State-Dependent Pricing and the Paradox of Flexibility[*]

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Abstract

This paper studies the effects of price setting on shock amplification under a passive monetary policy rule (that keeps nominal interest rates constant, e.g. due to their lower bound). Price setting is crucial for the determination of expected inflation and the long-term real interest rate, and thus for shock propagation, e.g. of government spending shocks or monetary forward guidance. This mechanism involves a "paradox of flexibility": increased flexibility of nominal prices, magnifies, rather than reduce, the real effects of shocks. In state-dependent pricing models, shock propagation under a constant nominal interest rate policy turns out to be stronger than in the standard New Keynesian, time-dependent framework. For the same frequency of price adjustment, the government spending multiplier in state-dependent models is much larger under a constant interest rate. This reflects the fact that the degree of price flexibility in these models is not just a function of the frequency of price adjustment, but rather of a "selection effect" due to firm idiosyncratic shocks. The strong propagation under constant interest rates thus holds across price-setting models with different degrees of price flexibility, extending the paradox to a broad concept of price flexibility.

Keywords: firm heterogeneity, state-dependent pricing, constant interest rate

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1 Introduction

In the aftermath of the "Great Recession" of 2008, monetary policy rates were rapidly decreased to historically low levels in **many** advanced countries, and have been mostly kept unchanged since then. This unprecedented situation has spurred a host of studies investigating the implications of such a passive monetary policy stance. Building on the contributions of Krugman (1998) and Eggertsson and Woodford (2003), this literature is **largely** based on the workhorse New Keynesian (NK) model, commonly used in monetary policy analysis. A key result is that a passive monetary policy that keeps the nominal interest rate constant for a period of time, results in a **strong** amplification of shocks. Specifically, several recent papers have argued **that the government spending multiplier is very large** under a passive monetary policy (e.g. Christiano et. al., 2011, Eggertsson, 2011, Woodford, 2011, Werning, 2012). Similarly, announcements ("forward guidance") about the future path of nominal interest rates have very potent effects on current real activity and inflation (e.g. Levin et al. 2010 and Del Negro et al. 2013). These findings **are** especially relevant given that, by the end of 2008, in many countries the short-term nominal interest rate used as the main operating target for monetary policy reached its effective lower bound.[1]

The mechanism underlying the amplification relies on a powerful real interest rate channel. In particular, fiscal stimulus has a large effect on output if private consumption increases with government purchases. In standard models consumption depends inversely on the long-term real interest rate. When the nominal interest rate is constant for some time, the long-term real rate is a function of inflation expectations. To the extent that systematic monetary policy allows a sustained increase in future inflation in response to government purchases, the long-term rate falls and current consumption rises. Aggregate activity thus increases in excess of government spending.

Importantly, the above mechanism involves a "paradox of flexibility": increased flexibility of nominal prices and wages, and thus a larger response of expected inflation, magnifies, rather than reduce, the real effects of shocks. **The more flexible prices are, and thus the steeper the New Keynesian Phillips curve**, the more future inflation **is** expected to increase, **resulting** in a larger spending

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[1] The literature on the implications of the zero lower bound is large and increasing. See among others Braun et al. (2012), Christiano and Eichenbaum (2012), Fernandez-Villaverde et al. (2014), Kiley (2013), Mertens and Ravn (2014), and Wieland (2014).

multiplier.

Several recent contributions, **however,** have argued that such a powerful transmission mechanism is questionable (e.g. Ohanian 2011, Cochrane 2015) and that the "paradox of flexibility" points to problematic aspects of the NK model. At the center of much of the criticism of the standard NK model is its assumption of exogenous timing of price changes *a la* Calvo (1983). W**e know that** the specifics of price setting at the micro level is essential for the dynamics of the aggregate price level and output. For instance, compared to time-dependent pricing, state-dependent pricing in a fixed menu cost model produces very different inflation and output responses to a monetary policy shock (Golosov and Lucas, 2007; Alvarez, Le Bihan and Lippi 2014). Yet, to the best of our knowledge, a study of the fiscal multiplier and of shock amplification under a constant interest rate policy, is missing in the context of state-dependent pricing models. [[2]]

This paper shows that simply **allowing pricing by firms to be state-dependent does not by itself** resolve the puzzle of amplification. To the contrary, state-dependent pricing results in an even stronger propagation of shocks compared to Calvo pricing.[3] **In particular, w**e examine shock amplification under constant interest rates within a framework of "smoothly state dependent pricing" by firms, as in Costain and Nakov (2011b). Pricing in this model is dubbed "smoothly state-dependent" because the probability of adjustment is a smoothly increasing function of the adjustment gain, rather than the 0–1 step function it is in the menu cost model, or the constant it is in the Calvo-Yun model. The framework is general in the sense that it nests the menu cost model at one extreme and the Calvo–Yun model at the opposite extreme, and its intermediate version is empirically more plausible than either of the extremes (Costain and Nakov, 2011a). Specifically, the smoother intermediate model matches well observed histograms of price changes such as that found in AC Nielsen's retail price data documented by Midrigan (2007) — see Figure 1. In addition, under standard policy, it produces larger real effects of monetary shocks than the fixed menu cost model of Golosov and Lucas.

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[2] A main difference of this class of models of price setting, relative to time-dependent models, concerns the behavior of "reset price inflation" (the rate of change of all desired prices) relative to actual inflation. As documented by Bils et al. (2009, 2012), time-dependent models imply unrealistically high persistence and stability of reset price inflation in response to shocks, relative to the data, which is inherited by actual inflation. But the tight link between reset price inflation and future inflation dynamics is a key determinant of the multiplier under a constant nominal interest rate in the New Keynesian model.

[3] **We focus on the case of the government spending multiplier under passive monetary policy, but results easily extend to the effects of monetary forward guidance.**

To preview the key results, we find that in the benchmark calibration, for the same frequency of price adjustment, the impact government spending multiplier in state-dependent models is an order of magnitude larger than in the Calvo-Yun model under a constant interest rate. This reflects the fact that the degree of price flexibility in these models **is endogenous and not just an exogenous frequency parameter**. In particular, aggregate price flexibility depends crucially on the "selection effect" discovered by Caplin and Spulber and emphasized by Golosov and Lucas (2007). **Namely, firms that adjust prices are those for which adjustment is more valuable and these firms tend to make larger price changes.** This makes the aggregate price level more flexible in equilibrium than in the Calvo model, *even when holding the average frequency of adjustment constant across models.* In response to a government spending shock, given a constant interest rate, the faster increase in the price level means that the real interest rate falls by more, stimulating private consumption and leading to a larger increase in output than under Calvo.

Interacting with the price setting mechanism, the monetary policy reaction function is also crucial in shaping the formation of expectations of future inflation. Even under a constant nominal interest rate, it is well known that the specific form of the reaction function matters for the response of the economy to shocks. As shown by Eggertsson and Woodford (2003), Adam and Billi (2005) and Nakov (2008) for the case of a zero lower bound episode with stochastic duration, this especially concerns the interest rate lift-off periods in the aftermath of a ZLB episode. Therefore, we also explore the effects of different monetary policy rules.

After presenting the model in Section 2, Sections 3 and 4 provide several analytical results for the textbook Calvo–Yun model without firm-level shocks. Following Woodford (2011), we start by analyzing a standard Taylor rule such that the nominal interest rate reacts to deviations of inflation *and* of government spending from their respective steady-state levels. The degree of accommodation to government spending is captured by the corresponding reaction coefficient in this rule. In the Calvo–Yun version of our model, we derive analytical conditions under which an appropriately chosen coefficient exactly delivers a constant nominal interest rate in response to government spending shocks that can follow either Woodford's binomial or a standard AR(1) process. Such a rule replicates the results under the zero lower bound in Woodford (2011), including the property that the multiplier is *locally* increasing in the degree of price flexibility. We also study the case in which the interest rate is kept constant for a given period of time by a series of anticipated shocks to the Taylor rule (dubbed the "modest interventions" rule by Galí, 2012), e.g. as in Erceg and Linde (2014) and Werning (2012). This is the standard approach adopted in larger policy models with many state variables. Similarly to Carlstrom, Fuerst and Paustian (2014), we show that such a monetary policy delivers the largest multipliers, and indeed magnifies the "paradox of flexibility". But we also show that if the constant interest rate is instead the result of a money growth rule, which imparts strong mean reversion to the price level, then all paradoxes disappear — in line with arguments in Cochrane (2015).

Section 5 turns to the numerical analysis of the model with state-dependent pricing and idiosyncratic shocks. Exploring the implications of systematic monetary policy under state-dependent pricing is a further contribution of the paper (Golosov and Lucas, 2007 and others assume a money growth rule). First, we present results under a standard Taylor rule. When the rule is only allowed to respond to inflation, the multiplier is less than 1 across all models, and lowest under state-dependent pricing. Second, we consider the case of a constant interest rate for a deterministic period of time. In this case, the multiplier is larger than unity across all models, but the amplification is largest under state-dependent pricing. The strong propagation under constant interest rates thus holds across price-setting models, and the "paradox of flexibility" extends to a broad concept of price flexibility.

2 A simple macroeconomic model nesting time- and state-dependent pricing

The model embeds state-dependent pricing by firms in an otherwise-standard New Keynesian general equilibrium framework. Following Costain and Nakov (2011a,b), the framework nests a continuum of pricing models, with the Calvo model on one extreme, and the Golosov and Lucas (2007) fixed menu cost model on the other. Besides the firms, there is a representative household, a monetary authority that implements a Taylor rule, and a government that levies lumps sum taxes to finance an exogenous stream of spending on goods.

