Debt-Ridden Borrowers and Productivity Slowdown *

Keiichiro Kobayashi† and Daichi Shirai‡

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Abstract

Economic growth slows down for extended periods after a financial crisis, and now there is a growing concern that "secular stagnation" may follow the Great Recession of 2007–2009. We construct a model in which a one-time buildup of debt can depress the economy persistently even when there is no shock on financial technology. We explicitly consider the debt dynamics in which the initial debt is large and is repaid in multiple or infinite periods. Productive firms are subject to a borrowing constraint and repay debt within finite periods when the initial debt is small whereas when the initial debt hits a certain threshold they fall into the "debt-ridden" state where they can pay only the interest and the amount of debt and inefficiency of production stay high permanently. In the general equilibrium with endogenous growth, the mass emergence of debt-ridden firms tightens borrowing constraints not only for themselves but also for normal firms, and it may manifest itself as the "financial shocks" discussed in recent macroeconomic literature. Tightening of aggregate borrowing constraints lowers the aggregate productivity and deteriorates the labor wedge, leading to a persistent recession. This model implies that debt reduction for overly indebted agents may restore economic growth quickly in the aftermath of financial crises.

Keywords: Endogenous borrowing constraint, financial shocks, labor wedge, secular stagnation.

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†Faculty of Economics, Keio University; CIGS; and RIETI. Email: keiichirokbys@gmail.com

‡The Canon Institute for Global Studies. Email: shirai.daichi@gmail.com

1 Introduction

The decade after a financial crisis tends to be associated with low economic growth (Reinhart and Rogoff, 2009; Reinhart and Reinhart, 2010). It is also known that the growth of total factor productivity slows down or even becomes negative for a decade (Kehoe and Prescott, 2007). Relatedly, there is a growing concern about "secular stagnation" in the aftermath of the Great Recession that the US economy and/or the European economy may stagnate persistently (Summers, 2013; Eggertsson and Mehrotra, 2014). It is also pointed out that financial constraints were tightened during and after the Great Recession in 2007–2009. It is a natural question whether tightening of financial constraints can cause a persistent slowdown in economic growth. We propose a theoretical model, in which the emergence of debt-ridden borrowers lowers the aggregate productivity persistently.

We consider that an exogenous shock makes a substantial number of firms become to bear a huge stock of debt, whereas there is no change in structural parameters in production or financial technologies. The shock we consider can be understood as a redistribution shock. We consider the debt dynamics in which the huge stock of debt is repaid in multiple or possibly infinite number of periods. The model is based on Jermann and Quadrini (2006, 2012), which explicitly consider the bargaining process after a default to derive a borrowing constraint. We slightly modify the setting of the lender-borrower relationship in Jermann and Quadrini (2012), and show that the borrowers repay debt within finite periods when the amount of the initial debt is small whereas when the initial debt hits a certain threshold they fall into the "debt-ridden" state where they can pay only the interest every period and the remaining amount of debt stays high forever. As the borrowing constraints are tightened debt-ridden firms cannot raise sufficient amount of working capital and they continue inefficient production permanently. It is shown in the general equilibrium that borrowing constraints become tighter also for normal firms if the measure of debt-ridden firms is larger. If a substantial number of firms become debt-ridden, both the aggregate borrowing capacity and productivity decline persistently. After the Great Recession in 2007–2009, many authors argue that a shock in the financial sector can cause a severe recession (e.g., a risk shock in Christiano, Motto and Rostagno 2014, and a financial shock in Jermann and Quadrini 2012). In our model, emergence of a substantial number of debt-ridden firms manifests itself as a tightening of the aggregate borrowing constraint, which can be interpreted as a financial shock. Tighter borrowing constraint discourages R&D activity by the firms and makes the productivity growth persistently low. It also deteriorates the labor wedge persistently. These features of our model look consistent with the facts observed in persistent recessions after financial crises (see Section 2).

Our model has a policy implication distinct from those of the models of exogenous financial shocks that debt restructuring or debt forgiveness for overly indebted borrowers may restore the aggregate efficiency and enhance economic growth as the mass emergence of debt-ridden borrowers can cause persistent stagnation. If on the contrary the risk shock or the financial shock were exogenous, debt restructuring would not have any aggregate effects.

Related literature Our theory is related to the literature on debt overhang. Myers (1977) pointed out the suboptimality of debt in the corporate finance literature and Lamont (1995) applied the notion of debt overhang in macroeconomics.[1] The debt overhang problem typically occurs when a firm cannot borrow new money for productive projects because its existing debt is too large. Debt overhang arises if the existing debt holder is different from the potential lender who would lend new money. In this paper, we take a small step forward by proposing a new theory that inefficiency can arise even if the lender of new money *is* the existing debt holder. This paper is also close to Caballero, Hoshi and Kashyap (2008). They analyze "zombie lending," defined as a de facto subsidy to unproductive firms from banks. They argue that congesting zombie firms hinder the entry of highly productive firms and lower aggregate productivity. In this paper, we make a complementary point to their argument: even an intrinsically productive firm can become unproductive when it is debt-ridden. Our model is closely related to that in Kobayashi and Nakajima (2015), which analyzes endogenous borrowing constraints and nonperforming loans. In the macroeconomic literature, exogenous borrowing constraints are introduced by the seminal papers by Kiyotaki and Moore (1997). Endogenous borrowing constraints are introduced by Albuquerque and Hopenhayn (2004), and analyzed in the general equilibrium setting by Cooley, Marimon and Quadrini (2004). Different versions of endogenous borrowing constraints are studied by Jermann and Quadrini (2006, 2007, 2012). The modeling method in Kobayashi and Nakajima (2015) and this paper is mostly close to Jermann and Quadrini (2012), in which the lender can fully commit to the amount of debt, while she cannot commit to liquidate the borrower-firm when it defaults.[2] Persistent recession in the aftermath of a financial crisis is explained by Guerron-Quintana and Jinnai (2014). Our model is also close to theirs in that a temporary shock affects productivity growth permanently, though there is a stark difference in policy implications: as in our model emergence of debt-ridden borrowers due to a temporary redistribution shock causes persistent recession, debt restructuring or redistribution from creditors to

[1] See also Krugman (1988) on debt overhang in international finance.

[2] The latest model in the literature of endogenous borrowing constraints is Kovrijnykh (2013), who assumes the partial commitment that the lender cannot fully commit to the amount of debt, while she assumes that the lender can commit to liquidate the borrower-firm when it defaults. Our conjecture is that introduction of partial commitment to our model may strengthen our results, but not weaken them.

borrowers restores the aggregate efficiency, whereas debt restructuring has no effect in Guerron-Quintana and Jinnai because the financial shock is an exogenous technological shock in their model.[3] Another paper that is closely related to ours is Ikeda and Kurozumi (2014). They build a medium scale DSGE model with financial friction à la Jermann and Quadrini (2012) and endogenous productivity growth à la Comin and Gertler (2006). Distinction from ours is that Ikeda and Kurozumi (2014) also posit a financial crisis as an exogenous financial shock.

This paper is organized as follows. In the next section, we review the facts on persistent recessions after financial crises. Section 3 presents the partial-equilibrium model of lender-borrower relationship and analyze the financial contract. In Section 4, we construct the full model by embedding the model of the previous section into an endogenous growth model, and show that stagnation can continue persistently when there emerge substantial number of debt-ridden borrowers. In this section we also show that the model can account for the productivity slowdown in Japan during the last twenty years. Section 5 presents our concluding remarks.

2 Facts on persistent recessions after financial crises

Numerous examples of decade-long stagnation after a financial crisis have been observed. The most notable episode is the Great Depression in the 1930s in the US and similar depressions in that period in the major nations. Ohanian (2001) shows that there was a large productivity decline during the US Great Depression that is unexplained by capital utilization or labor hoarding. Kehoe and Prescott (2007) drew our attention to the fact that many countries experienced decade-long severe recessions, which they call the "great depressions" of the twentieth century. Papers in their book unanimously emphasize that declines in the growth rate of total factor productivity were the primary cause of these great depressions. Another example of decade-long recession is the 1990s in Japan. The growth rates of gross domestic product (GDP) and total factor productivity (TFP) in the 1990s were both lower than in the 1980s. Figure 1 shows the actual GDP and the potential GDP in Japan. The kink in the beginning of the 1990s is apparent, when huge asset-price bubbles burst in the stock and real estate markets. See Figure 2 for asset prices in Japan in the 1990s. Table 1 shows various estimates of the TFP growth rate in Japan. Hayashi and Prescott (2002) emphasize that the growth of TFP slowed down in the 1990s. Fukao and Miyagawa (2008) estimate TFP using a microeconomic dataset, called Japan Industrial Productivity (JIP) database, and confirm the substantial TFP slowdown in the 1990s.

[3] Kobayashi and Shirai (2015) analyze the effects of redistribution of wealth on the economy using an exogenous borrowing constraint model.

One notable feature in the 1990s in Japan is the significant decrease in entries and increase in exits of firms. See Figure 3 for a comparison of entry and exit of firms between Japan and the US. In the literature, the procyclicality of net entry is well known (Bilbiie, Ghironi and Melitz, 2012). Net entry also contributes significantly to TFP growth for US manufacturing establishments (Bartelsman and Doms, 2000).[4]

Another characteristic of Japan in the 1990s that may be related to productivity slowdown was the persistently lingering nonperforming loans (NPLs) in the banking sector. The NPLs were the excess debt of nonfinancial firms, mainly in real estate, wholesale, retail, and construction. Figure 4 shows the amount of NPLs in Japan from 1992 through 2009. The delayed disposal of huge NPLs was seen as a de facto subsidy to nonviable firms ("zombie lending"). This zombie lending has also been named as the cause of Japan's persistent recession (Peek and Rosengren 2005, and Caballero, Hoshi, and Kashyap 2008).

Recently, there is a growing literature on business cycle accounting (BCA; Chari, Kehoe and McGrattan 2007) that analyze various episodes of business fluctuations including decade-long stagnations. The BCA focuses on four wedges as the driving forces of business cycles: the efficiency wedge (EW), the labor wedge (LW), the investment wedge (IW), and the government wedge (GW). EW is the observed total factor productivity; LW is MRS/MPL, where MRS is the marginal rate of substitution between consumption and leisure and MPL is the marginal product of labor; IW is the wedge between the market rate of interest and the stochastic discount factor; and GW is the dead weight loss, which manifests itself as government consumption in a simple real business cycle model. Chari, Kehoe and McGrattan (2007) note that deteriorations of the efficiency wedge and the labor wedge are the two primal factors that account for the Great Depression of the 1930s. Kobayashi and Inaba (2006) and Otsu (2011) stress on the same factors for the lost decade of Japan in the 1990s. Macroeconomic literature has recently focused considerable attention on the effects of deterioration of the labor wedge in recessions (see Mulligan, 2002; Shimer, 2009). A sharp decline in the labor wedge was also observed in the US economy during the Great Recession period of 2008–2010 (Pescatori and Tasci, 2011).

3 Model of debt dynamics

In this section we consider the partial equilibrium model of a lender (a bank) and a borrower (a firm). We derive the borrowing constraint and analyze the debt dynamics under the exogenously given prices. We embed this model into the endogenous growth model in Section 4.

[4] Nishimura, Nakajima and Kiyota (2005) argue that the malfunctioning of entry and exit contributes substantially to a fall in Japan's TFP in the late 1990s.

3.1 Set up

The time is discrete and continues from 0 to infinity: $t = 0, 1, 2, \cdots, \infty$. There are three agents in this model: the bank (lender), the firm (borrower), and the household (worker). The main players are the bank and the firm, and the household just supplies labor at the market wage rate, w_t, and buys the consumer goods from the firm. The consumer goods is produced by the firm from the labor input. We focus on the case where there exists the initial debt stock $(1 + r_0)d_{-1}$ at $t = 0$, where d_{-1} is to be taken as debt outstanding at the end of the previous period.

Suppose that the firm owes d_t to the bank at the end of period t and it holds the internal reserve a_t.[5] The reserve is invested in safe asset.[6] The debt d_t is observable and verifiable, and the bank can commit to d_t, meaning that the bank can legitimately require the firm to repay arbitrary amount, which is no greater than d_t. The internal reserve a_t is subject to moral hazard by the firm employees, who can use the $(1 - \theta)$ fraction of the reserve for their private purposes, which do not enhance the corporate value of the firm. To model this moral hazard, we assume that $(1 - \theta)a_t$ is just lost and only the remaining θa_t can be used for the business of the firm, where $0 < \theta < 1$.[7] The debt and the asset evolve from period t to period $t+1$ at the market rate of interest, r_{t+1}. Thus the amounts of debt and asset at the beginning of period $t + 1$ are $(1 + r_{t+1})d_t$ and $(1 + r_{t+1})\theta a_t$, respectively.

In period $t+1$, the firm employ labor l_{t+1} from the household and produce the consumer goods, $y_{t+1} = Al_{t+1}^{\eta}$, where $0 < \eta < 1$. The cost of labor input for the firm is $w_{t+1}l_{t+1}$, where w_{t+1} is the market rate of wage. The firm needs to borrow $w_{t+1}l_{t+1}$ from the bank and pay the wage to the worker in advance of production. Now we assume that the prices $\{r_t, w_t\}_{t=0}^{\infty}$ are given. When the firm completes production, it holds output, Al_{t+1}^{η}, and asset, $(1 + r_{t+1})\theta a_t$, and owes $(1 + r_{t+1})d_t + w_{t+1}l_{t+1}$ to the bank. At this stage, the firm decides to pay $b_{t+1} + w_{t+1}l_{t+1}$ to the bank during period $t + 1$, and to carry over the end-of-period debt stock, d_{t+1}. After the firm and the bank agree on $(b_{t+1},\ d_{t+1})$, the firm pays $w_{t+1}l_{t+1} + b_{t+1}$ to the bank and holds the remaining debt, d_{t+1}. Then the remaining cash flow, $Al_{t+1}^{\eta} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t$, is divided into the internal reserve, a_{t+1}, and the dividend to the firm-owner, $Al_{t+1}^{\eta} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t - a_{t+1}$. After the payment of period $t + 1$ is done, the firm and the corresponding variety of the intermediate good die with probability ρ. So the firm survives and moves on to period $t + 2$ with probability $1 - \rho$. If the firm dies with probability ρ, all the internal reserve

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[5] As we discuss in the following, the firm is subject to the borrowing constraint and earns a positive cash flow every period, which is divided into the dividend to the firm-owner and the internal reserve.

