

State-Dependent Pricing and the Paradox of Flexibility*

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Abstract

This paper studies the effects of price setting on shock amplification under a passive monetary policy rule (that keeps nominal interest rates constant, e.g. due to their lower bound). Price setting is crucial for the determination of expected inflation and the long-term real interest rate, and thus for shock propagation, e.g. of government spending shocks or monetary forward guidance. This mechanism involves a “paradox of flexibility”: increased flexibility of nominal prices, magnifies, rather than reduce, the real effects of shocks. In state-dependent pricing models, shock propagation under a constant nominal interest rate policy turns out to be stronger than in the standard New Keynesian, time-dependent framework. For the same frequency of price adjustment, the government spending multiplier in state-dependent models is much larger under a constant interest rate. This reflects the fact that the degree of price flexibility in these models is not just a function of the frequency of price adjustment, but rather of a “selection effect” due to firm idiosyncratic shocks.

Keywords: firm heterogeneity, state-dependent pricing, constant interest rate

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1 Introduction

In the aftermath of the “Great Recession” of 2008, monetary policy rates were rapidly decreased to historically low levels in many advanced countries, and have been mostly kept unchanged since then. This unprecedented situation has spurred a host of studies investigating the implications of such a passive monetary policy stance. A key finding is that a passive monetary policy, that keeps the nominal interest rate constant for a period of time, results in a strong amplification of shocks. In particular, fiscal stimulus can be very powerful in expanding economic activity and employment under such a passive policy. These findings are especially relevant given that in many countries the short-term nominal interest rate used as the main operating target for monetary policy has reached its effective lower bound.

Building on the contributions of Krugman (1998) and Eggertsson and Woodford (2003), several recent papers have argued that the government spending multiplier is very large under a passive monetary policy (e.g. Christiano et. al., 2011, Eggertsson, 2011, Woodford, 2011, Werning, 2012). Similarly, announcements (“forward guidance”) about the future path of nominal interest rates, following a period in which they are kept constant, should greatly affect current real activity and inflation (e.g. Levin et al. 2010 and Del Negro et al. 2013).¹

The mechanism underlying the amplification relies on a powerful real interest rate channel at the core of the workhorse New Keynesian (NK) model, on which this literature is largely based (and commonly used in monetary policy analysis). Focusing for the sake of clarity on fiscal stimulus, the latter may have a large effect on output if private consumption increases with government purchases instead of being crowded out. In standard models consumption depends inversely on the long-term real interest rate. When the nominal interest rate is constant for some time, the long-term real rate is essentially a function of inflation expectations, and in particular of the long-run price level. To the extent that nominal prices or wages are not fully flexible, and systematic monetary policy allows a sustained increase in future inflation and the expected long-run price level, the long-term rate falls and current consumption rises. Aggregate activity thus increases in excess of government spending

¹The literature on the implications of the zero lower bound is large and increasing. See among others Braun et al. (2012), Christiano and Eichenbaum (2012), Fernandez-Villaverde et al. (2014), Kiley (2013), Mertens and Ravn (2014), and Wieland (2014).

resulting in multipliers well above one.

Importantly, the above mechanism involves a “paradox of flexibility”: increasing the responsiveness of nominal prices and wages (while obviously falling short of full flexibility) magnifies, rather than reduce, the real effects of shocks. The more flexible prices are assumed to be, and thus the steeper the New Keynesian Phillips curve, the more future inflation is expected to increase, resulting in a larger spending multiplier.

This paper studies shocks propagation under a constant interest rate monetary policy in a simple macroeconomic model that departs from the workhorse NK framework in one but crucial aspect. Namely, nominal price flexibility is allowed to be endogenous (i.e. state-dependent), rather than exogenous (i.e. time-dependent). Several recent contributions have questioned the powerful transmission mechanism in the NK model (e.g. Ohanian 2011, Cochrane 2015). At the center of much of this criticism is the assumption of exogenous timing of price changes *a la* Calvo (1983) - Yun (1996). It is well known that the specifics of price setting at the micro level is essential for the dynamics of the aggregate price level and output. For instance, compared to (exogenous) time-dependent pricing, (endogenous) state-dependent pricing in a fixed menu cost model produces very different inflation and output responses to a monetary policy shock (Golosov and Lucas, 2007; Alvarez, Le Bihan and Lippi 2014).² Yet, to the best of our knowledge, a study of the fiscal multiplier and of shock amplification under a constant interest rate policy, is missing in the context of state-dependent pricing models.

The main finding of the paper is that allowing pricing by firms to be state-dependent results in an even stronger propagation of shocks compared to Calvo-Yun pricing. Under the same average frequency of price adjustment, the impact government spending multiplier in state-dependent pricing models is an order of magnitude larger than in the Calvo-Yun model, with a constant interest rate. This reflects the fact that the degree of price flexibility in these models is endogenous and not just an exogenous frequency parameter.³

²A main difference of this class of models of price setting, relative to time-dependent models, concerns the behavior of “reset price inflation” (the rate of change of all desired prices) relative to actual inflation. As documented by Bils et al. (2009, 2012), time-dependent models imply unrealistically high persistence and stability of reset price inflation in response to shocks, relative to the data, which is inherited by actual inflation. But the tight link between reset price inflation and future inflation dynamics is a key determinant of the multiplier under a constant nominal interest rate in the New Keynesian model.

³We focus on the case of the government spending multiplier under a constant interest rate, but results easily extend to the effects of monetary forward guidance or other shocks under a similarly passive monetary policy.

In particular, the paper adopts the general framework of “smoothly state dependent pricing” by firms subject to idiosyncratic shocks, as in Costain and Nakov (2011b). Pricing in this model is dubbed “smoothly state-dependent” because the probability of adjustment is a smoothly increasing function of the adjustment gain, rather than the 0–1 step function it is in the menu cost model, or the constant it is in the Calvo–Yun model. The framework is general in the sense that it nests the menu-cost model at one extreme and the Calvo–Yun model at the opposite extreme. However, its intermediate version is empirically more plausible than either of the extremes (Costain and Nakov, 2011a). The smoother intermediate model matches well observed histograms of firm-level price changes such as that found in AC Nielsen’s retail price data documented by Midrigan (2007) — see Figure 1. Importantly, under a standard (active) monetary policy rule, it virtually produces the same effects of a government spending shock as its Calvo–Yun extreme with the same average frequency of price adjustment.⁴

In such a state-dependent framework, aggregate price flexibility depends crucially on the “selection effect” emphasized by Golosov and Lucas (2007). Namely, firms that change prices are those for which adjustment is more valuable, and which tend to make larger price changes. This makes the aggregate price level more flexible in equilibrium than in the Calvo–Yun framework, *even when holding the average frequency of adjustment constant across models*. In response to a government spending shock, given a constant interest rate, the faster increase in the price level means that the real interest rate falls by more, stimulating private consumption and leading to a larger increase in output than under Calvo.