2.1 Representative household

The household's period utility is $\frac{1}{1-\gamma}C_t^{1-\gamma} - \frac{\chi}{1+\eta}N_t^{1+\eta} + \nu \log(M_t/P_t)$, where C_t denotes consumption, N_t labor supply, and M_t/P_t real money balances. Utility is discounted by β. Consumption is a CES aggregate of differentiated products C_{it}, with elasticity of substitution ϵ:

$$C_t = \left\{ \int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

The household's nominal period budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + R_t^{-1}B_t = W_tN_t + M_{t-1} + T_t + B_{t-1}, \tag{2}$$

where $\int_0^1 P_{it}C_{it}di$ is total nominal consumption. B_t is nominal bond holdings purchased at t, and paying interest rate $R_t - 1$ at time $t+1$. T_t is a nominal lump-sum transfer consisting of seigniorage revenues from the central bank plus dividend payments from the firms, net of lump-sum taxes to finance government spending on goods.

Households choose C_{it}, N_t, B_t, and M_t to maximize expected discounted utility, subject to the budget constraint (2). Optimal consumption across the differentiated goods implies

$$C_{it} = (P_t/P_{it})^{\epsilon}C_t, \tag{3}$$

where $P_t \equiv \left[\int_0^1 {P_{it}}^{1-\epsilon}di\right]^{\frac{1}{1-\epsilon}}$ is the relevant price index.

Optimal labor supply, consumption, and money use imply the following first-order conditions:

$$\begin{aligned} \chi C_t^{\gamma}N_t^{\eta} &= W_t/P_t, && (4) \\ 1 &= \beta R_t E_t\left[P_t C_{t+1}^{-\gamma}/\left(P_{t+1}C_t^{-\gamma}\right)\right], && (5) \\ M_t/P_t &= \nu C_t^{\gamma} R_t/\left(R_t - 1\right). && (6) \end{aligned}$$

2.2 Monopolistic firms

Firms are monopolistic competitors. Each firm i produces output Y_{it} using labor N_{it} as the only input, under a linear technology: $Y_{it} = A_{it}N_{it}$. Firm's productivity A_{it} is an idiosyncratic process, AR(1) in logs:

$$\log A_{it} = \rho_a \log A_{it-1} + \varepsilon^a_{it}, \tag{7}$$

where $0 \leq \rho_a < 1$ and $\epsilon^a_{it} \sim i.i.d.N(0, \sigma^2_a)$. Firm i charges a price P_{it} and faces demand from two sources, $Y_{it} = C_{it} + G_{it}$, where C_{it} is demand for goods by the households, and G_{it} is demand by the government. The government's consumption basket is also a CES aggregator with elasticity of substitution ϵ:

$$G_t = \left\{ \int_0^1 G_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{8}$$

Optimal allocation of expenditure across goods on the part of households and the government implies that firm i faces the demand curve $Y_{it} = (C_t + G_t)P_t^{\epsilon}P_{it}^{-\epsilon}$. The firm fulfills all demand at its posted price. It hires in a competitive labor markets at wage rate W_t, generating period profits

$$U_{it} = P_{it}Y_{it} - W_t N_{it}. \tag{9}$$

Costain and Nakov (2011) derive the following value function for a firm which produces with productivity A and sells at nominal price P when the aggregate state is Ω:

$$V(P, A, \Omega) = U(P, A, \Omega) + \beta E \left\{ \left. \tfrac{P(\Omega)C(\Omega')^{-\gamma}}{P(\Omega')C(\Omega)^{-\gamma}} \left[V(P, A', \Omega') + G(P, A', \Omega') \right] \right| A, \Omega \right\}, \tag{10}$$

where

$$G(P, A', \Omega') \equiv \lambda \left(\frac{\max_P V(P, A', \Omega') - V(P, A', \Omega')}{W(\Omega')} \right) \left(\max_P V(P, A', \Omega') - V(P, A', \Omega') \right) \tag{11}$$

is the *expected gain* from adjustment, and $\lambda \left(\frac{\max_P V(P,A',\Omega') - V(P,A',\Omega')}{W(\Omega')} \right) \in [0, 1]$ is a mapping from the gain from price adjustment to the probability of adjustment, which is detailed in the following subsection (2.3).

Price stickiness means that the individual price process associated with the Bellman equation is

$$P\prime = \begin{cases} \arg\max_P V(P, A', \Omega') & \text{with probability } \lambda(P, A', \Omega') \\ P & \text{with probability } 1 - \lambda(P, A', \Omega') \end{cases} . \tag{12}$$

2.3 Nesting alternative price-setting schemes

Following Costain and Nakov (2011a,b) we assume that the probability of price adjustment $\lambda(L)$, increases with the gain from adjustment L. Thus, the function $\lambda(L) \in [0, 1]$ that governs this probability is taken as a primitive of the model.[4] In particular, we postulate the following functional form:

$$\lambda\left(L\right) \equiv \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda})\left(\alpha/L\right)^{\xi}} \tag{13}$$

where $L \equiv \left[\max_P V(P, A', \Omega') - V(P, A', \Omega')\right]/W(\Omega')$ is the relevant endogenous state, with α and ξ positive, and $\bar{\lambda} \in [0, 1]$. This function is concave for $\xi \leq 1$ and S-shaped for $\xi > 1$ (see Fig. 1). Parameter ξ controls the degree of state dependence. In the limit $\xi = 0$, (13) nests Calvo (1983) so that $\lambda\left(L\right) = \bar{\lambda}$ regardless of L. At the opposite extreme, as $\xi \to \infty$, $\lambda\left(L\right)$ becomes the indicator function $\mathbf{1}\left\{L \geq \alpha\right\}$, which equals 1 whenever $L \geq \alpha$ and is zero otherwise. This is the fixed menu cost model which implies "extreme state dependence", in the sense that the adjustment probability jumps from 0 to 1 as soon as the state L passes the threshold α. For intermediate values $0 < \xi < \infty$, the hazard increases "smoothly" with the state L. We call this intermediate version "smoothly state-dependent pricing" (SSDP) model.

2.4 Monetary policy and aggregate consistency

In our baseline model monetary policy follows a Taylor rule,

$$r_t - \overline{r} = \phi_\pi \pi_t + \sum_{j=1}^{T} \varepsilon^r_{t-j} \tag{14}$$

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[4] Alternatively, λ can be viewed as an exogenously specified distribution of menu costs from which firms make random draws every period.

where r_t is the net nominal interest rate, $\overline{r} = E(r_t)$, $\pi_t = \log(P_t/P_{t-1})$ and $\phi_\pi > 1$. The shocks ε_t^r are *anticipated* and are set so that, following a spending shock, the nominal interest is constant at its steady state value for T periods. This is a standard approach for studying the effects of constant interest rates, see e.g. Erceg and Linde (2014). Galí (2012) dubs it "modest interventions" to keep the interest rate constant. This is equivalent to instead picking shocks, e.g. to the rate of time preference, so that the zero lower bound would constrain the nominal interest rate for T periods, while government spending increases following some exogenous process.

Further, we assume that government spending G_t follows an AR(1) process in logs:

$$\log\left(G_t/G^*\right) = \rho \log\left(G_{t-1}/G^*\right) + \varepsilon_t^g, \tag{15}$$

with persistence $0 < \rho < 1$ and $\epsilon_t^g \sim i.i.d.N(0, \sigma_g^2)$.

Seigniorage revenues are paid to the household as a lump-sum transfer, and the government budget is balanced each period. Because the timing of lump-sum taxes is irrelevant, bond market clearing is simply $B_t = 0$. When supply equals demand for each good i, total labor supply and demand satisfy

$$N_t = \int_0^1 \frac{Y_{it}}{A_{it}} di = P_t^\epsilon Y_t \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di \equiv \Delta_t Y_t. \tag{16}$$

where $\Delta_t \equiv P_t^\epsilon \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$ is a measure of price dispersion which takes into account heterogeneous productivity.

Costain and Nakov (2011b) show that in this case the aggregate state of this economy is summarized by $\Omega_t \equiv (G_t, \Phi_{t-1})$, where $\Phi_{t-1}(P, A)$ is the lagged distribution of firms over prices and productivities.

2.5 Computation and parameterization

The equilibrium is computed following the two-step algorithm of Reiter (2009). Reiter's method is especially well suited to contexts such as this model, in which idiosyncratic shocks are large, but aggregate shocks are small. In a first step, the aggregate steady state is computed on a finite grid, using backwards induction. Second, the stochastic aggregate dynamics are computed by linearization around each grid point. Thus, the Bellman equation is treated as a large system of expectational difference equations, instead of a functional equation. However, the size of the aggregate shock cannot be so large as to imply that the optimal firm choices fall outside of the grid.[5]

We calibrate the three models (Calvo, fixed menu cost, and SSDP) to match salient features of the microdata on price changes, such as those found by Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Midrigan (2007, 2011). In particular, we seek price adjustment and productivity processes which are consistent with the histogram of price changes in AC Nielsen's monthly data documented by Midrigan (2007).

The model is one of "regular" price changes, excluding temporary "sales", and the working frequency is monthly. We set the growth rate of money to 0%, consistent with the zero average price change found in AC Nielsen's data.[6] Other macro parameters are set to standard values in the RBC literature. Thus, the discount factor is $\beta = 1.04^{-1/12}$. Consumption utility is CRRA, $u(C) = \frac{1}{1-\gamma}C^{1-\gamma}$, with $\gamma = 2$, while labor disutility is $\chi \frac{N^{1+\eta}}{1+\eta}$, with $\eta = 1$ and $\chi = 11$, which implies that 35% of the time is dedicated to work. The elasticity of substitution in the consumption aggregator is $\epsilon = 7$. The utility of real money holdings is logarithmic, $v(m) = \nu \log(m)$, with $\nu = 1$. And we set $G^* = 0.07$, consistent with an average share of government spending of 20% of GDP.

