

Monetarism rides again? US monetary policy in a world of Quantitative Easing

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Abstract

This paper gives money a role in providing cheap collateral in a model of banking; this means that, besides the Taylor Rule, monetary policy can affect the risk-premium on bank lending to firms by varying the supply of M0 in open market operations, so that even when the zero bound prevails monetary policy is still effective; and fiscal policy under the zero bound still crowds out investment via the risk-premium. A simple rule for making M0 respond to credit conditions can substantially enhance the economy's stability. Both price-level and nominal GDP targeting rules for interest rates would combine with this to stabilise the economy further. With these rules for monetary control, aggressive and distortionary regulation of banks' balance sheets becomes redundant.

Keywords: DSGE model; Financial Frictions; Crises; Indirect Inference; money supply; QE; monetary policy; fiscal multiplier; zero bound

JEL classification: E3; E44; E52; C1;

The financial crisis of 2007–2011 has challenged our previous understanding of the monetary system, with its assumptions that asset markets are complete and that money injections work solely through the setting of interest rates on safe short-term government bonds. Instead it now seems more promising to assume that financial assets, and specifically bonds and credit, are imperfect substitutes and that money substitutes with a wide variety of financial and real assets in rather different ways; in such a world we can find a role for Quantitative Easing (QE) — Open Market Operations — that has now become a major instrument of monetary policy. This world harks back to that of Friedman's Quantity Theory restatement (Friedman, 1956) and Brunner and Meltzer's papers on the banking system (e.g. Brunner and Meltzer, 1963) as a transmission mechanism for money.

In this paper our aim is to construct a DSGE model in which this imperfect substitution occurs between financial assets and in which therefore money has a role beyond merely setting the interest rate on short-term government bonds, 'bank rate' for short. To do this we borrow from available models of the economy, banking and collateral to create out of them what could be called a 'neo-monetarist' model. Another element we inject is the possibility of hitting the zero bound on the bank rate; we do this quite simply by suspending the Taylor Rule when bank rate solves for this level or below and replacing it with this exogenous lower bound; this does not undermine inflation determinacy because this situation cannot continue indefinitely since the shocks to interest rates must die out in time. We test this model against the key features of US macroeconomic data and compare its performance with other models that do not have these new elements; we find that our augmented model gets somewhat closer to the data's behaviour.

The model has clear welfare and policy implications. In the economy there are two main interest rates, the safe rate on short-term bonds and the risky rate on bank credit. The first regulates consumption, while the second regulates investment. From a welfare viewpoint one may consider the first as designed to smooth consumption, while the credit premium should be smoothed at its friction-minimising level and the instrument that can do this is the monetary base (QE) which provides (monetary) collateral for banked investment projects.

There is also a role for regulation in this model, that is parallel with QE: it too may be used to stabilise the credit-premium, by loosening regulation when credit is expensive and vice versa. However, it can only have such a role if in steady state regulation is pitched at a distorting level, to raise the credit premium above its optimal, no-friction, rate; in this case, regulation can be lowered when the

credit premium is high (in a downturn) and raised when it is low (in an upturn). This cannot however be optimal because the steady state intervention is distorting. If the steady state level is pitched at a non-distorting level, then regulation can only be raised to offset a falling credit premium; it cannot be lowered any further with any effect since it is having no effect already.

Plainly, this would not be a problem if there was some other reason to set regulation at some high level — e.g. to prevent future banking crises. Yet we show later in this paper that banking alone cannot create crises according to this model; crises require non-banking shocks and banking shocks merely contribute to some worsening of crises when they happen. This points to the need for stabilising policies that are non-distortive, in the face of all such shocks; it does not justify distortive regulation unless such policies cannot do the job. Yet we also show later that monetary policy is a powerful stabiliser when augmented to include the use of the monetary base.

Finally, once money is introduced in the way we have done here, fiscal policy no longer has a strong effect with a fixed money supply even when the zero bound knocks out the Taylor Rule. The fiscal multiplier is the same whether interest rates are at the zero bound or not. The reason is that fiscal policy crowds out investment via the credit premium even though the safe interest rate does not move.

Thus our aim in this paper is to bring data to bear on the important policy issues identified here through the means of an estimated and tested DSGE model of the US economy. We are aware that there are other ways to bring data to bear on these issues: thus generalised VAR estimation may show the effects of different sorts of factors — as Stock and Watson (2012) — and the narratives of the economic history of the Great Recession may also shed light. Nevertheless these ways have their own drawbacks; the difficulties of identifying structural shocks in VARs and the possibilities of subjective bias in narrative both put limits on these methods; it is not easy to refute or confirm any causal processes they suggest. If we turn to alternative ways of estimating and testing this model structurally, we could have used Bayesian methods with strong priors which then dominate the results but the difficulty here lies in the selection of such priors when controversy surrounds most elements in our model; we could have chosen flat priors and thus moved to pure Maximum Likelihood estimation but here the problem is rather flat likelihood surfaces under small samples with these models (Canova and Sala, 2009), small-sample estimation bias, and rather weak power in the resulting Likelihood Ratio tests (Le et al, 2014). The strength of the indirect inference method we will be using is that we can identify a particular model, and, even though this model is highly nonlinear because of the zero bound switch, estimate it with only minor bias and perform a test that has substantial power. Thus we can be challenged by the normal methods of science in future work. Meanwhile our model has clear implications which we can use for policy analysis.

In the rest of this paper we begin by describing the new model; in section 2 we explain our testing procedure; in section 3 we test the model against the key data features; in section 4 we look at its policy and other implications along the lines just discussed; section 5 concludes.

1 The model

Our starting point in this paper is the work of Le et al. (2011) in finding a version of the Smets and Wouters (2007, SW) model of the US that could fit key US data features from the early 1980s. This New Keynesian model, which largely follows the specification of Christiano et al. (2005), is widely considered to get reasonably close to US data; as is familiar, it embodies habit-persistence for consumers, adjustment costs in capital, variable capacity utilisation, price/wage setting via Calvo contracts plus indexation and a Taylor Rule setting interest rates. Le et al. nevertheless found that further modifications were necessary to get it to replicate US data features. They showed that the original New Keynesian (NK) model was rejected. In addition, they examined an alternative version in which prices and wages were fully flexible but there was a simple one-period information delay for labour suppliers. This ‘New Classical (NC)’ version was also rejected. They also proposed a hybrid model that merged the NK and NC models by assuming that wage and price setters find themselves supplying labour and intermediate output partly in a competitive market with price/wage flexibility, and partly in a market with imperfect competition. They assumed that the size of each sector depended on the facts of competition and did not vary in the sample but they allowed the coverage of imperfect competition to differ between labour and product markets. The basic idea was that economies consist of product sectors where rigidity prevails and others where prices are flexible, reflecting the degree of competition in these sectors. Similarly with labour markets; some are much more competitive than others. An economy may be more or less dominated by competition and therefore more or less flexible in its wage/price-setting. The price and wage setting equations in the hybrid model are assumed to be a weighted average of the corresponding NK and NC equations.

It turned out that this combined model got much closer to the data for the full sample, when the rigidity was quite limited. Essentially, the NK model generated too little nominal variation while the NC model delivered too much. However the hybrid model was able to reproduce the variances of the data; and it is this key feature that enables it to match the data behaviour overall more closely. Further changes were made by Le et al. (2013) to incorporate a banking sector, following Bernanke et al. (BGG, 1999).

With this addition the model divides the production side into three distinct participants: as in SW, retailers and intermediate goods producers (now called entrepreneurs for a reason described later) and in addition, capital producers. Retailers function in the same way as before, operating in perfect competition to produce final goods by aggregating differentiated intermediate products using the Dixit-Stiglitz technology. With the assumption that retail output is made up of a fixed proportion of intermediate goods in an imperfectly competitive market and intermediate goods sold competitively, the aggregate price is a weighted average of prices received in the two types of market. As a result, the aggregate price equation is unchanged. Capital producers operate in a competitive market and take prices as given. They buy final consumption goods and transform them into capital to be sold on to entrepreneurs.

The difference of BGG from SW lies in the nature of entrepreneurs. They still produce intermediate goods, but now they do not rent capital from households (who do not buy capital but only buy bonds or deposits) but must buy it from capital producers and in order to buy this capital they have to borrow from a bank which converts household savings into lending. On their production side, entrepreneurs face the same situation as in Le et al. (2011). They hire labour from households for wages that are partly set in monopolistic, partly in competitive labour markets; and they buy capital from capital producers at prices of goods similarly set in a mixture of monopolistic and competitive goods markets. Thus the production function, the labour demand and real marginal cost equations are unchanged. It is on their financing side that there are major changes. Entrepreneurs buy capital using their own net worth, pledged against loans from the bank, which thus intermediates household savings deposited with it at the risk-free rate of return. The net worth of entrepreneurs is kept below the demand for capital by a fixed death rate of these firms $(1 - \theta)$; the stock of firms is kept constant by an equal birth rate of new firms. Entrepreneurial net worth therefore is given by the past net worth of surviving firms plus their total return on capital minus the expected return (which is paid out in borrowing costs to the bank) on the externally financed part of their capital stock — equivalent to

$$n_t = \theta n_{t-1} + \frac{K}{N} (cy_t - E_{t-1}cy_t) + E_{t-1}cy_t + enw_t \quad (1)$$

where $\frac{K}{N}$ is the steady state ratio of capital expenditures to entrepreneurial net worth, θ is the survival rate of entrepreneurs and enw_t is a net worth shock. Those who die will consume their net worth, so that entrepreneurial consumption is equal to $(1 - \theta)$ times net worth. In logs this implies that this consumption varies in proportion to net worth so that:

$$c_t^e = n_t \quad (2)$$

In order to borrow, entrepreneurs have to sign a debt contract prior to the realisation of idiosyncratic shocks on the return to capital: they choose their total capital and the associated borrowing before the shock realisation. The optimal debt contract takes a state-contingent form to ensure that the expected gross return on the bank's lending is equal to the bank opportunity cost of lending. When the idiosyncratic shock hits, there is a critical threshold for it such that for shock values above the threshold, the entrepreneur repays the loan and keeps the surplus, while for values below it, he would default, with the bank keeping whatever is available. From the first order conditions of the optimal contract, the external finance premium is equated with the expected marginal product of capital which under constant returns to scale is exogenous to the individual firm (and given by the exogenous technology parameter); hence the capital stock of each entrepreneur is proportional to his net worth, with this proportion increasing as the expected marginal product rises, driving up the external finance premium. Thus the external finance premium increases with the amount of the firm's capital investment that is financed by borrowing:

$$E_t cy_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (qq_t + k_t - n_t) + epr_t \quad (3)$$

where the coefficient $\chi > 0$ measures the elasticity of the premium with respect to leverage. Entrepreneurs leverage up to the point where the expected return on capital equals the cost of borrowing from financial intermediaries. The external finance premium also depends on an exogenous premium shock,

epr_t . This can be thought of as a shock to the supply of credit: that is, a change in the efficiency of the financial intermediation process, or a shock to the financial sector that alters the premium beyond what is dictated by the current economic and policy conditions.

Entrepreneurs buy capital at price qq_t in period t and uses it in $(t + 1)$ production. At $(t + 1)$ entrepreneurs receive the marginal product of capital rk_{t+1} and the ex-post aggregate return to capital is cy_{t+1} . The capital arbitrage equation (Tobin's Q equation) becomes:

$$qq_t = \frac{1 - \delta}{1 - \delta + R_*^K} E_t qq_{t+1} + \frac{R_*^K}{1 - \delta + R_*^K} E_t rk_{t+1} - E_t cy_{t+1} \quad (4)$$

The resulting investment by entrepreneurs is therefore reacting to a Q-ratio that includes the effect of the risk-premium. There are as before investment adjustment costs. Thus, the investment Euler equation and capital accumulation equations are unchanged from Le et al. (2011). The output market-clearing condition becomes:

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} inn_t + R_*^K k_y \frac{1 - \psi}{\psi} rk_t + c_y^e c_t^e + eg_t \quad (5)$$

1.1 Modifications to the BGG model to allow effects of Quantitative Easing and Bank Regulation

In the years since the crisis there have been key developments in the monetary scene. The first has been the zero bound on official interest rates, as central banks have driven the rate at which they will lend to banks down virtually to zero. The second development has been aggressive open market operations ('Quantitative Easing'), intended to inject liquidity into the banking system and spur greater credit creation. The third has been more intrusive regulation of banks, via increased capital and liquidity ratios. It seems important to us to introduce into the model here a tool to deal with each of these developments.