However, when the degree of flexibility of the aggregate price level is allowed to be endogenous, its response may quickly converge to a situation close to full flexibility under a passive monetary policy. Namely, even for small aggregate shocks, virtually all firms find it optimal to change their prices, under a sufficiently long period with constant interest rates.⁵ This result thus casts some doubt on the very large propagation mechanism of business cycle sized shocks found in the literature assuming an exogenous degree of flexibility. An investigation of this important issue is however beyond the scope

⁴It also produces larger real effects of monetary shocks than the fixed menu cost model of Golosov and Lucas.

⁵As we explain below, we solve our model using Reiter’s algorithm, combining projections and perturbations methods. Therefore, we are restricted to small enough aggregate shocks that do not trigger adjustment by all firms. In practice, we find that these shocks have to be quite small when interest rates are constant for a period of 2 or 3 years, as typically analyzed in the literature.

of this paper.

Interacting with the price setting mechanism, the monetary policy reaction function is also crucial in shaping the formation of expectations of future inflation. Even under a constant nominal interest rate, it is well known that the specific form of the reaction function matters for the response of the economy to shocks. As shown by Eggertsson and Woodford (2003), Adam and Billi (2005) and Nakov (2008) for the case of a zero lower bound episode with stochastic duration, this especially concerns the interest rate lift-off periods in the aftermath of a ZLB episode (see also Cochrane (2015)). An exploration of the effects of different monetary policy rules confirms the result that a large shock propagation occurs only when monetary policy allows the long-run response of the price level to be nonstationary.

After presenting the model in Section 2, Sections 3 and 4 provide several analytical results for the textbook Calvo–Yun model (abstracting from firm idiosyncratic shocks). First, the results in Woodford (2011) on the spending multiplier under a binomial process for government expenditure at the zero lower bound are extended to the case of a standard AR(1) process with a constant interest rate. In both cases, when the duration of the constant interest rate period is stochastic, the multiplier is shown to be only *locally* increasing in the degree of price flexibility. Second, the analysis considers the case of an interest rate that is kept constant for a given period of time by a series of anticipated shocks to the Taylor rule (dubbed the “modest interventions” rule by Galí, 2012), e.g. as in Erceg and Linde (2014) and Werning (2012). This is the standard approach adopted in large policy models with many state variables. Similarly to the results in Carlstrom, Fuerst and Paustian (2014), a constant interest rate of deterministic duration delivers the largest multipliers, and indeed magnifies the “paradox of flexibility”. But if the constant interest rate is instead the result of a money growth rule, which imparts strong mean reversion to the price level, then all paradoxes disappear — in line with arguments in Cochrane (2015).

Section 5 then turns to the numerical analysis of the model with state-dependent pricing and idiosyncratic shocks. Exploring the implications of systematic monetary policy under state-dependent pricing is a further contribution of the paper (Golosov and Lucas, 2007 and others assume a money growth rule). First, under a standard interest rate rule that is only allowed to depend on inflation, the

multiplier is less than 1 across all price-setting frameworks, and lowest under state-dependent pricing. Second, when the interest rate is instead constant for a deterministic period of time the multiplier is larger than unity across all models. Moreover, amplification is largest under state-dependent pricing. The strong propagation under constant interest rates thus holds across price-setting models, and the “paradox of flexibility” extends to a broad concept of price flexibility.

2 A simple macroeconomic model nesting time- and state-dependent pricing

The model embeds state-dependent pricing by firms in an otherwise-standard New Keynesian general equilibrium framework. Following Costain and Nakov (2011a,b), the framework nests a continuum of pricing models, with the Calvo model on one extreme, and the Golosov and Lucas (2007) fixed menu cost model on the other. Besides the firms, there is a representative household, a monetary authority that implements a Taylor rule, and a government that levies lumps sum taxes to finance an exogenous stream of spending on goods.

2.1 Representative household

The household’s period utility is $\frac{1}{1-\gamma}C_t^{1-\gamma} - \frac{\chi}{1+\eta}N_t^{1+\eta} + \nu \log(M_t/P_t)$, where C_t denotes consumption, N_t labor supply, and M_t/P_t real money balances. Utility is discounted by β . Consumption is a CES aggregate of differentiated products C_{it} , with elasticity of substitution ϵ :

$$C_t = \left\{ \int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \quad (1)$$

The household’s nominal period budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + R_t^{-1}B_t = W_tN_t + M_{t-1} + T_t + B_{t-1}, \quad (2)$$

where $\int_0^1 P_{it}C_{it}di$ is total nominal consumption. B_t is nominal bond holdings purchased at t , and

paying interest rate $R_t - 1$ at time $t + 1$. T_t is a nominal lump-sum transfer consisting of seigniorage revenues from the central bank plus dividend payments from the firms, net of lump-sum taxes to finance government spending on goods.

Households choose C_{it} , N_t , B_t , and M_t to maximize expected discounted utility, subject to the budget constraint (2). Optimal consumption across the differentiated goods implies

$$C_{it} = (P_t/P_{it})^\epsilon C_t, \quad (3)$$

where $P_t \equiv \left[\int_0^1 P_{it}^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$ is the relevant price index.

Optimal labor supply, consumption, and money use imply the following first-order conditions:

$$\chi C_t^\gamma N_t^\eta = W_t/P_t, \quad (4)$$

$$1 = \beta R_t E_t [P_t C_{t+1}^{-\gamma} / (P_{t+1} C_t^{-\gamma})], \quad (5)$$

$$M_t/P_t = \nu C_t^\gamma R_t / (R_t - 1). \quad (6)$$

2.2 Monopolistic firms

Firms are monopolistic competitors. Each firm i produces output Y_{it} using labor N_{it} as the only input, under a linear technology: $Y_{it} = A_{it}N_{it}$. Firm's productivity A_{it} is an idiosyncratic process, AR(1) in logs:

$$\log A_{it} = \rho_a \log A_{it-1} + \varepsilon_{it}^a, \quad (7)$$

where $0 \leq \rho_a < 1$ and $\varepsilon_{it}^a \sim i.i.d.N(0, \sigma_a^2)$. Firm i charges a price P_{it} and faces demand from two sources, $Y_{it} = C_{it} + G_{it}$, where C_{it} is demand for goods by the households, and G_{it} is demand by the government. The government's consumption basket is also a CES aggregator with elasticity of substitution ϵ :

$$G_t = \left\{ \int_0^1 G_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \quad (8)$$

Optimal allocation of expenditure across goods on the part of households and the government implies that firm i faces the demand curve $Y_{it} = (C_t + G_t)P_t^\epsilon P_{it}^{-\epsilon}$. The firm fulfills all demand at its

posted price. It hires in a competitive labor markets at wage rate W_t , generating period profits

$$U_{it} = P_{it}Y_{it} - W_tN_{it}. \quad (9)$$

Costain and Nakov (2011) derive the following value function for a firm which produces with productivity A and sells at nominal price P when the aggregate state is Ω :

$$V(P, A, \Omega) = U(P, A, \Omega) + \beta E \left\{ \frac{P(\Omega)C(\Omega')^{-\gamma}}{P(\Omega')C(\Omega)^{-\gamma}} [V(P, A', \Omega') + G(P, A', \Omega')] \middle| A, \Omega \right\}, \quad (10)$$

where

$$G(P, A', \Omega') \equiv \lambda \left(\frac{\max_P V(P, A', \Omega') - V(P, A', \Omega')}{W(\Omega')} \right) \left(\max_P V(P, A', \Omega') - V(P, A', \Omega') \right) \quad (11)$$

is the *expected gain* from adjustment, and $\lambda \left(\frac{\max_P V(P, A', \Omega') - V(P, A', \Omega')}{W(\Omega')} \right) \in [0, 1]$ is a mapping from the gain from price adjustment to the probability of adjustment, which is detailed in the following subsection (2.3).