This leaves us with five parameters to pin down the aggregate steady-state: the persistence ρ_a and the volatility of σ_a^2 of the idiosyncratic productivity process; and the three parameters $(\bar{\lambda}, \xi, \alpha)$ of the adjustment function (13). We estimate these parameters by minimizing a distance criterion between the data and the model's steady state. The criterion sums two terms, scaled for comparability: the first is the absolute difference between the adjustment frequencies in the data and the simulation, while the second is the Euclidean distance between the frequency vectors associated with the histograms of nonzero price adjustments in the data and the simulation.[7] The left panel of figure 1 shows the fit of the three models to the histogram of price changes from AC Nielsen's data. The right panel shows the adjustment hazard function for different parameterizations. Clearly, the SSDP model provides the best fit to the observed histogram.

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[5] See Costain and Nakov (2011) for a detailed exposition of Reiter's method as applied to sticky price models.

[6] Reiter's solution method is not limited to zero steady-state inflation rates.

[7] See Costain and Nakov (2011a,b) for more details about the estimation.

To simulate the model's dynamics, we need to specify two additional sets of parameters. One is related to the monetary policy rule (14). For our benchmark model, we set $\phi_\pi = 2$ and $T = 24$. The other set of parameters calibrates the exogenous process for government spending (15). We set $\rho = 0.9$, which is similar to Erceg and Linde (2014), and pick the shock size so that, on impact, the increase in government spending equals 0.1% of GDP. This value ensures that our grid contains the endogenous choices of firms in response to the shock.

3 The spending multiplier and systematic monetary policy

In this section we show the link between the size of the government spending multiplier and movements in the price level. Consider first the log-linearized representative household Euler equation (5). Here small letters denote percentage deviations from steady state (with output and spending both in deviations from steady state output), and $\sigma = \gamma^{-1}$:

$$c_t = y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - \sigma\left(r_t - \overline{r} - E_t\pi_{t+1}\right). \tag{17}$$

Solving this equation forward for c_0, we obtain the following expression for current output in excess of government purchases:

$$c_0 = y_0 - g_0 = -\sigma E_0 \sum_{j=1}^{T+1}\left(r_{j-1} - \overline{r} - \pi_j\right) + E_0\left(y_{T+1} - g_{T+1}\right). \tag{18}$$

Under stationarity of output and the government spending shock, $\lim_{T\to\infty} E_0\left(y_{T+1} - g_{T+1}\right) = 0$. The response of output in excess of government purchases depends on whether the sum of expected short-term real rates, or the long-term real rate, is positive or negative (see Woodford, 2011). Under nominal rigidities, the degree to which systematic monetary policy reacts to the government spending shock and interacts with future inflation formation will determine the size of the multiplier. This insight extends to the propagation of different shocks.

For instance, if we assume that the Taylor rule (14) is such that $r_{t+j-1} - \overline{r} = \phi_\pi \pi_{t+j}$, $\phi_\pi > 1$, the expression above becomes:

$$y_0 - g_0 = -\sigma\left(\phi_\pi - 1\right) E_0 \sum_{j=1}^{\infty} \pi_j = \sigma\left(\phi_\pi - 1\right)\left(p_0 - \lim_{T\to\infty} E_0 p_{T+1}\right). \tag{19}$$

Since $\sigma\left(\phi_\pi - 1\right) > 0$, the size of the multiplier is *increasing* in the contemporaneous price level p_0, and *decreasing* in the long-run expected response of the price level, $E_0 p_{T+1}$. Under the interest rate rule considered, the price level will not return to its original value. As we show below, under Calvo–Yun price setting, the multiplier is generally smaller than one, since the future price level is above the current one when government spending follows an AR(1) process (see also Woodford (2011); Corsetti et al. (2010) analyze more general spending processes). We will show numerically that this result also holds under a more general price-setting mechanism.

Results, however, change when the monetary policy rule entails a nominal interest rate less responsive to inflation. Suppose that monetary policy is such that the nominal interest rate is held constant for T periods after the shock, $r_t = \overline{r}$ for $t = 1...T$; the long-term real rate will then depend only on expected future inflation as follows:

$$\begin{aligned} y_0 - g_0 &= -\sigma E_0 \sum_{j=1}^{T+1} \left(r_{j-1} - \overline{r} - \pi_j\right) + E_0\left(y_{T+1} - g_{T+1}\right) && (20) \\ &= \sigma\left(\lim_{T\to\infty} E_0 p_{T+1} - p_0\right). && (21) \end{aligned}$$

Now the multiplier is larger than one if the expected future price level *exceeds* the contemporaneous price response. Hence, whether systematic monetary policy allows for a drift in the price level is a key determinant of the size of the multiplier under a constant nominal rate.

As an example, consider the systematic monetary policy analyzed under the zero lower bound by e.g. Christiano et al. (2011) and Woodford (2011), which stabilizes the price level at its value at lift-off of the interest rate. This monetary policy implies that the price level has a positive drift and exceeds the current price level, resulting in a large multiplier in the Calvo–Yun model. In this case, the more "flexible" prices are, the bigger the difference between the future and the current price level, as we show below. This is because inflation is determined by the rate of change of desired prices ("reset inflation"). Specifically, in the Calvo–Yun model, actual (and expected) inflation is proportional to reset inflation, $\pi_t^* \equiv \log\left(p_t^*/p_{t-1}^*\right)$:

$$\pi_t = \bar{\lambda}\pi_t^* + \left(1 - \bar{\lambda}\right)\pi_{t-1}, \tag{22}$$

where $\bar{\lambda}$ is the probability of price changes, and the desired price p_t^* in response to a government spending shock is given by:

$$\log\left(p_t^*/P_t\right) = \beta E_t \log\left(p_{t+1}^*/P_{t+1}\right) + \left(1 - \left(1 - \bar{\lambda}\right)\beta\right)(\gamma + \eta)\left(y_t - \frac{\gamma}{\gamma + \eta} g_t\right). \tag{23}$$

Notice that the size of the desired price change, and the evolution of inflation both depend on the frequency of price adjustment.

Conversely, as shown by Bils et al. (2009, 2012), the link between actual and reset inflation is quite different under state-dependent pricing. Endogenous price changing, and especially selection of changers, breaks the simple proportionality between π_t^* and π_t in time-dependent models. On the one hand, reset inflation tends to react more strongly to shocks on impact. This is because firms hit by larger idiosyncratic shocks are more likely to adjust their prices, other things equal. On the other hand, reset inflation is much less stable and persistent, resulting in different properties of cumulated actual inflation. Even assuming the same average frequency of price adjustment as in a time-dependent model, state-dependent pricing can have dramatic implications for the effects of government purchases on aggregate economic activity under a constant interest rate.

Likewise, when the constant interest rate results from a rule implying a stationary price level, i.e. $\lim_{T\to\infty} E_t p_{T+1} = 0$, such as a price level target, the price setting mechanism will affect the size of the multiplier. As we show below, the latter is in general smaller than unity, and decreasing in the degree of price flexibility, as the response of current inflation is larger the more flexible prices are. In this case, state-dependent pricing implies a smaller multiplier than time-dependent pricing, for the same average frequency of price adjustment.

4 Constant nominal interest rates, price flexibility and the size of the spending multiplier

We have argued that the interaction between monetary policy and the price setting mechanism is of paramount importance to understanding the propagation of shocks when the nominal interest rate is constant or at its lower bound. In this section we derive some analytical results for the version of our economy with Calvo–Yun pricing for the case of government spending shocks.

4.1 The multiplier under a constant interest rate of stochastic duration

Consider the log-linearized Calvo–Yun model in Woodford (2011):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(y_t - \Gamma g_t \right) \tag{24}$$

$$y_t - g_t = E_t \left(y_{t+1} - g_{t+1} \right) - \gamma^{-1} \left(r_t - \overline{r} - E_t \pi_{t+1} \right) \tag{25}$$

where

$$\kappa = \left(\gamma + \eta \right) \left[1 - \left(1 - \bar{\lambda} \right) \beta \right] \bar{\lambda} / \left(1 - \bar{\lambda} \right), \tag{26}$$

$$\Gamma = \frac{\gamma}{\gamma + \eta}, \tag{27}$$

with $0 < \Gamma < 1$ denoting the multiplier under flexible prices. Observe that g_t denotes government spending, in deviation from steady state, scaled by the steady state level of GDP. It follows the exogenous AR(1) process in (15). We will characterize the size of the (local) multiplier, $\gamma_y = \frac{\partial y_t}{\partial g_t}$, for different monetary policy responses to the spending shock.