[6] In the general equilibrium model in Section 4, a_t is invested in the physical capital.

[7] In the general equilibrium model of Section 4, we assume that $(1 - \theta)a_t$ is directly given to the representative household as a lump-sum transfer.

a_{t+1} is given to the firm-owner and the remaining debt d_{t+1} is automatically defaulted. Thus, the risk of sudden death imposes the following constraint on $(b_{t+1},\ d_{t+1})$ because the bank agrees only if the expected value of repayment is no less than $(1+r_{t+1})d_t$:

$$(1+r_{t+1})d_t \leq b_{t+1} + (1-\rho)d_{t+1}. \tag{1}$$

The payment is subject to the borrowing constraint:

$$w_{t+1}l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^{\eta} + (1+r_{t+1})\theta a_t + (1-\rho)\max\{\xi V_{nt+1} - d_{t+1},\ 0\}, \tag{2}$$

which will be derived in the next subsection.

Now we can describe the optimization problem for the firm. Denoting the value of the firm with asset a_t and debt stock d_t by $V_t(a_t, d_t)$, the firm's problem is written as the following Bellman equation:

$$V_t(a_t, d_t) = \max_{l_{t+1}, a_{t+1}, b_{t+1}} \frac{1}{1+r_{t+1}} \left[A l_{t+1}^{\eta} - w_{t+1}l_{t+1} - b_{t+1} + (1+r_{t+1})\theta a_t - a_{t+1} + (1-\rho)V_{t+1}(a_{t+1}, d_{t+1}) \right], \tag{3}$$

subject to the law of motion for debt, (1), the borrowing constraint, (2), and

$$\begin{aligned} &a_{t+1} \in \{0\} \cup \Gamma, \\ &\text{where} \quad \Gamma = \{a |\ 0 \leq a \leq A l_{t+1}^{\eta} - w_{t+1}l_{t+1} - b_{t+1} + (1+r_{t+1})\theta a_t\}. \end{aligned} \tag{4}$$

We use the last constraint because we allow the cash flow, $A l_{t+1}^{\eta} - w_{t+1}l_{t+1} - b_{t+1} + (1+r_{t+1})\theta a_t$, to be negative when $a_{t+1} = 0$.

3.2 Endogenous borrowing constraint

In this subsection, we describe the financial contract between the bank and the firm, and derive the borrowing constraint, (2).

3.2.1 Counterfactual default and renegotiation

As we described in the previous subsection, the firm pays $w_{t+1}l_{t+1} + b_{t+1}$ at the end of period $t+1$. We assume that the firm can choose to default on this payment and if it defaults the renegotiation on the amount of payment takes place at the end of period $t+1$. Suppose counterfactually that the firm defaults on $w_{t+1}l_{t+1} + b_{t+1}$. Note that when the firm defaults on the short-term debt it does not necessarily default on the long-term debt, d_{t+1}. The firm and the bank renegotiate on the repayment at $t+1$, given that they still have the agreement of the long-term debt, d_{t+1}. We also assume that if the bank liquidates the firm at the end of period $t+1$, she can recover the liquidation value, $(1-\rho)\xi V_{nt+1}$, where $0 < \xi < 1$ and V_{nt+1} is the value of the firm who does not owe any debt, which is specified in Section 3.2.3. Only when the firm is liquidated, the long-term debt, d_{t+1}, is finally defaulted. The liquidation value is $(1-\rho)\xi V_{nt+1}$ because the bank takes over the firm's business when she liquidates it, and successfully operates it by herself with probability ξ; if the bank successfully operates the firm by herself, the bank obtains V_{nt+1}; therefore, the expected value is ξV_{nt+1}. Although default is an event in off-the-equilibrium path, we describe what follows the default and derive the condition for the firm choosing not to default, which gives the borrowing constraint, (2).

Once the firm defaults on the payment of $w_{t+1}l_{t+1}+b_{t+1}$, the following events happen.

- The bank seizes the ϕ fraction of the firm's output and the internal reserve, $\phi A l_{t+1}^{\eta} + (1+r_{t+1})\theta a_t$, unconditionally, where $0<\phi<1$.
- Then, the bank and the firm start renegotiation on the amount f to pay instead of $w_{t+1}l_{t+1}+b_{t+1}$.
  - If they agree on f, the firm can continue to the next period. Thus, the firm's payoff is $(1-\phi)Al_{t+1}^{\eta} - f + (1-\rho)\{V_{t+1}(a_f, d_{t+1}) - a_f\}$, where a_f is the solution to

    $$\max_a -a + V_{t+1}(a, d_{t+1}),$$
    $$\text{s. t. } a \leq (1-\phi)Al_{t+1}^{\eta} - f,$$

    and the bank's payoff is $\phi A l_{t+1}^{\eta} + (1+r_{t+1})\theta a_t + f + (1-\rho)d_{t+1}$.
  - If they disagree on f, the bank can choose unilaterally whether to liquidate the firm or to forgive it.
    * If the bank liquidates the firm, it loses the continuation value. Thus the firm's payoff is $(1-\phi)Al_{t+1}^{\eta}$, and the bank's payoff is $\phi A l_{t+1}^{\eta} + (1+r_{t+1})\theta a_t + (1-\rho)\xi V_{nt+1}$.
    * If the bank forgives the firm, it can continue with remaining debt d_{t+1}. Thus the firm's payoff is $(1-\phi)Al_{t+1}^{\eta} + (1-\rho)\{V_{t+1}(a_f, d_{t+1}) - a_f\}$, and the bank's payoff is $\phi A l_{t+1}^{\eta} + (1+r_{t+1})\theta a_t + (1-\rho)d_{t+1}$.

Note that this renegotiation process after the counterfactual default is almost identical to that in Jermann and Quadrini (2012) if we assume $\phi = 0$.

3.2.2 Derivation of the borrowing constraint

The borrowing constraint is given by the condition that the bank is willing to accept $w_{t+1}l_{t+1}+b_{t+1}$. The bank accept this amount when either she is indifferent to the default because she can obtain no less than $w_{t+1}l_{t+1}+b_{t+1}$ when the firm defaults (the indifference condition) or the firm has no incentive to default (the no-default condition). We can derive the borrowing constraint by analyzing the renegotiation outcome. We follow Jermann and

Quadrini (2012) and assume for simplicity that the firm has all the bargaining power. Here we define $\hat{a}_0$ as a_f in the case where $f = 0$, and $\hat{a}_1$ as a_f in the case where $f = (1-\rho)(\xi V_{nt+1} - d_{t+1})$. Note that both the firm and the bank take $\{d_{t+1}, \xi V_{nt+1}, V(\hat{a}_0, d_{t+1}) - \hat{a}_0, V(\hat{a}_1, d_{t+1}) - \hat{a}_1\}$ as given in the renegotiation. There are two cases of renegotiation outcome, corresponding to different values of $\{d_{t+1}, \xi V_{nt+1}, V(\hat{a}_0, d_{t+1}) - \hat{a}_0, V(\hat{a}_1, d_{t+1}) - \hat{a}_1\}$.

Case 1: $d_{t+1} > \xi V_{nt+1}$. The bank always forgives the firm, when the renegotiation breaks down. The renegotiation outcome is that they agree that $f = 0$ and the firm continues, because the firm has all the bargaining power. Thus the payoff for the firm is $(1-\phi)Al^{\eta}_{t+1} - \hat{a}_0 + (1-\rho)V(\hat{a}_0, d_{t+1})$ if it defaults, and is $Al^{\eta}_{t+1} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t + (1-\rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\}$ if it does not default. If $w_{t+1}l_{t+1} + b_{t+1} \leq \phi Al^{\eta}_{t+1} + (1 + r_{t+1})\theta a_t$, then the bank obtains no less than $w_{t+1}l_{t+1} + b_{t+1}$ when the firm defaults. So in this case the indifference condition is satisfied, and it is easily proven that the no-default condition is also satisfied.[8] Suppose that $w_{t+1}l_{t+1} + b_{t+1} > \phi Al^{\eta}_{t+1} + (1 + r_{t+1})\theta a_t$. In this case, the bank strictly prefers no default. But the no-default condition for the firm is violated because the firm's payoff when defaults is strictly larger than when it does not default.[9] Therefore, the bank agrees on $w_{t+1}l_{t+1} + b_{t+1}$ only if the following constraint is satisfied:

$$w_{t+1}l_{t+1} + b_{t+1} \leq \phi Al^{\eta}_{t+1} + (1 + r_{t+1})\theta a_t.$$

Case 2: $\xi V_{nt+1} - \{V(\hat{a}_1, d_{t+1}) - \hat{a}_1\} \leq d_{t+1} \leq \xi V_{nt+1}$. The bank always liquidate the firm, when the renegotiation breaks down. On this premise, the renegotiation outcome is that they agree that $f = (1-\rho)\{\xi V_{nt+1} - d_{t+1}\}$ because the firm makes this offer in order to equalize the bank's payoff in the case of agreement (i.e., $\phi Al^{\eta}_{t+1} + (1+r_{t+1})\theta a_t + f + (1-\rho)d_{t+1}$) with that in the case of disagreement (i.e., $\phi Al^{\eta}_{t+1} + (1+r_{t+1})\theta a_t + (1-\rho)\xi V_{nt+1}$). The payoff for the firm is $(1-\phi)Al^{\eta}_{t+1} + (1-\rho)\{-\xi V_{nt+1} + d_{t+1} + V(\hat{a}_1, d_{t+1}) - \hat{a}_1\}$ if it defaults, and is $Al^{\eta}_{t+1} - w_{t+1}l_{t+1} - b_{t+1} + (1-\rho)\{V_{t+1}(a_{t+1}, d_{t+1}) - a_{t+1}\}$ if it does not

[8] Note that if $w_{t+1}l_{t+1} + b_{t+1} \leq \phi Al^{\eta}_{t+1} + (1 + r_{t+1})\theta a_t$, then $V(a_{t+1}, d_{t+1}) - a_{t+1} \geq V(\hat{a}_0, d_{t+1}) - \hat{a}_0$ because a_{t+1} is the solution to

$$\max_{a} -a + V_{t+1}(a, d_{t+1}),$$
$$\text{s. t. } a \leq Al^{\eta}_{t+1} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t.$$

It is straightforward from this to show that the firm is better-off by choosing not to default.

[9] The firm's payoff when defaults is strictly larger than when it does not default, because

$$\begin{aligned}(1-\phi)Al^{\eta}_{t+1} + (1-\rho)\{V(\hat{a}_0, d_{t+1}) - \hat{a}_0\} &\geq (1-\phi)Al^{\eta}_{t+1} + (1-\rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\} \\ &> Al^{\eta}_{t+1} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t + (1-\rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\}.\end{aligned}$$

default. If $w_{t+1}l_{t+1}+b_{t+1} \leq \phi A l_{t+1}^{\eta}+(1+r_{t+1})\theta a_t+(1-\rho)\{\xi V_{nt+1}-d_{t+1}\}$, then the bank obtains no less than $w_{t+1}l_{t+1}+b_{t+1}$ when the firm defaults. So in this case the indifference condition is satisfied, and it is easily proven that the no-default condition for the firm is also satisfied. Suppose that $w_{t+1}l_{t+1}+b_{t+1} > \phi A l_{t+1}^{\eta}+(1+r_{t+1})\theta a_t+(1-\rho)\{\xi V_{nt+1}-d_{t+1}\}$. In this case, the bank strictly prefers no default. But the no-default condition for the firm is violated because the firm's payoff when defaults is strictly larger than when it does not default.[10] Therefore, the bank agrees on $w_{t+1}l_{t+1}+b_{t+1}$ only if the following borrowing constraint is satisfied:

$$w_{t+1}l_{t+1}+b_{t+1} \leq \phi A l_{t+1}^{\eta}+(1+r_{t+1})\theta a_t+(1-\rho)\{\xi V_{nt+1}-d_{t+1}\}.$$

This Case 2 borrowing constraint becomes almost identical to that in Jermann and Quadrini (2012), if we set $\phi=0$, $\xi=1$, and $0<\rho<1$.

