kind rule. In other words, they investigate the effects of policies based on simple interest rules and thus show that specifications that are determinate under rationality may exhibit explosive or multiple equilibria in the case of bounded rationality.

Our paper extends the Branch and McGough's (2009) model to compute the optimal monetary policy. We use the linear-quadratic (LQ) approach developed and refined by Rotemberg and Woodford (1997), Woodford (2003: Chapter 6), and Benigno and Woodford (2012). We derive a second-order approximation of the policy objective from the consumers, who are assumed to have the same specification for the utility functions, while different agents only differ in the way they form their beliefs. We then use our consistent welfare measure to investigate the effects of bounded rationality on optimal monetary policy, and we consider the optimal response of the endogenous variables of the model assuming that the central bank optimizes either under commitment or under discretion. In the first case, the monetary authority, by commitment to a policy plan, affects the private sector's expectations. In the second case, the central bank, taking private agent's forecasts as given, minimizes welfare losses in each period.

We find that the introduction of bounded rationality in the New Keynesian framework matters as the existence of a group of non-rational agents who form their forecasts by adaptive or extrapolative mechanisms implies that optimal policies must be designed to stabilize the cross-variability of heterogeneous expectations. The rationale of our result are based on the fact that heterogeneous agents add a new dimension to the central bank's optimization problem, that of consumption inequality. As long as different individual consumption plans depend on different expectation paths, a central bank aiming to reduce the variability in individual consumption should minimize the cross-sectional variability of expectations.

The central bank problem thus consists of three dimensions: *i)* minimization of the variability of aggregate consumption, *ii)* minimization of the cost associated with price dispersion and *iii)* minimization of the cross-sectional variance of consumption. The latter requires stabilizing the expectation variability. However, the traditional trade-off between the first two is also affected by heterogeneity. In our context, the cost of price dispersion increases with the size of the group of boundedly rational agents, but as long as it continues to grow, the emphasis of stabilizing inflation versus output declines. This occurs because the dynamics of price dispersion are more complex than those in the standard case, where the dynamics rely only on inflation. Here, price dispersion also depends on output stabilization.

The paper is organized as follows. Section 2 presents the Branch and McGough's (2009) axioms that generalize the New Keynesian model to include non-homogeneous expectations and provides the resulting analytically tractable reduced form model that encompasses the representative rational agent benchmark as a special case. Section 3 derives the welfare criterion as a second-order approximation of the policy objective, assuming that the steady state is not distorted. Section 4 illustrates the properties of optimal policies under bounded rationality, compares these to the canonical policies and Taylor rules and discusses the implications of agents' heterogeneity for commitment and discretion. Section 5 concludes the paper.

2 The economy

We consider a simple yeoman-farmer economy. The economy is populated by a continuum of mass one of infinitely lived households who produce and consume. A fraction α of them (rational households) has rational expectations. The remaining fraction (non-rational households) form expectations according to a mechanism of bounded rationality. The two kinds of households are indexed by $\mathcal{R}$ and $\mathcal{B}$, which refer to rational and boundedly rational households, respectively. Each household produces a differentiated good by using its own labor and consumes a "bundle," a composite good composed of all products. We assume price stickiness by assuming that in every period each yeoman-farmer faces an exogenous constant probability of being able to reset its price. The model, borrowed from Branch and McGough (2009), is described in the following sub-sections. The axiomatic approach used to model heterogeneous expectations is presented in the next sub-section, while the remaining sub-sections describe the model equations, the private sector's first-order conditions, and the log-linearized economy, respectively.[5]

2.1 The axiomatic approach to heterogeneous expectations

Heterogeneous expectations are introduced by following the axiomatic approach developed by Branch and McGough (2009). Formally, denoting by $\mathcal{E}^i$ a generic (subjective) expectations operator (i.e., $\mathcal{E}^i_t x_{t+1}$ is the time t expectation on the value assumed by variable x at $t+1$ formed by an agent of type i), we impose the following assumptions: *i)* each expectation operator, $\mathcal{E}^i$, fixes observables; *ii)* all agents' beliefs coincide in the steady state; *iii)* $\mathcal{E}^i$ is a linear operator;[6] *iv)* $\mathcal{E}^i$ satisfies the law of iterated expectations; *v)* if x is a variable forecasted by agents at time t and time $t+k$ such that $\mathcal{E}^i_t \mathcal{E}^j_{t+k} x_{t+k} = \mathcal{E}^i_t x_{t+k}$, $i \neq j$; and *vi)* all agents have common

[5] We refer to Branch and McGough (2009) for details.

[6] It is worth noticing that we only need to assume that if x, y, αx, $x+y$ are forecasted, $\mathcal{E}^i(\alpha x) = \alpha \mathcal{E}^i x$ and $\mathcal{E}^i(x+y) = \mathcal{E}^i x + \mathcal{E}^i y$. Moreover, if for all $k \geqslant 0$, x_{t+k} and $\sum_k \beta^{t+k} x_{t+k}$ are forecasted by agents then $\mathcal{E}^i_t \sum_k \beta^{t+k} x_{t+k} = \sum_k \beta^{t+k} \mathcal{E}^i_t x_{t+k}$.

expectations on expected differences in limiting wealth.

As discussed by Branch and McGough (2009), assumptions *i)* to *iv)* are consistent with reasonable specifications of agent behavior. As such, these assumptions imply that the forecast of a known quantity should be the known quantity, that there exists some continuity in beliefs, that agents incorporate some economic structure into their forecasting model, and that agents' beliefs satisfy the law of iterated expectations at an individual level.

Assumptions *v)* to *vi)* are necessary for aggregation. The former implies that agents' forecasts satisfy the law of iterated expectations at an aggregate level. Therefore, it requires that an individual of type i's belief regarding the future expectations of agents of type j are the same as i's expectation. In other words, we impose a particular structure on higher-order beliefs. The axiom that agents agree on limiting wealth distributions does not allow for wealth distribution dynamics that otherwise affect the formulation of forecasts based on expectation type, thus causing a problem for aggregation. This allows us to remain close in form to the homogeneous case.[7]

By using the above assumptions, we define the aggregate expectations as a weighted average of group expectations, such that $\mathcal{E}_t x_t = (1-\alpha)\,\mathcal{E}_t^{\mathcal{B}} x_t + \alpha \mathcal{E}_t^{\mathcal{R}} x_t$, where $\mathcal{R}$ and $\mathcal{B}$ identify the expectation operator for rational and boundedly rational households, respectively.

Our economy is populated by rational and adaptive agents. Therefore, we assume that $\mathcal{E}_t^{\mathcal{R}} x_{t+1} = E_t x_{t+1}$, i.e., rational agents have a one-step ahead perfect foresight on economic variables. However, it is further noted that they are not fully rational as they are not able to correctly understand the forecasts of boundedly rational agents. That is, they have wrong second-order beliefs (see assumption *iv*). With respect to non-rational individuals and consistent with the literature, we assume that such individuals form their beliefs on the basis of a simple perceived linear law of motion, i.e., $x_t = \theta x_{t-1}$. Therefore, $\mathcal{E}_t^{\mathcal{B}} x_t = \theta x_{t-1}$, i.e., the operator $\mathcal{E}^{\mathcal{B}}$ is a form of adaptive ($\theta < 1$) or extrapolative ($\theta > 1$) expectations, where θ is defined as the adaption operator. We refer to $\theta = 1$ as the case of naive expectations. Applying the law of iterated expectations, we obtain $\mathcal{E}_t^{\mathcal{B}} x_{t+1} = \theta^2 x_{t-1}$.[8]

[7] For further details, see Branch and McGough (2009).