Since the focus here is on the case of a constant nominal interest rate, it is useful to recall the key results under the zero lower bound (ZLB). Under a binomial shock inducing a binding ZLB, which lasts an additional period with probability p (e.g. as in Eggertsson and Woodford, 2003 or Christiano et al., 2011), with the zero-inflation steady state being an absorbing state, the multiplier for a spending shock with the same binomial process is given by:

$$\gamma_y = 1 + \frac{(1-\Gamma)\, p\kappa}{(1-p)(1-\beta p)\gamma - p\kappa}.$$

In order to have a unique bounded solution it must be that the denominator in the above expression is positive, $(1-p)(1-\beta p)\gamma/p > \kappa$.[8] In words, the condition requires that the probability of the interest rate remaining constant (at its lower bound), p, is sufficiently smaller than the probability of not adjusting prices, $1-\bar{\lambda}$. Due to the linearity of the model, for this result it is immaterial whether the interest rate is constant at its lower bound e.g. because of a shock to the natural rate, or it is constant at its steady state value because of a monetary policy shock that is anticipated to exactly offset the effects of government spending on the nominal interest rate.[9]

The multiplier is larger than unity (and larger than its flexible-price counterpart Γ), and in fact can be much larger. In particular, it is increasing in κ, and thus in the degree of price flexibility $\bar{\lambda}$. This is the so-called "paradox of flexibility" stressed by Werning (2012) and Eggertsson and Krugman (2012). The multiplier is also *decreasing* in Γ, and in particular equal to one when $\Gamma = 1$, which is the case with linear labor disutility.[10]

The intuition for this result is straightforward. With a constant nominal interest rate, the long term real interest rate is a function of expected inflation, which in turn is increasing in the slope of

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[8] This can be verified easily by checking that both eigenvalues of the characteristic polynomial of the following system of difference equations are larger than 1 when the condition is satisfied,

$$\begin{aligned} \pi_t &= \beta\left[p\pi_t + (1-p)\cdot 0\right] + \kappa\left(y_t - \Gamma g\right) \\ y_t - g &= p\left(y_t - g\right) + \gamma^{-1}\left(p\pi_t + r\right) \end{aligned}$$

where $r < 0$, $g > 0$ such that that $r_t = 0$.

[9] The specific monetary policy rule prevailing when the economy is at the zero inflation steady state with probability $1-p$ is immaterial. What matters is that there is an expected drift in the price level, as we show below — see also Cochrane (2015). Conversely, in the fully stochastic cases examined by Adam and Billi (2005) and Nakov (2008), it makes a great deal of difference whether upon exit from a ZLB monetary policy is chosen optimally, even under discretion, or follows a Taylor rule.

[10] In this case the only allocation consistent with a constant nominal interest rate coincides with that under flexible prices with a constant real rate. This is the benchmark specification in the study of money growth shocks in Golosov and Lucas (2007).

the NKPC. Specifically, it is easy to show that the price level has a positive drift:

$$\begin{aligned} E_0 \sum_{j=1}^{\infty} \pi_j &= E_0 \lim_{T \to \infty} p_T - p_0 = \frac{\kappa\left(\gamma_y - \Gamma\right)}{(1-\beta p)} g_0 \sum_{j=1}^{\infty} p^j = \frac{\kappa\left(\gamma_y - \Gamma\right)}{(1-\beta p)} \frac{p}{1-p} g_0, \qquad (28) \\ \lim_{T \to \infty} p_T &= \frac{1}{1-p} \frac{\kappa\left(\gamma_y - \Gamma\right)}{(1-\beta p)} g_0, \; p_0 = \pi_0 = \frac{\kappa\left(\gamma_y - \Gamma\right)}{(1-\beta p)} g_0 \qquad (29) \end{aligned}$$

where

$$\gamma_y - \Gamma = \frac{(1-p)(1-\beta p)\gamma}{\Delta}(1-\Gamma). \qquad (30)$$

The latter expression is positive if the multiplier is larger than Γ, its flexible price counterpart, which is the case for $\Delta = (1-p)(1-\beta p)\gamma - p\kappa > 0$. Therefore, in the Calvo model under a constant interest rate (or ZLB) of stochastic duration, the multiplier increases in $\bar{\lambda}$, and thus in the slope of the NKPC because expected inflation is proportional to current inflation. The latter is in turn proportional to "reset inflation", and thus the more responsive to shocks, such as government spending, the steeper the Phillips curve.

We find it useful to consider what happens under a simple Taylor rule in which the interest rate is allowed to respond to government spending that follows an AR(1) process with persistence ρ:

$$r_t - \overline{r} = \phi_\pi \pi_t - \phi_g g_t. \qquad (31)$$

Guessing the following decision rules under the unique bounded solution (for $\phi_\pi > 1$),

$$\begin{aligned} \pi_t &= \gamma_\pi g_t, \qquad (32) \\ y_t &= \gamma_y g_t, \qquad (33) \end{aligned}$$

it is straightforward to find the solutions for the coefficients:[11]

$$\begin{aligned}\gamma_\pi &= \frac{\kappa\left(\gamma_y-\Gamma\right)}{1-\beta\rho}, && (34)\\ \gamma_y-1 &= \rho\left(\gamma_y-1\right)+\sigma\left[\phi_g-\left(\phi_\pi-\rho\right)\frac{\kappa\left(\gamma_y-\Gamma\right)}{1-\beta\rho}\right]. && (35)\end{aligned}$$

Importantly, the last expression implies that the multiplier is larger than one only to the extent that the expected deviation from steady-state of the short-term real interest rate,

$$\left(r_t-\overline{r}-E_t\pi_{t+1}\right)=\left[\left(\phi_\pi-\rho\right)\frac{\kappa\left(\gamma_y-\Gamma\right)}{1-\beta\rho}-\phi_g\right], \qquad (36)$$

is *negative*.

Solving explicitly for γ_y yields:

$$\gamma_y=1+\frac{\phi_g-\left(1-\Gamma\right)\dfrac{\phi_\pi-\rho}{1-\beta\rho}\kappa}{\left(1-\rho\right)\gamma+\dfrac{\phi_\pi-\rho}{1-\beta\rho}\kappa}. \qquad (37)$$

Note, first, that the multiplier is larger than one if and only if $\phi_g>\left(1-\Gamma\right)\dfrac{\phi_\pi-\rho}{1-\beta\rho}\kappa>0$, where it can be shown that the latter expression is indeed proportional to the real interest rate that would prevail with a unitary multiplier. For instance, for a rule with $\phi_g=0$, it is clear that the multiplier is always smaller than unity. As argued above, the response of the expected future price level is larger than that of the current price level, resulting in a positive long-term real interest rate.

Second, for $\gamma_y\geq 1$ the multiplier is increasing in Γ and ρ (to the extent that $\phi_\pi>\beta^{-1}$), and decreasing in ϕ_π, κ, and thus especially in the frequency of price adjustment $\bar{\lambda}$. Therefore, the steeper the Phillips Curve, e.g. the more flexible prices (the larger $\bar{\lambda}$), the smaller the multiplier.

Interestingly, the parameter ϕ_g in the Taylor rule can be chosen as to replicate the (ZLB) re-

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[11] This can be verified easily by checking that both eigenvalues of the characteristic equation of the following difference equation in inflation are larger than 1:

$$\left(1+\kappa\sigma\phi_\pi\right)\pi_t-\left(1+\beta+\kappa\sigma\right)E_t\pi_{t+1}+\beta E_t\pi_{t+2}=0.$$

The assumption of a Taylor rule is immaterial, as the same allocation could be decentralized with an appropriate money growth rule that responds to the government spending shock.

sults under an AR(1) government spending shock with persistence ρ instead of a binomial shock.[12] Specifically, it can be verified that setting

$$\phi_g = \frac{(1-\Gamma)(1-\rho)\kappa}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa}\phi_\pi,$$

would result in a constant nominal interest rate in response to the spending shock, and a multiplier equal to the one under the stochastic binomial ZLB, namely

$$\gamma_y - 1 = \frac{\phi_g - (1-\Gamma)\dfrac{\phi_\pi - \rho}{1-\beta\rho}\kappa}{(1-\rho)\gamma + \dfrac{\phi_\pi - \rho}{1-\beta\rho}\kappa} = (1-\Gamma)\frac{\rho\kappa}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa}.$$

However, in contrast with the binomial case above, the multiplier is defined also when $\Delta = (1-\rho)(1-\beta\rho)\gamma - \rho\kappa < 0$. In this case it is smaller than unity, and even negative.

4.2 The multiplier with a constant nominal rate for T periods

We now turn to the case of a sequence of anticipated shocks which keep the interest rate constant for T periods after the government spending shock, consistent with (14) above. As of $T+1$, monetary policy returns to whatever rule it followed before.[13] This is a standard approach to approximating the effects of the ZLB, see e.g. Werning (2012), especially also in models with non-trivial dynamics due to the presence of many state variables, in which the ZLB or even a constant interest rate with a stochastic duration cannot be implemented easily. However, similarly to Carlstrom et al. (2014), here we show that this approach in general does not provide the same results as in the binomial case of Woodford (2011) and Christiano et al. (2011), nor those under the AR(1) spending shock. Specifically, under this approach the "paradox of flexibility" is actually magnified, and the multiplier can be very large even when the counterpart of the condition above, $(1-\rho)(1-\beta\rho)\gamma > \kappa\rho$, is violated.

It is straightforward to show that over the period for which the interest rate is anticipated to be

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[12] The persistence of the spending shock ρ plays the same role of the probability p under the binomial ZLB example above.

[13] As noted above, this is equivalent to picking shocks, e.g. to the rate of time preference, so that the zero lower bound constrains the nominal interest rate for T periods, while government spending increases following an AR(1) process.

constant, the dynamics of the economy under an AR(1) spending shock are fully described by the following second order difference equation,

$$\beta\left(y_{t+1}-g_{t+1}\right)-\left(1+\beta+\kappa\gamma^{-1}\right)\left(y_t-g_t\right)+\left(y_{t-1}-g_{t-1}\right)=\kappa\gamma^{-1}\left(1-\Gamma\right)\rho g_t, \tag{38}$$

with general solution given by

$$\begin{aligned}\left(y_t-g_t\right) &= \frac{\left(1-\Gamma\right)\kappa\rho}{\left(1-\rho\right)\left(1-\beta\rho\right)\gamma-\rho\kappa}g_t+a_1\lambda_1^t+a_2\lambda_2^t, &(39)\\ \lambda_i &= \frac{1+\beta+\kappa\gamma^{-1}\pm\sqrt{\left(1+\beta+\kappa\gamma^{-1}\right)^2-4\beta}}{2\beta}, &(40)\end{aligned}$$

for $T-1\geq t\geq 0$, where the eigenvalues satisfy $0<\lambda_1<1<\lambda_2$, for $\kappa\gamma^{-1}>0$. Thus, the coefficient on government spending in the particular solution is the same as under the constant interest rate with stochastic duration studied above. However, as shown by Werning (2012), the deterministic duration of the constant interest rate now implies the presence of the exponential decay terms associated with λ_1 and λ_2. In particular, while under a stochastic duration the constants a_1 and a_2 are set to zero to rule out any explosive dynamics due to both eigenvalues being larger than 1, now they need to be determined using other appropriate terminal conditions.