Price stickiness means that the individual price process associated with the Bellman equation is

$$P_t = \begin{cases} \arg \max_P V(P, A', \Omega') & \text{with probability } \lambda(P, A', \Omega') \\ P & \text{with probability } 1 - \lambda(P, A', \Omega') \end{cases}. \quad (12)$$

2.3 Nesting alternative price-setting schemes

Following Costain and Nakov (2011a,b) we assume that the probability of price adjustment $\lambda(L)$, increases with the gain from adjustment L . Thus, the function $\lambda(L) \in [0, 1]$ that governs this probability is taken as a primitive of the model.⁶ In particular, we postulate the following functional form:

$$\lambda(L) \equiv \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda})(\alpha/L)^\xi} \quad (13)$$

⁶Alternatively, λ can be viewed as an exogenously specified distribution of menu costs from which firms make random draws every period.

where $L \equiv [\max_P V(P, A', \Omega') - V(P, A', \Omega')] / W(\Omega')$ is the relevant endogenous state, with α and ξ positive, and $\bar{\lambda} \in [0, 1]$. This function is concave for $\xi \leq 1$ and S-shaped for $\xi > 1$ (see Fig. 1). Parameter ξ controls the degree of state dependence. In the limit $\xi = 0$, (13) nests Calvo (1983) so that $\lambda(L) = \bar{\lambda}$ regardless of L . At the opposite extreme, as $\xi \rightarrow \infty$, $\lambda(L)$ becomes the indicator function $\mathbf{1}\{L \geq \alpha\}$, which equals 1 whenever $L \geq \alpha$ and is zero otherwise. This is the fixed menu cost model which implies “extreme state dependence”, in the sense that the adjustment probability jumps from 0 to 1 as soon as the state L passes the threshold α . For intermediate values $0 < \xi < \infty$, the hazard increases “smoothly” with the state L . We call this intermediate version “smoothly state-dependent pricing” (SSDP) model.

2.4 Monetary policy and aggregate consistency

In our baseline model monetary policy follows a Taylor rule,

$$r_t - \bar{r} = \phi_\pi \pi_t + \sum_{j=1}^T \varepsilon_{t-j}^r \quad (14)$$

where r_t is the net nominal interest rate, $\bar{r} = E(r_t)$, $\pi_t = \log(P_t/P_{t-1})$ and $\phi_\pi > 1$. The shocks ε_t^r are *anticipated* and are set so that, following a spending shock, the nominal interest is constant at its steady state value for T periods. This is a standard approach for studying the effects of constant interest rates, see e.g. Erceg and Linde (2014). Galí (2012) dubs it “modest interventions” to keep the interest rate constant. This is equivalent to instead picking shocks, e.g. to the rate of time preference, so that the zero lower bound would constrain the nominal interest rate for T periods, while government spending increases following some exogenous process.

Further, we assume that government spending G_t follows an AR(1) process in logs:

$$\log(G_t/G^*) = \rho \log(G_{t-1}/G^*) + \varepsilon_t^g, \quad (15)$$

with persistence $0 < \rho < 1$ and $\varepsilon_t^g \sim i.i.d.N(0, \sigma_g^2)$.

Seigniorage revenues are paid to the household as a lump-sum transfer, and the government budget

is balanced each period. Because the timing of lump-sum taxes is irrelevant, bond market clearing is simply $B_t = 0$. When supply equals demand for each good i , total labor supply and demand satisfy

$$N_t = \int_0^1 \frac{Y_{it}}{A_{it}} di = P_t^\epsilon Y_t \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di \equiv \Delta_t Y_t. \quad (16)$$

where $\Delta_t \equiv P_t^\epsilon \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$ is a measure of price dispersion which takes into account heterogeneous productivity.

Costain and Nakov (2011b) show that in this case the aggregate state of this economy is summarized by $\Omega_t \equiv (G_t, \Phi_{t-1})$, where $\Phi_{t-1}(P, A)$ is the lagged distribution of firms over prices and productivities.

2.5 Computation and parameterization

The model is solved following the two-step algorithm of Reiter (2009). In a first step, the steady state in which aggregate variables are constant but firms are subject to idiosyncratic shocks is computed on a finite grid, using backwards induction. Second, the stochastic dynamics in response to aggregate shocks are computed by linearization around each grid point. Thus, the Bellman equation is treated as a large system of expectational difference equations, instead of a functional equation. However, a limitation is that the size of the aggregate shock cannot be so large as to imply that the optimal firm choices fall outside of the grid.⁷

We calibrate the three models (Calvo, fixed menu cost, and SSDP) to match salient features of the microdata on price changes, such as those found by Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Midrigan (2007, 2011). In particular, we seek price adjustment and productivity processes which are consistent with the histogram of price changes in AC Nielsen’s monthly data documented by Midrigan (2007).

The model is one of “regular” price changes, excluding temporary “sales”, and the working frequency is monthly. We set the growth rate of money to 0%, consistent with the zero average price change found in AC Nielsen’s data.⁸ Other macro parameters are set to standard values in the RBC literature. Thus, the discount factor is $\beta = 1.04^{-1/12}$. Consumption utility is CRRA, $u(C) = \frac{1}{1-\gamma} C^{1-\gamma}$,

⁷See Costain and Nakov (2011) for a detailed exposition of Reiter’s method as applied to sticky price models.

⁸Reiter’s solution method is not limited to zero steady-state inflation rates.

with $\gamma = 2$, while labor disutility is $\chi \frac{N^{1+\eta}}{1+\eta}$, with $\eta = 1$ and $\chi = 11$, which implies that 35% of the time is dedicated to work. The elasticity of substitution in the consumption aggregator is $\epsilon = 7$. The utility of real money holdings is logarithmic, $v(m) = \nu \log(m)$, with $\nu = 1$. And we set $G^* = 0.07$, consistent with an average share of government spending of 20% of GDP.

This leaves us with five parameters to pin down the aggregate steady-state: the persistence ρ_a and the volatility of σ_a^2 of the idiosyncratic productivity process; and the three parameters $(\bar{\lambda}, \xi, \alpha)$ of the adjustment function (13). We estimate these parameters by minimizing a distance criterion between the data and the model's steady state. The criterion sums two terms, scaled for comparability: the first is the absolute difference between the adjustment frequencies in the data and the simulation, while the second is the Euclidean distance between the frequency vectors associated with the histograms of nonzero price adjustments in the data and the simulation.⁹ The left panel of figure 1 shows the fit of the three models to the histogram of price changes from AC Nielsen's data. The right panel shows the adjustment hazard function for different parameterizations. Clearly, the SSDP model provides the best fit to the observed histogram.