Summing up these two cases, we can rewrite the borrowing constraint as (2).[11]

3.2.3 Characterization of the liquidation value

The liquidation value ξV_{nt+1} is characterized by the solution to the optimization problem for a firm with no debt stock:

$$V_{nt}=\max_{l_{t+1}} \frac{1}{1+r_{t+1}}\left[A l_{t+1}^{\eta}-w_{t+1}l_{t+1}+(1-\rho)V_{nt+1}\right],$$

subject to the borrowing constraint, $w_{t+1}l_{t+1} \leq \phi A l_{t+1}^{\eta}+(1-\rho)\xi V_{nt+1}$. This problem defines V_{nt} recursively. In the case where the prices are constant over time, i.e., $r_t=r$ and $w_t=w$ for all t, the value of V_{nt} is also a constant and characterized as follows, given that the borrowing constraint is nonbinding.[12]

$$V_n=\frac{(1-\eta)}{r+\rho} A l_n^{\eta},$$

where the labor is given by

$$l_n=\left(\frac{\eta A}{w}\right)^{\frac{1}{1-\eta}}.$$

[10] The firm's payoff when defaults is strictly larger than when it does not default, because

$$\begin{aligned}&(1-\phi)A l_{t+1}^{\eta}+(1-\rho)\{-\xi V_{nt+1}+d_{t+1}+V(\hat{a}_1,d_{t+1})-\hat{a}_1\}\\ &\quad\geq (1-\phi)A l_{t+1}^{\eta}+(1-\rho)\{-\xi V_{nt+1}+d_{t+1}+V(a_{t+1},d_{t+1})-a_{t+1}\}\\ &\quad> A l_{t+1}^{\eta}-w_{t+1}l_{t+1}-b_{t+1}+(1+r_{t+1})\theta a_t+(1-\rho)\{V(a_{t+1},d_{t+1})-a_{t+1}\}.\end{aligned}$$

[11] To be more precise, there may exist Case 3 where $d_{t+1}<\xi V_{nt+1}-V(\hat{a}_1,d_{t+1})+\hat{a}_1$. As we show in Appendix A, it turns out that $V(a_{t+1},d_{t+1})-a_{t+1}$ must be equal to $V_{nt+1}-d_{t+1}$ in Case 3, which contradicts to $d_{t+1}<\xi V_{nt+1}-V(a_{t+1},d_{t+1})+a_{t+1}$ as $\xi<1$, implying that Case 3 never happens.

[12] In the numerical simulation in Section 4, the borrowing constraint is binding even for the normal firms, given that they conduct not only production of output but also R&D activity.

3.3 Debt dynamics and emergence of debt-ridden firms

We derive debt dynamics in the case where prices are constant: $w_t = w$ and $r_t = r$ for all t. The results in this section can be easily generalized in the case where prices vary over time. The initial value of internal reserve is zero: $a_{-1} = 0$. We analyze the response of the economy to the exogenously given initial debt: $d_{-1} > 0$. Define $b_n = \phi A l_n^\eta - w l_n$ and $d_n = b_n/(r+\rho)$.

One noticeable observation is that when the borrowing constraint is not binding and thus the production is efficient, i.e., $l_t = l_n$, then the firm never chooses to accumulate the internal reserve, a_t, because $(1-\theta)a_t$ will be just lost. So if $l_t = l_n$, then $a_t = 0$ in equilibrium.

3.3.1 Debt dynamics in the case with a small ξ

Here we consider the case where the parameters satisfy

$$d_n > \xi V_n \qquad \text{or, equivalently,} \qquad \xi < \frac{\phi - \eta}{1-\eta}. \tag{5}$$

When the initial debt d_{-1} is small such that $d_{-1} < \frac{1}{1+r}[\phi A l_n^\eta - w l_n + (1-\rho)\xi V_n]$, the borrowing constraint is

$$w l_n + (1+r)d_{-1} \leq \phi A l_n^\eta + (1-\rho)\xi V_n,$$

and the production is always efficient: $l_t = l_n$ for all $t \geq 0$.[13]

When the initial debt is large, i.e., $\frac{1}{1+r}[\phi A l_n^\eta - w l_n + (1-\rho)\xi V_n] < d_{-1} \leq d_n$, there exists T (≥ 0) such that the borrowing constraint is $w l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^\eta$ for $-1 < t < T$, and is $w l_{t+1} + b_{t+1} + (1-\rho)d_{t+1} = w l_{t+1} + (1+r)d_t \leq \phi A l_{t+1}^\eta + (1-\rho)\xi V_n$ for $t \geq T$. The repayment is

$$b_t = b_n$$

for $0 \leq t \leq T$, and d_T satisfies that $d_T \leq \frac{1}{1+r}[\phi A l_n^\eta - w l_n + (1-\rho)\xi V_n]$. The value of T increases as the initial debt d_{-1} is closer to d_n, and $T = +\infty$ when $d_{-1} = d_n$. The production is efficient: $l_t = l_n$ for all $t \geq 0$ if $d_{-t} \leq d_n$.

When the initial debt satisfies that $d_n < d_{-1} \leq d_z$, where $d_z = \frac{b_z}{r+\rho}$, $b_z = [\phi + (1-\rho)(1-\phi)\theta]A l_z^\eta - w l_z$, and $l_z = [\{\phi + (1-\rho)(1-\phi)\theta\}\eta A/w]^{\frac{1}{1-\eta}}$ $(< l_n)$, the borrowing constraint is binding and therefore it is

$$w l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^\eta + (1+r)\theta a_t,$$

for all t. As is shown in Appendix B, the firm chooses $a_{t+1} = 0$ if $\mu_{t+1} < \theta^{-1} - 1$, a_{t+1} $(\in [0, A l_{t+1}^\eta - w l_{t+1} - b_{t+1} + (1+r)\theta a_t])$ is indeterminate if $\mu_{t+1} = \theta^{-1} - 1$, and

[13] The firm is indifferent whether to repay all debt or to repay partially because the value of the firm at $t = -1$ is $V_n - d_{-1}$ irrespective of the repayment path.

$a_{t+1} = Al_{t+1}^{\eta} - wl_{t+1} - b_{t+1} + (1+r)\theta a_t$ if $\mu_{t+1} > \theta^{-1} - 1$, where μ_{t+1} is the Lagrange multiplier associated with the borrowing constraint. In Appendix B, we also show that d_z is the maximum repayable debt.

The dynamics is given as follows. For a given value of d (> 0), we define l_d as the solution to $(1+r)d = \phi Al_d^{\eta} - wl_d + (1+r)(1-\phi)\theta Al_d^{\eta}\mathbf{1}(\mu - (\theta^{-1} - 1)) + (1-\rho)d_n$, where μ is the time-invariant Lagrange multiplier associated with the borrowing constraint that is determined by $(1+\mu)wl_d = [1 + \phi\mu + (1-\phi)\theta\mu\mathbf{1}(\mu - (\theta^{-1} - 1))]\eta Al_d^{\eta}$, and $\mathbf{1}(x)$ is a step function that is defined by

$$\mathbf{1}(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ 1 & \text{if } x > 0. \end{cases}$$

The first order conditions (FOCs) and the envelope conditions imply that the debt dynamics are given as follows. There exists a certain d and T such that[14]

$$V_d(a_t, d_t) = -(1+\mu), \quad \text{for} \quad -1 \leq t \leq T, \tag{6}$$

$$V_d(a_t, d_t) = -1, \quad \text{for} \quad t \geq T+1, \tag{7}$$

$$l_{t+1} = l_d, \quad \text{for} \quad -1 \leq t \leq T, \tag{8}$$

$$l_{t+1} = l_n, \quad \text{for} \quad t \geq T+1, \tag{9}$$

$$b_{t+1} = \phi Al_d^{\eta} - wl_d + (1+r)(1-\phi)\theta Al_d^{\eta}\mathbf{1}(\mu - (\theta^{-1} - 1)), \quad \text{for} \quad -1 \leq t \leq T-1, \tag{10}$$

$$b_{T+1} + (1-\rho)d_{T+1} = (1+r)d, \tag{11}$$

$$b_{t+1} + (1-\rho)d_{t+1} \leq \phi Al_n^{\eta} + (1-\rho)\xi V_n, \quad \text{for} \quad t \geq T+1, \tag{12}$$

$$a_{t+1} = (1-\phi)Al_{t+1}^{\eta}\mathbf{1}(\mu - (\theta^{-1} - 1)), \tag{13}$$

where $V_d(a,d) = \frac{\partial V(a,d)}{\partial d}$.[15] The firm decides the values of T and d to maximize $V(0, d_{-1})$ subject to

$$d > \xi V_n.$$

We can show the following lemma.

Lemma 1. *The firm chooses a finite T ($< +\infty$) if the initial debt d_{-1} is small. There exists $\bar{d} \leq d_z$ such that the firm chooses $T = +\infty$ if the initial debt satisfies $d_{-1} \in [\bar{d},\ d_z]$.*

[14] The firm's problem at $t = T$ is

$$V(d_t) = \max \frac{1}{1+r}[Al_{t+1}^{\eta} - wl_{t+1} - (1+r)d_t + (1-\rho)V_n],$$

$$\text{s.t.} \quad \begin{cases} wl_{t+1} + b_{t+1} \leq \phi Al_{t+1}^{\eta}, \\ d_{t+1} = \frac{(1+r)d_t - b_{t+1}}{1-\rho} \leq d_n, \end{cases}$$

where $V(d_{t+1}) = V_n - d_{t+1}$.

[15] Equations (10) and (13) are not accurate, and if $\mu = \theta^{-1} - 1$, then a_{t+1} and b_{t+1} are indeterminate as long as $0 \leq a_{t+1} \leq (1-\phi)Al_{t+1}^{\eta}$, $\phi Al_d^{\eta} - wl_d \leq b_{t+1} \leq \phi Al_d^{\eta} - wl_d + (1+r)(1-\phi)\theta Al_d^{\eta}$, and b_{t+1} satisfies the law of motion for debt, given d_{-1}.

See Appendix C for the proof. When $d_n < d_{-1} < d_z$, the values of $\{l_d, T\}$ evolve as follows as d_{-1} increases: T is monotonically increasing in d_{-1} but changes its value discretely, while l_d is decreasing in d_{-1} when T stays constant and jumps up when T increases its value discretely by one. For example, when d_{-1} is only slightly larger than d_n, then l_0 is decided by $(1+r)d_{-1} = \phi A l_0^\eta - wl_0 + (1-\rho)d_n$ and $l_t = l_n$ for $t \geq 1$, while $T = -1$. As d_{-1} increases l_0 decreases and T stays constant at $T = -1$. As d_{-1} exceeds a certain threshold, then l_0 jumps up and T changes to $T = 0$, with $d_0 > \xi V_n$. As d_{-1} increases and converges to d_z, the value of l_t $(0 \leq t \leq T+1)$ converges to l_z and T converges to ∞. T reaches at ∞ as $d_{-1} \geq \bar{d}$, where $\bar{d}$ $(\leq d_z)$ is given in Lemma 1. The maximum repayable debt is d_z.

If the initial debt exceeds d_z, the bank has no other choice than to forgive $d_{-1} - d_z$ and make the firm continue inefficient production $l_t = l_z$ every period.[16] Therefore, in this parameter region, the borrower is made a debt-ridden firm who chooses inefficient production, $l_t = l_z$, forever, if the initial debt d_{-1} is sufficiently large.

3.3.2 Debt dynamics in the case with a large ξ

Now we consider the parameter region where

$$d_n < \xi V_n < d_z \qquad \text{or, equivalently,} \qquad \frac{\phi - \eta}{1 - \eta} < \xi < [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}. \tag{14}$$

When the initial debt d_{-1} is small, i.e., $d_{-1} \leq \frac{1}{1+r}\{\phi A l_n^\eta - wl_n + (1-\rho)\xi V_n\}$, then the borrowing constraint is $(1+r)d_{-1} \leq \phi A l_n^\alpha - wl_n + (1-\rho)\xi V_n$ and the production is efficient: $l_t = l_n$ for all $t \geq 0$.

When the initial debt satisfies $\frac{1}{1+r}\{\phi A l_n^\eta - wl_n + (1-\rho)\xi V_n\} < d_{-1}$, there exists T (≥ -1) such that, for $-1 \leq t < T$, the borrowing constraint is

$$wl_{t+1} + b_{t+1} \leq \phi A l_{t+1}^\eta + (1+r)\theta a_t,$$

whereas for $t = T$, the borrowing constraint is

$$wl_{t+1} + b_{t+1} + (1-\rho)d_{t+1} \leq \phi A l_{t+1}^\eta + (1+r)\theta a_t + (1-\rho)\xi V_n,$$

and for all $t > T$, the borrowing constraint never bind: $wl_n + b_{t+1} + (1-\rho)d_{t+1} = wl_n + (1+r)d_t \leq \phi A l_n^\eta + (1-\rho)\xi V_n$. For a given value of d (> 0), we define l_d as the solution to $(1+r)d = \phi A l_d^\eta - wl_d + (1+r)(1-\phi)\theta A l_d^\eta \mathbf{1}(\mu - (\theta^{-1} - 1)) + (1-\rho)\xi V_n$, where μ is the time-invariant Lagrange multiplier associated with the borrowing constraint that is determined by $(1+\mu)wl_d = [1 + \phi\mu + (1-\phi)\theta\mu\mathbf{1}(\mu - (\theta^{-1} - 1))]\eta A l_d^\eta$. The first order conditions and the envelope conditions imply that the debt dynamics are determined by

[16] Kobayashi and Nakajima (2015) analyzes the possibility that the bank does not reduce the debt and continues to hold the nonperforming loans.

(6)–(12) as those in the case with a small ξ. The firm decides the values of T and d to maximize $V(0,\ d_{-1})$ subject to $d > \xi V_n$. As we show in Appendix C that Lemma 1 also holds in this case, the firm chooses a finite T $(< +\infty)$ if $d_{-1} < \bar{d}$, and the firm chooses $T = +\infty$ if $d_{-1} \in [\bar{d},\ d_z]$. The maximum repayable debt is d_z. If the initial debt exceeds d_z, the bank has no other choice than to forgive $d_{-1} - d_z$. In this parameter region, the borrower is also made a debt-ridden firm who chooses inefficient production, $l_t = l_z$, forever, if the initial debt satisfies $d_{-1} \geq \bar{d}$.

3.3.3 Debt dynamics in the case with a larger ξ

In the case where ξ is larger such that

$$d_z \leq \xi V_n \qquad \text{or, equivalently,} \qquad [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}} \leq \xi, \tag{15}$$

the borrowing constraint is always

$$wl_{t+1} + b_{t+1} \leq \phi A l_{t+1}^{\eta} + (1-\rho)\{\xi V_n - d_{t+1}\}.$$

The proof is given in Appendix D.