[8] See Branch and McGough (2009) for details. See also Pesaran (1987), Brock and Hommes (1997, 1998), Branch and McGough (2005) for empirical support and further considerations.

2.2 The model

Each household i produces, as a monopolist, its own differentiated product and directly purchases a composite good seeking to maximize the expected value of the following utility function:

$$\mathcal{E}_0^i \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^i\right) - \nu(Y_t(i))\right] \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and $\mathcal{E}^i \in \left\{\mathcal{E}^{\mathcal{R}}, \mathcal{E}^{\mathcal{B}}\right\}$ denotes a generic (subjective) expectation operator of agent i who can belong either to the rational or boundedly rational subset. The terms $u\left(C_t^i\right)$ and $\nu(Y_t(i))$ indicate the utility from consuming the composite good (C_t^i) and the disutility from producing the differentiated product $(Y_t(i))$, respectively.[9]

In each period, a number of randomly selected agents are allowed to change their prices. Each firm may reset its price only with the probability $1-\xi_p$ in any given period and independent of the time elapsed since the last adjustment occurred. Thus, in each period, a proportion $1-\xi_p$ of producers reset their prices, while a fraction ξ_p keep their prices unchanged. As a result, the average duration of a price is given by $(1-\xi_p)^{-1}$.

As optimal prices depend on the expectations regarding future marginal costs, they differ between the two types of agents. Moreover, the Calvo lottery also implies heterogeneity within each type of agent as only a fraction of them reset their prices. Acting as price setters, individual agents face the risk associated with the Calvo lottery. Following a common procedure among heterogeneous agent models to somehow limit heterogeneity between types, it is assumed that agents are engaged in a form of risk sharing to protect themselves from the risk of Calvo price setting. A benevolent financial regulator collects all income and then redistributes to each type of household the average income of that agent's type.[10]

It follows that individuals are fully insured against the risk associated with the possibility that they will be not able to adjust prices. Because agents have different expectations, the insurance contracts are designed to guarantee different amounts of expected real income: $\Omega_t^{\mathcal{B}} = \int_\alpha^1 \frac{P_t(i)Y_t(i)}{(1-\alpha)P_t} di$ and $\Omega_t^{\mathcal{R}} = \int_0^\alpha \frac{P_t(i)Y_t(i)}{\alpha P_t} di$. Aggregating each agent's expected real income, we

[9] There exists a continuum of goods represented by an interval $[0,1]$, $C_t^i \equiv \left(\int_0^1 C_t^i\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ is then a Dixit-Stiglitz consumption basket and $C_t^i\left(j\right)$ is the quantity of good j consumed by household i in period t. The consumer price index is defined as $P_t \equiv \left[\int_0^1 P_t\left(i\right)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$.

[10] Among others, the same risk-sharing mechanism is used by Kocherlakota (1996), Shi (1999) and Mankiw and Reis (2007). It is entirely standard in models with heterogeneous agents. Alternatives are also discussed by Branch and McGough (2009).

obtain the real output of the economy:

$$Y_t = \alpha \Omega_t^{\mathcal{R}} + (1-\alpha)\Omega_t^{\mathcal{B}} \tag{2}$$

Moreover, because of the insurance mechanism, according to the agent's type $\tau \in \{\mathcal{R}, \mathcal{B}\}$, the real budget constraint of household i is:

$$C_t^i + B_t^i = \frac{1+i_{t-1}}{1+\pi_t} B_{t-1}^i + \Omega_t^{\tau} \tag{3}$$

where $B_t^i P_t$ is the quantity of one period, nominal riskless discount bonds purchased in period t and maturing in $t+1$ held by agent i –each bond pays one unit of money at maturity and its price is $Q_t = (1+i_t)^{-1}$. The term $1+i_t$ indicates the gross nominal interest rate on a riskless one period bond purchase in period t, and $1+\pi_t$ defines the gross inflation rate.

Finally, the existence of fully enforceable contracts requires the agents to behave as if they will receive their full marginal revenue when producing more by assuming that agents should choose price and output as if they faced their perceived trade-off. In fact, each agent's income is independent of his efforts because of the presence of the insurance company. Thus, without enforceable contracts, any agent would choose an effort equal to zero due to a free-riding behavior.

2.3 First-order conditions

Each household must decide its optimal consumption (saving) plan, i.e., allocating its consumption expenditures among the different goods and choosing the optimal price (i.e., effort) if selected in a Calvo lottery.

The optimal consumption plan of household i is obtained by the maximization of (1) subject to (3) and a solvency constraint. The optimal plan is the same among agents belonging to the same type, and it can be obtained by a simple variational argument. As a result, agents of each type $\tau \in \{\mathcal{R}, \mathcal{B}\}$ make choices about consumption with respect to the intertemporal Euler equation:

$$\frac{1}{1+i_t} = \beta \mathcal{E}_t^{\tau} \left[\frac{P_t}{P_{t+1}} \frac{u_c\left(C_{t+1}^{\tau}\right)}{u_c\left(C_t^{\tau}\right)} \right] \qquad \tau \in \{\mathcal{R}, \mathcal{B}\} \tag{4}$$

The optimal allocation of household consumption expenditures among different goods, $C_t(j)$, requires that the consumption index C_t is maximized for any level of expenditure $\int_0^1 P_t(i)\, C_t(j)\, dj$. Solving the intratemporal goods allocation problem, the set of demand equations for each type

$\tau \in \{\mathcal{R}, \mathcal{B}\}$ of household is:

$$Y_t^{\tau}(j) = \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon} \left(\Omega_t^{\tau} + (1 + i_{t-1}) \frac{B_{t-1}^{\tau}}{P_t} - \frac{B_t^{\tau}}{P_t}\right) \tag{5}$$

In aggregate terms and given the bond market clearing condition, we derive the standard demand for the good j:

$$\begin{aligned} Y_t(j) &= \alpha \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon} \Omega_t^{\mathcal{R}} + (1-\alpha) \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon} \Omega_t^{\mathcal{B}} \\ &= \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon} Y_t \end{aligned} \tag{6}$$

Each producer j belonging to type $\tau \in \{\mathcal{R}, \mathcal{B}\}$ chooses the price (P_t^j), thus solving:

$$\max_{P_t^j} \mathcal{E}_0^{\tau} \sum_{i=0}^{\infty} \left(\beta \xi_p\right)^i \left[(1-T) \lambda_{t+i}(P_t^j) Y_{t+i}(j) - \nu\left(Y_{t+i}(j)\right) \right] \tag{7}$$

subject to (6). The first term of the sum (7) is the marginal utility of additional nominal income, which can be interpreted as the contribution to utility derived by sales revenues. The second term is the production cost in terms of effort. A production subsidy T is introduced to eliminate distortions in the steady state.