To simulate the model's dynamics, we need to specify two additional sets of parameters. One is related to the monetary policy rule (14). For our benchmark model, we set $\phi_\pi = 2$ and $T = 24$. The other set of parameters calibrates the exogenous process for government spending (15). We set $\rho = 0.9$, which is similar to Erceg and Linde (2014), and pick the shock size so that, on impact, the increase in government spending equals 0.1% of GDP. This value ensures that our grid contains the endogenous choices of firms in response to the shock in all three model specifications.

3 The spending multiplier and systematic monetary policy

In this section we discuss the link between the size of the government spending multiplier and movements in the price level. Consider first the log-linearized representative household Euler equation (5). Here small letters denote percentage deviations from steady state (with output and spending both in

⁹See Costain and Nakov (2011a,b) for more details about the estimation.

deviations from steady state output), and $\sigma = \gamma^{-1}$:

$$c_t = y_t - g_t = E_t(y_{t+1} - g_{t+1}) - \sigma(r_t - \bar{r} - E_t\pi_{t+1}). \quad (17)$$

Solving this equation forward for c_0 , we obtain the following expression for current output in excess of government purchases:

$$c_0 = y_0 - g_0 = -\sigma E_0 \sum_{j=1}^{T+1} (r_{j-1} - \bar{r} - \pi_j) + E_0(y_{T+1} - g_{T+1}). \quad (18)$$

Under stationarity of output and the government spending shock, $\lim_{T \rightarrow \infty} E_0(y_{T+1} - g_{T+1}) = 0$. The response of output in excess of government purchases depends on whether the sum of expected short-term real rates, or the long-term real rate, is positive or negative (see Woodford, 2011). Under nominal rigidities, the degree to which systematic monetary policy reacts to the government spending shock and interacts with future inflation formation will determine the size of the multiplier. This insight extends to the propagation of different shocks.

For instance, if we assume that the Taylor rule (14) is such that $r_{t+j-1} - \bar{r} = \phi_\pi \pi_{t+j}$, $\phi_\pi > 1$, the expression above becomes:

$$y_0 - g_0 = -\sigma(\phi_\pi - 1) E_0 \sum_{j=1}^{\infty} \pi_j = \sigma(\phi_\pi - 1) \left(p_0 - \lim_{T \rightarrow \infty} E_0 p_{T+1} \right). \quad (19)$$

Since $\sigma(\phi_\pi - 1) > 0$, the size of the multiplier is *increasing* in the contemporaneous price level p_0 , and *decreasing* in the long-run expected response of the price level, $E_0 p_{T+1}$. Under the interest rate rule considered, the price level will not return to its original value. As we show below, under Calvo–Yun price setting, the multiplier is generally smaller than one, since the future price level is above the current one when government spending follows an AR(1) process (see also Woodford (2011); Corsetti et al. (2010) analyze more general spending processes). Below we show numerically that this result also holds under a more general price-setting mechanism.

Matters, however, are different when the monetary policy rule entails a nominal interest rate less responsive to inflation. Suppose that monetary policy is such that the nominal interest rate is held

constant at its steady state level for T periods after the shock, $r_t = \bar{r}$ for $t = 1 \dots T$; the long-term real rate will then depend only on expected future inflation as follows:

$$y_0 - g_0 = -\sigma E_0 \sum_{j=1}^{T+1} (r_{j-1} - \bar{r} - \pi_j) + E_0 (y_{T+1} - g_{T+1}) \quad (20)$$

$$= \sigma \left(\lim_{T \rightarrow \infty} E_0 p_{T+1} - p_0 \right). \quad (21)$$

Now the multiplier is larger than one if the expected future price level *exceeds* the impact price response. Hence, whether systematic monetary policy allows for a drift in the price level is a key determinant of the size of the multiplier, after a period in which interest rates are constant (including at their lower bound).

As an example, consider the systematic monetary policy analyzed under the zero lower bound by e.g. Christiano et al. (2011) and Woodford (2011), which stabilizes the price level at the value assumed on the date of lift-off of the interest rate. This monetary policy implies that the price level has a positive drift and exceeds the current price level, resulting in a large multiplier in the Calvo–Yun model. In this case, the more “flexible” prices are, the bigger the difference between the future and the current price level, as we show below. This is because inflation is determined by the rate of change of desired prices (“reset inflation”). Specifically, in the Calvo–Yun model, actual (and expected) inflation is proportional to reset inflation, $\pi_t^* \equiv \log(p_t^*/p_{t-1}^*)$:

$$\pi_t = \bar{\lambda} \pi_t^* + (1 - \bar{\lambda}) \pi_{t-1}, \quad (22)$$

where $\bar{\lambda}$ is the probability of price changes, and the desired price p_t^* in response to a government spending shock is given by:

$$\log(p_t^*/P_t) = \beta E_t \log(p_{t+1}^*/P_{t+1}) + (1 - (1 - \bar{\lambda}) \beta) (\gamma + \eta) \left(y_t - \frac{\gamma}{\gamma + \eta} g_t \right). \quad (23)$$

Notice that the size of the desired price change, and the evolution of inflation both depend on the frequency of price adjustment.

Conversely, as shown by Bilal et al. (2009, 2012), the link between actual and reset inflation is

quite different under state-dependent pricing. Endogenous price changing, and especially selection of changers, breaks the simple proportionality between π_t^* and π_t in time-dependent models. On the one hand, reset inflation tends to react more strongly to shocks on impact. This is because firms hit by larger idiosyncratic shocks are more likely to adjust their prices, other things equal. On the other hand, reset inflation is much less stable and persistent, resulting in different properties of cumulated actual inflation. Even assuming the same average frequency of price adjustment as in a time-dependent model, state-dependent pricing can have dramatic implications for the effects of government purchases on aggregate economic activity under a constant interest rate.

Likewise, when the constant interest rate results from a rule implying a stationary price level, i.e. $\lim_{T \rightarrow \infty} E_t p_{T+1} = 0$, such as a price level target, the price setting mechanism will affect the size of the multiplier. As we show below, the multiplier is in general smaller than unity, and decreasing in the degree of price flexibility, as the response of current inflation is larger the more flexible prices are. In this case, state-dependent pricing implies a smaller multiplier than time-dependent pricing, for the same average frequency of price adjustment.