The firm repays debt as much as possible in period 0 so that production can be inefficient only in period 0. If the initial debt satisfies $d_{-1} \leq \phi A l_n^{\eta} - wl_n + (1-\rho)\xi V_n$, then $l_t = l_n$ for all $t \geq 0$. We define $\underline{l} = [\phi\eta A/w]^{\frac{1}{1-\eta}}$. The maximum repayable debt is $\phi A\underline{l}^{\eta} - w\underline{l} + (1-\rho)\xi V_n$. If d_{-1} satisfies $\phi A l_n^{\eta} - wl_n + (1-\rho)\xi V_n < d_{-1} \leq \phi A\underline{l}^{\eta} - w\underline{l} + (1-\rho)\xi V_n$, the labor input in period 0 is given as the solution to $wl_0 + (1+r)d_{-1} \leq \phi A l_0^{\eta} + (1-\rho)\xi V_n$, where $l_0 < l_n$, and $l_t = l_n$ for all $t \geq 1$. If the initial debt exceeds $\phi A\underline{l}^{\eta} - w\underline{l} + (1-\rho)\xi V_n$, the bank has no other choice than to forgive $d_{-1} - [\phi A\underline{l}^{\eta} - w\underline{l} + (1-\rho)\xi V_n]$.

4 Full model

We consider a closed economy in which the final good is produced competitively from capital input and varieties of intermediate goods. The firms are monopolistic competitors and produce respective varieties of intermediate goods from labor input. The model is a version of the expanding variety model, in which new entry of firms increases aggregate productivity. The expanding variety model was proposed by Rivera-Batiz and Romer (1991) and simplified by Acemoglu (2009). We follow Benassy (1998) in that labor is used to produce the intermediate goods and also to conduct research and development (R&D) activity that expands the variety. We assume that the monopolistically competitive firms produce the intermediate goods and also conduct R&D activity, which are subject to the borrowing constraints.

4.1 Basic setup

A representative household owns a mass of firms, indexed by $i \in [0, N_t]$, that produce intermediate goods, where N_t is the measure of the varieties of intermediate goods in period t. Firm i produces the variety i monopolistically, and can borrow fund from the bank, which is also owned by the representative household. In what follows we omit the bank for simplicity and consider the household as the lender. The final good is produced competitively from the intermediate goods x_{it}, $i \in [0, N_t]$, and capital by the following production function:

$$Y_t = \left(\int_0^{N_t} x_{it}^{\eta} di \right)^{\frac{\alpha}{\eta}} K_t^{1-\alpha},$$

where $0 < \alpha < 1$ and $0 < \eta < 1$. Because the final good producer maximizes $Y_t - \int_0^{N_t} p_{it} x_{it} di - r_t^K K_t$, where p_{it} is the real price of the intermediate good i and r_t^K is the rental rate of capital, perfect competition in the final good market implies that

$$r_t^K = (1-\alpha)\frac{Y_t}{K_t},$$
$$p_{it} = p(x_{it}) = \alpha Y_t^{1-\frac{\eta}{\alpha}} K_t^{\frac{\eta}{\alpha}-\eta} x_{it}^{\eta-1}.$$

Firm i produces the intermediate good i from labor input l_{it} by the following production function:

$$x_{it} = l_{it}.$$

Firm i, where $0 \leq i \leq N_t$, chooses $x_{it} (= l_{it})$ to maximize $p(x_{it})x_{it} - w_t l_{it}$, where w_t is the wage rate, and they are owned by a representative household. In this subsection we only consider the case where the firms do not owe any debt, while we consider in Section 4.3 the case where some of the firms with measure Z_t have a sufficiently large amount of debt so that they are debt-ridden and the others with measure $N_t - Z_t$ have no debt. Each firm i employs labor h_{it}, produces intermediate goods $x_{it} = l_{it}$, and conduct R&D activity with input $h_{it} - l_{it}$ (≥ 0). The labor input $h_{it} - l_{it}$ (≥ 0) in R&D activity creates $\kappa \bar{N}_t \{h_{it} - l_{it}\}$ units of new varieties of intermediate goods, where $\bar{N}_t$ is the social level of the variety, which represents the externality from the stock of knowledge on the R&D activity. The ρ fraction of the varieties of intermediate goods and corresponding firms die every period.

A representative household solves the following problem:

$$\max_{C_t, H_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t [\ln C_t + \gamma \ln(1 - H_t)]$$

subject to the budget constraint

$$C_t + K_{t+1} - (1-\delta)K_t + \kappa \bar{N}_t (\bar{H}_t - \bar{L}_t) V_{nt} \leq w_t H_t + r_t^K K_t + \int_0^{N_t} \pi_{it} di + (1-\rho) \int_0^{N_t} V_{nt} di,$$

where β is the subjective discount factor, C_t is consumption, H_t is the total labor supply, K_t is capital stock, δ is the depreciation rate of capital, and π_{it} is the dividend from firm $i \in [0, N_t]$. The value of the firm is determined by

$$V_{it} = \max_{x_{t+1}, h_{t+1}, l_{t+1}} \beta \frac{\lambda_{t+1}}{\lambda_t} [\pi_{it+1} + (1-\rho) V_{it+1}],$$

$$\text{s. t.} \quad \begin{cases} \pi_{it+1} = p(x_{t+1}) x_{t+1} - w_{t+1} h_{t+1} + \kappa \bar{N}_{t+1} V_{nt+1} (h_{t+1} - l_{t+1}), \\ w_{t+1} h_{t+1} \leq \phi p(x_{t+1}) x_{t+1} + (1-\rho) \xi V_{nt+1}, \end{cases}$$

where λ_t is the Lagrange multiplier associated with the budget constraint for the representative household, which is given as follows by the FOC:

$$\lambda_t = \frac{1}{C_t}.$$

On the internal reserve: The firm can set aside a part of its cash flow π_{it+1} as the internal reserve a_{it+1}, which is invested in the physical capital. The internal reserve is subject to a severe moral hazard by the firm employees such that they can take $(1-\theta) a_{it+1}$ privately to their households as lump-sum transfers and the firm can use only θa_{it+1} as the reserve. We use the parameter values that make the borrowing constraint binding along the balanced growth path (BGP), but the Lagrange multiplier associated with the borrowing constraint μ_t satisfies $\mu_t < \theta^{-1} - 1$ so that the firm chooses $a_{it+1} = 0$ for all t along the BGP.

The market clearing conditions are

$$C_t + K_{t+1} - (1-\delta) K_t = Y_t,$$

$$\int_0^{N_t} h_{it} di = H_t,$$

$$\int_0^{N_t} l_{it} di = L_t,$$

$$N_{t+1} = (1-\rho) N_t + \kappa \bar{N}_t (H_t - L_t),$$

$$\bar{N}_t = N_t.$$

4.2 Balanced growth path without debt-ridden firms

In this subsection, we characterize the BGP without debt-ridden firms. We assume that the parameter values are set such that the borrowing constraint is binding in the BGP.[17]

In the BGP, the labor and the growth rate are constant: $H_t = H$ and $L_t = L$ and $N_{t+1}/N_t = g$. We define g_Y by

$$g_Y = g^{\frac{1-\eta}{\eta}}.$$

[17] There are of course parameter values that make the borrowing constraint nonbinding in the BGP. We do not consider those parameter values in this paper.

We guess that $Y_t = Y \times N_t^{(1-\eta)/\eta}$, $C_t = C \times N_t^{(1-\eta)/\eta}$, $K_t = K \times N_t^{(1-\eta)/\eta}$, $w_t = w \times N_t^{(1-\eta)/\eta}$, $h_t = H/N_t$, $l_t = L/N_t$ and $V_t = V \times N_t^{(1-2\eta)/\eta}$. The FOCs and constraints imply that the BGP is given by the following system of equations:

$$
\begin{aligned}
& g_Y = g^{\frac{1-\eta}{\eta}}, \\
& Y = K^{1-\alpha} L^{\alpha}, \\
& C + (g_Y - 1 + \delta)K = Y, \\
& w = \frac{\gamma C}{1-H}, \\
& r^K = (1-\alpha)\frac{Y}{K}, \\
& g_Y = \beta(r^K + 1 - \delta), \\
& g = 1 - \rho + \kappa(H - L), \\
& V_n = \frac{(1-\phi)\alpha\beta K^{1-\alpha} L^{\alpha}}{g - \beta(1-\rho)(1-\xi) - \beta\kappa(H-L)}, \\
& wH = \phi\alpha K^{1-\alpha} L^{\alpha} + (1-\rho)\xi V_n, \\
& w = \frac{\kappa V_n}{1+\mu}, \\
& \kappa V_n = (1+\phi\mu)\eta\alpha K^{1-\alpha} L^{\alpha-1},
\end{aligned}
$$

where μ is the Lagrange multiplier associated with the borrowing constraint.

Numerical example of the BGP is given in Table 3, whereas the parameter values are given in Table 2. Note that we set $\beta = 0.98$ as it is an annual model.[18] The growth rate of TFP on the BGP is 2.1%, i.e., $g_Y = g^{\frac{\eta}{1-\eta}} = 1.021$.

4.3 Low growth equilibrium with debt-ridden firms

Now we consider the equilibrium where some firms owe a large amount of debt to the representative household through banks, and show that the economy falls into persistently low growth if the measure of debt-ridden firms is large. We assume that firms $i \in [0, Z_t]$ have identical debt stock d_t and firms $i \in (Z_t, N_t]$ have no debt. Due to symmetry we can assume without loss of generality that all firms with debt choose identical labor input $l_{it} = l_{zt}$ and repayment $b_{it} = b_t$ for $i \in [0, Z_t]$ and that all firms with no debt also choose identical labor input $l_{it} = l_{nt}$ for $i \in (Z_t, N_t]$. In this case the representative household maximize her utility by solving the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t [\ln C_t + \gamma \ln(1 - H_t)]$$

[18] Hayashi and Prescott (2002) estimate $\beta = 0.976$ and Sugo and Ueda (2008) use $\beta = 0.98$ for annual discount rate of the Japanese economy.

subject to

$$C_t + K^C_{t+1} - (1-\delta)K^C_t + \kappa\bar{N}_t(\bar{H}_t - \bar{L}_t)V_{nt}$$
$$\leq w_t H_t + r^K_t K^C_t + \int_0^{Z_t} \pi_{zt} di + \int_{Z_t}^{N_t} \pi_{nt} di + \int_0^{Z_t} b_{it} di + (1-\rho)\int_0^{N_t} V_{it} di,$$

where K^C_t may be less than the total amount of capital stock because the debt-ridden firms may hold capital stock as their internal reserves. The initial debt for the Z_t debt-ridden firms is larger than the maximum repayable debt (d_z in Section 3) and the lender imposes the debt-ridden firm to pay the repayment, b_t, which is optimal for the lender, every period. For simplicity we assume that

$$\xi = 0,$$

so that the borrowing constraint is always $w_{t+1}l_{t+1} + b_{t+1} \leq \phi\alpha Y_{t+1}^{1-\frac{\eta}{\alpha}} K_{t+1}^{\frac{\eta}{\alpha}-\eta} l_{t+1}^{\eta} + \frac{\lambda_t}{\beta\lambda_{t+1}}\theta a_t$.

The problem for the owner of a debt-ridden firm is to maximize V_{zt}, given b_t:

$$\max_{l_{zt+1}} V_{zt} = \frac{\beta\lambda_{t+1}}{\lambda_t}\Big[\alpha Y_{t+1}^{1-\frac{\eta}{\alpha}} K_{t+1}^{\frac{\eta}{\alpha}-\eta} l_{zt+1}^{\eta} - w_{t+1}h_{zt+1} - b_{t+1} + \kappa\bar{N}_{t+1}V_{nt+1}\{h_{zt+1} - l_{zt+1}\}$$
$$+ \frac{\lambda_t}{\beta\lambda_{t+1}}\theta a_t - a_{t+1} + (1-\rho)V_{zt+1}\Big],$$

subject to

$$\begin{cases} a_{t+1} \leq \alpha Y_{t+1}^{1-\frac{\eta}{\alpha}} K_{t+1}^{\frac{\eta}{\alpha}-\eta} l_{zt+1}^{\eta} - w_{t+1}h_{zt+1} - b_{t+1} + \kappa\bar{N}_{t+1}V_{nt+1}\{h_{zt+1} - l_{zt+1}\} + \frac{\lambda_t}{\beta\lambda_{t+1}}\theta a_t, \\ w_{t+1}h_{zt+1} + b_{t+1} \leq \phi\alpha Y_{t+1}^{1-\frac{\eta}{\alpha}} K_{t+1}^{\frac{\eta}{\alpha}-\eta} l_{zt+1}^{\eta} + \frac{\lambda_t}{\beta\lambda_{t+1}}\theta a_t, \\ l_{zt+1} \leq h_{zt+1}. \end{cases}$$