Let us begin with bank regulation: what this does is to raise the cost of lending to entrepreneur-firms, or 'firms' for short — this is the credit friction. The regulations insist banks hold as counterpart funds for the credit assets they hold, not purely deposits that have low cost but also in particular capital; the latter is more expensive because shareholders putting up such equity require an appropriate premium to compensate them for the risk the banks' losses will lose this capital. We do not model the regulations explicitly through these balance sheet quantities but for simplicity put into the model an addition to the credit friction, ξ , reflecting these requirements — and also the costs of other regulative intrusions¹.

Next, we consider the role of QE. To deal with this, we note that in BGG firms put up no collateral. Net worth by construction is all invested in plant, machinery and other capital. However, once so invested, this amount cannot be recovered at original value plainly: it will have less value as second hand sales when the firm goes bankrupt because it has become specialised to the firm's activities. The cost of bankruptcy recovery (costly state verification) applies to the valuation of the activity this capital still allows.

It is in fact normal for banks to request an amount of collateral from the firms to which they lend. This gives firms more 'skin in the game' and so more incentive to avoid bankruptcy. (Some models underpin bank contracting entirely on the basis that banks will only lend against collateral — Kiyotaki and Moore (1997) — however we do not adopt this extreme position here.) We therefore supplement the BGG model by the assumption that banks require firms to put up the amount of collateral, c , as a fraction of their net worth. We also assume that recovery of this typical collateral costs a proportion, δ , of its original value when posted — we can think of the example of a house being put up and it costing this proportion in fees and forced-sale losses to sell the house and recover its value in cash. We modify the workings of the model according to these two assumptions — these modifications are shown in an appendix.²

¹Such as 'ringfencing' different activities and imposing high liquidity ratios

²The posting of collateral actually lowers firms' profits from borrowing for given net worth and leverage; this is because collateral has no yield and could be sold for higher profitable investment. Hence it seems to be puzzling that banks demand collateral in a contract designed to maximise firms' profits subject to the constraints of truth-telling and bank zero net profits due to competition. However, undoubtedly collateral is a routine precaution taken by banks engaged in arm's length lending. The natural interpretation of collateral in this context is that it ensures that the borrower does not abscond; in the event of absconding the collateral is directly seizable. Equivalent amounts are taken in numerous financial transactions as 'deposits', 'margin' and so on.

It is at this point we introduce the idea of cash as collateral. If a firm holds some cash on its balance sheet, this can be recovered directly with no loss of value and no verification cost; thus it eliminates the cost δ . We show in the appendix that the elimination of this cost lowers the credit premium for given leverage; it therefore permits firms to increase leverage and so raise their expected returns. We therefore assume that banks and firms have an interest in firms holding as much cash as can be acquired for collateral. Thus as M0 is issued we assume that it is acquired by firms from banks to be held as collateral. This effect of the monetary base on collateral echoes Williamson (2013) in a search model.

The government/central bank issues this cash through open market operations (QE) to households in exchange for government bonds they hold. They deposit this cash with the banks. Firms wish to acquire as much of this cash as possible for their collateral needs. We can think of them as investing their net worth in cash (to the maximum available), with the rest going into other collateral and capital. In practice of course their profits (which create their net worth) are continuously paid out as dividends to the banks which provide them with credit, so they have nothing with which to acquire these assets if they do not collaborate with banks. So they achieve this balance sheet outcome by agreeing with the banks that, as a minimum counterpart to the credit advanced they will hold the maximum cash collateral available, which is M0. Thus all of M0 at once finds its way into firms' balance sheets, where it is securely pledged to the banks in the event of bankruptcy (for example by being actually lodged with them); in practice as we explain below with respect to the sectoral balance sheets it would be held as a counterpart deposit by firms and the M0 held by the banks.

Finally, the short-term interest rate is set by the central bank according to some rule, such as the Taylor Rule. In our model here only firms hold M0; households have no use for it and deposit it at once in banks where as we have seen it is lent to firms to hold as collateral, in effect M0's only use. In New Keynesian models it is implicitly assumed that the Taylor Rule is enforced by open market operations of some sort, presumably in money and Treasury Bills. Here we make the assumption that it is enforced by open market operations in public debt; households hold part of their savings in government bonds, the rest in bank deposits, which pay the short term interest rate also obtainable on Treasury Bills (treated here as an equivalent asset). The Taylor Rule represents the short term interest rate at which the government debt office will borrow; hence it sets the Treasury Bill rate and so the bank deposit rate.³

This now gives our monetary authorities three instruments: ξ , M0 and r . Accordingly, apart from their interest rate setting rule they will need two other operating rules for these instruments. We will discuss these shortly.

First, we set out the balance sheets of the agents in the economy and discuss how they are altered by acts of policy (see Table 1).

Firms		Banks		Households		Govt/central bank	
A	L	A	L	A	L	A	L
$COLL - exM0 -$							
$(COLL - CDEP(M0) +)$	(NW)	$(CR +)$	$(\frac{DEP +}{CDEP(M0) +})$	$(\frac{DEP +}{GB -})$	$(CUMSAV)$	$(CUMDEF)$	$(\frac{GB -}{M0 +})$
$K +$							

Table 1: Balance Sheets of the Agents of the Economy

where COLL= collateral (exM0=held as non-monetary; M0=held as money); K=capital investment; NW=net worth; CR=credit; DEP=deposits; GB=government bonds; CUMSAV=stock of private savings; CUMDEF=accumulated government borrowing; M0=monetary base. For simplicity we have written as if the firms hold M0 directly; in practice of course they would hold it indirectly as a marked deposit with the bank, and the bank would hold the M0 on its behalf- ready to seize it as collateral in the event of bankruptcy. This is shown in the sectoral balance sheets as CDEP(M0) which is an asset of firms corresponding to their M0 deposit; in turn it is a liability of banks, which hold the corresponding M0 as an asset. Thus injections of M0 by the central bank wind up being held as liquid collateral by banks to back up their credit operations.

Consider now how an open market operation (QE) by buying GB for M0 would change these balance sheets — as indicated by + and — in this table. Households place the extra cash on deposit; the banks then lend it to firms who are able to use it as collateral in a future lending deal with the banks, so that a larger part of collateral is held as M0. With collateral cheaper (δ falls) the bank credit premium falls

³Thus in effect this rate determines household consumption (directly as the safe rate on deposits) and (indirectly via the bank credit rate, together with other effects on this) firms' investment. Private plus public spending in turn sets total demand which is satisfied by suppliers; inflation then responds to the output gap, as suppliers reset prices, so clearing the goods market.

(which will induce a future rise in investment and leverage); the other collateral is converted into capital stock. These are the partial equilibrium or direct effects, which then lead to further general equilibrium changes in response to the fall in the credit premium.

To adjust the model for these additional features, we need to introduce the effect of M0 on the credit premium via its effect on the cost of liquidating collateral, δ ; and we need to add ξ , the macro-prudential instrument directly raising the credit friction, into the credit premium equation. We can think of ξ as being like a buffer of M0 that the banks need to hold for reasons of liquidity, and that is hence unavailable for use as collateral; hence it is equivalent to negative M0. The credit premium equation now has additional terms in m ($=\ln M0$) and ξ , as follows:

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = s_t = \chi(qq_t + k_t - n_t) - \psi m_t + \xi_t + epr_t \quad (6)$$

where ψ is the elasticity of the premium to M0 via its collateral role. This effect comes about, conditional on leverage ($k - n$), through the willingness of banks (under their zero profit condition) to reduce the credit premium for given leverage. Now that they will recover more in the event of bankruptcy, the equilibrium contract, for given leverage, now has a lower bankruptcy threshold and a lower required rate of return on firm assets. Both produce a lower credit premium for given leverage.

We now need equations for the supply of M0 and for the setting of ξ . QE programmes have sought to raise money supply growth (and implicitly therefore credit growth); before these programmes M0 seems to have been set to accommodate the supply of credit/broad money generated at the interest rates set by the Taylor Rule.

Macro-prudential measures have been built on the Basel Agreements nos 1 and 2; clearly they have been made more harsh over this period in response to the crisis, which was unpredicted by officials. Before that there was a gradual tightening of regulation at least in the Agreements, if not always in practical application by individual countries.

What these considerations suggest, as argued above, is that the supply of M0 was supplied via the discount window, before the crisis when interest rates were above the zero bound, as required to support the supply of money (M in logs); after the crisis, when interest rates were at the zero bound (which we take to be 0.25% p.a.), M0 (i.e. QE) seems to have been targeting the credit premium around its steady state, s^* , aiming to bring credit conditions back to normal. For macro-prudential measures the above suggests that they have evolved as an exogenous $I(1)$ time-series process, with the crisis acting as an exogenous shock to the process.

So we write the equation for M0 in two parts:

$$m_t = \psi_0 + \psi_1 M_t + errm_{2t} \text{ for } r_t > 0.0625\% \text{ and } \Delta m_t = \psi_2 (s_t - s^*) + errm_{2t} \text{ for } r_t \leq 0.0625\%$$

where ψ_1, ψ_2 are both positive. The credit premium tends to be correlated inversely with the broad money supply, so that one may think of this approximately as a policy of money targeting; however the money element in the banks' balance sheet fluctuates with other things and so from a welfare viewpoint it is the credit premium that should be targeted with as much information on it as can be amassed, including that from M itself.

We write the equation for the macro-prudential instrument as:

$$\Delta \xi_t = errxi_t$$

Given that we have very poor data on these macro-prudential measures we have to this point simply included these in the error epr_t .

Finally, we need an equation now additionally for the supply of money, which we define as equal to deposits (= credit) + M0. Here we simply use the firms' balance sheet ($M = CR + M0 = K + COLL - NW + M0$) which can be written in loglinearised form as:

$$M_t = (1 + \nu - c - \mu)K_t + \mu m_t - \nu n_t$$

where M, K, m, n are respectively the logs of Money, capital, M0 and net worth, we have omitted the constant (which includes collateral, assumed fixed as a proportion of money); ν, μ, c are respectively the ratios of net worth, M0 and collateral to money.⁴

⁴Notice that in this model the demand for money is simply the demand for deposits as a savings vehicle. Savings in the model are equal to investment by market-clearing, so that any additional investment requiring additional bank supply of leverage is equal to the additional supply of savings.

We treat the above model with money as a third version. The final step is to allow for the zero bound on the government short-term bond rate. To do this we note that when this interest rate solves above 0.25%, the Taylor Rule sets interest rates so that inflation is determined. The operative M0 equation is the one supporting the broad money supply.

However when interest rate solves for 0.25% or below it can fall no further than 0.25% and so it is set at this level and the Taylor Rule is rendered inoperative; when this happens, the model retains inflation determinacy because at some point in the future the model emerges from the zero bound and the Taylor Rule is again operative. The operative M0 equation is the one targeting the credit premium.

The full model is given in Appendix 2.

2 The method of Indirect Inference

We evaluate the model's capacity in fitting the data using the method of Indirect Inference originally proposed in Meenagh et al. (2009a) and subsequently with a number of refinements by Le et al. (2011) who evaluate the method using Monte Carlo experiments. The approach employs an auxiliary model that is completely independent of the theoretical one to produce a description of the data against which the performance of the theory is evaluated indirectly. Such a description can be summarised either by the estimated parameters of the auxiliary model or by functions of these; we will call these the descriptors of the data. While these are treated as the 'reality', the theoretical model being evaluated is simulated to find its implied values for them.

Indirect inference has been widely used in the estimation of structural models (e.g., Smith, 1993, Gregory and Smith, 1991, 1993, Gourieroux et al., 1993, Gourieroux and Monfort, 1995 and Canova, 2005). Here we make a further use of indirect inference, to evaluate an already estimated or calibrated structural model. The common element is the use of an auxiliary time series model. In estimation the parameters of the structural model are chosen such that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the actual data. The optimal choices of parameters for the structural model are those that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. Common choices of this function are the actual coefficients, the scores or the impulse response functions. In model evaluation the parameters of the structural model are taken as given. The aim is to compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of a structural model — which is taken as a true model of the economy, the null hypothesis — with the performance of the auxiliary model when estimated from the actual data. If the structural model is correct then its predictions about the impulse responses, moments and time series properties of the data should statistically match those based on the actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates.