4 Constant nominal interest rates, price flexibility and the size of the spending multiplier

We have argued that the interaction between monetary policy and the price setting mechanism is of paramount importance to understanding the propagation of shocks when the nominal interest rate is constant or at its lower bound. In this section we collect some useful analytical results for the version of our economy with Calvo–Yun pricing for the case of government spending shocks. First, we show that under a period of constant nominal interest rates of stochastic duration the multiplier (and shock propagation in general) is large if the degree of price flexibility is such that the persistence of inflation is close to but lower than the persistence of the spending shock. Second, we show that when the nominal interest rate is kept constant for a deterministic period of time, the multiplier is just larger the more flexible prices are, resulting in a pervasive paradox of flexibility

4.1 The multiplier under a constant interest rate of stochastic duration

Consider the log-linearized Calvo–Yun model in Woodford (2011), comprising in addition to the Euler equation shown above,

$$y_t - g_t = E_t(y_{t+1} - g_{t+1}) - \gamma^{-1}(r_t - \bar{r} - E_t\pi_{t+1}),$$

the standard New-Keynesian Phillips curve (NKPC):

$$\pi_t = \beta E_t\pi_{t+1} + \kappa(y_t - \Gamma g_t). \quad (24)$$

The slope κ is the following function of deep parameters

$$\kappa = (\gamma + \eta) [1 - (1 - \bar{\lambda}) \beta] \bar{\lambda} / (1 - \bar{\lambda}), \quad (25)$$

$$\Gamma = \frac{\gamma}{\gamma + \eta}, \quad (26)$$

where $0 < \Gamma < 1$ denotes the multiplier under flexible prices. Observe that g_t denotes government spending, in deviation from steady state, scaled by the steady state level of GDP. It follows the exogenous AR(1) process in (15). We will characterize the size of the (local) multiplier, $\gamma_y = \frac{\partial y_t}{\partial g_t}$, for different monetary policy responses to this spending shock.

Since the focus in this paper is on the case of a constant nominal interest rate, it is useful to recall the key results under the zero lower bound (ZLB). Under a binding ZLB which lasts an additional period with probability p (e.g. as in Eggertsson and Woodford, 2003 or Christiano et al., 2011), assuming that the zero-inflation steady state is an absorbing state, the multiplier for a spending shock (following a similar binomial process reverting with probability $1 - p$ to its absorbing, steady state value) is given by:¹⁰

$$\gamma_y = 1 + \frac{(1 - \Gamma) p \kappa}{(1 - p)(1 - \beta p) \gamma - p \kappa}.$$

In order to have a unique bounded solution it must be that the denominator in the above expression

¹⁰See e.g. Woodford (2011), where the ZLB binds because of such a binomial shock to the natural interest rate in the Euler equation.

is positive, $(1 - p)(1 - \beta p)\gamma/p > \kappa$.¹¹ In words, the condition requires that the probability p of the interest rate remaining constant at its lower bound (and of spending g_t remaining above its steady state value) is sufficiently smaller than the probability of not adjusting prices, $1 - \bar{\lambda}$. Due to the linearity of the model, for this result it is immaterial whether the interest rate is constant at its lower bound (e.g. because of a shock to the natural interest rate), or it is constant at its steady state value (because of a monetary policy shock that is anticipated to exactly offset the effects of government spending on the nominal interest rate).

The multiplier is larger than unity (and larger than its flexible-price counterpart Γ), and in fact can be much larger. In particular, it is increasing in κ , and thus in the (exogenous) degree of price flexibility $\bar{\lambda}$. This is the so-called “paradox of flexibility” stressed e.g. by Werning (2012) and Eggertsson and Krugman (2012). The multiplier is also *decreasing* in Γ , and in particular equal to one when $\Gamma = 1$, which is the case with linear labor disutility.¹²

The intuition for this result is straightforward. With a constant nominal interest rate, the long term real interest rate is a function of expected cumulated inflation, which in turn is increasing in the slope of the NKPC. Specifically, it is easy to show that the price level has a positive drift:¹³

$$E_0 \sum_{j=1}^{\infty} \pi_j = E_0 \lim_{T \rightarrow \infty} p_T - p_0 = \frac{\kappa(\gamma_y - \Gamma)}{(1 - \beta p)} g_0 \sum_{j=1}^{\infty} p^j = \frac{\kappa(\gamma_y - \Gamma)}{(1 - \beta p)} \frac{p}{1 - p} g_0, \quad (27)$$

$$\lim_{T \rightarrow \infty} p_T = \frac{1}{1 - p} \pi_0, \quad \pi_0 = p_0 = \frac{\kappa(\gamma_y - \Gamma)}{(1 - \beta p)} g_0, \quad (28)$$

¹¹This can be verified easily by checking that both eigenvalues of the characteristic polynomial of the following system of difference equations are larger than 1 when the condition is satisfied,

$$\begin{aligned} \pi_t &= \beta [p\pi_t + (1 - p) \cdot 0] + \kappa (y_t - \Gamma g) \\ y_t - g &= p(y_t - g) + \gamma^{-1} (p\pi_t + r) \end{aligned}$$

where $r < 0$, $g > 0$ such that that $r_t = 0$.

¹²In this case the only allocation consistent with a constant nominal interest rate coincides with that under flexible prices with a constant real rate. This is the benchmark specification in the study of money growth shocks in Golosov and Lucas (2007).

¹³The specific monetary policy rule prevailing when the economy is back at the zero inflation steady state is immaterial, to the extent that it allows for an expected drift in the price level — see also Cochrane (2015). Conversely, in the fully stochastic cases examined by Adam and Billi (2005) and Nakov (2008), it makes a great deal of difference whether upon exit from a ZLB monetary policy is chosen optimally, even under discretion, or follows a Taylor rule.

where the spending multiplier in excess of its flexible price counterpart, $\gamma_y - \Gamma$, is given by:

$$\gamma_y - \Gamma = \frac{(1-p)(1-\beta p)\gamma}{\Delta} (1-\Gamma). \quad (29)$$

The latter expression is positive if the multiplier γ_y is larger than Γ , which is the case for $\Delta = (1-p)(1-\beta p)\gamma - p\kappa > 0$. Therefore, in the Calvo model under a ZLB (or a constant interest rate policy) of stochastic duration, the multiplier increases in $\bar{\lambda}$, and thus in the slope of the NKPC because expected inflation is proportional to current inflation. The latter in turn is the more responsive to shocks, such as government spending, the steeper the Phillips curve.

We find it useful to consider next what happens under a simple Taylor rule in which the interest rate is allowed to respond to a government spending shock that follows an AR(1) process with persistence ρ :

$$r_t - \bar{r} = \phi_\pi \pi_t - \phi_g g_t. \quad (30)$$

Guessing the following decision rules under the unique bounded solution (for $\phi_\pi > 1$),¹⁴

$$\pi_t = \gamma_\pi g_t, \quad (31)$$

$$y_t = \gamma_y g_t, \quad (32)$$

it is straightforward to find the solutions for the coefficients:

$$\gamma_\pi = \frac{\kappa(\gamma_y - \Gamma)}{1 - \beta\rho}, \quad (33)$$

$$\gamma_y - 1 = \rho(\gamma_y - 1) + \sigma \left[\phi_g - (\phi_\pi - \rho) \frac{\kappa(\gamma_y - \Gamma)}{1 - \beta\rho} \right]. \quad (34)$$

Intuitively, the last expression implies that the multiplier is larger than one only to the extent that

¹⁴This can be verified easily by checking that both eigenvalues of the characteristic equation of the following difference equation in inflation are larger than 1:

$$(1 + \kappa\sigma\phi_\pi)\pi_t - (1 + \beta + \kappa\sigma)E_t\pi_{t+1} + \beta E_t\pi_{t+2} = 0.$$

The assumption of a Taylor rule is immaterial, as the same allocation could be decentralized with an appropriate money growth rule that responds to the government spending shock in a similar way.

the expected deviation from steady-state of the short-term real interest rate,

$$(r_t - \bar{r} - E_t \pi_{t+1}) = \left[(\phi_\pi - \rho) \frac{\kappa (\gamma_y - \Gamma)}{1 - \beta\rho} - \phi_g \right], \quad (35)$$

is *negative*.