We change the variables as follows: $V_{zt} = \tilde{V}_{zt}N_t^{(1-2\eta)/\eta}$, $Y_t = \tilde{Y}_t N_t^{(1-\eta)/\eta}$, $C_t = \tilde{C}_t N_t^{(1-\eta)/\eta}$, $K_t = \tilde{K}_t N_t^{(1-\eta)/\eta}$, $b_t = \tilde{b}_t N_t^{(1-2\eta)/\eta}$, $x_{zt} = \tilde{x}_{zt}/N_t$, $x_{nt} = \tilde{x}_{nt}/N_t$, $l_{zt} = \tilde{l}_{zt}/N_t$, $l_{nt} = \tilde{l}_{nt}/N_t$, $h_{zt} = \tilde{h}_{zt}/N_t$, $h_{nt} = \tilde{h}_{nt}/N_t$, $w_t = \tilde{w}_t N_t^{\frac{1-\eta}{\eta}}$, $a_t = \tilde{a}_t N_t^{\frac{1-2\eta}{\eta}}$, $d_{zt} = \tilde{d}_{zt}N_t^{(1-2\eta)/\eta}$, and $z_t = Z_t/N_t$. Then, the firm's problem can be rewritten as follows. Given $\tilde{b}_t$,

$$\tilde{V}_{zt} = \max \frac{\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}}\Big[\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{l}_{zt+1}^{\eta} - \tilde{w}_{t+1}\tilde{h}_{zt+1} - \tilde{b}_{t+1} + \kappa\tilde{V}_{nt+1}\{\tilde{h}_{zt+1} - \tilde{l}_{zt+1}\}$$
$$+ \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta\tilde{a}_t - \tilde{a}_{t+1} + (1-\rho)\tilde{V}_{zt+1}\Big],$$

subject to

$$\begin{cases} \tilde{a}_{t+1} \leq \alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{zt+1}^{\eta} - \tilde{w}_{t+1}\tilde{h}_{zt+1} - \tilde{b}_{t+1} + \kappa\tilde{V}_{nt+1}\{\tilde{h}_{zt+1} - \tilde{l}_{zt+1}\} + \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta\tilde{a}_t, \\ \tilde{w}_{t+1}\tilde{h}_{zt+1} + \tilde{b}_{t+1} \leq \phi\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{zt+1}^{\eta} + \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta\tilde{a}_t, \\ \tilde{l}_{zt+1} \leq \tilde{h}_{zt+1}. \end{cases}$$

As the borrowing constraint should be tight for the debt-ridden firm, it chooses to accumulate the internal reserve as much as possible and the constraints imply that

$$\tilde{a}_{t+1} = (1-\phi)\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{zt+1}^{\eta} + \kappa\tilde{V}_{nt+1}\{\tilde{h}_{zt+1} - \tilde{l}_{zt+1}\}.$$

The lender solves the following Ramsey problem to decide the repayment, $\tilde{b}_{t+1}$:

$$\tilde{d}_{zt} = \max_{\tilde{h}_{zt},\tilde{l}_{zt}} \frac{\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}}[\tilde{b}_{t+1} + (1-\rho)\tilde{d}_{zt+1}],$$

subject to

$$\begin{cases} \tilde{w}_{t+1}\tilde{h}_{zt+1} + \tilde{b}_{t+1} \leq \phi\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{zt+1}^{\eta} + \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta[(1-\phi)\alpha\tilde{Y}_t^{1-\frac{\eta}{\alpha}}\tilde{K}_t^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{zt}^{\eta} + \kappa\tilde{V}_{nt}\{\tilde{h}_{zt} - \tilde{l}_{zt}\}], & (\mu_{t+1}) \\ \tilde{l}_{zt+1} \leq \tilde{h}_{zt+1}. & (\nu_{t+1}) \end{cases}$$

The solution is given by the following:

$$\tilde{h}_{zt+1} = \tilde{l}_{zt+1} = \arg\max \phi\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{l}^{\eta} - \tilde{w}_{t+1}\tilde{l} = \left[\frac{\{\phi + (1-\phi)(1-\rho)\theta\}\alpha\eta\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}}{\tilde{w}_{t+1}}\right]^{\frac{1}{1-\eta}}.$$

Note that when $\tilde{w}_{t+1} > \theta\kappa\tilde{V}_{nt+1}$, the above solution is unique, and that when $\tilde{w}_{t+1} = \theta\kappa\tilde{V}_{nt+1}$, the value of $\tilde{h}_{zt+1}$ is indeterminate and we can set that $\tilde{h}_{zt+1} = \tilde{l}_{zt+1}$ without loss of generality.

Normal firms without debt solve the following problem, which is similar to the above one.

$$\tilde{V}_{nt} = \max \frac{\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}}\Big[\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{l}_{nt+1}^{\eta} - \tilde{w}_{t+1}\tilde{h}_{nt+1} + \kappa\tilde{V}_{nt+1}\{\tilde{h}_{nt+1} - \tilde{l}_{nt+1}\}$$
$$+ \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta\tilde{a}_t - \tilde{a}_{t+1} + (1-\rho)\tilde{V}_{nt+1}\Big],$$

subject to

$$\text{s. t.} \qquad \tilde{w}_{t+1}\tilde{h}_{nt+1} \leq \phi\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{l}_{nt+1}^{\eta} + \frac{g_t\tilde{C}_{t+1}}{\beta\tilde{C}_t}\theta\tilde{a}_t. \tag{16}$$

We denote the Lagrange multiplier associated with (16) by μ_{nt+1}. The FOCs imply that if $\mu_{nt+1} < \theta^{-1} - 1$, then the firm chooses not to accumulate the internal reserve, $\tilde{a}_{nt+1} = 0$, whereas if $\mu_{nt+1} = \theta^{-1} - 1$, then the firm chooses to accumulate the internal reserve, $0 < \tilde{a}_{nt+1} \leq (1-\phi)\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{\frac{\eta}{\alpha}-\eta}\tilde{l}_{nt+1}^{\eta} + \kappa\tilde{V}_{nt+1}\{\tilde{h}_{nt+1} - \tilde{l}_{nt+1}\}$. The amount of a_{nt+1} is indeterminate and the aggregate amount $\int_{Z_t}^{N_t} a_{nt+1}di$ is decided by the aggregate supply of the capital stock.

Zero growth path with $z = 1$: When all firms are debt-ridden, i.e., the initial $z = 1$, then there is no firm that conducts R&D activity and consequently there is no productivity growth. The economy converges to the zero growth path (ZGP) which is specified by $L = l_z = [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}} l_n$, $l_n = \left(\frac{\alpha\eta Y^{1-\frac{\eta}{\alpha}}K^{(1-\alpha)\frac{\eta}{\alpha}}}{w}\right)^{\frac{1}{1-\eta}}$, $g_Y = g^{\frac{1-\eta}{\eta}}$, $g = 1-\rho$, $Y = L^{\alpha}K^{1-\alpha}$, $C + (g_Y - 1 + \delta)K = Y$, $w = \frac{\gamma C}{1-L}$, $g_Y = \beta(r^K + 1 - \delta)$, and

$r^K = (1-\alpha)\frac{Y}{K}$. There is no growth in productivity nor output. Given the parameter values of Table 2, the variables of the ZGP are shown in Table 3. In the example, the aggregate labor is $H = 0.24$ in the ZGP, which is smaller than $H = 0.33$ in the BGP. Although the example of $z = 1$ is an extreme case, it demonstrates the aggregate effect of emergence of debt-ridden firms on the economy qualitatively. Comparison of the ZGP with the historical episodes follows:

- Productivity slowdown: As there is no agent who conducts R&D activity if $z = 1$, the productivity stops growing in the ZGP in this example. This feature is consistent with decade-long slowdown of productivity observed in the aftermath of a financial crisis.

- Decrease in net entry: As $g_t = N_{t+1}/N_t = 1 - \rho$, there is no new entry of firms in the ZGP with $z = 1$. This feature is consistent with observations on entry and exit of firms in Japan in the 1990s.

- Buildup of nonperforming loans: In the ZGP of our model, all firms are debt-ridden and their debt stays at an inefficiently high level, that makes the firms continue inefficient production permanently. This result may explain the persistent stagnation of the economy with overly indebted firms and/or households, such as Japan in the 1990s.

- Labor-wedge deterioration: In this example, we can calculate the labor wedge $1-\tau$, which is 0.96 in the BGP where $z = 0$ and 0.13 in the ZGP where $z = 1$.[19] This result implies that the persistent deterioration of the labor wedge observed in the aftermath of a financial crisis can be accounted for by emergence of debt-ridden firms, which lowers the labor wedge by tightening the borrowing constraints on working capital loans.

Equilibrium dynamics with $0 < z < 1$: We can calculate the equilibrium dynamics by a full nonlinear method numerically in the case where the initial $z_0 = Z_0/N_0$ is less than one. Linearization is not necessary for the deterministic simulation. See Appendix E for the details of the dynamics. Figure 5 shows the results of the numerical simulation in which the economy is initially on the BGP where $z_t = 0$ and an unexpected shock hits the economy at period 10 that makes $z_{10} = 0.38$. The parameter values are given in Table 2. The features of the equilibrium path shown in Figure 5 are similar to those of the ZGP that we discussed above: Productivity, $TFP_t = Y_t/(K_t^{1-\alpha}H_t^{\alpha})$, and net entry,

[19] As Chari, Kehoe and McGrattan (2007) posit, the labor wedge, $1-\tau_t$, is defined by $1-\tau_t = \frac{MRS_t}{MPL_t}$, where $MRS_t = \frac{\gamma C_t}{1-H_t} = w_t$ and $MPL_t = \frac{\alpha Y_t}{H_t}$ in our model. Thus the labor wedge can be calculated by $1-\tau_t = \frac{w_t H_t}{\alpha Y_t}$.

$g_t = N_{t+1}/N_t$, both slow down; output, labor and consumption stagnate persistently; and the labor wedge, $LW_t = w_t H_t/(\alpha Y_t)$, also deteriorates for extended periods. As long as there exist normal firms, they conduct R&D activity so that N_t increases, z_t decreases, and the economy eventually converges to the BGP. An interesting feature in the case where z_t (> 0) is larger is that the borrowing constraint for normal firms, (16), becomes tighter and the aggregate level of R&D activity is lowered, leading to the slower growth of productivity and output.

Figure 6 compares the observed total factor productivity of the numerical experiment with the actual TFP of the last two decades in the Japanese economy. The simulated TFP is identical to that in Figure 5. We set period 10 of the simulation at the year of 1990, when the asset-price bubble collapsed. The figure shows that the model fits the data fairly well. Figure 7 compares economic growth in the simulation with the actual and predicted growth in the United States and Europe.[20] Predictions of the growth rates are set at the average growth rates during the period between 2009 and 2014. Similarly in Japan, an unexpected shock hits the economy in 2008 that makes $z_{2008} > 0$. Parameters ($\kappa,\ \gamma,\ z_{2008}$) are calibrated such that the BGP growth rates equal the average growth rates before 2008 and the BGP labor supplies equal 1/3 in the US and the EU economy, respectively: $(\kappa,\ \gamma,\ z_{2008}) = (1.35,\ 1.78,\ 0.43)$ for the US and $(\kappa,\ \gamma,\ z_{2008}) = (1.24,\ 1.77,\ 0.6)$ for the EU. Figure 7 shows that the model can capture the long-term or predicted features of the US and EU economies. There is the short-run discrepancy in the comparison with the US economy. This discrepancy can be attributed to a mechanism outside of this model, such as the bank-run effect of sudden liquidity dry-up (see Kobayashi and Nakajima, 2014).

5 Conclusion

Decade-long recessions with low productivity growth are often observed after financial crises. In particular, the "secular stagnation" hypothesis has drawn much attention of researchers after the Great Recession (e.g., Summers 2013). We proposed a hypothesis that the emergence of debt-ridden borrowers causes a persistent productivity slowdown. Economic agents become overly indebted sometimes for reasons such as the boom and bust of asset-price bubbles. Analyzing the endogenous borrowing constraint, we show that borrowers with a large initial debt fall into the debt-ridden state, where they are subject to tighter borrowing constraints than normal firms and continue inefficient production persistently.

The emergence of a substantial number of debt-ridden borrowers lowers the aggregate productivity through tightening the aggregate borrowing constraint. Tightening of

[20]The EU-15 consists of Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and United Kingdom.

aggregate borrowing constraints owing to the mass emergence of debt-ridden borrowers may manifest itself as a "financial shock" during or after a financial crisis. We show that the growth rate of aggregate productivity is lowered persistently, if the measure of debt-ridden firms exceeds a certain threshold level. This result has a significant policy implication that the mass debt restructuring for overly indebted borrowers may restore the efficiency in production and enhance economic growth when the economy falls into persistent stagnation.

Appendix A: Nonexistence of Case 3-borrowing constraint

In Section 3.2.2, we derive the endogenous borrowing constraint in Case 1 where $d_{t+1} > \xi V_{nt+1}$ and Case 2 where $\xi V_{nt+1} - V(\hat{a}_1, d_{t+1}) + \hat{a}_1 \leq d_{t+1} \leq \xi V_{nt+1}$. In this appendix we show the nonexistence of Case 3 where $d_{t+1} < \xi V_{nt+1} - V(\hat{a}_1, d_{t+1}) + \hat{a}_1$.