By substituting the demand function into the household's objective function and calculating with respect to P_t^j , we obtain:

$$\mathcal{E}_0^{\tau} \sum_{i=0}^{\infty} \left(\beta \xi_p\right)^i \left[\frac{\partial u\left(C_{t+i}^{\tau}\right)}{\partial C_{t+i}^{\tau}} \left(\frac{P_t^j}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i} - \left(\frac{P_t^j}{P_{t+i}}\right)^{-\varepsilon-1} Y_{t+i} \frac{\partial \nu\left(Y_{t+i}(j)\right)}{\partial Y_{t+i}(j)}\right] = 0 \tag{8}$$

where we used the fact that $T = -\frac{1}{\varepsilon - 1}$ to implement the optimal steady state and $\lambda_t^j P_t = \partial u(C_t^j)/\partial C_t^j$.

2.4 The log-linear economy

By log-linearization of (4) and after some substitutions, we obtain

$$c_t^{\tau} = \mathcal{E}_t^{\tau} c_{t+1}^{\tau} - \sigma(i_t - \mathcal{E}_t^{\tau} \pi_{t+1}) \qquad \tau \in \{\mathcal{R}, \mathcal{B}\} \tag{9}$$

where $\sigma^{-1} \equiv -\bar{C}\ u_{CC}/u_C > 0$ is the inverse of the intertemporal elasticity of substitution of consumption, i.e., the coefficient of relative risk aversion. We define the log deviations of the variables from their steady state values with lower case letters, i.e., $c_t^\tau = \log(C_t^\tau/\bar{C})$.

By using the budget constraint, $c_t^\tau = \omega_t^\tau \equiv \left(\beta^{-1}\log(B_{t-1}^\tau) - \log(B_t^\tau)\right)/\bar{Y} + \log(\Omega_t^\tau/\bar{\Omega})$,[11] the log-linearized consumer Euler equation (9) is then rewritten in terms of wealth as:

$$\omega_t^\tau = \mathcal{E}_t^\tau \omega_{t+1}^\tau - \sigma(i_t - \mathcal{E}_t^\tau \pi_{t+1}) \qquad \tau \in \{\mathcal{R}, \mathcal{B}\} \tag{10}$$

As the above condition holds for both types of agents, it is possible to derive the usual IS relation by aggregating (10) provided that the axioms *i)* to *vii)* are satisfied by the expectational operator $\mathcal{E}_t^\tau$.

Specifically, iterating (10) forward we obtain

$$\omega_t^\tau = \omega_\infty - \sigma \mathcal{E}_t^\tau \sum_{k\geq 0}(i_{t+k} - \pi_{t+k+1}) \qquad \tau \in \{\mathcal{R}, \mathcal{B}\} \tag{11}$$

where ω_∞ is the limiting wealth that is common between types.

By using the bond market clearing condition, $\alpha\log(B_t^{\mathcal{R}}) + (1-\alpha)\log(B_t^{\mathcal{B}}) = 0$, and aggregating the budget constraints, we obtain:

$$y_t = \alpha\omega_t^{\mathcal{R}} + (1-\alpha)\omega_t^{\mathcal{B}} \tag{12}$$

Finally, combining (11) with (12), and providing $\omega_\infty^{\mathcal{R}} = \omega_\infty^{\mathcal{B}} = \omega_\infty$, we obtain an IS curve that is similar to the relation derived in the standard New Keynesian framework with the exception of the conditional expectation operator, which is substituted by a convex combination of the heterogeneous expectation operators of the two types of agents. Formally,

$$y_t = \mathcal{E}_t y_{t+1} - \sigma\left(i_t - \mathcal{E}_t \pi_{t+1}\right) \tag{13}$$

where $\mathcal{E}_t y_{t+1} = \alpha\mathcal{E}_t^{\mathcal{R}} y_{t+1} + (1-\alpha)\mathcal{E}_t^{\mathcal{B}} y_{t+1}$ and $\mathcal{E}_t \pi_{t+1} = \alpha\mathcal{E}_t^{\mathcal{R}} \pi_{t+1} + (1-\alpha)\mathcal{E}_t^{\mathcal{B}} \pi_{t+1}$.

Regarding the supply side, by solving forward (8) and making use of assumptions *iii)* to *v)*, the log-linear version of the optimal price equation of agent j belonging to type $\tau \in \{\mathcal{R}, \mathcal{B}\}$

[11] The relationship is obtained by exploiting the fact that $B^{\mathcal{R}} = B^{\mathcal{B}} = 0$. See Branch and McGough (2009: 1040).

becomes:

$$\mathcal{E}_0^\tau \sum_{i=0}^{\infty} \left(\beta\xi_p\right)^i \left[\log\left(\frac{P_t^j}{P_{t+i}}\right) - \frac{\sigma^{-1}}{1+\eta\varepsilon}c_{t+i} - \frac{\eta}{1+\eta\varepsilon}y_{t+i}\right] = 0 \tag{14}$$

where $\eta = \nu_{YY}\bar{Y}/\nu_Y$. Therefore,

$$\log\left(\frac{P_t^j}{P_t}\right) = \xi_p\beta\mathcal{E}_t^\tau\pi_{t+1} + \left(1-\beta\xi_p\right)\left[\frac{\sigma^{-1}}{1+\eta\varepsilon}\omega_t^\tau + \frac{\eta}{1+\eta\varepsilon}y_t\right] + \beta\xi_p\mathcal{E}_t^\tau \log\left(\frac{P_{t+1}^j}{P_{t+1}}\right) \tag{15}$$

Aggregating (15) for different types of agents and combining the definition of the aggregate price dynamics yields an AS curve similar to the relation derived in the standard New Keynesian framework with the same expectation operator used for (13):

$$\pi_t = \beta\mathcal{E}_t\pi_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)\left(\eta+\sigma^{-1}\right)}{\xi_p\left(1+\varepsilon\eta\right)}y_t \tag{16}$$

i.e.,

$$\pi_t = \alpha\beta E_t\pi_{t+1} + (1-\alpha)\beta\theta^2\pi_{t-1} + \kappa y_t + e_t \tag{17}$$

where $\kappa = \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)\left(\eta+\sigma^{-1}\right)}{\xi_p(1+\varepsilon\eta)}$. Note that (17) has been augmented by a supply disturbance.

The Phillips curve (17) shows that the inclusion of boundedly rational agents in our New Keynesian framework implies inflation persistence. Providing the proportion of non-rational agents increases, the importance of the backward component relative to the forward component in the Phillips curve also increases. The effect of a change in the share of boundedly rational individuals on the backward term of (17) is magnified by the extrapolative expectations, i.e., $\partial\pi_{t-1}/\partial\alpha\theta < 0$).[12]

3 Welfare criterion

We compute a quadratic Taylor series approximation of utility for household i. The first term of (1) is approximated as

$$\tilde{u}\left(C_t^i\right) = \bar{C}u_C\left(c_t(i) + \frac{1-\sigma^{-1}}{2}c_t^2(i)\right) + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right) \tag{18}$$

Note that in the steady state, $\bar{C} = \bar{C}^{\mathcal{R}} = \bar{C}^{\mathcal{B}}$ by assumption; the term $\mathcal{O}\left(\left\|\xi^3\right\|\right)$ indicates the terms of order greater than two, and $t.i.p.$ collects the terms independent of policy. Integrating

[12] Recall that the portion of non-rational agents is decreasing in α.