Solving explicitly for the multiplier γ_y yields the following:

$$\gamma_y = 1 + \frac{\phi_g - (1 - \Gamma) \frac{\phi_\pi - \rho}{1 - \beta\rho} \kappa}{(1 - \rho) \gamma + \frac{\phi_\pi - \rho}{1 - \beta\rho} \kappa}. \quad (36)$$

The following considerations are in order. First, the multiplier is larger than one if and only if $\phi_g > (1 - \Gamma) \frac{\phi_\pi - \rho}{1 - \beta\rho} \kappa > 0$ (where it can be shown that the latter expression is indeed proportional to the real interest rate that would prevail with a unitary multiplier). For instance, for a rule with $\phi_g = 0$, it is clear that the multiplier is always smaller than unity. As argued above, the response of the expected future price level is larger than that of the current price level, resulting in a positive long-term real interest rate.

Second, for $\gamma_y \geq 1$ the multiplier is increasing in Γ and ρ (to the extent that $\phi_\pi > \beta^{-1}$), and decreasing in ϕ_π , κ , and thus in the frequency of price adjustment $\bar{\lambda}$. Therefore, the steeper the Phillips Curve, e.g. the more flexible prices (the larger $\bar{\lambda}$), the smaller the multiplier, seemingly in contrast with the results under the ZLB obtained above.

However, it is possible to set the parameter ϕ_g in the Taylor rule as to replicate the (ZLB) results with a binomial shock also under an AR(1) government spending shock with persistence ρ . Specifically, it can be verified that setting

$$\phi_g = \frac{(1 - \Gamma) (1 - \rho) \kappa}{(1 - \rho) (1 - \beta\rho) \gamma - \rho\kappa} \phi_\pi,$$

would result in a constant nominal interest rate in response to the AR(1) spending shock, and a

multiplier equal to the one under the stochastic binomial ZLB, namely:¹⁵

$$\gamma_y - 1 = \frac{\phi_g - (1 - \Gamma) \frac{\phi_\pi - \rho}{1 - \beta\rho} \kappa}{(1 - \rho) \gamma + \frac{\phi_\pi - \rho}{1 - \beta\rho} \kappa} = (1 - \Gamma) \frac{\rho \kappa}{(1 - \rho) (1 - \beta\rho) \gamma - \rho \kappa}.$$

Interestingly, in contrast with the binomial case, the multiplier is defined also when $\Delta = (1 - \rho) (1 - \beta\rho) \gamma - \rho \kappa < 0$. The restriction on $\Delta > 0$ is not needed as a unique bounded equilibrium always exists in this case. However, when $\Delta < 0$, which is the case when the degree of price flexibility is sufficiently high for a given persistence ρ , the multiplier is smaller than unity, and can even be negative.

Therefore, it emerges that when the duration of a policy of nominal interest rates is stochastic, a large multiplier in the workhorse NK model can arise only if the degree of price flexibility is sufficiently low.

4.2 The multiplier with a constant nominal rate for T periods

We now turn to the case of a sequence of anticipated shocks which keep the interest rate constant for T periods after the government spending shock, consistent with (14) above. As of $T + 1$, monetary policy returns to whatever rule it followed before.¹⁶ This is a common approach to approximating the effects of the ZLB in the workhorse NK model, see e.g. Werning (2012). But it is also the standard approach followed in models with non-trivial dynamics due to the presence of many state variables, in which the ZLB or even a constant interest rate with a stochastic duration cannot be implemented easily. However, similarly to Carlstrom et al. (2014), here we show that this approach in general does not provide the same results as in the stochastic binomial case of Woodford (2011) and Christiano et al. (2011), nor those under the AR(1) spending shock. Specifically, under this approach the “paradox of flexibility” is actually magnified, and the multiplier can be very large even when the counterpart of the condition above, $(1 - \rho) (1 - \beta\rho) \gamma > \kappa\rho$, is violated.

It is straightforward to show that over the period for which the interest rate is anticipated to be

¹⁵The persistence of the spending shock ρ plays the same role of the probability p under the binomial ZLB example above.

¹⁶As noted above, this is equivalent to picking shocks, e.g. to the rate of time preference, so that the zero lower bound constrains the nominal interest rate for T periods, while government spending increases following an AR(1) process.

constant, the dynamics of the economy under an AR(1) spending shock are fully described by the following second order difference equation,

$$\beta (y_{t+1} - g_{t+1}) - (1 + \beta + \kappa\gamma^{-1}) (y_t - g_t) + (y_{t-1} - g_{t-1}) = \kappa\gamma^{-1} (1 - \Gamma) \rho g_t, \quad (37)$$

with general solution for $T - 1 \geq t \geq 0$, given by

$$(y_t - g_t) = \frac{(1 - \Gamma) \kappa \rho}{(1 - \rho)(1 - \beta \rho) \gamma - \rho \kappa} g_t + a_1 \lambda_1^t + a_2 \lambda_2^t, \quad (38)$$

$$\lambda_i = \frac{1 + \beta + \kappa\gamma^{-1} \pm \sqrt{(1 + \beta + \kappa\gamma^{-1})^2 - 4\beta}}{2\beta}, \quad (39)$$

where the eigenvalues satisfy $0 < \lambda_1 < 1 < \lambda_2$, for $\kappa\gamma^{-1} > 0$. Observe that λ_1 is decreasing in κ .

The coefficient on government spending in the particular solution, $\frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}g_t$, is the same as under the constant interest rate with stochastic duration studied above. However, as shown by Werning (2012), the deterministic duration of the constant interest rate now implies the presence of the exponential decay terms associated with λ_1 and λ_2 . Recall that under a stochastic duration both eigenvalues are larger than 1, so that the constants a_1 and a_2 are set to zero to ensure a bounded solution. But now as T is finite, they need to be determined using other appropriate terminal conditions.

Specifically, the constants a_1 and a_2 are now determined by the following two terminal conditions for T and $T - 1$:

$$\gamma (y_T - g_T) = \gamma (y_{T+1} - g_{T+1}) + \pi_{T+1} \quad (40)$$

$$\begin{aligned} \gamma (y_{T-1} - g_{T-1}) &= \gamma (y_T - g_T) + \pi_T \\ &= \gamma (1 + \kappa\gamma^{-1}) (y_{T+1} - g_{T+1}) + (1 + \beta + \kappa\gamma^{-1}) \pi_{T+1} + (1 - \Gamma) \kappa \rho g_{T-1}. \end{aligned} \quad (41)$$

These constants are thus uniquely determined if after the interest rate is free to move the equilibrium is unique, as to pin down $(y_{T+1} - g_{T+1})$ and π_{T+1} .