The testing procedure thus involves first constructing the errors implied by the previously estimated/calibrated structural model and the data. These are called the structural errors and are backed out directly from the equations and the data⁵. These errors are then bootstrapped and used to generate for each bootstrap new data based on the structural model. An auxiliary time series model is then fitted to each set of data and the sampling distribution of the coefficients of the auxiliary time series model is obtained from these estimates of the auxiliary model. A Wald statistic is computed to determine whether functions of the parameters of the time series model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

The auxiliary model should be a process that would describe the evolution of the data under any relevant model. It is known that for non-stationary data the reduced form of a macro model is a VARMA where non-stationary forcing variables enter as conditioning variables to achieve cointegration (i.e. ensuring that the stochastic trends in the endogenous vector are picked up so that the errors in the VAR are stationary). This in turn can be approximated as a VECM (Appendix 3). So following Meenagh et al. (2012) we use as the auxiliary model a VECM which we reexpress as a VAR(1) for the three macro variables (interest rate, output and inflation, with a time trend and with the productivity

⁵Some equations may involve calculation of expectations. The method we use here to initiate the tests is the robust instrumental variables estimation suggested by McCallum (1976) and Wickens (1982): we set the lagged endogenous data as instruments and calculate the fitted values from a VAR(1) – this also being the auxiliary model chosen in what follows. Once the search procedure (effectively indirect estimation) has converged on the best model parameters, we then move to generating the expectations exactly implied by the parameters and the data, and use these to calculate the errors, which are then the exact errors implied by the model and data. The reason we do not use this 'exact' method at the start is that initially when the model is far from the data, the expectations generated are also far from the true ones, so that the errors are exaggerated and the procedure may not converge.

residual entered as an exogenous non-stationary process (these two elements having the effect of achieving cointegration). Thus our auxiliary model in practice is given by: $y_t = [I - K]y_{t-1} + \gamma\bar{x}_{t-1} + gt + v_t$ where \bar{x}_{t-1} is the stochastic trend in productivity, gt are the deterministic trends, and v_t are the VECM innovations. We treat as the descriptors of the data all the VAR coefficients and the VAR error variances, so that the Wald statistic is computed from these. Thus effectively we are testing whether the observed dynamics, volatility and cointegrating relations of the chosen variables are explained by the simulated joint distribution of these at a given confidence level. The Wald statistic is given by:

$$(\Phi - \bar{\Phi})' \sum_{(\Phi\Phi)}^{-1} (\Phi - \bar{\Phi}) \quad (7)$$

where Φ is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with $\bar{\Phi}$ and $\sum_{(\Phi\Phi)}$ representing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively.

The joint distribution of the Φ is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is therefore an estimate of the small sample distribution⁶. Such a distribution is generally more accurate for small samples than the asymptotic distribution; it is also shown to be consistent by Le et al. (2011) given that the Wald statistic is ‘asymptotically pivotal’; they also showed it had quite good accuracy in small sample Monte Carlo experiments⁷.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones (H_0 , the null hypothesis); they can be derived from calibration, estimation, or both. However derived, the test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? Of course only one true model with one set of coefficients is possible. Nevertheless we may have chosen coefficients that are not exactly right numerically, so that the same model with other coefficient values could be correct. Only when we have examined the model with all coefficient values that are feasible within the model theory will we have properly tested it. For this reason we extend our procedure by a further search algorithm, in which we seek other coefficient sets that minimise the Wald test statistic — in doing this we are carrying out indirect estimation. The indirect estimates of the model are consistent and asymptotically normal, in common with FIML — see Smith (1993), Gregory and Smith (1991, 1993), Gouriéroux et al., (1993), Gouriéroux and Monfort (1995) and Canova (2005).

Thus we calculate the minimum-value Wald statistic for each model using a powerful algorithm based on Simulated Annealing (SA) in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the space⁸. The merit of this extended procedure is that we are comparing the best possible versions of each model type when finally doing our comparison of model compatibility with the data.

We use Indirect Inference rather than the now widely-used Bayesian approach to estimating our model here, because we wish to test the model as a whole against the data. Under the Bayesian approach one assumes that the prior distributions and the model structure are correct; but because of well-known controversies in macroeconomics, this is not an assumption we can (yet) make. Even a major model like the Smets-Wouters (2007) model of the U.S., that was carefully estimated by Bayesian methods, was rejected by our indirect inference test, see Le et al. (2011). Under uncertainty about the priors any model ranking or probability assessment we made would be affected by the choice of priors. Thus our aim is to find a model that we can say is not rejected by the data features, so that we can rely on it for discussions of policy; at a later stage of analysis when such a model has been found to be generally reliable, one could progress to the use of Bayesian methods in refining it.

For overall testing we could also have used the common Likelihood Ratio (LR) test instead of the Wald test — a Bayesian test with flat priors (which might be used given lack of agreement on priors)

⁶The bootstraps in our tests are all drawn as time vectors so contemporaneous correlations between the innovations are preserved.

⁷Specifically, they found on stationary data that the bias due to bootstrapping was just over 2% at the 95% confidence level and 0.6% at the 99% level. Meenagh et al (2012) found even greater accuracy in Monte Carlo experiments on nonstationary data.

⁸We use a Simulated Annealing algorithm due to Ingber (1996). This mimics the behaviour of the steel cooling process in which steel is cooled, with a degree of reheating at randomly chosen moments in the cooling process—this ensuring that the defects are minimised globally. Similarly the algorithm searches in the chosen range and as points that improve the objective are found it also accepts points that do not improve the objective. This helps to stop the algorithm being caught in local minima. We find this algorithm improves substantially here on a standard optimisation algorithm. Our method used our standard testing method: we take a set of model parameters (excluding error processes), extract the resulting residuals from the data using the LIML method, find their implied autoregressive coefficients ($AR(1)$ here) and then bootstrap the implied innovations with this full set of parameters to find the implied Wald value. This is then minimised by the SA algorithm.

amounts in effect to an LR test. This test is examined carefully in Le et al. (2012) who find that the two methods test quite different properties of the model in their check on its misspecification: the LR test is based on a model’s in-sample current forecasting ability whereas the Wald is based on the ability of the model, including its implied errors, to replicate the behaviour of the data, as found in the VAR coefficients and the data variances. In effect the Wald test asks in a parsimonious way whether the model can replicate the impulse response functions found in the data. This reflects our aims for the use of the model, to assess the impulse response functions of policy shocks.

It turns out that the Wald has far greater power than the LR in the context of a macro model like the one here; this property is demonstrated for the Smets-Wouters model by Le et al. (2012) whose comparative table we reproduce next. Table 2 shows a Monte Carlo experiment with the SW model treated as true, generating stationary data; this model is mis-specified by changing the parameters alternately by $\pm x\%$ (see the original for full details). The rejection rate for the Wald rises sharply with x . Table 3 shows the same exercise on non-stationary data that the power of the Wald remains very similar under non-stationarity as we have in the model here.

Percent Mis-specified	Wald	LR
True	5.0	5.0
1	19.8	6.3
3	52.1	8.8
5	87.3	13.1
7	99.4	21.6
10	100.0	53.4
15	100.0	99.3
20	100.0	99.7

Table 2: Rejection Rates for Wald and Likelihood Ratio for 3 Variable VAR(1) with stationary data

3 variable VAR(1)-non-stationary data	
Level of falseness	Rejection rate
1%	34.1%
3%	55.2%
5%	90.5%
7%	100%

Table 3: Rejection Rates for Wald and Likelihood Ratio for 3 Variable VAR(1) with non-stationary data

3 Testing the model against the data

We now evaluate two versions of this model: one with the BGG banking sector; and one which further adds the monetary base as collateral and switch between regimes without and with the zero lower bound. Each model is reestimated by indirect estimation before its final test statistic is determined. It can be seen in Table 4 that both models fail to be rejected at 95%, with p-values in excess of 0.05. Adding the extra features slightly improves the p-value; hence bringing the model more closely into alignment with real-world features does not damage the model’s coherence with the data.

In a later section we show key IRFs for the final model (i.e. in the final column) that we have developed here. We must emphasise that because of the differing monetary responses in the ‘non-crisis’ (no zero bound) regime and the ‘crisis’ regime (zero bound) the responses under the two regimes differ materially. Under the first, interest rate policy is active and M0 policy is supportive (i.e. M0 expands as needed to accommodate rising M2, and vice versa); under the second interest rate policy is enforcedly inactive at the zero bound and M0 policy is active in controlling the credit premium. Also under the first regulative policy was inactive (and largely ineffective in the control of monetary conditions since M0 is supportive of M2; under the second regulative policy was active though it is only modelled here as an exogenous error process and not a reactive policy).

We will argue that welfare would be improved if monetary policy combined an interest rate rule (whenever feasible) with an active M0 rule and if regulative policy were dropped. So one regime we will examine closely in what follows is where these things are done — we will call it the ‘Monetary Reform’

Model Coefficients: 1984Q1-2011Q4			
		Model with Banking alone	Model with Banking and Money
Elasticity of capital adjustment	φ	7.376	6.814
Elasticity of consumption	σ_c	1.476	1.700
External habit formation	λ	0.646	0.714
Probability of not changing wages	ξ_w	0.813	0.627
Elasticity of labour supply	σ_L	4.915	2.683
Probability of not changing prices	ξ_p	0.882	0.973
Wage indexation	ι_w	0.943	0.354
Price indexation	ι_p	0.202	0.168
Elasticity of capital utilisation	ψ	0.053	0.104
Share of fixed costs in production (+1)	Φ	1.400	1.761
Taylor Rule response to inflation	r_p	4.266	2.375
Interest rate smoothing	ρ	0.786	0.737
Taylor Rule response to output	r_y	0.036	0.025
Taylor Rule response to change in output	$r_{\Delta y}$	0.055	0.021
Share of capital in production	α	0.156	0.178
Proportion of sticky wages	ω^w	0.444	0.442
Proportion of sticky prices	ω^r	0.066	0.090
Elasticity of the premium with respect to leverage	χ	0.012	0.032
Money response to premium	ψ_2	<i>N/A</i>	0.065
Elasticity of the premium to M0	ψ	<i>N/A</i>	0.055
Money response to credit growth	ψ_1	<i>N/A</i>	0.043
WALD (Y, π, R)		21.263	21.9037
*Fixed parameters p -value		0.0656	0.0686

Table 4: Coefficient Estimates (1984Q1-2011Q4)

regime. Improvement would also come if the interest rate rule were targeting the price level, or even better nominal GDP.

4 Accounting for the Banking Crisis — how much could policy setting avoid?

It is widely argued that there needs to be regulative intervention in order to prevent future crises by preventing banks from lending excessively. We look at this argument in two ways through the prism of the model we have constructed here. First we consider the actual causes of this particular crisis according to the model. Second, we do a stochastic analysis in which we create many pseudo-histories to see how often purely banking shocks (which we assume could be prevented by regulation) have produced a crisis; and also how much difference banking shocks and banking transmission (which again we assume could have been controlled by regulation) have made to the crises we could have had.

We analyse what the model says should happen in the economy during and after the Great Recession. We use that charts that follow for main macro variables: output, inflation and interest rate.