(18) over i, we obtain

$$\int_0^1 \tilde{u}\left(C_t^i\right) di = \bar{C} u_C \left\{ c_t - \frac{1}{2} var_i c_t(i) + \frac{1-\sigma^{-1}}{2} \left[c_t^2 + var_i\left(c_t(i)\right)\right] \right\} + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right) \quad (19)$$

where we use the relation $var_i(c_t(i)) = E_i c_t(i)^2 - (E_i c_t(i))^2$ and the fact that up to a second-order $c_t = E_i c_t(i) + \frac{1}{2} var_i c_t(i)$. Note also that the cross-sectional variance of the consumption is equal to $var_i(c_t(i)) = \alpha(1-\alpha)\left(c_t^{\mathcal{R}} - c_t^{\mathcal{B}}\right)^2$.[13]

Regarding effort, it is noted that each agent potentially supplies a different quantity of output. This depends on the agent's type and on whether he is or is not extracted in a Calvo lottery. A second-order approximation of the second term of (1) leads to

$$\tilde{\nu}(Y_t(i)) di = u_N \bar{N} \left(y_t(i) + \frac{1+\eta}{2} y_t^2(i) \right) + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right) \quad (20)$$

and, after integration, we obtain

$$\int_0^1 \tilde{\nu}(Y_t(i)) di = u_N \bar{N} \left(y_t - var_i(y_t(i)) + \frac{1+\eta}{2} \left(\left[E_i y_t(i)\right]^2 + var_i(y_t(i)) \right) \right) + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right) \quad (21)$$

where $\int_0^1 \bar{N}(i) = \bar{N}$ as the price dispersion is zero in the steady state. Then, by considering $Y(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$, the cross-sectional variance of $y_t(i)$ can be expressed as $var_i(y_t(i)) = \varepsilon^2 var_i(p_t(i))$. Thus, in the non-distorted steady state, where the equality $\bar{C} u_C / u_N = \bar{N}$ holds, the above expression is rewritten as

$$\int_0^1 \tilde{\nu}(Y_t(i)) di = \bar{C} u_C \left(y_t + \frac{\varepsilon^2 \eta}{2} var_i(p_t(i)) + \frac{1+\eta}{2} y_t^2 \right) + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right) \quad (22)$$

Combining (19) and (22), the approximated intertemporal utility can be expressed as

$$\textstyle\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{U}_t}{\bar{C} u_c} = - \textstyle\sum_{t=0}^{\infty} \beta^t \left[L_t + t.i.p. + \mathcal{O}\left(\left\|\xi^3\right\|\right)\right] \quad (23)$$

where the instantaneous loss is

$$L_t = \frac{1}{2} \left[\left(\eta + \frac{1}{\sigma} \right) y_t^2 + \left(\varepsilon^2 \eta \right) var_i(p_t(i)) + \frac{1}{\sigma} var_i(c_t(i)) \right] \quad (24)$$

and[14]

[13] It follows from $E_i\left(c_t^2(i)\right) = \alpha\left(c_t^{\mathcal{R}}\right)^2 + (1-\alpha)\left(c_t^{\mathcal{B}}\right)^2$ and $(E_i c_t(i))^2 = c_t^2$.

[14] Derivations of price and consumption dispersions are provided in the Appendix.

$$var_i(\log p_t(i)) = \delta\pi_t^2 + \frac{\delta\xi_p(1-\alpha)}{\alpha}\left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa\left(\frac{c_t^{\mathcal{B}} + \eta\sigma y_t}{1+\eta\sigma}\right)\right]^2 \quad (25)$$

$$var_i(c_t(i)) = \alpha(1-\alpha)\left(c_t^{\mathcal{R}} - c_t^{\mathcal{B}}\right)^2 \quad (26)$$

where $\delta = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}$.

Unlike the traditional case, welfare (24) is composed of three components when heterogeneous expectations are introduced in a New Keynesian framework by assuming that a fraction of agents form their expectations according to a mechanism of bounded rationality.

As in the textbook case, the first two components of (24) relate costs associated with consumption variability (y_t^2) and price dispersion ($var_i(p_t(i))$. However, here, the dispersion of prices (25) not only depends on current inflation but also has a more complex structure. Note also that the dispersion of prices positively depends on the proportion of boundedly rational agents. Everything being equal, it increases in the fraction of agents who form their expectations in a non-rational way.

Furthermore, an additional third term captures the cost linked to inequality in the consumption of the two types of agents ($var_i(c_t(i))$. This cost is not linear in the degree of bounded rationality. Rather, it has a peak when $\alpha = 0.5$ because, in this case, *coeteris paribus*, the distribution of agents exhibits the highest dispersion (see (26)). It is noted that inequality is increasing in the variability of expectations across types as the different beliefs drive different choices.

4 Optimal policies under bounded rationality

4.1 Calibration

We calibrate the model to the U.S. economy. The time unit is one quarter. The calibration of the structural parameters are chosen to equal those estimated or calibrated by Rotemberg and Woodford (1997) by using structural vector auto-regression (SVAR) methodology and microeconomic evidence, respectively. We assume that the subjective discount rate β is 0.99 such that $(\beta^{-1} - 1)$ equals the long-run average real interest rate. In the goods market, the intratemporal elasticity of substitution between the differentiated goods (price elasticity of demand) is set equal to 7.84, thus implying a markup of 15%. The parameter ξ_p, which represents the frequency of price adjustment, is set at 0.66, and thus, prices are fixed, on average, for three quarters. Finally, η, which is the elasticity of the marginal disutility of producing output with respect to an increase in output, is set on the basis of data regarding labor costs.[15] Accordingly, calibration is summarized in the following table.

Table 1 – Baseline calibration

$\beta = 0.99$	$\sigma^{-1} = 0.16$	$\varepsilon = 7.84$	$\eta = 0.47$	$\xi_p = 0.66$

Regarding the parameters characterizing heterogeneity (α, θ), we consider different calibrations. We assume a baseline value where α is equal to 0.7, thus implying that 30% of households form their expectations using a mechanism of bounded rationality, whereas 70% of the households are rational. We explore the effects of α for a range $\alpha \in [0.5, 1]$.[16] The model for $\alpha = 1$ clearly represents the homogeneous-rational agents standard New Keynesian model.

The baseline value for the adaption parameter θ is set equal to one, which implies that boundedly rational households recognize, with one-period lag, changes in inflation and the output gap (naive expectations). Again, we consider different specifications to test the robustness of our results, and we report numerical simulations for a range between 0.8 and 1.2. It is noted that different values of θ imply different "rules of thumb" for non-rational households when setting their expectations. Lower values of θ involve mean-reverting strategies according to which deviations from the average are expected to revert it. For example, if inflation is below the average, increases in inflation are expected. Higher values of θ entail trend-following behaviors, and thus, deviations from the average are expected to be confirmed, such that when the current inflation trend is upward (above the average), the expectation is that the inflation will continue to follow that trend.

4.2 Optimal monetary policy, expectations stabilization and determinacy

We begin the analysis by investigating the behavior of our baseline model, which is characterized by bounded rationality ($\alpha = 0.7$ and $\theta = 1$), when the economy is perturbed by a stochastic disturbance. In the figure below, we plot the impulse response functions (IRFs) for the inflation following a cost-push shock when a timeless perspective commitment is implemented. In particular, the dynamic response of the aggregate inflation joint with the expected inflation for both

[15] See Rotemberg and Woodford (1997) for details.