Specifically, the constants a_1 and a_2 are now determined by the following two terminal conditions for T and $T-1$:

$$\gamma\left(y_T-g_T\right)=\gamma\left(y_{T+1}-g_{T+1}\right)+\pi_{T+1} \tag{41}$$

$$\begin{aligned}\gamma\left(y_{T-1}-g_{T-1}\right) &= \gamma\left(y_T-g_T\right)+\pi_T\\ &= \gamma\left(1+\kappa\gamma^{-1}\right)\left(y_{T+1}-g_{T+1}\right)+\left(1+\beta+\kappa\gamma^{-1}\right)\pi_{T+1}+\left(1-\Gamma\right)\kappa\rho g_{T-1}, &(42)\end{aligned}$$

yielding the following solution

$$\begin{aligned}
a_1 &= -\frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{1-\rho\beta\lambda_1}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_1}\right)^T g_0+\frac{1}{1-\beta\lambda_1^2}\frac{(1-\beta\lambda_1)\gamma(y_{T+1}-g_{T+1})+\pi_{T+1}}{\gamma\lambda_1^T},\\
a_2 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{\beta\lambda_1(\lambda_1-\rho)}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_2}\right)^T g_0+\frac{\beta\lambda_1}{1-\beta\lambda_1^2}\frac{(1-\lambda_1)\gamma(y_{T+1}-g_{T+1})-\lambda_1\pi_{T+1}}{\gamma\lambda_2^T}.
\end{aligned}$$

These constants are uniquely determined if after the interest rate is free to move the equilibrium is uniquely defined, as to pin down $(y_{T+1}-g_{T+1})$ and π_{T+1}. Since $\lambda_2 > 1 > \rho$, a_2 will play a small role in the initial dynamics, especially when T is large. Conversely, since $0 < \lambda_1 < 1$, when the equilibrium after liftoff is such that $(y_{T+1}-g_{T+1})$ and π_{T+1} are not small, or when $\rho > \lambda_1$, a_1 could be quite large. Importantly, $\rho > \lambda_1$ if and only if $\Delta = (1-\rho)(1-\beta\rho)\gamma-\rho\kappa < 0$, where recall that λ_1 is decreasing in κ and thus in the frequency of price adjustment. A sufficiently large κ then implies that $\rho > \lambda_1 > 0$.

To better appreciate the consequences for the spending multiplier, write out the solution for the impact response of output as follows,

$$\begin{aligned}
y_0-g_0 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}g_0+a_1+a_2 \qquad (43)\\
&= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\left[1-\frac{(1-\rho\beta\lambda_1)\left(\frac{\rho}{\lambda_1}\right)^T-\beta\lambda_1(\lambda_1-\rho)\left(\frac{\rho}{\lambda_2}\right)^T}{1-\beta\lambda_1^2}\right]g_0+\\
&\quad \frac{1}{1-\beta\lambda_1^2}\left\{\left[1-\beta\lambda_1^2\left(\frac{\lambda_1}{\lambda_2}\right)^T\right]\frac{\pi_{T+1}}{\gamma\lambda_1^T}+\left[1-\beta\lambda_1\left(1-(1-\lambda_1)\left(\frac{\lambda_1}{\lambda_2}\right)^T\right)\right]\frac{(y_{T+1}-g_{T+1})}{\lambda_1^T}\right\}.
\end{aligned}$$

The following considerations are in order.[14] First, the multiplier is very similar to its counterpart under the constant interest rate of stochastic duration when, in addition to $(1-\rho)(1-\beta\rho)\gamma > \rho\kappa$ and $\rho < \lambda_1$, T is sufficiently large. In particular, assume that as of $T+1$ monetary policy implements

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[14] It is straightforward to show that very similar expressions would obtain for the case of monetary forward guidance, whereas a monetary expansion is announced to take place after T periods, during which the interest rate is kept constant. Namely, the impact effects are given by:

$$\begin{aligned}
y_0 &= \frac{(1-\beta\lambda_1)\gamma y_{T+1}+\pi_{T+1}}{\gamma(1-\beta\lambda_1^2)}\lambda_1^{-T}+\beta\lambda_1\frac{(1-\lambda_1)\gamma y_{T+1}-\lambda_1\pi_{T+1}}{\gamma(1-\beta\lambda_1^2)}\lambda_2^{-T}\\
\pi_0 &= \frac{(1-\beta\lambda_1)\gamma y_{T+1}+\pi_{T+1}}{(1-\beta\lambda_1^2)}\lambda_1^{-T}+\beta\lambda_1\frac{(1-\lambda_1)\gamma y_{T+1}-\lambda_1\pi_{T+1}}{(1-\beta\lambda_1^2)}\lambda_2^{-T}
\end{aligned}$$

the flexible price solution. Then $\pi_{T+1} = 0$ and $(y_{T+1} - g_{T+1}) = -(1-\Gamma)\rho^T g_0$, so that for T sufficiently large $\left(\frac{\rho}{\lambda_i}\right)^T \to 0, i = 1, 2$ and the same result as under the stochastically constant interest rate obtains:

$$y_0 - g_0 \simeq \frac{\rho\kappa}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa}(1-\Gamma) g_0.$$

When prices become more flexible and κ increases, in the range of parameters for which the denominator in the above equation is positive, and $\rho < \lambda_1 < 1$, the multiplier increases.

Secondly, similarly to the stochastic case with an AR(1) shock analyzed above, the multiplier is also defined for parameterizations such that $\Delta < 0$ ($<=>\rho > \lambda_1$). This occurs in particular for our parameterization, in which effectively we posit $\rho = 1-\overline{\lambda}$. Thus the denominator Δ in the above equation is negative, $(1-\rho)(1-\beta\rho)\gamma - \rho\kappa = -\left(1-\overline{\lambda}\right)\left(1-\beta\left(1-\overline{\lambda}\right)\right)\eta = -(1-\rho)(1-\beta\rho)$. Nevertheless, for T sufficiently large, the multiplier can also in this case be positive and very large, increasing in κ and thus in the frequency of price adjustment. Namely, the paradox of flexibility arises for any value of $\kappa < \infty$ (i.e. $\overline{\lambda} < 1$), resulting in an ever growing multiplier, the steeper the Phillips curve and the more flexible prices. Assume again $\pi_{T+1} = 0$ and $(y_{T+1} - g_{T+1}) = -(1-\Gamma)\rho^T g_0$, for T sufficiently large that $\left(\frac{\rho}{\lambda_2}\right)^T \simeq 0$, the impact response of output is given by the following:

$$y_0 - g_0 \simeq \frac{\rho\kappa}{\rho\kappa - (1-\rho)(1-\beta\rho)\gamma}\left\{\frac{1-\rho}{1-\beta\lambda_1^2}\frac{\rho\kappa\beta\lambda_1 + (1-\beta\lambda_1)(1-\beta\rho)\gamma}{\kappa\rho}\left(\frac{\rho}{\lambda_1}\right)^T - 1\right\}(1-\Gamma) g_0.$$

When $\rho > \lambda_1$, the multiplier is again positive and very large.

Finally, monetary policy after liftoff of the constant rate policy can also affect the short-run multiplier, if it is chosen as to manipulate the values of output and inflation – see Del Negro et al. (2013) and Cochrane (2015). If after lift-off monetary policy follows the benchmark Taylor rule in (14), inflation and output are uniquely given by:

$$\begin{aligned} \pi_{T+1} &= (1-\Gamma)\frac{\kappa(1-\rho)\gamma}{(1-\rho)(1-\beta\rho)\gamma + (\phi_\pi - \rho)\kappa}\rho^{T+1} g_0 \\ y_{T+1} - g_{T+1} &= -(1-\Gamma)\frac{(\phi_\pi - \rho)\kappa}{(1-\rho)(1-\beta\rho)\gamma + (\phi_\pi - \rho)\kappa}\rho^{T+1} g_0; \end{aligned}$$

in turn this yields the following expression for the impact multiplier:

$$\begin{aligned}
y_0 - g_0 &\simeq \frac{(1-\Gamma)\,\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa}\left[\frac{\left(1-\beta\lambda_1^2\right) - (1-\rho\beta\lambda_1)\left(\frac{\rho}{\lambda_1}\right)^T}{1-\beta\lambda_1^2}\right] g_0 + \frac{1}{1-\beta\lambda_1^2}\left\{\frac{\pi_{T+1}}{\gamma\lambda_1^T} + \frac{(y_{T+1}-g_{T+1})}{\lambda_1^T}\right\} \\
&\simeq \frac{(1-\Gamma)\,\kappa\rho}{1-\beta\lambda_1^2}\left[\frac{\left(1-\beta\lambda_1^2\right) - (1-\rho\beta\lambda_1)\left(\frac{\rho}{\lambda_1}\right)^T}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa} - \frac{(\phi_\pi - 1)\left(\frac{\rho}{\lambda_1}\right)^T}{(1-\rho)(1-\beta\rho)\gamma + (\phi_\pi - \rho)\,\kappa}\right] g_0.
\end{aligned}$$

It is obvious that the bigger ϕ_π, the smaller the impact multiplier; in particular for $\phi_\pi \to \infty$ it will take on the value derived above under $\pi_{T+1} = 0$.