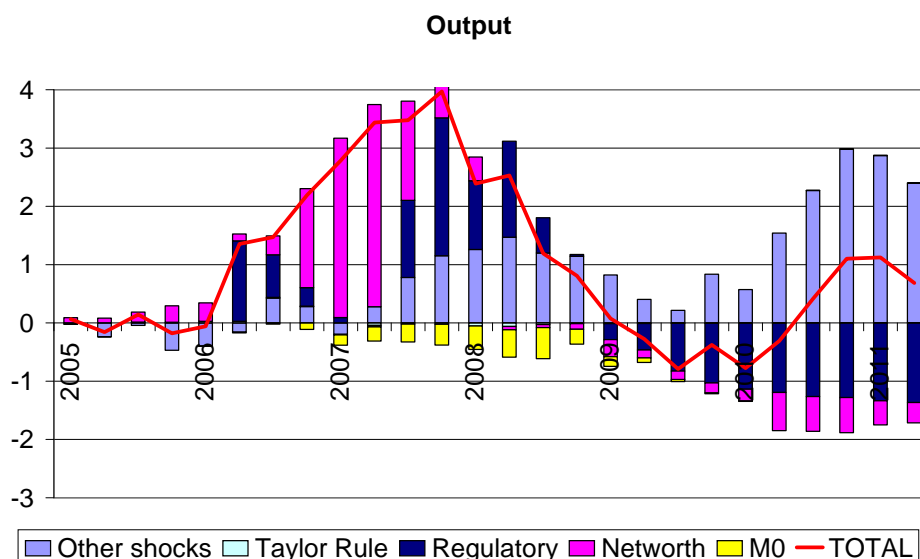


Figure 1: Shock decomposition for output for the period 2005-2011

In these charts we isolate ‘monetary factors’ in the great recession and in particular

- two factors that were operating through banking transmission: net worth and the regulative shock to the credit premium,
- two factors that were due purely to monetary policy: the Taylor Rule and M0 and finally
- all other factors (the Rest) which can be considered the normal causes of the ‘business cycle’.

Output (Figure 1) is dominated by net worth (creating bank leverage), regulatory shocks to banks (deregulation before, followed by sharp tightening), and general business cycle shocks. Monetary policy proper plays little part, by contrast with the standard model.

Interest rates (Figure 2) until the zero bound hits are dominated by the Taylor Rule and the general business cycle, with smaller contributions from regulative and net worth shocks. The decomposition is telling us that in the upswing before the crisis the credit boom was little restrained by the rise in interest rates.

Inflation (Figure 3) is dominated by business cycle shocks. During this period inflation did not fluctuate much except in response for raw material price shocks. The Fed’s implicit inflation target appears to have anchored domestically generated inflation with great solidity.

The overall picture is of an economy enjoying a strong banking boom until the crisis, at which point the same forces driving the boom went into reverse.

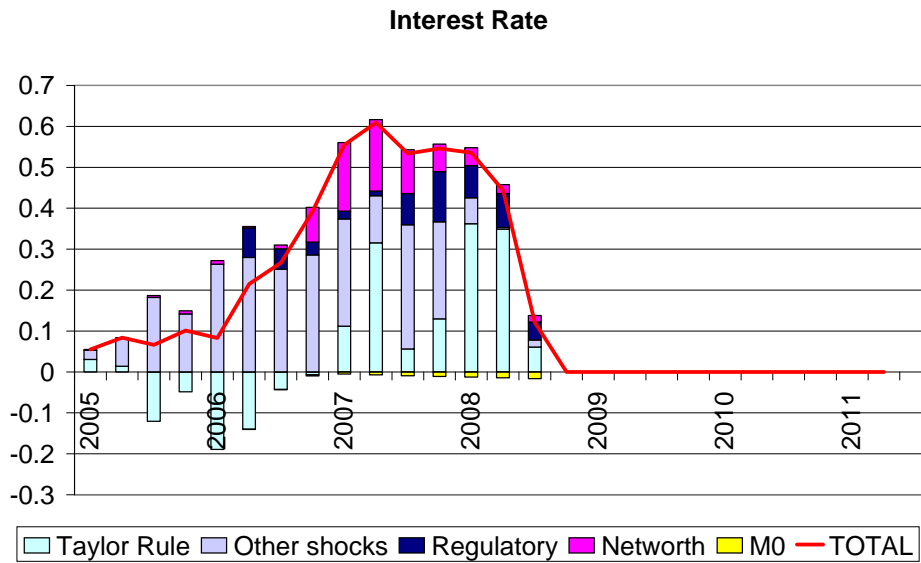


Figure 2: Shock decomposition for interest rate for the period 2005-2011

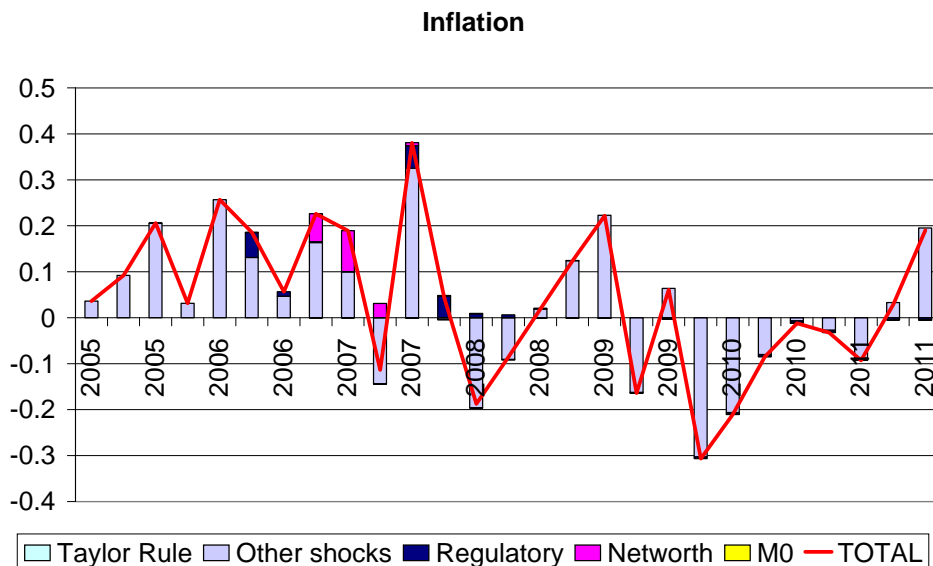


Figure 3: Shock decomposition for inflation for the period 2000-2011

4.1 The causes of crisis

We can ask a further question: what is the nature of a crisis and what causes it, according to our analysis of this sample? We define a ‘crisis’ as a severe interruption in output growth for at least three years and a financial crisis as a crisis in which there is also a binding zero lower bound on the nominal interest rate. First, we examine this question by inspecting the bootstrap experience (potential scenarios over the period) from the model and its shocks drawn from the period 1984–2007, during which massive financial shocks are absent — we call these ‘standard shock scenarios’. Then we look at the bootstrap simulations with the full sample of shocks 1984–2011 — we call these ‘crisis-inclusive shock scenarios’.

We find the following results:

1) Crisis is a normal part of the US economy: Crises are regular results from ‘standard’ shock scenarios. The shocks from the financial crisis period are not necessary conditions for big economic recessions. Figures 4 and 5 show some examples of the bootstrap simulations.

We find that on average crises appear every 52 years. With crisis-inclusive shock scenarios, crises become more frequent — every 48 years. However, qualitatively their behaviour is similar to the standard shock scenarios illustrated below. The inclusion of crisis shocks adds to the shock volatility.

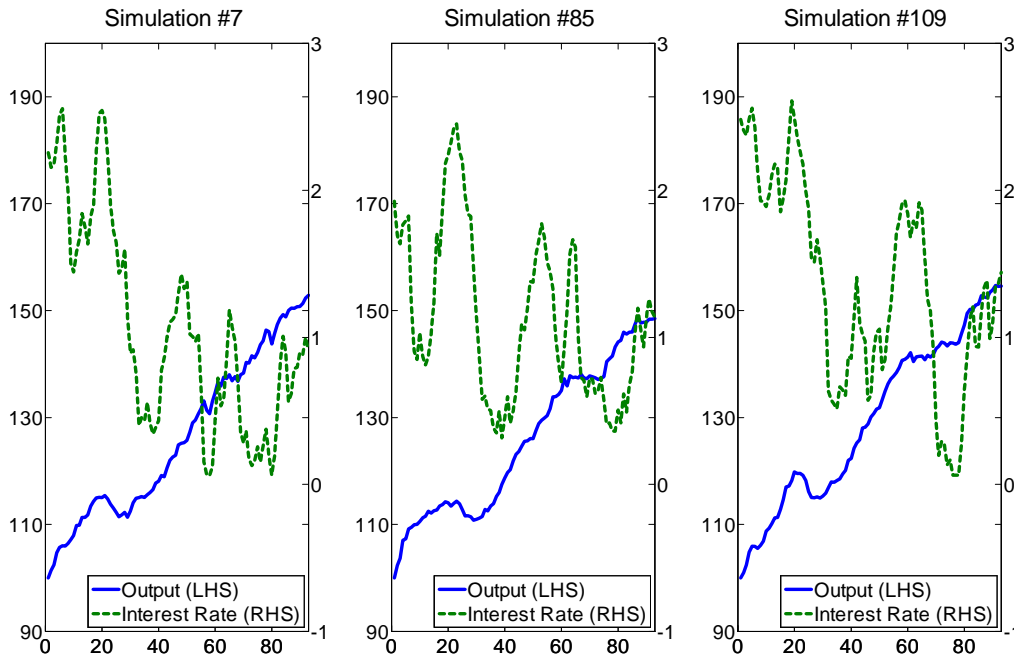


Figure 4: Crises without financial crisis

2) When there is a crisis, about half the time there is also a financial crisis where the nominal interest rate is driven down to the zero lower bound — the charts above show some illustrations of each with standard shocks. This ratio is the same when crisis-inclusive shocks are used.

3) Figure (5) shows that an extreme financial shock is not required to produce a financial crisis. This follows from the fact that we generated financial crises simply using the standard shock scenarios. Including crisis shocks simply adds to the frequency of crises as we have seen.

4) Financial shocks on their own are not sufficient to produce crisis. We created bootstrap scenarios using only financial shocks but including the crisis. These scenarios did not produce any economic crisis.

What we see here is rather in line with the conclusions of Stock and Watson (2012) from a VAR factor analysis of the period: that the financial crisis period did not have a bigger share of financial shocks generating it than previous US business cycles. The shocks were just generally bigger. Thus we find the Great Recession was a crisis and a financial crisis: but its triggers were the usual mixture of real and financial shocks.

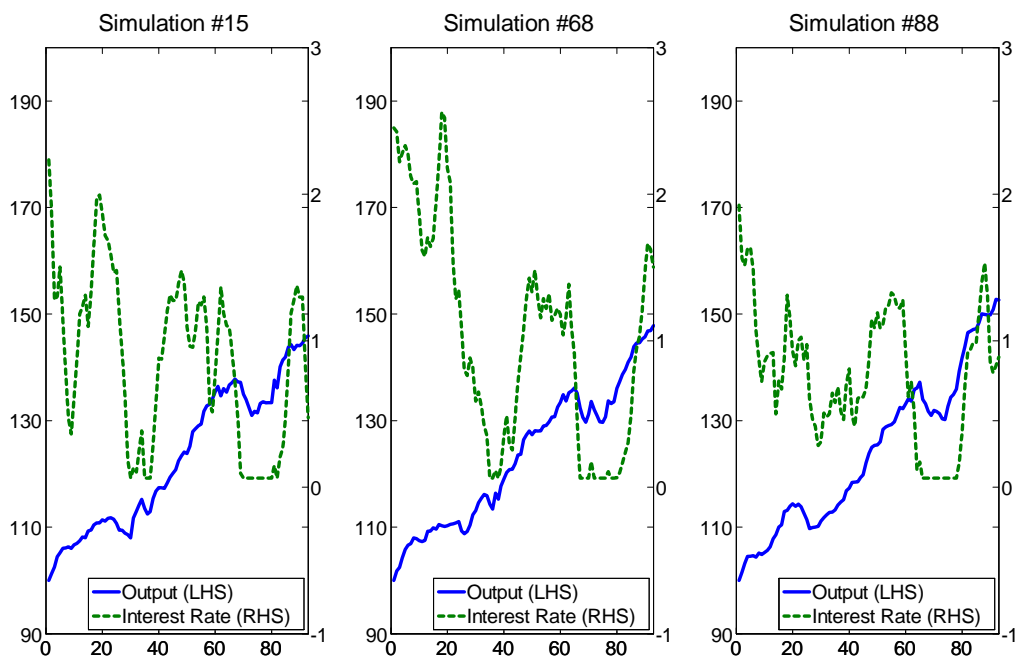


Figure 5: Crises with financial crisis

5 Implications of the model for policy

We begin with fiscal policy which many have argued holds the key to responding to crisis conditions. We interpret it here as a government spending shock. Then we look at monetary policy and regulative policy. Finally we consider possible changes to the monetary regime, which we divide into the initiation of a separate rule for active M0 setting via open market operations, side by side with an interest rate rule — we call this ‘Monetary Reform’; and modifications of the interest rate rule where we consider Price Level Targeting (PLT) and Nominal GDP Targeting (NGDPT).