[16] We explored the model properties and robustness results for the whole existence field of α. Results are available upon request.

categories of agents (boundedly rational and rational) is illustrated in Figure 1.

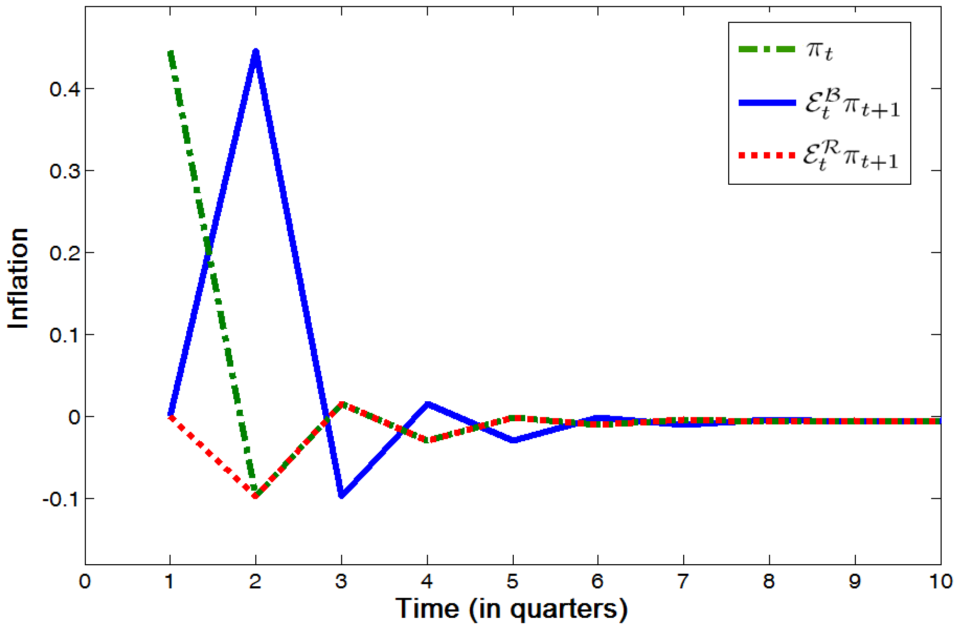

Figure 1 - Inflation IRF to a cost-push shock (commitment regime).

In contrast to the standard case, where all the agents are perfectly rational (i.e., $\alpha = 1$), when a certain degree of heterogeneity is introduced, the optimal stabilization path of the inflation exhibits fluctuations before returning to its steady state. To understand the intuition about this dynamic, consider the role of the expectations in a framework characterized by bounded rationality. Assume that at time $t = 1$ a shock hits the economy. The policymaker then raises the interest rate to compress inflationary pressure, thereby creating a small deflation. Thus, a fraction of boundedly rational agents form their expectations assuming that inflation is equal to its previous value. However, their belief is biased, and as a consequence, they overestimate the inflation, as it was brought down by the monetary contraction pursued by the central banker. In the next period, aggregate inflation exhibits a slight recovery, but again, inflation forecasts of the non-rational agents are wrong as they now are underestimating the inflation level. The policy maker must now adjust the nominal interest rate up and down to stabilize inflation expectations. Obviously, except for the first period, the inflation forecasts of the rational agents are always correct. This is the rationale that underpins the dynamics depicted in Figure 1.

Figure 2 illustrates the IRFs for inflation and for the output gap when the economy is hit by a cost-push shock. We compare the dynamics obtained from our baseline framework (solid lines) to those stemming from the standard model where all the agents are rational, i.e., $\alpha = 1$ (dashed lines). The figure reports the dynamics under both commitment and discretion in the left and right panels, respectively.

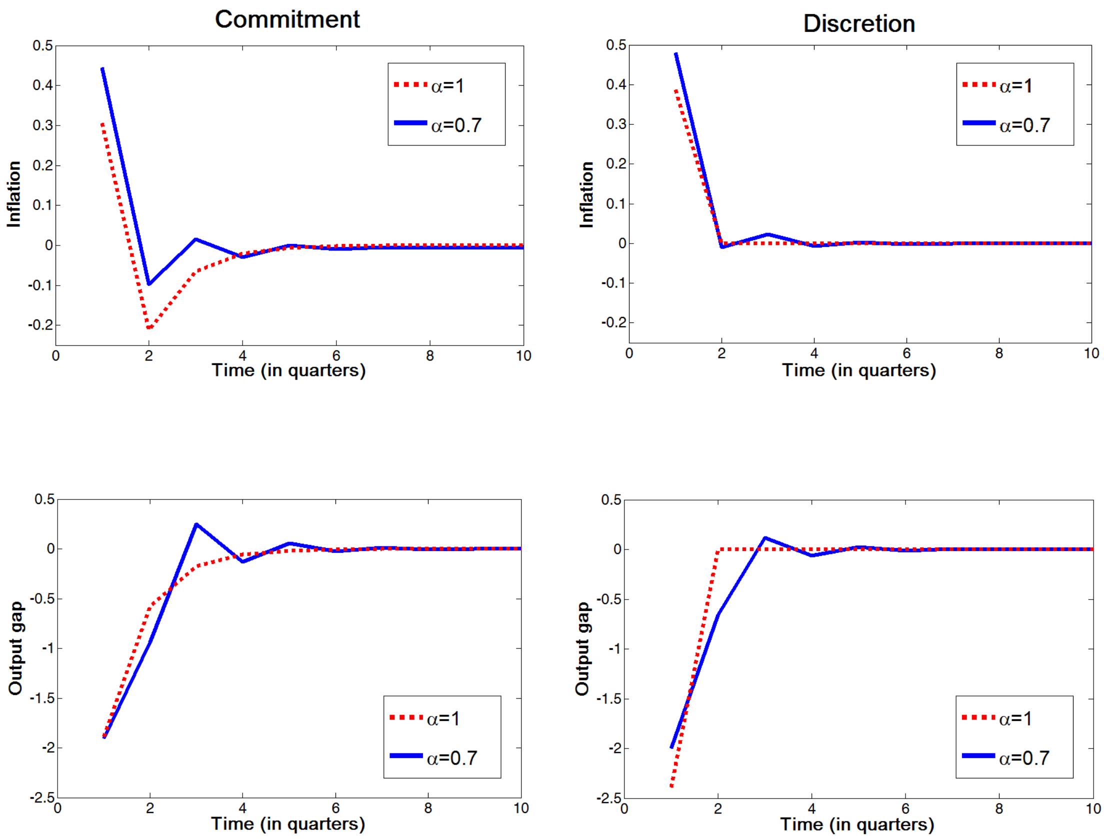

Figure 2 - Optimal stabilization policy for both regime (cost-push shock).

Because of the previously described mechanism, the inflation and output gap exhibit fluctuating dynamics in both regimes when some degree of bounded rationality is allowed. Moreover, for $\alpha = 0.7$, under discretion the output gap is more stabilized with respect to the standard fully rational case. This is because price dispersion is affected by y_t (see (25)). Hence, by reducing the variability of the output gap, the policymaker is able to exploit a better inflation-output gap trade-off.