However, systematic monetary policy after lift-off can be chosen as to rule out the exponential dynamics associated with the eigenvalue λ_1, as argued by Cochrane (2015). In particular, we conclude our discussion by showing what would happen when monetary policy follows a constant money growth rule. In the case of the log-log money demand function assumed in our specification,

$$m_t = \gamma\,(y_t - g_t) - \frac{\beta^2}{1-\beta}\,(r_t - \overline{r}), \tag{44}$$

a constant money growth rule also delivers a constant nominal interest rate:

$$\gamma\mu_t = \pi_t + \gamma\Delta\,(y_t - g_t) = 0 = r_t - \overline{r}. \tag{45}$$

This is the monetary policy reaction function studied by Golosov and Lucas (2007), which would thus imply an unconditionally constant nominal interest rate, but in the case of a money growth shock.

Using the fact that $\pi_t = -\gamma\Delta\,(y_t - g_t)$, and substituting out the NKPC into the Euler equation we get the usual second-order difference equation in y_t:

$$\beta E_t\,(y_{t+1} - g_{t+1}) - (1+\beta+\kappa\sigma)\,(y_t - g_t) + (y_{t-1} - g_{t-1}) = \kappa\sigma\,(1-\Gamma)\,g_t, \tag{46}$$

which now has the following general solution *for any* t:

$$y_t = \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1} + a_2\lambda_2^t, \tag{47}$$

where again $0 < \lambda_1 < 1 < \lambda_2$, as derived above. Hence, for $a_2 = 0$ there will be a unique bounded solution, given by:

$$\begin{aligned} y_t &= \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1}, && (48) \\ \gamma_0^y - 1 &= -\frac{\kappa\sigma\,(1-\Gamma)}{\beta\,(1-\rho) + 1 - \lambda_1 + \kappa\sigma}, && (49) \\ \gamma_1^y &= -\frac{1-\beta\lambda_1\rho}{\beta\,(1-\rho) + 1 - \lambda_1 + \kappa\sigma}. && (50) \end{aligned}$$

Despite the fact that the nominal interest rate is constant, the impact multiplier, γ_0^y, although still larger than Γ is now always lower than 1, and decreasing in the slope of the Phillips curve κ. In particular, the impact multiplier converges to its flexible price counterpart Γ, for κ that diverges.[15]

The intuition for this result is straightforward: the monetary policy rule, despite implementing a constant nominal rate, is akin to a price-level target, i.e. $p_t = -\gamma\,(y_t - g_t)$. This entails a long-run real interest rate that is now much higher than under the interest rate rule considered in the previous section, in which an inflation target was assumed. Namely, since under this rule the price level is stationary, it is easy to show that

$$\gamma\,(y_0 - g_0) = E_0 \sum_{j=1}^{\infty} \pi_j = E_0 \left(\lim_{T\to\infty} p_{T+1} - p_0 \right) = -p_0, \qquad (51)$$

as $\lim_{T\to\infty} E_0 p_{T+1} = -\lim_{T\to\infty} E_0 \gamma\,(y_{T+1} - g_{T+1}) = 0$. The multiplier is then decreasing in the contemporaneous response of the price level, so that a steeper NKPC will always result in a smaller multiplier. Likewise, the more "flexible" are prices across price setting models, the lower is the multiplier.[16]

To summarize, a key driver of the stimulus effect of government spending (and of shock propagation with sticky prices) is the response of the long-run real interest rate. When the monetary policy rule is such that the nominal interest rate reacts to endogenous variables, e.g. as in the case of a standard Taylor rule, the real interest rate response will be a function of the current and future effects of

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[15] Cochrane (2015) obtains a similar result under a ZLB of deterministic duration, rather than for the case of a constant interest rate. But the key property of the assumed monetary policy after lift-off from the ZLB is the same. Namely, monetary policy must be such that the price level goes back to its initial value, before the onset of the ZLB.

[16] Likewise, in the case of monetary forward guidance, no amplification nor paradox would arise if the monetary expansion is such that the price level increase is only temporary.

government spending. To the extent that the interest rate responds to current inflation, the steeper the Phillips Curve (the more flexible prices), the smaller the decrease in the real interest rate. This is because the response to current inflation ($\phi_\pi > 1$) necessary to ensure determinacy is larger than the persistence of inflation expectations.

Conversely, when the monetary policy rule is such that the interest rate does not (or cannot under the ZLB) respond to endogenous variables, and is thus constant, as is the case under conditions studied above, the real interest rate is purely a function of expected inflation. While the monetary policy reaction function will determine whether the price level is stationary or not, the relation between current and future prices will be shaped by the model of price setting. The Calvo–Yun model implies a very tight and specific relation between the two concepts of inflation: this is related to the concept of reset inflation introduced by Bils et al. (2009, 2012). However, this property of reset inflation and of the process of expected inflation does not seem a general one that would apply to other, more plausible models of price stickiness, with relevant consequences for the size of the multiplier. This is what we study in the rest of the paper focusing on the case of government spending shocks.

5 Simulation results under state-dependent pricing

Price setting and the spending multiplier under a Taylor rule We start by looking at the effects of a government spending shock under a standard interest rate rule. The expenditure shock propagates through the usual channel in New Keynesian models: it raises inflation and inflation expectations. Given an active Taylor rule this increases the (long-run) real interest rate faced by households and consumption drops.[17]

The results are presented in Figure 2. The figure plots the responses to an AR(1) government spending shock with $\rho = 0.9$, under a rule that only responds to current inflation with $\phi_\pi = 2$, of nine variables: the government spending process, consumption, GDP, the price level, the nominal interest rate, the short-term real interest rate (all annualized), the fraction of adjusting firms, the “selection

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[17] While the effects of a government spending shock have been analyzed in time-dependent models such as Calvo (Christiano et. al., 2011) or pre-set prices (Woodford, 2011), our interest here is in how the fiscal multiplier changes if pricing is instead state-dependent.

effect", computed as in Costain and Nakov (2011), and the fiscal multiplier. The red lines with dots plot the responses with Calvo pricing, the green lines with circles plot those under fixed menu costs, and the blue lines with squares plot those under the SSDP model. The shock to government spending is scaled so that on impact, government expenditure increases by 0.1% of GDP.

In all three models the government spending shock raises inflation and the real interest rate, and depresses consumption. However, some quantitative differences emerge when looking across the three models. For instance, the price level rises by a cumulative 0.8% in the fixed menu cost model, while it rises only by 0.5% in the Calvo model. Under the SSDP model the rise in the price level is in-between the Calvo and the FMC models. The difference in the behavior of the price level is just the "selection effect" emphasized by Golosov-Lucas (2007): in the menu cost model the firms that adjust are those for which adjustment is most valuable, they tend to make large price changes and the aggregate price level is more flexible than in the Calvo model. Importantly, this effect operates even when the average fraction of adjusting firms remains virtually constant across models. This is visible in the last row, showing that the selection effect contributes much to inflation in the menu cost model, while the fraction of adjusting firms barely moves given the monetary policy rule. [18]

Given the active Taylor rule, the stronger rise of the real interest rate means that consumption falls by more under fixed menu costs than it does under Calvo pricing. As a result, the impact multiplier in the menu cost model is closest to the flexible-price benchmark of $\Gamma = 2/3$. Under Calvo, although it is still less than one (as anticipated in the previous section), the fiscal multiplier exceeds the flexible price counterpart. Overall, these experiments confirm that under a standard Taylor rule the endogenous response of inflation is larger in the state-dependent model, leading to smaller real effects of shocks and that the fiscal multiplier is less than one. We now turn to the case of a constant interest rate, for which the paradox of flexibility has been established under time-dependent pricing.

The multiplier under constant interest rates We report results under a constant interest rate, which we assume is anticipated not to react to the shock for 24 months. In Figure 3, the blue lines with squares plot the responses under SSDP, and the red lines with dots plot those under Calvo

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[18] Note that the selection effect is absent by construction in the Calvo model.

pricing. Figure 4 plots the responses under fixed menu costs in green lines with circles. The shock to government spending is again scaled so that on impact, government expenditure increases by 0.1% of GDP.

Across all models, the fact that monetary policy does not react produces a persistent expansionary effect. This is consistent with the nominal interest rate not increasing with the rise in expected future inflation, so that the *long-run* ex-ante real interest rate drops in response to the shock. The result is that consumption rises and thus the multiplier exceeds unity in all three models. Strikingly, the fixed menu cost model now delivers a much larger multiplier than the Calvo model, an order of magnitude larger. Obviously this is reflected in a very large response of inflation and in a dramatic fall in the short term real rate. Intuitively, under a constant interest rate, more flexible prices lead to a larger response of the expected price level upon exit. As we have shown in the previous section, contrary to the stochastic ZLB case analyzed by Woodford (2011), when the interest rate is kept constant as in our experiment, an equilibrium is defined for any admissible flexibility of prices. Under fixed menu costs, the effects of a constant interest rate are much bigger than under Calvo pricing with the same average frequency of price adjustment. The "paradox of flexibility" thus holds under state-dependent pricing and accounts for these very large responses.

To see that the reason is indeed a strengthening of the "selection effect" in the sense of Golosov and Lucas (2007), we decompose the inflation response. To construct the decomposition, define the conditionally optimal price level $p_t^{*k} \equiv \text{argmax}_p v_t(p, a^k)$, and also $x_t^{*jk} \equiv \log(p_t^{*k}/p^j)$, the desired log price adjustment of a firm at time t with productivity a^k and real price p^j. We can then write the average desired adjustment as $\overline{x}_t^* = \sum_{j,k} x_t^{*jk} \widetilde{\Psi}_t^{jk}$, and write the fraction of firms adjusting as $\overline{\lambda}_t = \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}$.