Suppose that Case 3 arises, i.e., parameters satisfy $d_{t+1} < \xi V_{nt+1} - V(\hat{a}_1, d_{t+1}) + \hat{a}_1$. In this case, the renegotiation surplus (i.e., $d_{t+1} + V(\hat{a}_1, d_{t+1}) - \hat{a}_1 - \xi V_{nt+1}$) is negative and the bank's participation constraint can never be satisfied by any offer that the firm can make in the renegotiation. Thus the bank always liquidate the firm when it defaults. The indifference condition for the bank is $w_{t+1}l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^{\alpha} + (1 + r_{t+1})\theta a_t + (1 - \rho)\{\xi V_{nt+1} - d_{t+1}\}$, while the no-default condition for the firm is not necessarily satisfied. Thus we need to derive the no-default condition explicitly. The payoff for the firm is $(1 - \phi) A l_{t+1}^{\alpha}$ if it defaults, and is $A l_{t+1}^{\alpha} - w_{t+1}l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t + (1 - \rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\}$ if it does not default. The no-default condition is that the former is no greater than the latter, and implies the following borrowing constraint:

$$w_{t+1}l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^{\alpha} + (1 + r_{t+1})\theta a_t + (1 - \rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\}. \tag{17}$$

This Case 3 borrowing constraint is almost identical to that in Albuquerque and Hopenhayn (2004). Given that the borrowing constraint is (17), the firm's problem in period t is written as follows:

$$V(a_t, d_t) = \max_{l_{t+1}, b_{t+1}} \frac{1}{1 + r}\{A l_{t+1}^{\eta} - w l_{t+1} - b_{t+1} + (1 + r_{t+1})\theta a_t + (1 - \rho)[V(a_{t+1}, d_{t+1}) - a_{t+1}]\}$$

$$\text{s.t.} \quad \begin{cases} w l_{t+1} + b_{t+1} \leq \phi A l_{t+1}^{\eta} + (1 + r_{t+1})\theta a_t + (1 - \rho)\{V(a_{t+1}, d_{t+1}) - a_{t+1}\}, \\ b_{t+1} + (1 - \rho) d_{t+1} = (1 + r) d_t \end{cases}$$

The FOCs and the envelope condition imply that

$$\begin{aligned} V_d(a_{t+1}, d_{t+1}) &= -1, \\ V_d(a_t, d_t) &= -(1 + \mu_{t+1}), \\ w &= (1 + \mu_{t+1}\phi)\eta A l_{t+1}^{\eta - 1}, \end{aligned}$$

where $V_d(a, d) \equiv \frac{\partial V(a,d)}{\partial d}$ and μ_{t+1} (≥ 0) is the Lagrange multiplier associated with the borrowing constraint. This result implies that $\mu_0 > 0$ and $\mu_t = 0$ for all $t \geq 1$. Therefore, $l_0 < l_n$ and $l_t = l_n$ for all $t \geq 1$. Thus, $V(0, d_{-1}) < V_n - d_{-1}$ and $V(a_t, d_t) = V_n + a_t - d_t$ for all $t \geq 0$. As we assume $\xi < 1$, it is obvious that $d_t + V(a_t, d_t) - a_t = V_n > \xi V_n$ for $t \geq 0$, which contradicts to the condition for Case 3. Therefore, for any structural parameters, Case 3 borrowing constraint arises only for $t = 0$ and it never arises for $t \geq 1$ in our model.[21]

---

[21] If we assume the limited liability constraint: $w_t l_t + b_t \leq A l_t^{\eta}$ as Albuquerque and Hopenhayn (2004) do, then Case 3-borrowing constraint can arise for $t \geq 1$ in our model.

Appendix B: Accumulation of the internal reserve and the maximum repayable debt with the internal reserve

In Section 3.3.1 we consider the firm's problem with the binding borrowing under time-invariant prices. The problem is

$$
\begin{aligned}
&V(a_t, d_t) = \\
&\frac{1}{1+r}[Al_{t+1}^{\eta} - wl_{t+1} - b_{t+1} + (1+r)\theta a_t - (1-\rho)a_{t+1} + (1-\rho)V(a_{t+1}, d_{t+1})], \\
&\text{s. t.} \begin{cases} a_{t+1} \geq 0, & (\zeta_{t+1}) \\ a_{t+1} \leq Al_{t+1}^{\eta} - wl_{t+1} - b_{t+1} + (1+r)\theta a_t, & (\nu_{t+1}) \\ wl_{t+1} + b_{t+1} \leq \phi Al_{t+1}^{\eta} + (1+r)\theta a_t, & (\mu_{t+1}) \end{cases}
\end{aligned}
$$

The FOC with respect to a_{t+1} and the envelope condition with respect to a_t imply that

$$
\begin{cases} a_{t+1} = 0, & \text{if } \mu_{t+1} < \theta^{-1} - 1, \\ a_{t+1} \in [0, Al_{t+1}^{\eta} - wl_{t+1} - b_{t+1} + (1+r)\theta a_t], & \text{if } \mu_{t+1} = \theta^{-1} - 1, \\ a_{t+1} = Al_{t+1}^{\eta} - wl_{t+1} - b_{t+1} + (1+r)\theta a_t, & \text{if } \mu_{t+1} > \theta^{-1} - 1. \end{cases}
$$

Given that $\mu_{t+1} > 0$ and $\nu_{t+1} > 0$, the value of a_{t+1} is written as $a_{t+1} = (1-\phi)Al_{t+1}^{\eta}$. The borrowing constraint is reduced to

$$
wl_{t+1} + b_{t+1} \leq \phi Al_{t+1}^{\eta} + (1+r)(1-\phi)\theta Al_t^{\eta}. \tag{18}
$$

The maximum repayable debt d_{zt} is given as the solution to the following Ramsey problem for the bank:

$$
d_{zt} = \max_{l_{t+1}} \frac{1}{1+r}[b_{t+1} + (1-\rho)d_{zt+1}],
$$

subject to (18). The solution is

$$
\begin{aligned}
&l_t = l_z \equiv \{[\phi + (1-\phi)(1-\rho)\theta]\eta A/w\}^{\frac{1}{1-\eta}}, \text{ for all } t \geq 0, \\
&b_0 = \phi Al_z^{\eta} - wl_z, \\
&b_t = [\phi + (1+r)(1-\phi)\theta]Al_z^{\eta} - wl_z.
\end{aligned}
$$

The maximum debt is $d_z = d_{z,-1} = \frac{1}{1+r}[b_0 + (1-\rho)\hat{d}_z]$, where $\hat{d}_z = \frac{1}{1+r}[b_t + (1-\rho)\hat{d}_z]$. Thus,

$$
d_z = \frac{1}{r+\rho}\{[\phi + (1-\rho)(1-\phi)\theta]Al_z^{\eta} - wl_z\}. \tag{19}
$$

Appendix C: Proof of Lemma 1

We denote the production function by $y_t = f(l_t)$. Suppose that firm owes $d = \frac{b}{r+\rho}$ initially. We focus on the case where

$$
d > \xi V_n.
$$

Define l_d and μ by $b = [\phi + (1-\phi)(1-\rho)\theta]f(l_d) - wl_d$ and $(1+\mu)wl_d = [1 + \phi\mu + (1-\phi)\theta\mu\mathbf{1}(\mu - (\theta^{-1} - 1))]\eta A l_d^{\eta}$.

The case where the initial debt is small such that $\mu < \theta^{-1} - 1$

In this subsection, we prove Lemma 1 in the case where the initial debt is not large so that $\mu < \theta^{-1} - 1$. If the firm chooses to pay b every period, then the Lagrange multiplier associated with the borrowing constraint μ is small ($\mu < \theta^{-1} - 1$) so that the firm chooses not to accumulate the internal reserve, as shown in Appendix B. In this case the value of the firm becomes

$$V_\infty = \frac{1}{1+r}[f(l_d) - wl_d - b + (1-\rho)V_\infty] = \frac{1}{r+\rho}(1-\phi)f(l_d),$$

Suppose that the firm chooses to pay $b+\varepsilon$, where ε is a very small positive number, every period for $0 \le t \le T-1$, and to repay all debt in T. We denote the firm's value in this case by V_f. The values of T and ε must satisfy the following relationship:

$$\frac{b}{r+\rho} = \frac{1 - \left(\frac{1-\rho}{1+r}\right)^T}{r+\rho}(b+\varepsilon) + \frac{1}{1+r}\left(\frac{1-\rho}{1+r}\right)^T D,$$

where $D = \phi f(\tilde{l}_d) - wl_d + (1-\rho)X$ and $\tilde{l}_d$ is given by $b + \varepsilon = \phi f(\tilde{l}_d) - wl_d$, where

$$X = \begin{cases} d_n & \text{for } \xi < \frac{\phi-\eta}{1-\eta}, \\ \xi V_n & \text{for } \frac{\phi-\eta}{1-\eta} \le \xi \le [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}. \end{cases}$$

We can show

$$\left(\frac{1+r}{1-\rho}\right)^{T+1} = 1 + \frac{b - (r+\rho)X}{\varepsilon}$$

Then

$$\begin{aligned} V_f = & \frac{1}{1+r}\frac{1 - \left(\frac{1-\rho}{1+r}\right)^T}{1 - \frac{1-\rho}{1+r}}(1-\phi)f(\tilde{l}_d) \\ & + \left(\frac{1-\rho}{1+r}\right)^{T+1}[(1-\phi)f(\tilde{l}_d) + (1-\rho)(V_n - X)] \\ = & \frac{1 - \left(\frac{1-\rho}{1+r}\right)^{T+1}}{r+\rho}(1-\phi)f(\tilde{l}_d) + \left(\frac{1-\rho}{1+r}\right)^{T+1}(V_n - X). \end{aligned}$$

The condition for the firm to optimally choose the finite path is that there exists $\tilde{l}_d$ ($> l_n$) such that $V_\infty < V_f$, which is rewritten as

$$(1-\phi)f(l_d) < \left(1 - \left(\frac{1-\rho}{1+r}\right)^{T+1}\right)(1-\phi)f(\tilde{l}_d) + \left(\frac{1-\rho}{1+r}\right)^{T+1}(r+\rho)(V_n - X).$$

We define x by $\tilde{l}_d = (1+x)l_d$. The Taylor expansion of $b + \varepsilon = \phi f(\tilde{l}_d)$ implies that

$$l_d x = \frac{\varepsilon}{\phi f'(l_d)}.$$

The Taylor expansion implies that $(1-\phi)f(\tilde{l_d}) \approx (1-\phi)f(l_d) + (1-\phi)f'(l_d)l_d x$. Then the above condition ($V_\infty < V_f$) can be approximated by

$$(1-\phi)f(l_d) < \left[\left(\frac{1+r}{1-\rho}\right)^{T+1} - 1\right](1-\phi)f'(l_d)l_d x + (r+\rho)(V_n - X)$$

Using $\left(\frac{1+r}{1-\rho}\right)^{T+1} = 1 + \frac{b-(r+\rho)X}{\varepsilon}$ and $l_d x = \frac{\varepsilon}{\phi f'(l_d)}$, this condition can be rewritten as

$$(1-\phi)wl_d < (r+\rho)(\phi V_n - X).$$

- When $\xi < \frac{\phi-\eta}{1-\eta}$, it is the case that $X = d_n$ and we can show that the left-hand side equals the right-hand side for $l_d = l_n$. So for $l_d < l_n$, it is the case that $V_\infty < V_f$. Therefore the firm chooses a finite T.

- When $\frac{\phi-\eta}{1-\eta} \le \xi \le [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}$, it is the case that $X = \xi V_n$. It is easily shown that the above condition is violated for $l_d = l_n$. The above condition can be rewritten as follows for $l_d = l_z$:

  $$[\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}\eta < (\phi - \xi)(1-\eta).$$

  This condition is violated for all $\xi \in [\frac{\phi-\eta}{1-\eta},\ \ [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}]$, if the following condition is satisfied.

  $$[\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}} > (1-\phi). \tag{20}$$

  If condition (20) is satisfied, then $V_\infty > V_f$ for all (l_d, μ) that satisfy $\mu < \theta^{-1} - 1$ and the firm chooses $T = \infty$.

The case where the initial debt is large so that $\mu > \theta^{-1} - 1$

We focus on the case where the initial debt is very large so that $\mu > \theta^{-1} - 1$. In this case the firm chooses to accumulate the internal reserve as much as possible: $a_t = (1-\phi)f(l_t)$. Given the initial debt d, we redefine l_d by

$$d = \frac{[\phi + (1-\phi)(1-\rho)\theta]f(l_d) - wl_d}{r+\rho}.$$

If the firm chooses to hold the debt permanently, the repayments are

$$\begin{aligned} b_0 &= \phi f(l_d) - wl_d, \\ b_t &= \phi f(l_d) - wl_d + (1+r)(1-\phi)\theta f(l_d), \quad \text{for all } t \ge 1. \end{aligned}$$

The value of the firm is given by

$$V_\infty = \frac{(1-\phi)\rho}{r+\rho}f(l_d).$$

Suppose that the firm chooses to repay the debt in finite periods, and changes l_d to $\tilde{l}_d = (1+x)l_d$, where $x < 0$, for $0 \leq t \leq T$. The repayment becomes

$$\begin{aligned}
&\tilde{b}_0 = \phi f(\tilde{l}_d) - w\tilde{l}_d, \\
&\tilde{b}_t = \phi f(\tilde{l}_d) - w\tilde{l}_d + (1+r)(1-\phi)\theta f(\tilde{l}_d), \quad \text{for } 1 \leq t \leq T-1, \\
&D \equiv \tilde{b}_T + (1-\rho)\tilde{d}_T = \phi f(\tilde{l}_d) - w\tilde{l}_d + (1+r)(1-\phi)\theta f(\tilde{l}_d) + (1-\rho)X.
\end{aligned}$$

The debt d is expressed as $d = \frac{1}{1+r}[\tilde{b}_0 + \left(\frac{1-\rho}{1+r}\right)\tilde{b} + \cdots + \left(\frac{1-\rho}{1+r}\right)^{T-1}\tilde{b} + \left(\frac{1-\rho}{1+r}\right)^{T} D]$. This can be rewritten as

$$\begin{aligned}
d =& \frac{1}{r+\rho}\{[\phi + (1-\phi)(1-\rho)\theta]f(\tilde{l}_d) - w\tilde{l}_d\} \\
&- \frac{1-r-\rho}{r+\rho}\left(\frac{1-\rho}{1+r}\right)^{T}\{[\phi + (1+r)(1-\rho)\theta]f(\tilde{l}_d) - w\tilde{l}_d\} + \left(\frac{1-\rho}{1+r}\right)^{T}(1-\rho)X.
\end{aligned}$$