Solving explicitly yields the following solution:

$$\begin{aligned}
a_1 &= -\frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{1-\rho\beta\lambda_1}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_1}\right)^T g_0 + \frac{1}{1-\beta\lambda_1^2}\frac{(1-\beta\lambda_1)\gamma(y_{T+1}-g_{T+1})+\pi_{T+1}}{\gamma\lambda_1^T}, \\
a_2 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{\beta\lambda_1(\lambda_1-\rho)}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_2}\right)^T g_0 + \frac{\beta\lambda_1}{1-\beta\lambda_1^2}\frac{(1-\lambda_1)\gamma(y_{T+1}-g_{T+1})-\lambda_1\pi_{T+1}}{\gamma\lambda_2^T}.
\end{aligned}$$

Since $\lambda_2 > 1 > \rho$, a_2 will play a small role in the initial dynamics under the constant interest rate, especially when T is large. Conversely, since $0 < \lambda_1 < 1$, a_1 could be quite large when the equilibrium after liftoff is such that both $(y_{T+1} - g_{T+1})$ and π_{T+1} are not small, or when $\rho > \lambda_1$. Crucially, $\rho > \lambda_1$ if and only if $\Delta = (1 - \rho)(1 - \beta\rho)\gamma - \rho\kappa < 0$, where recall that λ_1 is decreasing in κ and thus in the frequency of price adjustment. A sufficiently large κ then implies that $\rho > \lambda_1$.

To better appreciate the consequences for the spending multiplier, write out the solution for the impact response of output as follows:

$$\begin{aligned}
y_0 - g_0 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}g_0 + a_1 + a_2 \tag{42} \\
&= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\left[1 - \frac{(1-\rho\beta\lambda_1)\left(\frac{\rho}{\lambda_1}\right)^T - \beta\lambda_1(\lambda_1-\rho)\left(\frac{\rho}{\lambda_2}\right)^T}{1-\beta\lambda_1^2}\right]g_0 + \\
&\quad \frac{1}{1-\beta\lambda_1^2}\left\{\left[1 - \beta\lambda_1^2\left(\frac{\lambda_1}{\lambda_2}\right)^T\right]\frac{\pi_{T+1}}{\gamma\lambda_1^T} + \left[1 - \beta\lambda_1\left(1 - (1-\lambda_1)\left(\frac{\lambda_1}{\lambda_2}\right)^T\right)\right]\frac{(y_{T+1}-g_{T+1})}{\lambda_1^T}\right\}.
\end{aligned}$$

The following considerations are in order.¹⁷ First, the multiplier is very similar to its counterpart under the constant interest rate of stochastic duration when, in addition to $(1 - \rho)(1 - \beta\rho)\gamma > \rho\kappa$ and $\rho < \lambda_1$, T is sufficiently large. In particular, assume that as of $T + 1$ monetary policy implements the flexible price solution. Then $\pi_{T+1} = 0$ and $(y_{T+1} - g_{T+1}) = -(1 - \Gamma)\rho^T g_0$, so that for T sufficiently

¹⁷It is straightforward to show that very similar expressions would obtain for the case of monetary forward guidance, whereas a monetary expansion is announced to take place after T periods, during which the interest rate is kept constant. Namely, the impact effects in this case are given by:

$$\begin{aligned}
y_0 &= \frac{(1-\beta\lambda_1)\gamma y_{T+1} + \pi_{T+1}}{\gamma(1-\beta\lambda_1^2)}\lambda_1^{-T} + \beta\lambda_1\frac{(1-\lambda_1)\gamma y_{T+1} - \lambda_1\pi_{T+1}}{\gamma(1-\beta\lambda_1^2)}\lambda_2^{-T} \\
\pi_0 &= \frac{(1-\beta\lambda_1)\gamma y_{T+1} + \pi_{T+1}}{(1-\beta\lambda_1^2)}\lambda_1^{-T} + \beta\lambda_1\frac{(1-\lambda_1)\gamma y_{T+1} - \lambda_1\pi_{T+1}}{(1-\beta\lambda_1^2)}\lambda_2^{-T}
\end{aligned}$$

large $\left(\frac{\rho}{\lambda_i}\right)^T \rightarrow 0, i = 1, 2$ and the same result as under the stochastically constant interest rate obtains, namely:

$$y_0 - g_0 \simeq \frac{\rho\kappa}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa} (1-\Gamma) g_0.$$

When prices are assumed to be more flexible so that κ is larger, in the range of parameters for which the denominator in the above equation is positive, and $\rho < \lambda_1 < 1$, the multiplier increases.

Secondly, similarly to the stochastic case with an AR(1) shock analyzed above, the multiplier is also defined for parameterizations such that $\Delta < 0$ ($\Leftrightarrow \rho > \lambda_1$). This occurs in particular for our chosen parameterization, in which effectively we posit $\rho = 1 - \bar{\lambda} = 0.9$. Thus the denominator Δ in the above equation is negative, $(1-\rho)(1-\beta\rho)\gamma - \rho\kappa = -(1-\bar{\lambda})(1-\beta(1-\bar{\lambda}))\eta = -(1-\rho)(1-\beta\rho)$. Nevertheless, for T sufficiently large, the multiplier can also in this case be positive and very large, increasing in κ and thus in the frequency of price adjustment. Namely, the paradox of flexibility arises for any value of $\kappa < \infty$ (i.e. $\bar{\lambda} < 1$), resulting in an ever growing multiplier, the steeper the Phillips curve and the more flexible prices. Assume again $\pi_{T+1} = 0$ and $(y_{T+1} - g_{T+1}) = -(1-\Gamma)\rho^T g_0$; for T sufficiently large that $\left(\frac{\rho}{\lambda_2}\right)^T \simeq 0$, the impact response of output is given by the following:

$$y_0 - g_0 \simeq \frac{\rho\kappa}{\rho\kappa - (1-\rho)(1-\beta\rho)\gamma} \left\{ \frac{1-\rho}{1-\beta\lambda_1^2} \frac{\rho\kappa\beta\lambda_1 + (1-\beta\lambda_1)(1-\beta\rho)\gamma}{\kappa\rho} \left(\frac{\rho}{\lambda_1}\right)^T - 1 \right\} (1-\Gamma) g_0.$$

When $\rho > \lambda_1$, the multiplier is again positive and very large.