The above results are robust for different calibrations regarding the adaptation parameter and the share of boundedly rational agents. In a reasonable range, we obtain IRFs that are qualitatively similar to those represented in Figure 2.[17]

Our additional simulations are also used to test the equilibrium determinacy. In our context, we find that when the monetary authority behaves following an optimal rule (under both a commitment and a discretion policy regime), the model is always determined, regardless of the fraction of boundedly rational agents. This result is in accordance with the findings of Clarida *et al.* (1999), who show that implementing optimal policy is sufficient to guarantee the determinacy of the model. It is noted that in the same setup, but assuming that monetary policy is set by a Taylor rule, Branch and McGough (2009) find that the determinacy region hinges both on the share of boundedly rational agents and the adaption parameter.

4.3 Gains from commitment and bounded rationality

We now examine how heterogeneous beliefs affect the relative gains of commitment over discretion for different combinations of α and θ (other parameters are calibrated as indicated in Table 1.) Our results, summarized in Table 2, indicate that welfare loss is associated with both discretion and commitment. The table also reports the percentage loss of the former regime over the latter.

[17] Further results are available upon request.

Table 2 - Welfare loss and commitment gains

		α		
θ	Policy regime	0.5	0.7	0.9
	commitment	177.433	111.929	70.676
1.2	discretion	181.820	115.565	75.495
	% loss	2.472	3.248	6.819
	commitment	155.301	99.308	65.643
1.0	discretion	164.588	106.691	73.648
	% loss	5.980	7.435	12.194
	commitment	123.180	81.775	58.778
0.8	discretion	157.712	103.279	73.481
	% loss	28.034	26.296	25.015

The table illustrates that, with other factors being the same, a larger share of non-rational agents always involves higher welfare losses. As α decreases, in fact, the economy becomes more persistent (see equation (17)) and, moreover, price dispersion rises. As a consequence, inflation stabilization costs more. Parameter θ also affects welfare losses as a higher θ (i.e., extrapolative expectations) increases the degree of inflation persistence, thereby magnifying the welfare losses.

As expected, a commitment rule always leads to welfare gains compared to discretion. However, the proportion of boundedly rational individuals plays a key role in influencing the relative gains of commitment. As explained above, a larger share of non-rational agents makes the economy more persistent and induces greater price dispersion. These two types of distortion have opposite effects on marginal commitment gains.

1. Inertia entails more backward looking behavior of the Phillips curve, thereby reducing the efficacy of the commitment (persistence effect).

2. Commitment performs better relative to discretion as the price dispersion rises (dispersion effect).

As shown by, e.g., Steinsson (2003), gains from a commitment are strictly related to the ability of a policy maker to manipulate private agents' expectations. This benefit tends to vanish as the backward looking component becomes predominant with respect to the forward one.

As the share of boundedly rational individuals increases, for $\theta \geq 1$, the persistence effect dominates the dispersion effect and commitment gains decline. By contrast, when forecasts are based on adaptive rules, i.e., $\theta < 1$, the dispersion effect prevails over the persistence effect and commitment gains rise as α decreases. This is because when $\theta \geq 1$, inflation inertia is magnified, whereas when $\theta < 1$, the value attached to the lag component progressively drops.

Finally, we consider the effect of θ for a given α on the relative gains of commitment. In this case, gains are linear such that the smaller the value of θ, the higher the gains are. The explanation is that a smaller θ reduces the degree of backward looking behavior, allowing the policy maker to exploit a better manipulation of the expectations.

4.4 Optimal policies vs. Taylor rules

It is often argued that Taylor rules well describe the conduct of monetary policy. Moreover, under certain circumstances, they also mimic optimal discretionary policies. In the canonical model, a simple Taylor rule, responding only to inflation according to the Taylor principle, may lead to similar dynamics compared to optimal discretionary policies. We check the robustness of this result by examining how the gains from discretion of the Taylor rule are affected by the degree of bounded rationality.

We consider two model specifications that differ only in how the interest rate is set. In one case, the policy maker acts under discretion, whereas in the other, the central bank adjusts the nominal interest rate according to the following Taylor rule:

$$i_t = \phi_\pi \pi_t \tag{27}$$

where $\phi_\pi = 1.5$.

In the table below, we provide the results of our numerical simulations for several values of α and θ. The remaining parameters are calibrated as reported in Table 1.

Table 3 - Optimal policy vs. Taylor rule

		$1-\alpha$			
θ	Policy regime	0	0.1	0.3	0.5
1.2	Discretion		75.495	115.565	181.820
	Taylor rule		130.059	269.864	415.770
	% gain		72.274	133.565	128.671
1.0	Discretion	62.737	73.648	106.691	164.588
	Taylor rule	65.071	105.321	191.309	294.61
	% gain	3.720	43.005	79.311	78.998
0.8	Discretion		73.481	103.279	157.712
	Taylor rule		91.067	148.753	227.385
	% gain		23.932	44.030	44.177

In the standard framework where all the agents are rational (i.e., $1-\alpha = 0$), the Taylor rule is suboptimal, while at the same time, discretion gains are small (approximately 3.7%). Note that these gains are independent of θ. As we introduce heterogeneous beliefs, we observe that the welfare losses associated with a Taylor rule dramatically grow and that the relative gains of discretion are extremely large. The costs associated with Taylor rule-based policies are between 40% and 135% when $\alpha \leq 0.7$, depending on θ, and between 20% and 75% when α is approximately 0.9.

Our results indicate that, although the costs of a Taylor rule compared to optimal inflation targeting are small when the standard case is considered, they may become huge once some degree of bounded rationality is introduced. The increasing costs of the Taylor rules occur because they cannot stabilize the individual belief variability, and thus, they are associated with increasing costs in terms of inequality. Moreover, considering the canonical trade-off between price and aggregate consumption stabilization, as explained, the weight in the Taylor rule should decrease in α to mimic optimal policies.

In Figure 3, we plot the marginal gains of discretion, expressed in percentage terms, over a Taylor rule. According to Table 3, they are very close to zero when all the agents are rational; however, as the share of non-rational agents increases, acting following a simple Taylor rule induces exceptionally high marginal losses.

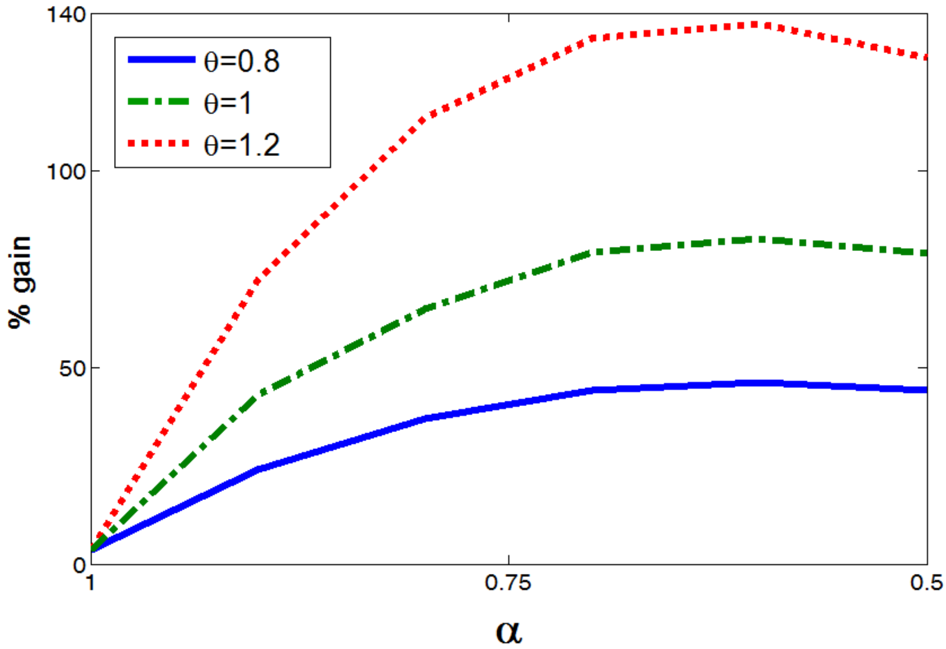

Figure 3 - Discretion gains over a Taylor rule.