5.1 Fiscal Policy (Responses to Government Spending Shock)

Figure 6 shows the fiscal multiplier effects in the standard non-crisis context where the Taylor Rule is operating. We see the usual effects: a rise in inflation and interest rates (via the Taylor Rule) which reduce (‘crowd out’) investment, while consumption rises very slightly on the prospect of higher future output/income. Real wages rise with higher output and labour hours rise. Basically the rise in net output is equal to the rise in government spending (other demand effects are very small) and is made possible with a roughly constant capital stock by rising labour supply, through higher wages, and by higher capital consumption (with rising capacity utilisation) which adds to gross GDP. With rising entrepreneurial net worth, the credit premium falls and credit rises. The resulting impact fiscal multiplier is 1.3; beyond the output directly produced to satisfy higher government demand the rise in capital consumption adds to gross GDP, which raises the multiplier from 1.0 (on net GDP) to 1.3 (on gross GDP).⁹ There is no difference between the multiplier before the crisis and that with the zero bound. Consumption rises very slightly more under the zero bound but the key impact in raising their consumption at all is the substitution effect of higher wages and labour supply which occur in order to permit the extra government spending to be converted into output; this is the same across both regimes.

⁹In recent empirical work a large multiplier has been found for tax cuts — eg Romer and Romer (2012). However, these effects may well be the result of ‘supply-side’ effects of tax structure or even effects of inter-temporal income-shifting rather than the ‘Keynesian multiplier’ effects we are discussing here. This model is not equipped to deal with those other effects.

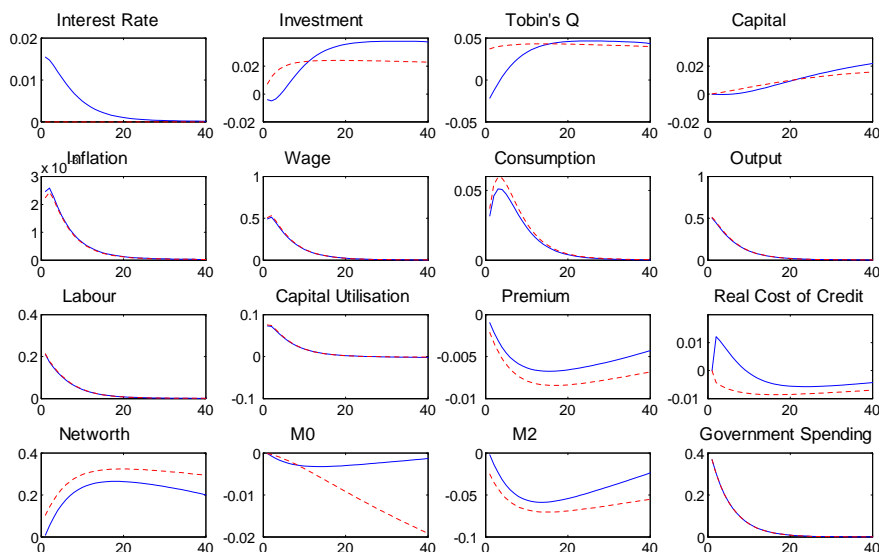


Figure 6: IRFs for a government spending shock to the non-crisis (solid) and crisis model (dashed)

5.2 Monetary Policy

5.2.1 Responses to Taylor Rule Shock.

Figure 7 shows the effects of an increase in the nominal interest rate. This is a quite standard picture, not dissimilar from most New Keynesian models. The real interest rate rises, discouraging both consumption and investment. This reduces output, hours and real wages. It also lowers entrepreneurs' net worth, so raising the risk premium which contributes to the further drop in investment; with lower net worth there is distress borrowing from banks which leads to an accommodating rise in M0.

5.2.2 Responses to Quantitative Easing Shock

Figure 8 shows the effects of a rise in M0 under normal and crisis regimes. Under the normal regime the effect is to lower the credit premium and set off an investment boom. This stimulates more output, higher real wages/labour supply and more consumption. This in turn generates inflation and a response from the Taylor Rule raising real interest rates and so limiting the rise in consumption. M0 accommodates the rise in credit.

Under the crisis regime, there is also a fall in the risk premium and investment rises. But the Taylor Rule response is cut out and this eliminates one offsetting mechanism to the stimulus. Instead, M0 gradually reverses the stimulus in response to the falling risk premium.

Hence a stimulus to M0 has positive effects on output with or without the Taylor Rule, revealing that there is more to monetary policy than the setting of the Fed funds rate: direct open market operations are effective in this model in their own right.

5.3 Regulative Policy

5.3.1 Responses to credit premium shock (Macro-prudential policy)

More regulations, such as Basel 2 and 3, raise the credit premium. The effects of imposing more regulations on banking businesses are approximated here by a credit premium shock. Figure 9 shows that a higher credit premium under either crisis or non-crisis regimes results in lower output, consumption, investment, net worth, capital, working hours and real wages. When the Taylor Rule is operating it offsets the impact of regulative tightening; when the economy is at the zero bound, M0 expansion offsets it somewhat more aggressively under the baseline model.

It is clear from these impulse responses that macro-prudential policy is feasible in principle. However we do not pursue it here as a potential reform because for it to operate as a stabilisation tool regulative

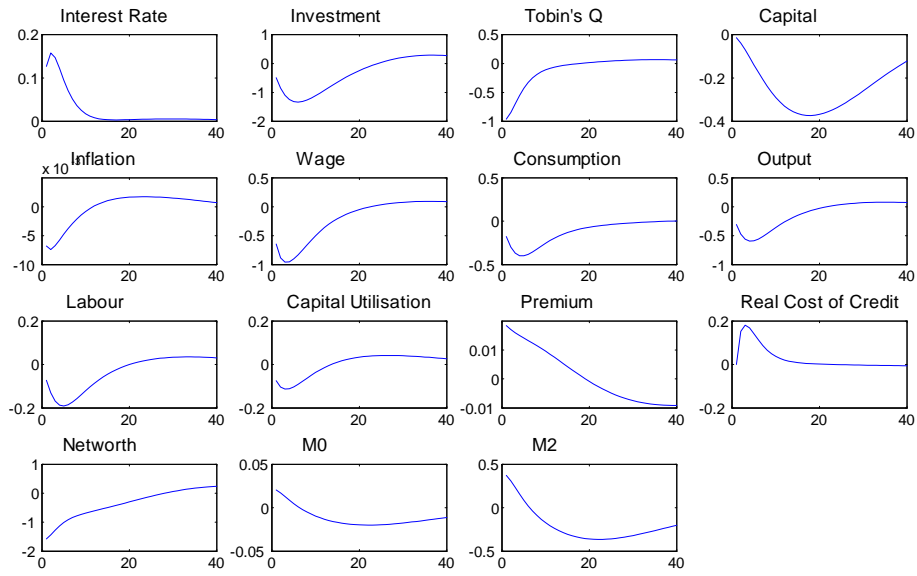


Figure 7: IRFs for a Taylor Rule shock to the non-crisis model

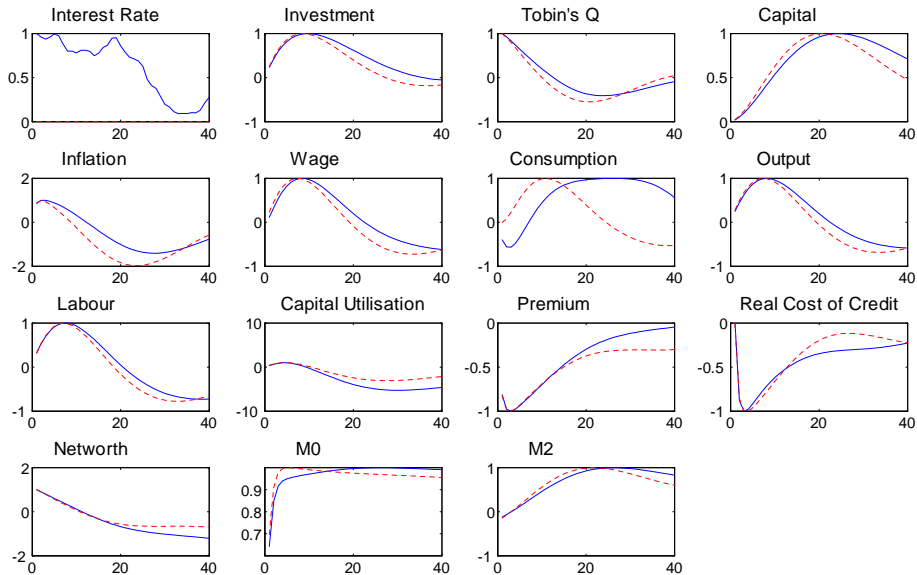


Figure 8: IRFs for a M0 shock to the non-crisis (solid) and crisis model (dashed)

policy needs to be set at a distortional level, so that its tightness can be both raised and lowered. The model does not readily supply a method for calculating its cost other than through the temporary effects (on investment and output) shown in these responses. The distortional cost comes through its long-term effects on productivity through inhibiting intermediation. However, we will see in what follows that there are alternative monetary regimes that can reduce instability; if so there is no case for using this distortional regime for this purpose.

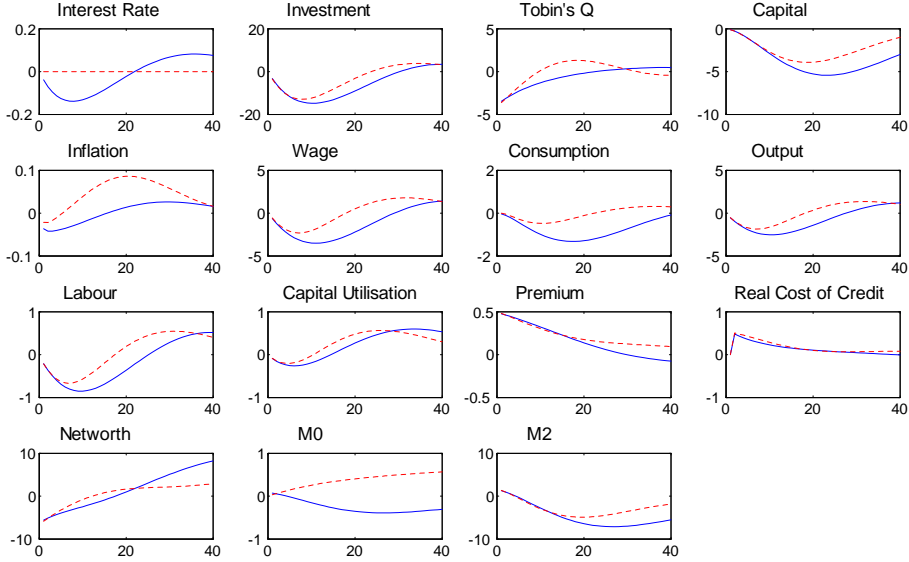


Figure 9: IRFs for a premium shock to the non-crisis (solid) and crisis model (dashed)

5.4 Changes in the monetary regime

The Great Recession showed that an economy with inflation targeting alone struggled to cope with big shocks to the economy and might even contribute to financial instability (Beckworth, 2014) and welfare losses because monetary policy was too tight (and may have been too loose in the boom that led up to it). As a result of cutting nominal interest rates dramatically in response to big economic shocks, many OECD countries are in the zero lower bound situation. New Keynesian economists have argued that this is a liquidity trap situation, but the model shows that there is a strong impact of quantitative easing under the zero bound. In this section, we discuss some possible changes to regulative and monetary regimes that could improve economic stability, compared with the baseline regime (embedded in the model) of inflation targeting, minimal regulation and a fairly weak M0 response to the credit premium.

Our main focus with these alternative regimes is their capacity to reduce the number of crises. We also look at the more conventional measure of business cycle welfare cost, as measured by the weighted sum of the variances (over the business cycle) of consumption and hours of work; we also look at a widely used alternative measure, the weighted sum of the cycle variances of output and inflation. These measures abstract from any changes in deterministic or stochastic trends, on the grounds that monetary policy cannot affect these trends. Here we follow the usual practice of proxying these trends with an HP filter (applied to each variable in each simulation).