Then inflation can be written as

$$\Pi_t = \sum_{j,k} x_t^{*jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}. \tag{52}$$

To a first-order approximation, we can decompose the deviation in inflation at time t as

$$\Delta\Pi_t = \overline{\lambda}\Delta\overline{x}_t^* + \overline{x}^*\Delta\overline{\lambda}_t + \Delta\sum_{j,k} x_t^{jk}(\lambda_t^{jk} - \overline{\lambda}_t)\widetilde{\Psi}_t^{jk}, \tag{53}$$

where terms without time subscripts represent steady states, and Δ represents a change relative to steady state.[19]

The "intensive margin", $\mathcal{I}_t \equiv \overline{\lambda}\Delta\overline{x}_t^*$, is the part of inflation due to changes in the average desired adjustment, holding fixed the fraction of firms adjusting. The "extensive margin", $\mathcal{E}_t \equiv \overline{x}^*\Delta\overline{\lambda}_t$, is the part due to changes in the fraction adjusting, assuming the average desired change among those who adjust equals the steady-state average in the whole population. The "selection effect", $\mathcal{S}_t \equiv \Delta\sum_{j,k} x_t^{jk}(\lambda_t^{jk} - \overline{\lambda}_t)\widetilde{\Psi}_t^{jk}$, is the inflation caused by redistributing adjustment opportunities from firms desiring small (or negative) price adjustments to firms desiring large (positive) adjustments, while fixing the total number adjusting. The middle panel in the last row of the figure reports the selection effect. We see that, indeed, the spike of inflation on impact in the FMC and SSDP models is a selection effect.

In fact, a very large multiplier is obtained also under the "intermediate" smoothly state-dependent pricing model (SSDP). Notice that in this model the response of inflation is larger than under Calvo pricing, and more persistent than in the fixed menu cost model, in which there is a large inflation spike on impact, that, however, dies out very quickly. This is because the SSDP model, due to its relatively smoother nature (that is, lower degree of state-dependence, see Costain and Nakov, 2011), exhibits inflation persistence similar to the Calvo model. This can been seen from the fact that the fraction of adjusting firms rises by much less than in the fixed menu cost (FMC) model. At the same time the SSDP model does feature an important selection effect, similar in nature to the FMC model. So the fall in the long-term real interest rate is quite strong in the SSDP model and the resulting fiscal multiplier is also large.

We also look at the case in which the interest rate is kept constant for 36 months, reported in Figure 5. From our analytical results in the previous section, in the simple Calvo framework the

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[19] See Costain and Nakov (2011B) for further discussion of this decomposition.

multiplier is increasing in the number of periods T over which monetary policy is unresponsive to the shock. This expectation is confirmed in the Calvo model with idiosyncratic shocks and in the SSDP model, but not in the FMC. In the latter the multiplier although still very large, is now smaller than its counterpart in Figure 4, and even smaller than that resulting from the SSDP model. This result stems from the fact that, as argued above, in the FMC model inflation is much less persistent than in the SSDP model. Therefore, the expected price level quickly stops increasing, and over a sufficiently long horizon, it can result in a lower expected inflation than in the SSDP model.

6 Conclusion

We have studied the effects of state-dependent pricing on the amplification of shocks under a constant interest rate, focusing on the government spending multiplier. Since the type of price setting by firms is crucial for the determination of the long-term real interest rate in the workhorse DSGE model, it is a primary determinant of the size of the spending multiplier. To have a multiplier substantially bigger than unity under a standard interest rate rule, both monetary policy has to be accommodative, and the aggregate price level has to be sufficiently sticky. Conversely, when the nominal interest is constant or at the zero lower bound, if the monetary policy is such that the price level has a positive drift, a paradox of flexibility arises: the multiplier, and thus shock amplification, is increasing in the degree of price flexibility. We thus find much larger multipliers in state-dependent models than under Calvo pricing, when the nominal interest rate is constant, for the same frequency of price adjustment.

In concluding, it is however important to remember that our results concern the size of policy multipliers in the workhorse New Keynesian model, which is the core of DSGE models also used to inform policy debates, but do not consider the quantitative importance of the many additional factors in the latter. These other factors, such as distortionary taxes, credit and financial constraints for households and firms, wage rigidities, information frictions, and many others, are important for specific quantitative answers and for a full welfare evaluation. Their role in a model with state-dependent pricing is left to future research.