5 Conclusions

Starting from well-documented observations that people form their expectations according to different mechanisms, this paper studied the impact of heterogeneous expectations on optimal monetary policies in a New Keynesian framework. The point of departure of our work is Branch and McGough (2009) who introduce bounded rationality in a small-scale New Keynesian DSGE model and provide an equilibrium determinacy analysis when the policymaker sets monetary policy according to a Taylor rule. We use their setup to investigate the implications of the heterogeneity of optimal monetary policy design. We also test the robustness of canonical inflation targeting policies under both discretion and commitment regimes and the equilibrium determinacy when some agents behave according to bounded rationality.

Our simulations show that optimal policies and welfare losses quantitatively and qualitatively depend on the share of boundedly rational agents. As the share of boundedly rational agents increases, welfare deteriorates because the economy becomes more persistent when the share of boundedly rational agents rises, and simultaneously, price dispersion increases, thereby inducing further welfare costs. In this context, optimal policies depart from those optimal in the canonical framework, which sometimes can be represented by appropriated Taylor rules. The assumption that some agents form their expectations using an adaptive or extrapolative mechanism in fact introduces a new dimension to the standard policy problem faced by the central bank, that of consumption inequality. Moreover, we find that optimal policies are always associated with equilibrium determinacy under both discretion and commitment, which differs from recent literature based on Taylor rules in the same context.

The assumption that some agents form their expectations using an adaptive or extrapolative mechanism implies that the central bank also needs to minimize the economy inequality (cross-sectional variance of consumption) in addition to the usual two dimensions of the central bank's policy problems (i.e., minimize the variability of aggregate consumption and the costs associated with price dispersion.) In other words, it now faces a three-dimensional problem.

To minimize the economy inequality, the central bank should stabilize expectation variability. As long as different individual consumption plans depend on different expectation paths, the variability in individual consumption is reduced by minimizing the cross-sectional variability of expectations. In turn, this requires that the central bank endogenously creates some fluctuations in the after-shock adjustment dynamics to keep expectations of the different kinds of agents closer. Regarding the usual trade-off between the stabilization of aggregate consumption and price dispersion, it, too, is affected by heterogeneity. The cost of price dispersion increases with the size of the group of boundedly rational agents, but the emphasis of stabilizing inflation versus output declines as long as heterogeneity increases, due to the more complex dynamics of price dispersion that, in this case, also depend on output stabilization.

Comparing welfare performances under different policy regimes, we find, as expected, that commitment always guarantees the lowest welfare losses. Then, following Steinsson (2003), we investigate the relative gains of commitment over discretion by considering different degrees of heterogeneity. In general, the relative gains of commitment depend on two different effects, which are higher when *i)* the forward-looking component of the Phillips curve progressively enhances compared to the backward one (persistence effect); *ii)* price dispersion is more costly (dispersion effect). We find that an increase in the share of boundedly rational subjects has an ambiguous effect on the relative gains of commitment as, on the one hand, it reduces the lead component of the Phillips curve, but, on the other hand, it raises the price dispersion. In our setup, commitment gains are more likely to be observed when agents form their expectations using an adaptive mechanism rather than an extrapolative mechanism. Note that extrapolative expectations magnify the persistence effect.

Finally, we highlight the importance of pursuing an optimal stabilization policy rather than following a simple exogenous interest rule. We show that in a world characterized by a fraction of non-rational agents, the costs of neglecting an optimal policy rule barely increase, leading to significant welfare losses. The rationale is that a Taylor rule is not able to mimic the optimal policy design because it overreacts to inflation and does not stabilize individual expectations. The former involves suboptimal choices in the trade-off between aggregate consumption and price dispersion stabilization, while the latter induces costs in terms of consumption inequality between individuals.

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Appendix – Price dispersion derivation

This appendix provides the derivation of the price dispersion for Branch and McGough's (2009) model. The discounted sum of the price dispersion evolves as follows:

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{1-\beta\xi_p}\sum_{t=0}^{\infty}\beta^t\left[\frac{\xi_p\left(\xi_p+\alpha-\alpha\xi_p\right)}{\alpha\left(1-\xi_p\right)}\pi_t^2+\frac{(1-\alpha)\,\xi_p^2}{\alpha\left(1-\xi_p\right)}\beta^2\theta^4\pi_{t-1}^2+\right.$$
$$+\frac{\left(1-\xi_p\right)(1-\alpha)\left(1-\xi_p\beta\right)^2}{\alpha}\left(\psi_a^2 c_t^{\mathcal{B}^2}+\psi_b^2 y_t^2+2\psi_a\psi_b c_t^{\mathcal{B}} y_t\right)+$$
$$+2\frac{(1-\alpha)\left(1-\xi_p\beta\right)\xi_p}{\alpha}\beta\theta^2\pi_{t-1}\left(\psi_a c_t^{\mathcal{B}}+\psi_b y_t\right)-2\frac{(1-\alpha)\,\xi_p^2}{\alpha\left(1-\xi_p\right)}\beta\theta^2\pi_t\pi_{t-1}+$$
$$\left.-2\frac{\xi_p(1-\alpha)\left(1-\xi_p\beta\right)}{\alpha}\pi_t\left(\psi_a c_t^{\mathcal{B}}+\psi_b y_t\right)\right] \tag{28}$$

where $\psi_a=\frac{\sigma^{-1}}{1+\eta\varepsilon}$ and $\psi_b=\frac{\eta}{1+\eta\varepsilon}$. Given that $\kappa=\frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)\left(\eta+\sigma^{-1}\right)}{\xi_p(1+\varepsilon\eta)}$, the undiscounted price dispersion can be alternatively written, after some algebraic manipulation, as

$$var_i\left(\log p_t(i)\right)=\frac{\xi_p}{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}\pi_t^2+\frac{(1-\alpha)\,\xi_p^2\left[\pi_t-\beta\theta^2\pi_{t-1}-\kappa\left(\frac{c_t^{\mathcal{B}}+\eta\sigma y_t}{1+\eta\sigma}\right)\right]^2}{\alpha\left(1-\beta\xi_p\right)\left(1-\xi_p\right)} \tag{29}$$

The above expression is derived by defining $P_t\equiv E_i\log p_t(i)$ and $\Delta_t=var_i\left(\log p_t(i)\right)$. In our context, the aggregate price level evolves as:

$$P_t=\left(1-\xi_p\right)(1-\alpha)\log p_t^{\mathcal{B}}+(1-\xi_p)\alpha\log p_t^{\mathcal{R}}+\xi_p\log p_{t-1}(i) \tag{30}$$

Subtracting P_{t-1} from both sides yields:

$$\pi_t=\left(1-\xi_p\right)(1-\alpha)\left(\log p_t^{\mathcal{B}}-P_{t-1}\right)+(1-\xi_p)\alpha\left(\log p_t^{\mathcal{R}}-P_{t-1}\right) \tag{31}$$

We can express (31) as:

$$\pi_t=\left(1-\xi_p\right)\left(P_t^*-P_{t-1}\right) \tag{32}$$

because $P_t^* = (1-\alpha)\log p_t^{\mathcal{B}} + \alpha \log p_t^{\mathcal{R}}$.