5.4.1 Monetary reform

One of the features of the run-up to the Great Recession was a substantial expansion of money and credit, permitted by the inflation targeting regime. This came about because inflation did not respond much to this expansion, anchored as it was by expectations that the inflation target would be effective. Yet since monetary expansion has a stimulative effect on the economy, supplementing the interest rate rule with a money supply rule could be helpful to stability. We now investigate how a monetary reform regime of this type might work; we assume it is implemented in both non-crisis and crisis times. Here we

supplement the Taylor Rule with a powerful M0 rule responding to the credit premium with a response much larger than in the baseline model. Figure 10 shows IRFs of model's variables to a credit premium shock with (dashed line) and without (thick line) the counteractive M0 rule. It can be seen that it stabilises investment and the credit premium.

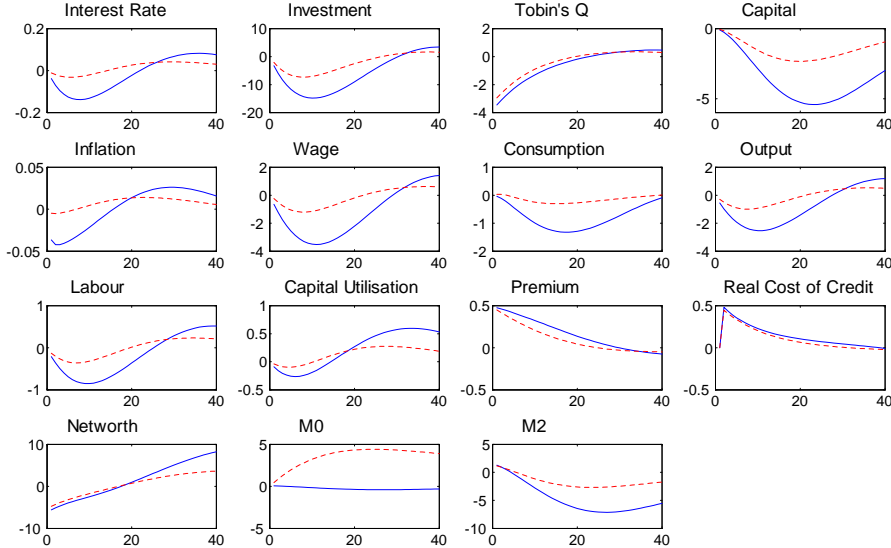


Figure 10: IRFs from the credit premium shock under noncrisis model with monetary reform (dashed) and non crisis model without monetary reform (solid)

The optimal monetary reform takes the following form

$$\Delta m_t = 0.106 (s_t - s^*)$$

To measure its effect in stabilising the economy we perform a large number of bootstrap simulations over the sample period and compute the average frequency of crisis as defined above, namely a drop of at least 3% in output where output does not recover to its previous peak for 3 years. This M0 rule brings down the frequency from every 48 years in the baseline case to every 151 years — tripling stability. On the more conventional measures of welfare it reduces them both substantially (see Table 5).

5.4.2 Price-level targeting regime

The zero lower bound situation and the recession associated with it has renewed interest in price level targeting (PLT) as a better alternative monetary policy that can achieve price stability while also reducing the impact of the zero lower bound (Wolman, 2005; Vestin, 2006; Nakov, 2008; and Dib et al, 2008; for a recent survey see Hatcher and Minford, 2014). Under PLT, inflation expectations adjust to stabilise the economy: if an unanticipated shock pushes the price level below the target, people will expect higher than average inflation in the future to bring the price level back to the target. PLT has two advantages over inflation targeting. First, due to the automatic adjustment in inflation expectations, the central bank does not need to move interest rates aggressively in response to shocks (Cover and Pecorino, 2005), thus it reduces the likelihood of hitting the zero bound. Second, PLT can generate positive inflation expectations in a deflationary situation, lowering real interest rates even at the zero bound and so strengthen recovery.

The PLT rule is specified as follows:

$$r_t = \rho_1 r_{t-1} + (1 - \rho_1) \{ \rho_\pi (p_t - \bar{p}) + \rho_y (y_t - y^*) + \rho_{\Delta y} [(y_t - y^*) - (y_{t-1} - y^*)] \} + er_t$$

Under the zero inflation steady state, the steady price level is assumed constant here and normalised as $\bar{p} = 0$.

We are looking for an optimal PLT specification that provides the least frequency of crisis under our bootstrap simulations. The following PLT

$$r_t = 0.545r_{t-1} + (1 - 0.545) \{1.745p_t + 0.021(y_t - y^*) + 0.026(y_t - y_{t-1})\} + er_t$$

reduces crisis frequency to 464 years from the baseline 48. On the conventional measures it also improves matters on a similar scale to monetary reform. Again it raises the variance of inflation, as intended (Table 5).

5.4.3 Nominal GDP targeting

A group known as Market Monetarists who run a widely-accessed blog on monetary policy, have been calling for monetary policy to target the level of nominal GDP (NGDP), rather than either a monetary aggregate or inflation (Sumner 2011, Nunes and Cole 2013). A similar proposal was made some time ago in a series of papers by McCallum (1988) and McCallum and Nelson (1999) who suggested a rule setting interest rates in response to deviations of nominal GDP growth from a target rate. McCallum argued that this rule would be superior to monetary targeting because of the large and unpredictable changes in payments technology and financial regulations. Compared with the later Taylor Rule McCallum’s rule has interest rates responding as strongly to output growth deviations as to inflation deviations. However, Market Monetarists argue for targeting the level of NGDP rather than its growth rate; the reasons are similar to those for PLT, except that in this case a expected future interest rate stimulus is triggered also by output falling below its trend (McCallum, 2011). A concern about this is that with a stochastic productivity trend monetary policy would be affected by permanent shifts in productivity; thus the NGDP rule we use here allows for changes in the model’s productivity trend — since this is hard for the central bank to estimate, the results for the NGDP rule shown here are ‘best case’. Nevertheless, if this best case can be assumed, the NGDP rule generates expectations of very strong monetary responses in conditions of prolonged recession — analogous to Roosevelt’s 1930s abandonment of the Gold Standard (Krugman, 2011; Carney, 2012 and Woodford, 2012).

Implementing the NGDP target, the central bank would specify an intermediate target for the official interest rate. The rule might be written as follows:

$$r_t = \rho_1 r_{t-1} + \rho_y (y_t + p_t - \bar{y}_t - \bar{p}) + er_t$$

where $\bar{y} + \bar{p}$ is the target for NGDP, where $\bar{p} = 0$ and \bar{y}_t follows the trend path in real output generated by productivity.

Given this general rule, we bootstrap our model and implied shocks to see whether implementing the NGDP targeting regime could help to stabilise the economy. We found that the rule in of form of

$$r_t = 0.859r_{t-1} + 1.827(p_t + y_t - \bar{y}_t) + er_t$$

reduces further the frequency of crisis to 541 years, from PLT’s 464. Measured by the conventional welfare measures NGDPT does better than PLT if one uses the one with output variance and worse if one uses the one with consumption variance (Table 5).

5.4.4 Combining PLT or NGDPT with monetary reform

We also consider the alternative monetary regimes where the central bank combines price level targeting or nominal GDP targeting with the M0 rule. Table 5 brings all these results together. It shows that crises are reduced further under either combination; of the two the PLT+Monetary-Reform combination outperforms the NGDPT combination on the standard welfare cost measure if one uses the measure with consumption variance and underperforms it on the measure with output variance.

There is no unambiguous winner from these suggested regimes. Both the PLT and NGDPT combinations offer a big rise in stability with a slight rise in inflation variance (the standard deviation of the annualised inflation rate rises by 0.2%). If one wishes to minimise inflation variance the best way to increase stability is the straight NGDPT regime on its own; but with NGDPT on its own or in combination the central bank needs to assess the productivity trend correctly, which may cause difficulties. In the last row we show the frequency of zero-bound episodes; this is not so much a measure of stability for the economy, as for the policy regime. The regime that minimises zero bound episodes is again NGDPT, with or without monetary reform; the frequency then comes down to one episode every 70 years.

We have shown below charts of several crisis episodes, to illustrate how these different regimes work to increase stability. As one can see, in all cases the new regimes inject stabilising action when the

economy surges or collapses. Also it is quite apparent (as it is from Table 5 of stability measures) that all the regimes operate in a broadly similar way, with similar quantitative effects. The reason they are so similar is that all of them are ‘integral control’ mechanisms; that is to say, they all react to a measure of the level of the economy’s state relative to its target level and their reaction in all cases is to move yields (either the government interest rate or the private credit rate or both) until the state gets close to target.

	Base case	Monetary Reform	PLT	NGDPT	PLT+ Mon. Reform	NGDPT+ Mon. Reform
Frequency of crisis (expected crises per 1000 years)	20.8	6.62	2.15	1.83	1.41	1.31
Exp welfare cost (1)*	1.43	0.65	0.662	0.657	0.646	0.663
<i>var</i> (cons)	0.498	0.151	0.121	0.121	0.117	0.124
<i>var</i> (hours)	0.932	0.499	0.541	0.536	0.529	0.539
Exp welfare cost (2)*	2.652	0.563	0.576	0.558	0.564	0.526
<i>var</i> (output)	2.619	0.544	0.522	0.54	0.512	0.474
<i>var</i> (inflation)	0.033	0.019	0.054	0.018	0.054	0.052
Frequency of zero bound (per 1000 years)	34.8	20.2	21.2	14.1	20.7	13.9

*equal weights for each variance

Table 5: Frequency of crisis and stability under different monetary regimes

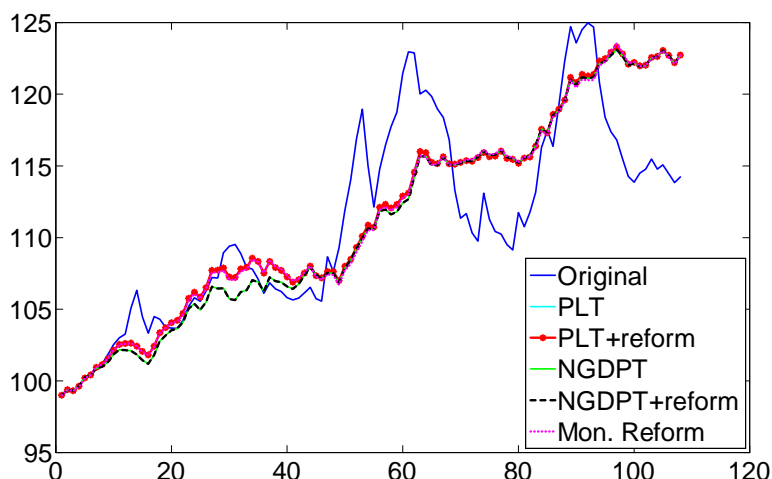


Figure 11: Simulated output under different rules (example 1)

6 Conclusion

In this paper we attempt to remedy the lack of a role for money in prevailing DSGE models. The route we choose is to give money a role in providing cheap collateral in a model of banking; this means that monetary policy can affect the risk-premium on bank lending to firms by varying the supply of M0 in open market operations. This effect comes on top of the usual rate-setting of the safe interest rate done via the Taylor Rule in normal times. But it implies that even when the zero bound prevails monetary policy is still effective; it also implies that fiscal policy, which has a modest multiplier effect in normal times, does not enjoy a ‘free lunch’ in crisis times due to the zero bound, as it still crowds out investment via the risk-premium which remains unbounded from below.