The Taylor expansion implies

$$\frac{1}{r+\rho}\{[\phi + (1-\phi)(1-\rho)\theta]f(\tilde{l}_d) - w\tilde{l}_d\} = d + \frac{1}{r+\rho}\{[\phi + (1-\phi)(1-\rho)\theta]f'(l_d) - w\}l_d x.$$

Therefore, it is approximated that

$$\left(\frac{1-\rho}{1+r}\right)^{T} = \frac{\frac{1}{r+\rho}\{[\phi + (1-\phi)(1-\rho)\theta]f'(l_d) - w\}l_d x}{\frac{1-r-\rho}{r+\rho}\{[\phi + (1+r)(1-\rho)\theta]f(l_d) - wl_d\} - (1-\rho)X}.$$

The firm's value when it repays debt in finite periods, V_f, is given by

$$\begin{aligned}
V_f =& \frac{1}{1+r}\left[(1-\phi)\rho f(\tilde{l}_d) + \left(\frac{1-\rho}{1+r}\right)(1-\phi)\rho f(\tilde{l}_d) + \cdots \right. \\
&\left. + \left(\frac{1-\rho}{1+r}\right)^{T-1}(1-\phi)\rho f(\tilde{l}_d) + \left(\frac{1-\rho}{1+r}\right)^{T}[(1-\phi)f(\tilde{l}_d) + (1-\rho)(V_n - X)]\right].
\end{aligned}$$

This is rewritten as

$$V_f = \tilde{V}_\infty + \left(\frac{1-\rho}{1+r}\right)^{T+1}\left[\frac{r}{r+\rho}(1-\phi)f(\tilde{l}_d) + V_n - X\right],$$

where $\tilde{V}_\infty = \frac{(1-\phi)\rho}{r+\rho}f(\tilde{l}_d)$. The Taylor expansion implies that

$$V_f = V_\infty + \frac{l_d x}{r+\rho}\left[(1-\phi)f'(l_d) + \frac{\frac{1-\rho}{1+r}\{[\phi + (1-\phi)(1-\rho)\theta]f'(l_d) - w\}\left[\frac{r}{r+\rho}(1-\phi)f(l_d) + V_n - X\right]}{\frac{1-r-\rho}{r+\rho}\{[\phi + (1+r)(1-\rho)\theta]f(l_d) - wl_d\} - (1-\rho)X}\right]$$

We focus on the case where the initial debt is large so that l_d is very close to $l_z = \arg\max_{l_d}[\phi + (1-\phi)(1-\rho)\theta]f(l_d) - wl_d$. As $[\phi + (1-\phi)(1-\rho)\theta]f'(l_z) - w = 0$, we can approximate $V_f = V_\infty + \frac{l_d x}{r+\rho}(1-\phi)f'(l_d) < V_\infty$ as $x < 0$, when l_d is sufficiently close to l_z. So there exists $\bar{l}_d$ such that $V_f < V_\infty$ if $l_d \in [l_z,\ \bar{l}_d]$. Define $\bar{d} = \frac{[\phi + (1-\phi)(1-\rho)\theta]f(\bar{l}_d) - w\bar{l}_d}{r+\rho}$. Then we can say that the firm chooses $T = \infty$ if the initial debt d_{-1} satisfies $d_{-1} \in [\bar{d}, d_z]$.

Appendix D: The borrowing constraint when $\xi \geq [\phi + (1 - \phi)(1 - \rho)\theta]^{\frac{1}{1-\eta}}$

We can prove the following lemma.

Lemma 2. *In the case where $\xi \geq [\phi+(1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}$, the borrowing constraint cannot be $wl_t + b_t \leq \phi A l_t^\alpha + (1+r)\theta a_{t-1}$ for any t.*

Proof. The fact that $\xi \geq [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}$ implies $d_z \leq \xi V_n$. Therefore, "the BC is $wl_t + b_t \leq \phi A l_t^\alpha + (1+r)\theta a_{t-1}$ for all t" cannot hold. Suppose that there exists $T \geq 0$ such that the borrowing constraint is $wl_t + b_t \leq \phi A l_t^\alpha + (1+r)\theta a_{t-1}$ for $0 \leq t \leq T$, and is $wl_t + b_t \leq \phi A l_t^\alpha + (1-\rho)(\xi V_n - d_t) + (1+r)\theta a_{t-1}$ for $t \geq T+1$. Then it should be the case that $d_{-1} > \xi V_n > d_z$.

Define b_{zt} as the repayment path corresponding to d_z. Because b_{zt} is the maximum repayable amount, b_t is (weakly) smaller than b_{zt} for $0 \leq t \leq T$. Therefore, it should be the case that $d_{t-1} \leq d_t$ for $0 \leq t \leq T$. Specifically, $d_T \geq d_{-1} > \xi V_n$. As the borrowing constraint is $wl_t + b_t \leq \phi A l_t^\alpha + (1+r)\theta a_{t-1}$, given $d_{t-1} = d_{-1}$, it must be also $wl_t + b_t \leq \phi A l_t^\alpha + (1+r)\theta a_{t-1}$, given $d_{t-1} = d_T$. It is a contradiction. Therefore, T must be -1. □

This lemma implies that the borrowing constraint is $wl_t + b_t \leq \phi A l_t^\alpha + (1-\rho)(\xi V_n - d_t) + (1+r)\theta a_{t-1}$ for all t (≥ 0) in the case where $\xi \geq [\phi + (1-\phi)(1-\rho)\theta]^{\frac{1}{1-\eta}}$.

Appendix E: Transition dynamics with $Z > 0$

In this appendix we describe the transition dynamics in the case where measure Z of debt-ridden firms are generated at time -1, whereas $N_{-1} - Z$ firms have no debt. Z is large and (16) is binding. The economy eventually converges to the BGP.

We use the normalized variables defined in the text: $V_{zt} = \tilde{V}_{zt} N_t^{(1-2\eta)/\eta}$, $V_{nt} = \tilde{V}_{nt} N_t^{(1-2\eta)/\eta}$, $Y_t = \tilde{Y}_t N_t^{(1-\eta)/\eta}$, $C_t = \tilde{C}_t N_t^{(1-\eta)/\eta}$, $K_t = \tilde{K}_t N_t^{(1-\eta)/\eta}$, $b_t = \tilde{b}_t N_t^{(1-2\eta)/\eta}$, $x_{zt} = \tilde{x}_{zt}/N_t$, $x_{nt} = \tilde{x}_{nt}/N_t$, $l_{zt} = \tilde{l}_{zt}/N_t$, $l_{nt} = \tilde{l}_{nt}/N_t$, $h_{zt} = \tilde{h}_{zt}/N_t$, $h_{nt} = \tilde{h}_{nt}/N_t$, $TFP_t = T\tilde{F}P_t N_t^{(1-\eta)\alpha/\eta}$ and $z_t = Z_t/N_t$.

We assume that all agents have the perfect foresight on the paths of the one-time shock with certainty. In this setting, we can apply a deterministic simulation with occasionally binding borrowing constraint (16) using Dynare (see Adjemian, Bastani, Karamé, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto and Villemot, 2011). This approach can solve a full nonlinear system of simultaneous equations even when constraints are occasionally binding using a modified Newton-Raphson algorithm. The details of the algorithm can be found in Juillard (1996). This algorithm solves $n \times T$ simultaneous equations, where n is the number of endogenous variables and T is the number of simulation periods. We set the number of periods of simulation 300, i.e., $T = 300$ and our model has 17 endogenous variables, i.e., $n = 17$.

We denote the Lagrange multiplier associated with (16) by μ_{nt}. Endogenous 17 variables $\{r_t, \tilde{w}_t, \tilde{V}_{nt}, \tilde{l}_{zt}, \tilde{l}_{nt}, \tilde{Y}_t, \tilde{C}_t, \tilde{K}_t, z_t, H_t, L_t, LW_t, g_t, g_{Yt}, T\tilde{F}P_t, \tilde{h}_{nt}, \mu_{nt}\}$ are calculated from the following 17 equations.

$$\tilde{l}_{zt} = \tilde{x}_{zt} = \left[\frac{\{\phi + (1-\phi)(1-\rho)\theta\}\alpha\eta\tilde{Y}_t^{1-\frac{\eta}{\alpha}}\tilde{K}_t^{(1-\alpha)\frac{\eta}{\alpha}}}{\tilde{w}_t}\right]^{\frac{1}{1-\eta}},$$

$$\tilde{Y}_t = \{z_t\tilde{x}_{zt}^{\eta} + (1-z_t)\tilde{x}_{nt}^{\eta}\}^{\frac{\alpha}{\eta}}\tilde{K}_t^{1-\alpha},$$

$$L_t = z_t\tilde{l}_{zt} + (1-z_t)\tilde{l}_{nt},$$

$$\tilde{C}_t + g_t^{\frac{1-\eta}{\eta}}\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t = \tilde{Y}_t,$$

$$\tilde{w}_t = \frac{\gamma\tilde{C}_t}{1-H_t},$$

$$r_t = (1-\alpha)\frac{\tilde{Y}_t}{\tilde{K}_t},$$

$$1 = \frac{\beta\tilde{C}_t}{g_t^{\frac{1-\eta}{\eta}}\tilde{C}_{t+1}}\left[r_{t+1} + 1 - \delta\right],$$

$$g_t = 1 - \rho + \kappa(H_t - L_t),$$

$$(1-\rho)z_t = g_t z_{t+1},$$

$$\tilde{V}_{nt} = \frac{\beta\tilde{C}_t}{g_t\tilde{C}_{t+1}}\left[\alpha\tilde{Y}_{t+1}^{1-\frac{\eta}{\alpha}}\tilde{K}_{t+1}^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{x}_{nt+1}^{\eta} - \kappa\tilde{V}_{nt+1}\tilde{l}_{nt+1} + (\kappa\tilde{V}_{nt+1} - \tilde{w}_{t+1})\tilde{h}_{nt+1} + (1-\rho)\tilde{V}_{nt+1}\right],$$

$$\mu_{nt} = \frac{\kappa\tilde{V}_{nt}}{\tilde{w}_t} - 1,$$

$$\begin{cases} \text{if } \mu_{nt} = \theta^{-1} - 1, \quad \text{then} \quad \tilde{w}_t\tilde{h}_{nt} > \phi\alpha\tilde{Y}_t^{1-\frac{\eta}{\alpha}}\tilde{K}_t^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{x}_{nt}^{\eta} + (1-\rho)\xi\tilde{V}_{nt}, \\ \text{if } \mu_{nt} < \theta^{-1} - 1, \quad \text{then} \quad \tilde{w}_t\tilde{h}_{nt} = \phi\alpha\tilde{Y}_t^{1-\frac{\eta}{\alpha}}\tilde{K}_t^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{x}_{nt}^{\eta} + (1-\rho)\xi\tilde{V}_{nt}, \end{cases}$$

$$\kappa\tilde{V}_{nt} = (1+\phi\mu_{nt})\eta\alpha\tilde{Y}_t^{1-\frac{\eta}{\alpha}}\tilde{K}_t^{(1-\alpha)\frac{\eta}{\alpha}}\tilde{x}_{nt}^{\eta-1},$$

$$H_t = (1-z_t)\tilde{h}_{nt} + z_t\tilde{l}_{zt},$$

$$g_{Yt} = g_t^{\frac{1-\eta}{\eta}}\frac{\tilde{Y}_{t+1}}{\tilde{Y}_t},$$

$$T\tilde{F}P_t = \frac{\tilde{Y}_t}{\tilde{K}_t^{1-\alpha}H_t^{\alpha}},$$

$$LW_T = \frac{\tilde{w}_t H_t}{\alpha\tilde{Y}_t}.$$

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	HP	KI	JIP2014
1971–1980	0.83		2.04
1981–1990	1.93	2.06	2.02
1991–2000	0.36	0.35	0.03
2001–2005		0.71	1.39
2006–2011			-0.28

Note: HP, KI, JIP2014 are from updated versions of Hayashi and Prescott (2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008), respectively. JIP2014

Table 1: TFP growth rate in Japan

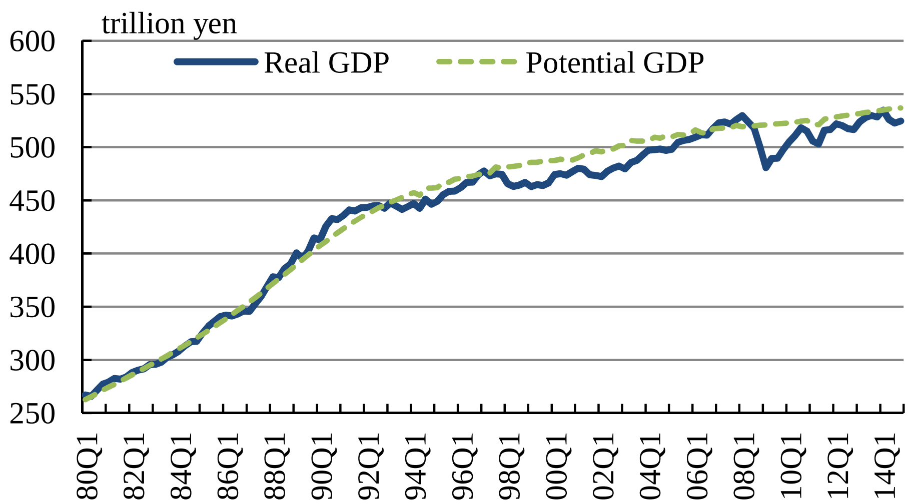
Parameter	Economic interpretation	value
α	the share of labor in production	2/3
β	the subjective discount factor	0.98
γ	the inverse of the elasticity of labor supply	1.78
δ	the depreciation rate of capital	0.1
η	the parameter for the aggregation function	0.67
κ	the efficiency of R&D	1.44
ϕ	the collateral ratio	0.95
ρ	the exit rate	0.1
θ	the efficiency of internal reserve	0.55