We now compute the variance:

$$\begin{aligned}\Delta_t &= var_i \left[\log p_t(i) - P_{t-1}\right] \\ &= E_i\left[\left(\log p_t(i) - P_{t-1}\right)^2\right] - \left[E_i \log p_t(i) - P_{t-1}\right]^2 \end{aligned} \tag{33}$$

The first term of the right-hand side can be rewritten as:

$$\xi_p E_i\left[\left(\log p_{t-1}(i) - P_{t-1}\right)^2\right] + \left(1-\xi_p\right)(1-\alpha)\left(\log p_t^{\mathcal{B}} - P_{t-1}\right)^2 + (1-\xi_p)\alpha\left(\log p_t^{\mathcal{R}} - P_{t-1}\right)^2 \tag{34}$$

The expectation of the boundedly rational agents with respect to the future aggregate price level is:

$$\mathcal{E}_t^{\mathcal{B}} \log p_{t+1} = \left(1-\xi_p\right)\mathcal{E}_t^{\mathcal{B}} \log p_{t+1}^{\mathcal{B}} + \xi_p P_t \tag{35}$$

Adding and subtracting $\left(1-\xi_p\right)P_{t+1}$ to (35) and exploiting the properties of the expectation operator of the non-rational agents we obtain:

$$\frac{\xi_p}{1-\xi_p}\theta^2\pi_{t-1} = \mathcal{E}_t^{\mathcal{B}}\left(\log p_{t+1}^{\mathcal{B}} - P_{t+1}\right) \tag{36}$$

From the model, we know that the optimal pricing rule of the boundedly rational agents is:

$$\log p_t^{\mathcal{B}} = \log p_t + \xi_p\beta\theta^2\pi_{t-1} + \left(1-\xi_p\beta\right)\left[\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right] + \xi_p\beta\mathcal{E}_t^{\mathcal{B}}\left(\log p_{t+1}^{\mathcal{B}} - P_{t+1}\right) \tag{37}$$

Substituting (36) in (37) yields:

$$\log p_t^{\mathcal{B}} - P_{t-1} = \pi_t + \frac{\xi_p\beta\theta^2}{1-\xi_p}\pi_{t-1} + \left(1-\xi_p\beta\right)\left[\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right] \tag{38}$$

Price for the rational agents is equal to:

$$\log p_t^{\mathcal{R}} = \frac{P_t^* - (1-\alpha)\log p_t^{\mathcal{B}}}{\alpha} \tag{39}$$

Combining (38) and (39) and then adding and subtracting P_{t-1}, the result is:

$$\log p_t^{\mathcal{R}} - P_{t-1} = \frac{1}{\alpha\left(1-\xi_p\right)}\pi_t - \frac{1-\alpha}{\alpha}\left\{\pi_t + \frac{\xi_p\beta\theta^2}{1-\xi_p}\pi_{t-1} + \left(1-\xi_p\beta\right)\left[\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right]\right\} \tag{40}$$

From the combination of (34), (38) and (40), we can rewrite (33) as:

$$\begin{aligned}\Delta_t \quad = \quad & \xi_p\Delta_{t-1} + \left(1-\xi_p\right)(1-\alpha)\left[\pi_t + \frac{\xi_p\beta\theta^2}{1-\xi_p}\pi_{t-1} + \left(1-\xi_p\beta\right)\left[\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right]\right]^2 + \\ & +(1-\xi_p)\alpha\left[\frac{1}{\alpha\left(1-\xi_p\right)}\pi_t - \frac{1-\alpha}{\alpha}\left\{\pi_t + \frac{\xi_p\beta\theta^2}{1-\xi_p}\pi_{t-1} + \left(1-\xi_p\beta\right)\left[\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right]\right\}\right]^2 - \pi_t^2\end{aligned}$$

This yields:

$$\begin{aligned}&\Delta_t = \xi_p\Delta_{t-1} + \frac{\xi_p\left(\xi_p+\alpha-\alpha\xi_p\right)}{\alpha\left(1-\xi_p\right)}\pi_t^2 + \frac{(1-\alpha)\,\xi_p^2\beta^2\theta^4}{\alpha\left(1-\xi_p\right)}\pi_{t-1}^2 + \\ &+\frac{\left(1-\xi_p\right)(1-\alpha)\left(1-\xi_p\beta\right)^2}{\alpha}\left(\psi_a^2 c_t^{\mathcal{B}^2} + \psi_b^2 y_t^2 + 2\psi_a\psi_b c_t^{\mathcal{B}} y_t\right) + \\ &+2\frac{(1-\alpha)\left(1-\xi_p\beta\right)\xi_p\beta\theta^2}{\alpha}\pi_{t-1}\left(\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right) + \\ &-2\frac{(1-\alpha)\,\xi_p^2\beta\theta^2}{\alpha\left(1-\xi_p\right)}\pi_t\pi_{t-1} - 2\frac{\xi_p(1-\alpha)\left(1-\xi_p\beta\right)}{\alpha}\pi_t\left(\psi_a c_t^{\mathcal{B}} + \psi_b y_t\right)\end{aligned} \tag{41}$$

With respect to the case of $\alpha = 0$ (standard homogenous-agent model), the Calvo price dispersion must be considered. Iterating (41) forward, the degree of price dispersion in any period $t \geq 0$ is given by:

$$\begin{aligned}&\Delta_t = \xi_p^{t+1}\Delta_{-1} + \sum_{s=0}^{t}\xi_p^{t-s}\left[\frac{\xi_p\left(\xi_p+\alpha-\alpha\xi_p\right)}{\alpha\left(1-\xi_p\right)}\pi_s^2 + \frac{(1-\alpha)\,\xi_p^2\beta^2\theta^4}{\alpha\left(1-\xi_p\right)}\pi_{s-1}^2 + \right.\\ &+\frac{\left(1-\xi_p\right)(1-\alpha)\left(1-\xi_p\beta\right)^2}{\alpha}\left(\psi_a^2 c_s^{\mathcal{B}^2} + \psi_b^2 y_s^2 + 2\psi_a\psi_b c_s^{\mathcal{B}} y_s\right) + \\ &+2\frac{(1-\alpha)\left(1-\xi_p\beta\right)\xi_p\beta\theta^2}{\alpha}\pi_{s-1}\left(\psi_a c_s^{\mathcal{B}} + \psi_b y_s\right) + \\ &\left.-2\frac{(1-\alpha)\,\xi_p^2\beta\theta^2}{\alpha\left(1-\xi_p\right)}\pi_s\pi_{s-1} - 2\frac{\xi_p(1-\alpha)\left(1-\xi_p\beta\right)}{\alpha}\pi_s\left(\psi_a c_s^{\mathcal{B}} + \psi_b y_s\right)\right]\end{aligned} \tag{42}$$

We now discount over all periods $t \geq 0$, thus yielding (28).