Though this model is based on standard DSGE foundations, its message is similar to that of models promulgated by ‘monetarists’ such as Friedman, Brunner and Meltzer: that the quantity of money

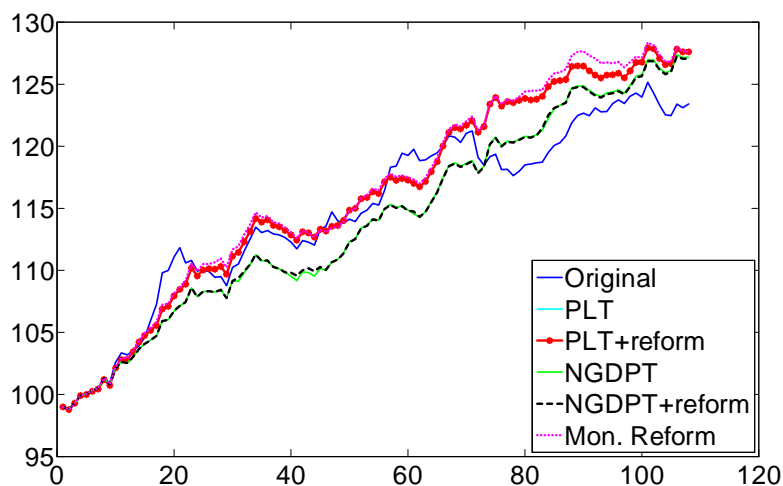


Figure 12: Simulated output under different rules (example 2)

matters and cannot be ignored in favour solely of an interest-rate-setting rule. For the ECB it justifies the existence of a ‘second pillar’. For central banks generally, it suggests that money supply growth needs to be controlled, over and above the setting of the safe interest rate for inflation targeting reasons, in order to maintain control of credit and avoid credit booms and busts. We suggest a simple rule for making M0 respond to credit conditions that can substantially enhance the economy’s stability. Both price-level and nominal GDP targeting rules for interest rates would combine with this to stabilise the economy further; of the two PLT offers the safest route as it is not dependent on assessing the productivity trend and it also achieves a similar scale of gain. If these rules for monetary control were instituted, they would make aggressive regulation of banks’ balance sheets, such as has recently been promoted, redundant; since such regulation involves raising the credit premium in a distortionary way, avoiding it in favour of monetary control would be welfare-enhancing.

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7 Appendix 1: Augmenting the BGG model for collateral and money

The assumptions added to BGG are that the banks demand collateral as a proportion of net worth of c ; and that liquidating this collateral costs δ per unit of collateral. The BGG model consists of three parts:

- a) a bankruptcy threshold at which firms will choose to default
- b) banks' zero profit condition (free entry drives profits to zero) — this condition gives us the banks' leverage offer curve.

c) firms' maximisation of utility subject to a) and b); this gives us the overall contract.

Now we look at each part in detail:

a) the bankruptcy threshold ($\bar{\omega}$; ω is the return obtained per unit of assets, distributed as a random variable with a mean of unity): this is such that the firm is indifferent between defaulting and staying in business. If it goes bankrupt, it loses $(1 + R^K)\bar{\omega}A + cN$ and it gains ZB . Here $Z=1+\text{credit rate}$ and $B=\text{bank borrowing}$; A is total investment, $R^K = \text{the firms' return on investment}$, and $N=\text{net worth of the firm}$. Thus at the threshold $ZB = (1 + R^K)\bar{\omega}A + cN$. Note also that the firms' balance sheet is $B = A - N + cN$; thus when this condition holds: $(1 + R^K)\bar{\omega}A + cN = Z(A - N + cN)$. Let $L = A/N = \text{Leverage}$. Divide the condition by N and obtain: $Z = \frac{(1+R^K)\bar{\omega}L+c}{L-1+c}$

b) banks' zero profit condition is given by

$$[1 - F(\bar{\omega})]ZB + (1 - \mu)G(\bar{\omega})(1 + R^K)A + cNF(\bar{\omega})(1 - \delta) = (1 + R)B$$

On the left hand side the first term is the probability of obtaining the loan proceeds (ZB), where $F(\bar{\omega})$ is the probability of going bankrupt. In the second term $G(\bar{\omega})$ is the expected value of the returns per unit asset to be made if the firm goes bankrupt times the probability of bankruptcy; this is reduced by the cost of collection, μ . Finally, there is the recovery of collateral in the event of bankruptcy minus its liquidation cost δ . On the right hand side is the cost of the funds the bank has received from depositors at the riskless rate, R .

Substitute from the bankruptcy threshold $ZB = (1 + R^K)\bar{\omega}A + cN$ in the first term of the LHS and on the RHS for B from firms' balance sheet $B = A - N + cN$. This gives:

$$[1 - F(\bar{\omega})](1 + R^K)\bar{\omega}A + (1 - \mu)G(\bar{\omega})(1 + R^K)A + cN(1 - \delta F(\bar{\omega})) = (1 + R)(A - N + cN)$$

Let $\Gamma(\bar{\omega}) = [1 - F(\bar{\omega})]\bar{\omega} + G(\bar{\omega})$. Divide by N to obtain:

$$[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + R^K)L = (1 + R)(L - 1) + c(1 + R - 1 + \delta F(\bar{\omega})) \text{ so that we obtain:}$$

$$L = \frac{1+R-c[R+\delta F(\bar{\omega})]}{1+R-\Psi(\bar{\omega})(1+R^K)} \text{ where } \Psi(\bar{\omega}) = \Gamma(\bar{\omega}) - \mu G(\bar{\omega})$$

This is the banks' leverage offer curve. It can be readily verified that it slopes upward and is convex in $[1 - \Gamma(\bar{\omega})]$ space — as shown in the diagram below.

Note that $dL/d\bar{\omega} > 0$, $dL/dR^K > 0$, $dL/d\delta < 0$

c) To obtain the overall contract firms' utility (returns), relative to their cost of funds, are maximised. These are given by:

$$\int_{\bar{\omega}}^{\infty} \frac{\{(1+R^K)\omega A + cN - ZB\}dF(\omega)}{N(1+R)}; \text{ now also note that from the bankruptcy threshold } ZB = (1 + R^K)\bar{\omega}A +$$

cN . So it can be seen that the firms' returns are unaffected by the existence of collateral, essentially because it remains as part of their gross return if they do not go bankrupt but also, for given total assets A , the borrowing costs at which they will choose to go bankrupt rise by the amount of this collateral.

Substituting into the returns from the bankruptcy threshold gives the overall returns as:

$$\int_{\bar{\omega}}^{\infty} \frac{\{(1+R^K)(\omega - \bar{\omega})A\}dF(\omega)}{N(1+R)} = \frac{(1+R^K)}{(1+R)}L[1 - \Gamma(\bar{\omega})] \text{ where the first two terms give the total expected return}$$

to the firm from its invested capital (A) as a proportion of its funds, $N(1+R)$ and the last term $[1 - \Gamma(\bar{\omega})]$

is the share of this that goes to the firm (the bank takes the loan costs if the firm survives and the returns below $\bar{\omega}$ if it does not).

This utility function gives indifference curves in $(\bar{\omega}, L)$ space, that are concave. An interior optimum is reached.

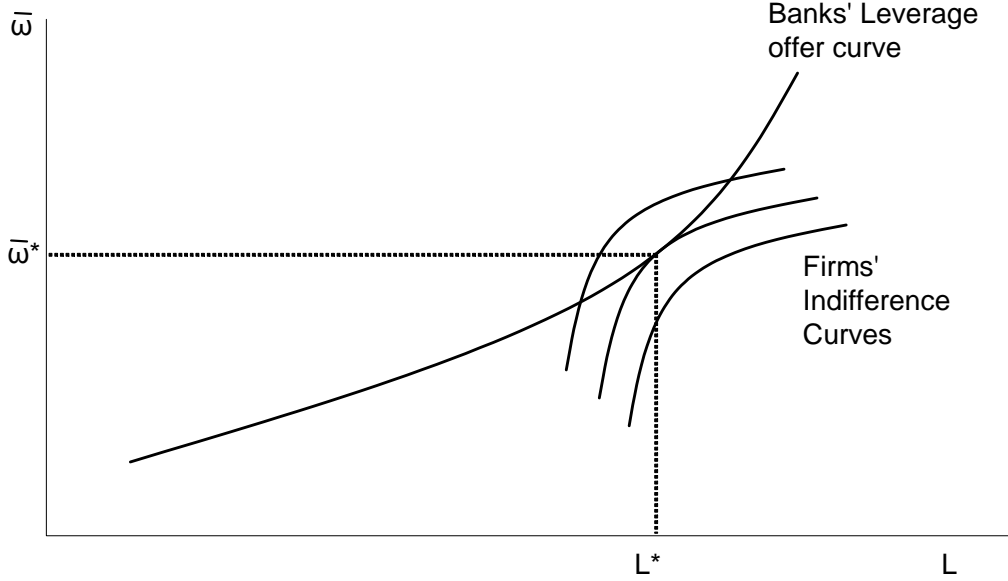


Figure 13: The optimum contract for $(\bar{\omega}^*, L^*)$ for given R^k, R, δ

We can compute this optimum by maximising $\frac{(1+R^K)}{(1+R)}L[1-\Gamma(\bar{\omega})]$ wrt $(\bar{\omega}, L)$ subject to the leverage offer curve from the banks $L = \frac{1+R-c[R+\delta F(\bar{\omega})]}{1+R-\Psi(\bar{\omega})(1+R^K)}$ (from b above). Solving for the implicit function this gives in $\bar{\omega}$ gives us finally the firm's optimum choice of $\bar{\omega}$ as the solution of:

$$\{1+R-c[R+\delta F(\bar{\omega})]\}\{1+R-\Phi'(1+R^K)\} = \left\{\frac{-c\delta F'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})}\right\}\{1+R-\Psi(\bar{\omega})(1+R^K)\}$$

$$\text{where } \Phi' = \frac{\Psi'(\bar{\omega})}{\Gamma'(\bar{\omega})} + \left(1 - \frac{\Psi'(\bar{\omega})}{\Gamma'(\bar{\omega})}\right)\Psi(\bar{\omega}) \approx 1$$

In addition we have the leverage offer curve defining L in terms of $\bar{\omega}$ and so giving us the total $(\bar{\omega}, L)$ solution.

We can now create two equations in $(\bar{\omega}, L)$ from the firm's optimum and the banks' leverage offer. We can rewrite the firm's optimum choice using the banks' leverage offer as:

$$1) L\{1+R-\Phi'(1+R^K)\} = \left\{\frac{-c\delta F'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})}\right\}$$

and then we can add the banks' leverage offer:

$$2) L = \frac{1+R-c[R+\delta F(\bar{\omega})]}{1+R-\Psi(\bar{\omega})(1+R^K)}$$

We now investigate the comparative static properties of changes around the equilibrium by taking the total differential of this two-equation system in $dL, d\bar{\omega}, d\delta$ and dR^K . We will evaluate the derivatives at an equilibrium where $\delta = 0$; we do this for convenience because we will be dealing with a heavily monetised collateral set-up where it is close to zero. Note that in the rest of the DSGE model $\ln L_t = k_t - n_t$ is determined while δ is determined by the provision of M0 as an alternative to illiquid collateral. Thus we can regard these as exogenous to this banking model subsector which then solves for $\bar{\omega}$ and R^K (the return on capital required to make the needed leverage possible). These two elements are internal to the bank contract decision and unobservable in the public domain but in turn from these we can solve for the observable cost of the bank credit, Z , from the bankruptcy threshold as $Z = \frac{((1+R^K)\bar{\omega}L+c)}{L-1+c}$.