Table 2: Parameter settings

	z	$r^K - \delta$	w	H	L	LW	TFP	C	K	Y	g_Y	g
BGP	0	0.042	0.68	0.33	0.23	0.96	0.79	0.26	0.84	0.36	1.021	1.044
ZGP	1	-0.031	2.09	0.24	0.24	0.13	4.84	4.34	27.62	5.70	0.949	0.900

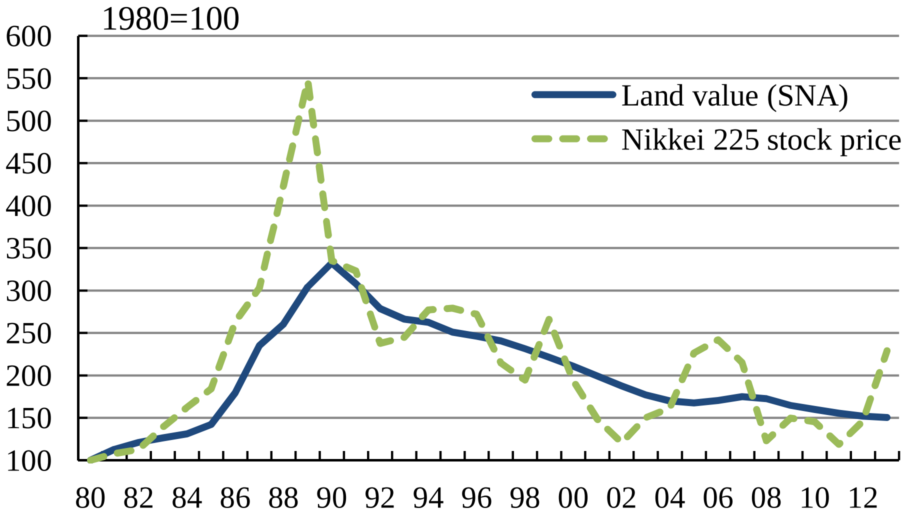
Note: BGP=Balanced growth path, ZGP=Zero growth path. Variables in BGP are detrended.

Table 3: Balanced growth path and zero growth path



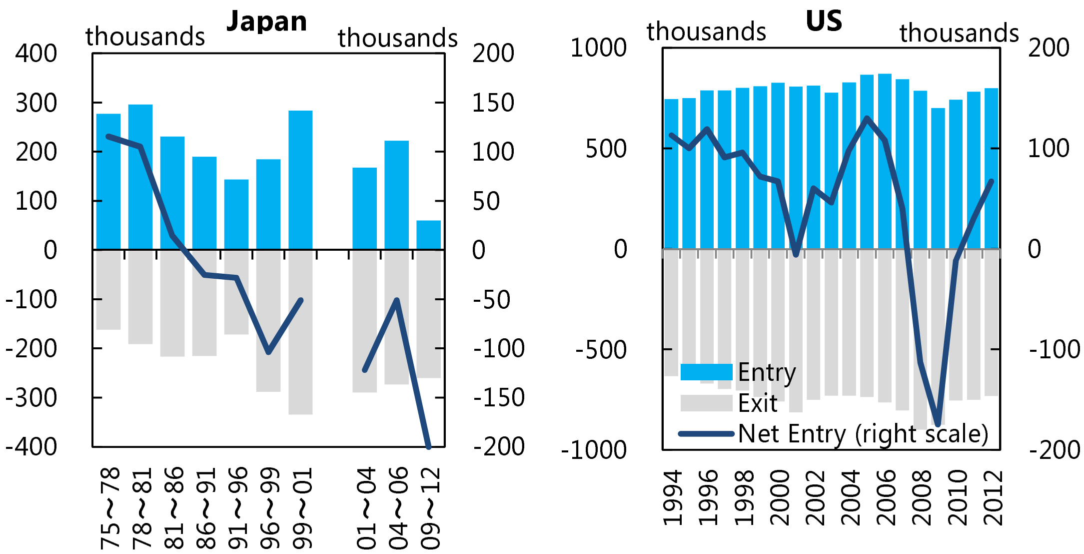
Source: Cabinet Office, Government of Japan, "Annual Report on National Accounts"

Figure 1: Real and potential GDP in Japan



Sources: Cabinet Office, Government of Japan; Nihon Keizai Shimbun.

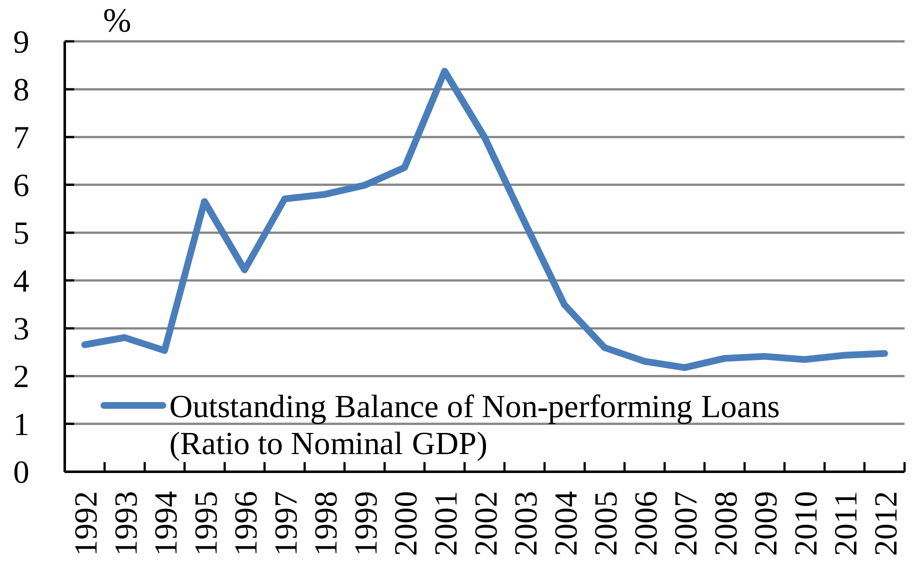
Figure 2: Land value and stock prices in Japan



Note: Japan's figures after 2001 are based on 1993-basis industry classification.

Sources: (Japan) Ministry of Internal Affairs and Communications, "Establishment and Enterprise Census"; (US) Bureau of Labor Statistics, "Business Employment Dynamics."

Figure 3: Entry and exit of private sector establishments: US and Japan



Note: The non-performing loans are the Risk Management Loans (RMLs) defined in the Banking Act in Japan. These consist of loans to bankrupt borrowers, delayed loans, three-month overdue loans, and loans with modified terms and conditions. RMLs do not include securitized loans.

Sources: Financial Services Agency,*Status of Non-Performing Loans*; Cabinet Office, Government of Japan,*Annual Report on National Accounts*.

Figure 4: Development of non-performing loans

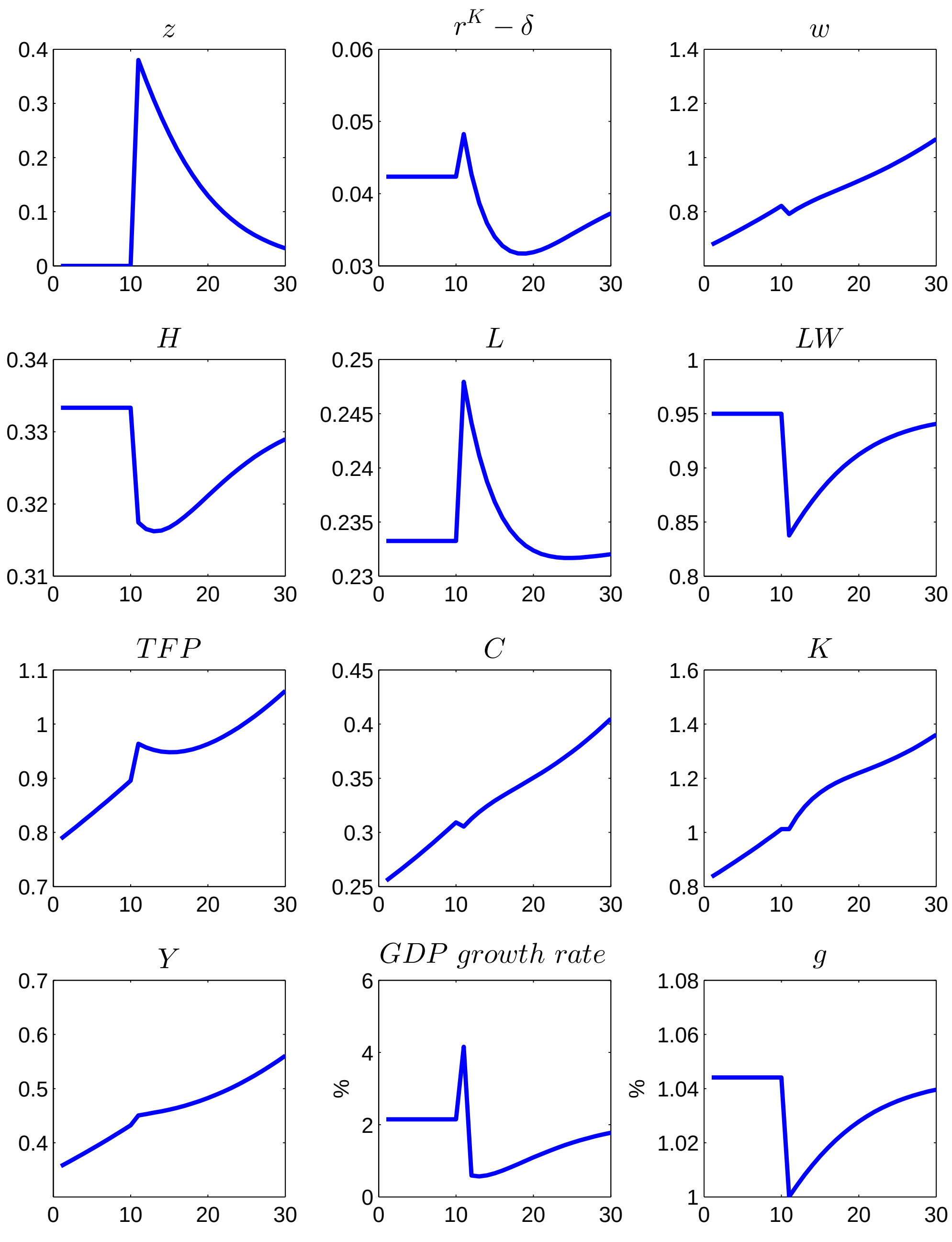
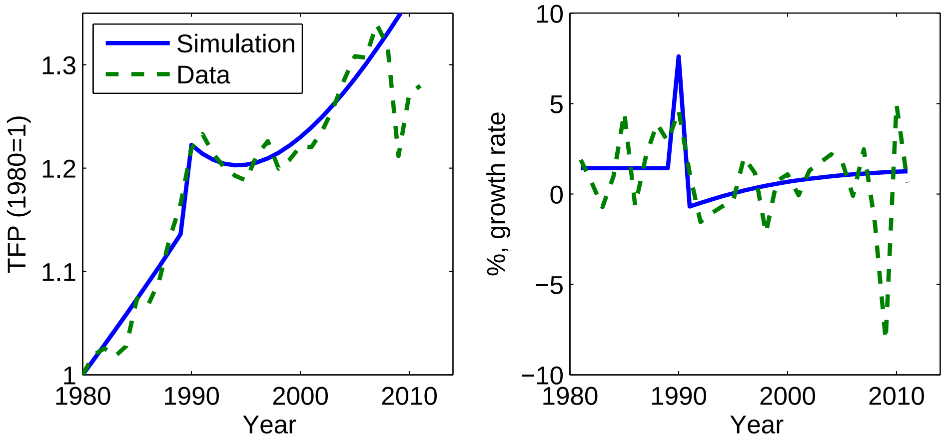


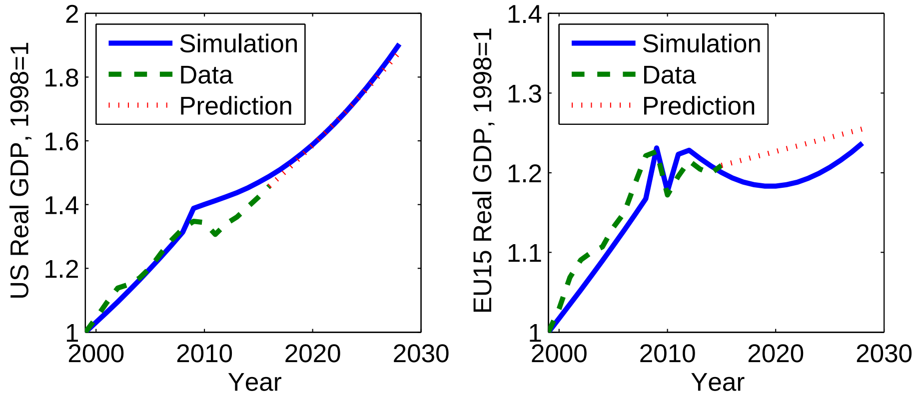
Figure 5: Responses to a financial shock



Note: Data is TFP in the "market economy" sectors that excludes education, medical services, government activities, and imputed house rent.

Sources: Our calculation; The Research Institute of Economy, Trade and Industry, *JIP 2014 database*.

Figure 6: Japan's TFP: comparison between data and simulation



Note: Prediction is the linear projection of the average growth rate between 2009 and 2014.

Sources: Our calculation; U.S. Bureau of Economic Analysis, *NIPA Tables*; Eurostat

Figure 7: US and EU15 GDP: comparison between data and simulation