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Appendix: figures

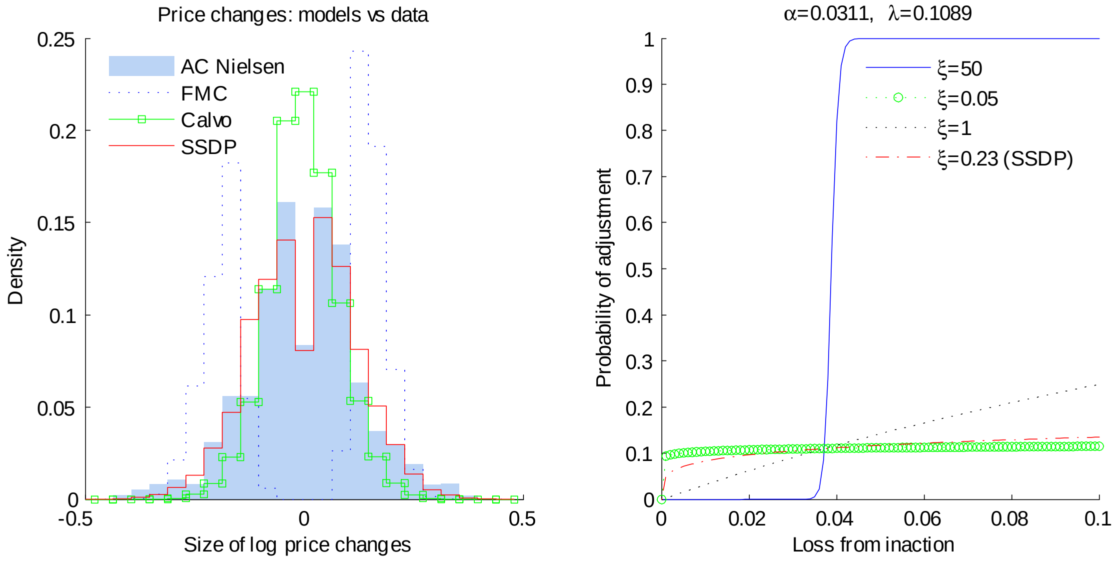

Fig.1: Adjustment probability functions and histograms of price changes

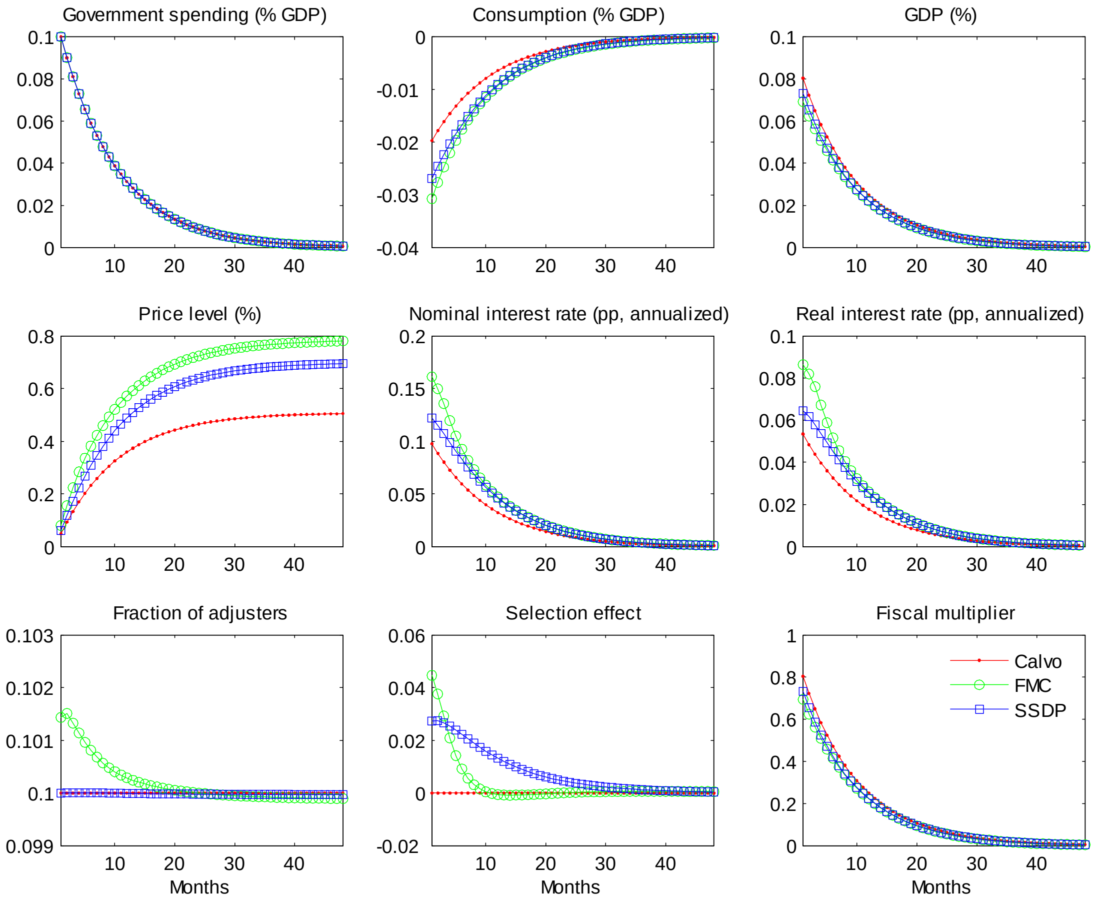

Fig.2: Responses to a government spending shock under a Taylor rule

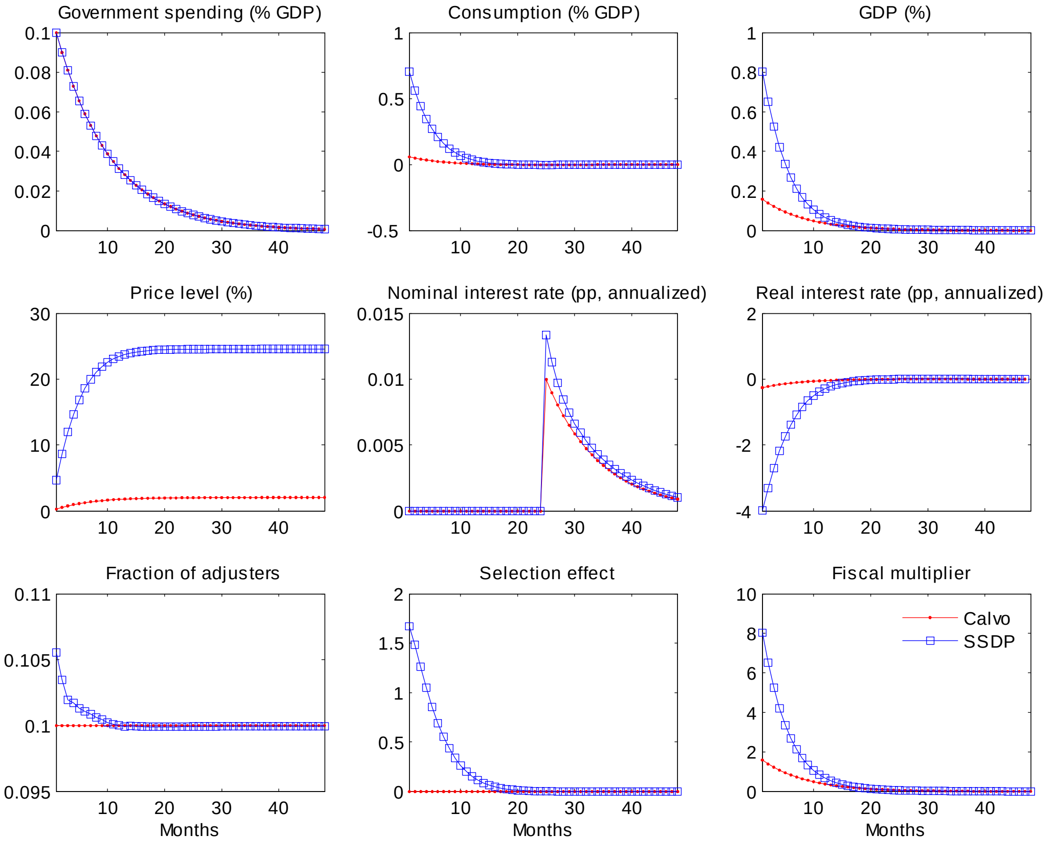

Fig.3: Responses to a government spending shock under a constant interest rate (Calvo, SSDP)

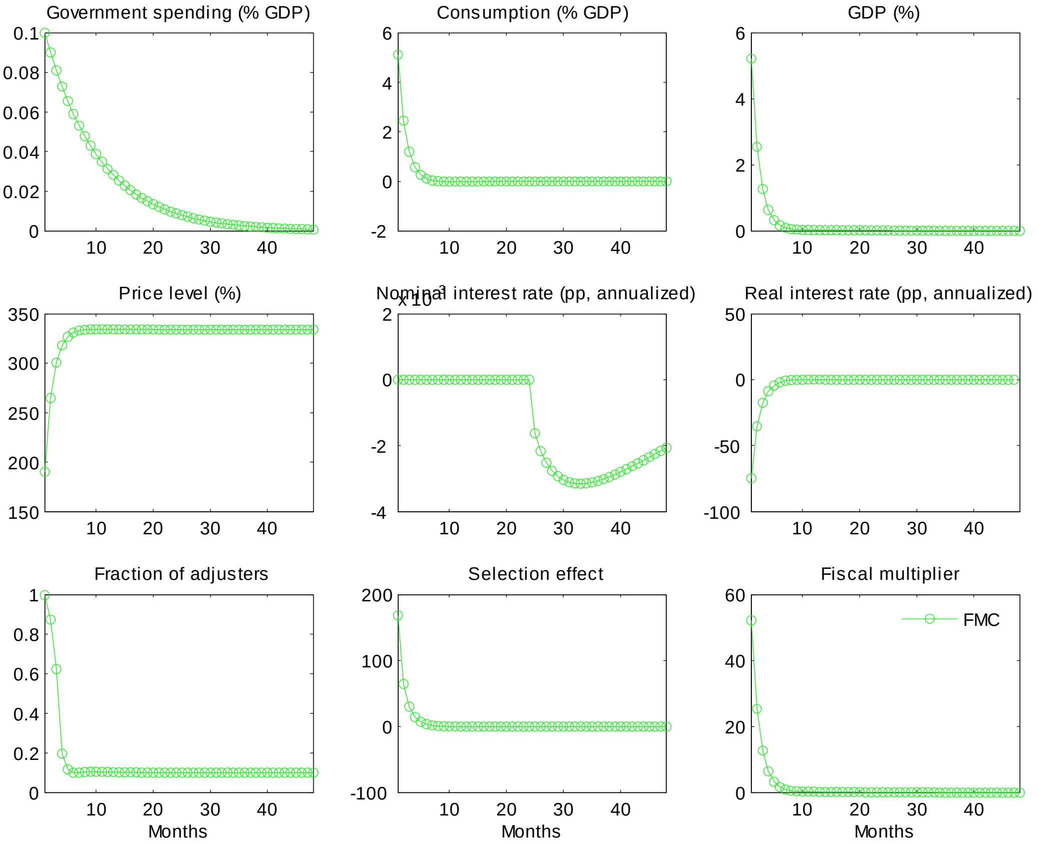

Fig.4: Responses to a government spending shock under a constant interest rate (FMC)

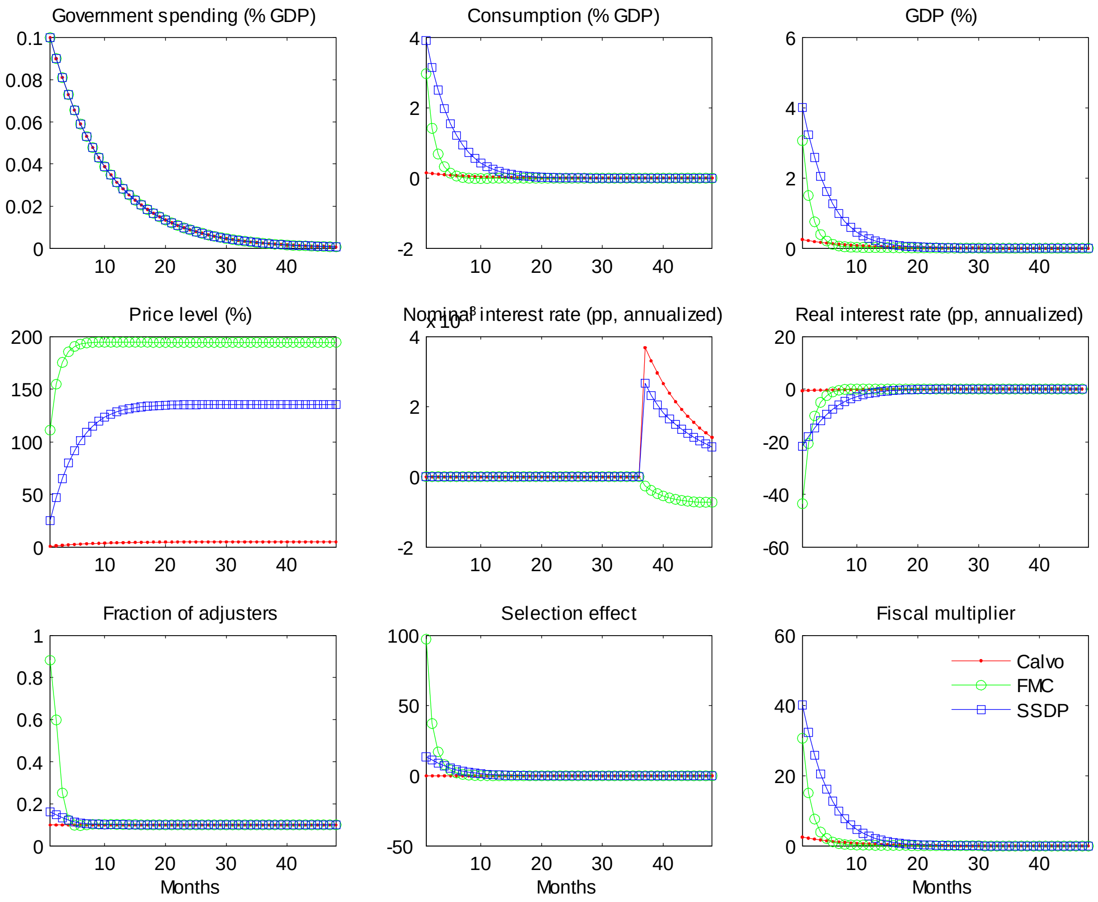

Fig.5: Responses to a government spending shock under a constant interest rate for 3 years