We write the total differential as:

$$1) \{1+R-\Phi'(1+R^K)\}dL + L(-\Phi')dR^K = (\text{derivative} = 0)d\bar{\omega} + \left\{\frac{-cF'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})}\right\}d\delta$$

and

$$2) dL = L\left\{\frac{\Psi'(\bar{\omega})(1+R^K)}{1+R-\Psi(\bar{\omega})(1+R^K)}\right\}d\bar{\omega} + L\left\{\frac{\Psi(\bar{\omega})}{1+R-\Psi(\bar{\omega})(1+R^K)}\right\}dR^K + \left\{\frac{-cF(\bar{\omega})}{1+R-\Psi(\bar{\omega})(1+R^K)}\right\}d\delta$$

Our interest lies in the effect of δ on the equilibrium values of R^K and \bar{w} , and thus on Z . We begin by noting from 1) that

$$\frac{dR^K}{d\delta} = \left\{ \frac{cF'(\bar{w})[1-\Gamma(\bar{w})]}{L\Phi'\Gamma'(\bar{w})} \right\} = \left\{ \frac{cF'(\bar{w})[1-\Gamma(\bar{w})]}{L\Phi'[1-F(\bar{w})]} \right\} > 0$$

and from 2) that:

$$\begin{aligned} \frac{d\bar{w}}{d\delta} &= \frac{d\bar{w}}{dR^K} \cdot \frac{dR^K}{d\delta} + \frac{d\bar{w}}{d\delta} = \left\{ \frac{-\Psi(\bar{w})}{\Psi'(\bar{w})(1+R^K)} \right\} \left\{ \frac{cF'(\bar{w})[1-\Gamma(\bar{w})]}{L\Phi'\Gamma'(\bar{w})} \right\} + \frac{cF(\bar{w})}{L\Psi'(\bar{w})(1+R^K)} = \frac{cF(\bar{w})}{L\Psi'(\bar{w})(1+R^K)} \left\{ 1 - \frac{F'(\bar{w})}{F(\bar{w})} \frac{\Psi(\bar{w})[1-\Gamma(\bar{w})]}{\Phi'\Gamma'(\bar{w})} \right\} \\ &= \frac{cF(\bar{w})}{L\Psi'(\bar{w})(1+R^K)} \left\{ 1 - \frac{F'(\bar{w})}{F(\bar{w})} \frac{\Psi(\bar{w})[1-\Gamma(\bar{w})]}{\Phi'[1-F(\bar{w})]} \right\} \text{ since we note that } \Gamma'(\bar{w}) = [1-F(\bar{w})]. \end{aligned}$$

The sign of the last total derivative is strictly ambiguous and needs to be computed numerically. Consider a bankruptcy rate around 2.3% and a standard normal distribution of $\ln w$ (i.e. with a standard deviation of unity, so that the bankruptcy threshold will be exactly two standard deviations below the mean). \bar{w} will then take the value of 0.135 ($=e^{-2}$); $F(\bar{w}) = 0.023$; $\frac{F'(\bar{w})}{F(\bar{w})} = 2.3$; $\Psi(\bar{w}) \simeq \Gamma(\bar{w}) = [1-F(\bar{w})]\bar{w} = 0.13x0.977 = 0.127$ since $G(\bar{w}) \simeq 0$; it also follows as noted above that $\Phi' = \frac{\Psi'(\bar{w})}{\Gamma'(\bar{w})}$ thus $\left\{ 1 - \frac{F'(\bar{w})}{F(\bar{w})} \frac{\Psi(\bar{w})[1-\Gamma(\bar{w})]}{\Phi'[1-F(\bar{w})]} \right\} = 0.73$ and so is clearly positive for any values around that size of bankruptcy rate and standard deviation. The reason essentially is that the banks' share of returns, $\Gamma(\bar{w})$, is under the assumed competitive nature of banks quite modest; and so a rise in the rate of return has only a modest effect on profits while a rise in the bankruptcy threshold has a much larger effect. Hence at zero profits with given leverage the trade-off of threshold given up for extra required rate of return is small.

Finally, we find

$$\frac{dZ}{d\delta} = \frac{L}{L-1+c} \left\{ [1+R^K] \frac{d\bar{w}}{d\delta} + \bar{w} \frac{dR^K}{d\delta} \right\} = \frac{c}{L-1+c} \left\{ \frac{F(\bar{w})}{\Psi'(\bar{w})} \left\{ 1 - \frac{F'(\bar{w})}{F(\bar{w})} \frac{\Psi(\bar{w})[1-\Gamma(\bar{w})]}{\Phi'[1-F(\bar{w})]} \right\} + \left\{ \frac{\bar{w}F'(\bar{w})[1-\Gamma(\bar{w})]}{\Phi'[1-F(\bar{w})]} \right\} \right\} > 0,$$

on the assumption that $\frac{d\bar{w}}{d\delta} > 0$ as above.

Thus finally since δ is reduced by M0 injections we can conclude that a rise in M0 will reduce the required return on capital and also the credit premium.

8 Appendix 2: Model Listing

Consumption Euler equation

$$c_t = \frac{\frac{\lambda}{\gamma}}{1 + \frac{\lambda}{\gamma}} c_{t-1} + \frac{1}{1 + \frac{\lambda}{\gamma}} E_t c_{t+1} + \frac{(\sigma_c - 1) \frac{W_* L_*}{C_*}}{\left(1 + \frac{\lambda}{\gamma}\right) \sigma_c} (l_t - E_t l_{t+1}) - \left(\frac{1 - \frac{\lambda}{\gamma}}{\left(1 + \frac{\lambda}{\gamma}\right) \sigma_c} \right) (r_t - E_t \pi_{t+1}) + e b_t \quad (8)$$

Investment Euler equation

$$inn_t = \frac{1}{1 + \beta\gamma(1-\sigma_c)} inn_{t-1} + \frac{\beta\gamma(1-\sigma_c)}{1 + \beta\gamma(1-\sigma_c)} E_t inn_{t+1} + \frac{1}{(1 + \beta\gamma(1-\sigma_c)) \gamma^2 \varphi} qqt + e inn_t \quad (9)$$

Tobin Q equation

$$qq_t = \frac{1 - \delta}{1 - \delta + R_*^K} E_t qq_{t+1} + \frac{R_*^K}{1 - \delta + R_*^K} E_t r k_{t+1} - E_t c y_{t+1} \quad (10)$$

Capital Accumulation equation

$$k_t = \left(\frac{1 - \delta}{\gamma} \right) k_{t-1} + \left(1 - \frac{1 - \delta}{\gamma} \right) inn_t + \left(1 - \frac{1 - \delta}{\gamma} \right) \left(1 + \beta\gamma(1-\sigma_c) \right) (\gamma^2) (\varphi) (e inn_t) \quad (11)$$

Price Setting equation

$$\begin{aligned} r k_t &= \omega^r \left[\begin{aligned} & \frac{1}{\alpha \left(\frac{1}{1 + \beta\gamma(1-\sigma_c) \iota_P} \right) \left(\frac{(1-\beta\gamma(1-\sigma_c)\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\epsilon_p+1)} \right)} \\ & \left(\pi_t - \frac{\beta\gamma(1-\sigma_c)\iota_P}{1 + \beta\gamma(1-\sigma_c)\iota_P} E_t \pi_{t+1} - \frac{\iota_P}{1 + \beta\gamma(1-\sigma_c)\iota_P} \pi_{t-1} + \left(\frac{1}{1 + \beta\gamma(1-\sigma_c)\iota_P} \right) \right) \\ & \left(\frac{(1-\beta\gamma(1-\sigma_c)\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\epsilon_p+1)} \right) ((1-\alpha) w_t - e a_t) - e p_t \end{aligned} \right] \\ &+ (1 - \omega^r) \left[\frac{e a_t}{\alpha} - \frac{1 - \alpha}{\alpha} w_t \right] \end{aligned} \quad (12)$$

Wage Setting equation

$$w_t = \omega^w \left[\begin{aligned} & \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} E_t w_{t+1} + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} w_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} E_t \pi_{t+1} - \frac{1+\beta\gamma^{(1-\sigma_c)} l_w}{1+\beta\gamma^{(1-\sigma_c)}} \pi_t \\ & + \frac{l_w}{1+\beta\gamma^{(1-\sigma_c)}} \pi_{t-1} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} \left(\frac{(1-\beta\gamma^{(1-\sigma_c)} \xi_w)(1-\xi_w)}{(1+\epsilon_w(\phi_w-1))\xi_w} \right) \\ & \left(w_t - \sigma_l l_t - \left(\frac{1}{1-\frac{\lambda}{\gamma}} \right) \left(c_t - \frac{\lambda}{\gamma} c_{t-1} \right) \right) + e w_t \end{aligned} \right] + (1-\omega^w) \left[\sigma_l l_t + \left(\frac{1}{1-\frac{\lambda}{\gamma}} \right) \left(c_t - \frac{\lambda}{\gamma} c_{t-1} \right) - (\pi_t - E_{t-1} \pi_t) + e w_t^S \right] \quad (13)$$

Labour demand

$$l_t = -w_t + \left(1 + \frac{1-\psi}{\psi} \right) r k_t + k_{t-1} \quad (14)$$

Market Clearing condition in goods market

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i n n_t + R_*^K k_y \frac{1-\psi}{\psi} r k_t + c_y^e c_t^e + e g_t \quad (15)$$

Aggregate Production equation

$$y_t = \phi \left[\alpha \frac{1-\psi}{\psi} r k_t + \alpha k_{t-1} + (1-\alpha) l_t + e a_t \right] \quad (16)$$

Taylor Rule

$$r_t = \rho r_{t-1} + (1-\rho) (r_p \pi_t + r_y y_t) + r_{\Delta y} (y_t - y_{t-1}) + e r_t \text{ for } r_t > 0.0625 \quad (17)$$

Premium

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = s_t = \chi (q q_t + k_t - n_t) - \psi m_t + \xi_t + e p r_t \quad (18)$$

Net worth

$$n_t = \frac{K}{N} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + \theta n_{t-1} + e n w_t \quad (19)$$

Entrepreneurial consumption

$$c_t^e = n_t \quad (20)$$

M0

$$\Delta m_t = \psi_1 \Delta M_t + e r r m_{2t} \text{ for } r_t > 0.0625 \text{ and } \Delta m_t = \psi_2 (s_t - c^*) + e r r m_{2t} \text{ for } r_t \leq 0.0625$$

M2

$$M_t = (1 + \nu - \mu) k_t + \mu m_t - \nu n_t \quad (21)$$

9 Appendix 3: VECM

Following Meenagh et al. (2012), we can say that after log-linearisation a DSGE model can usually be written in the form

$$A(L)y_t = B E_t y_{t+1} + C(L)x_t + D(L)e_t \quad (A1)$$

where y_t are p endogenous variables and x_t are q exogenous variables which we assume are driven by

$$\Delta x_t = a(L)\Delta x_{t-1} + d + c(L)\epsilon_t. \quad (A2)$$

The exogenous variables may contain both observable and unobservable variables such as a technology shock. The disturbances e_t and ϵ_t are both iid variables with zero means. It follows that both y_t and x_t are non-stationary. L denotes the lag operator $z_{t-s} = L^s z_t$ and $A(L)$, $B(L)$ etc. are polynomial functions with roots outside the unit circle.

The general solution of y_t is

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t. \quad (\text{A3})$$

where the polynomial functions have roots outside the unit circle. As y_t and x_t are non-stationary, the solution has the p cointegration relations

$$\begin{aligned} y_t &= [I - G(1)]^{-1}[H(1)x_t + f] \\ &= \Pi x_t + g. \end{aligned} \quad (\text{A4})$$

The long-run solution to the model is

$$\begin{aligned} \bar{y}_t &= \Pi \bar{x}_t + g \\ \bar{x}_t &= [1 - a(1)]^{-1}[dt + c(1)\xi_t] \\ \xi_t &= \sum_{i=0}^{t-1} \epsilon_{t-i} \end{aligned}$$

Hence the long-run solution to x_t , namely, $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$ has a deterministic trend $\bar{x}_t^D = [1 - a(1)]^{-1}dt$ and a stochastic trend $\bar{x}_t^S = [1 - a(1)]^{-1}c(1)\xi_t$.

The solution for y_t can therefore be re-written as the VECM

$$\begin{aligned} \Delta y_t &= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t \\ &= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t \\ \omega_t &= M(L)e_t + N(L)\epsilon_t \end{aligned} \quad (\text{A5})$$

Hence, in general, the disturbance ω_t is a mixed moving average process. This suggests that the VECM can be approximated by the VARX

$$\Delta y_t = K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \quad (\text{A6})$$

where ζ_t is an iid zero-mean process.

As

$$\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$$

the VECM can also be written as

$$\Delta y_t = K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t. \quad (\text{A7})$$

Either equations (A6) or (A7) can act as the auxiliary model. Here we focus on (A7); this distinguishes between the effect of the trend element in x and the temporary deviation from its trend. In our models these two elements have different effects and so should be distinguished in the data to allow the greatest test discrimination.

It is possible to estimate (A7) in one stage by OLS. Meenagh et al. (2012) do Monte Carlo experiments to check this procedure and find it to be extremely accurate.

Our DSGE model here is non-linear since it comprises two regimes between which the economy shifts according to the shocks hitting it. In this respect it is similar to the original models estimated by Smith and others under indirect inference, which was intended by them to overcome the difficulty that the model was non-linear and so could not be estimated by the usual direct inference methods. As they showed, an auxiliary model that is not the reduced form of the true model but simply some linear representation of the data behaviour yields an indirect inference estimator of the DSGE parameters that is still asymptotically normal and consistent.