Finally, monetary policy after liftoff of the constant rate policy can also affect the short-run multiplier, if it is chosen as to appropriately manipulate the values of output and inflation – see Del Negro et al. (2013) and Cochrane (2015). If after lift-off monetary policy follows the benchmark Taylor rule in (14), the unique bounded solution for inflation and output yields:

$$\begin{aligned} \pi_{T+1} &= (1-\Gamma) \frac{\kappa(1-\rho)\gamma}{(1-\rho)(1-\beta\rho)\gamma + (\phi_\pi - \rho)\kappa} \rho^{T+1} g_0 \\ y_{T+1} - g_{T+1} &= -(1-\Gamma) \frac{(\phi_\pi - \rho)\kappa}{(1-\rho)(1-\beta\rho)\gamma + (\phi_\pi - \rho)\kappa} \rho^{T+1} g_0. \end{aligned}$$

In turn this results in the following expression for the impact multiplier:

$$\begin{aligned}
y_0 - g_0 &\simeq \frac{(1 - \Gamma) \kappa \rho}{(1 - \rho)(1 - \beta \rho) \gamma - \rho \kappa} \left[\frac{(1 - \beta \lambda_1^2) - (1 - \rho \beta \lambda_1) \left(\frac{\rho}{\lambda_1}\right)^T}{1 - \beta \lambda_1^2} \right] g_0 + \frac{1}{1 - \beta \lambda_1^2} \left\{ \frac{\pi_{T+1}}{\gamma \lambda_1^T} + \frac{(y_{T+1} - g_{T+1})}{\lambda_1^T} \right\} \\
&\simeq \frac{(1 - \Gamma) \kappa \rho}{1 - \beta \lambda_1^2} \left[\frac{(1 - \beta \lambda_1^2) - (1 - \rho \beta \lambda_1) \left(\frac{\rho}{\lambda_1}\right)^T}{(1 - \rho)(1 - \beta \rho) \gamma - \rho \kappa} - \frac{(\phi_\pi - 1) \left(\frac{\rho}{\lambda_1}\right)^T}{(1 - \rho)(1 - \beta \rho) \gamma + (\phi_\pi - \rho) \kappa} \right] g_0.
\end{aligned}$$

It is obvious that the bigger ϕ_π , the smaller the impact multiplier; in particular for $\phi_\pi \rightarrow \infty$ it will take on the value derived above under $\pi_{T+1} = 0$.

However, systematic monetary policy after lift-off can also be chosen as to rule out the exponential dynamics associated with the eigenvalue λ_1 , as argued by Cochrane (2015). In particular, we conclude our discussion by showing what would happen when monetary policy follows a constant money growth rule. In the case of the log-log money demand function assumed in our specification,

$$m_t = \gamma(y_t - g_t) - \frac{\beta^2}{1 - \beta}(r_t - \bar{r}), \quad (43)$$

a constant money growth rule also delivers a constant nominal interest rate:

$$\gamma \mu_t = \pi_t + \gamma \Delta(y_t - g_t) = 0 = r_t - \bar{r}. \quad (44)$$

This is the monetary policy reaction function studied by Golosov and Lucas (2007), which would thus imply an unconditionally constant nominal interest rate, but in the case of a money growth shock.

Using the fact that $\pi_t = -\gamma \Delta(y_t - g_t)$, and substituting out the NKPC into the Euler equation we get the usual second-order difference equation in y_t :

$$\beta E_t(y_{t+1} - g_{t+1}) - (1 + \beta + \kappa \sigma)(y_t - g_t) + (y_{t-1} - g_{t-1}) = \kappa \sigma (1 - \Gamma) g_t, \quad (45)$$

which now has the following general solution for any t :

$$y_t = \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1} + a_2 \lambda_2^t, \quad (46)$$

where again $0 < \lambda_1 < 1 < \lambda_2$, as derived above. Hence, for $a_2 = 0$ there will be a unique bounded solution, given by:

$$y_t = \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1}, \quad (47)$$

$$\gamma_0^y - 1 = -\frac{\kappa\sigma(1-\Gamma)}{\beta(1-\rho) + 1 - \lambda_1 + \kappa\sigma}, \quad (48)$$

$$\gamma_1^y = -\frac{1 - \beta\lambda_1\rho}{\beta(1-\rho) + 1 - \lambda_1 + \kappa\sigma}. \quad (49)$$

Despite the fact that the nominal interest rate is (always!) constant, the impact multiplier, γ_0^y , although still larger than Γ is now always lower than 1. Moreover, its uniformly decreasing in the slope of the Phillips curve κ . In particular, the impact multiplier converges to its flexible price counterpart Γ , for κ that diverges.¹⁸

The intuition for this result is straightforward: the monetary policy rule, despite implementing a constant nominal rate in response to any shock, is akin to a price-level target, i.e. $p_t = -\gamma(y_t - g_t)$. This entails a long-run real interest rate that is now much higher than under the interest rate rule considered in the previous section, in which instead an *inflation* target was assumed. Formally, since under this rule the price level is stationary, it is easy to show that

$$\gamma(y_0 - g_0) = E_0 \sum_{j=1}^{\infty} \pi_j = E_0 \left(\lim_{T \rightarrow \infty} p_{T+1} - p_0 \right) = -p_0, \quad (50)$$

as $\lim_{T \rightarrow \infty} E_0 p_{T+1} = -\lim_{T \rightarrow \infty} E_0 \gamma(y_{T+1} - g_{T+1}) = 0$. The multiplier is then decreasing in the contemporaneous response of the price level, so that a steeper NKPC will always result in a smaller multiplier. Likewise, the more “flexible” are prices across price setting models, the lower is the multiplier.¹⁹

To summarize, a key driver of the stimulus effect of government spending (and of shock propagation with sticky prices) is the response of the long-run real interest rate. When the monetary policy rule is such that the nominal interest rate reacts to endogenous variables, e.g. as in the case of a standard

¹⁸Cochrane (2015) obtains a similar result under a ZLB of deterministic duration, rather than for the case of a constant interest rate. But the key property of the assumed monetary policy after lift-off from the ZLB is the same. Namely, monetary policy must be such that the price level reverts to its initial value, before the onset of the ZLB.

¹⁹By the same token, in the case of monetary forward guidance, no amplification nor paradox would arise if the monetary expansion is such that the price level increase is only temporary.

Taylor rule, the real interest rate response will be a function of the current and future effects of government spending. To the extent that the interest rate responds to current inflation, the steeper the Phillips Curve (the more flexible prices), the smaller the decrease in the real interest rate. This is because the response to current inflation ($\phi_\pi > 1$) necessary to ensure determinacy is larger than the persistence of inflation expectations.

Conversely, when the monetary policy rule is such that the interest rate does not (or cannot under the ZLB) respond to endogenous variables, and is thus constant, as is the case under conditions studied above, the real interest rate is purely a function of expected inflation. While the monetary policy reaction function will ultimately determine whether the price level is stationary or not after interest rates can move again, the relation between current and future prices will be shaped by the model of price setting. The Calvo–Yun model implies a very tight and specific relation between the two concepts of inflation: this is related to the concept of reset inflation introduced by Bils et al. (2009, 2012). However, this property of reset inflation and of the process of expected inflation does not seem a general one that would apply to other, more plausible models of price stickiness, with relevant consequences for the size of the multiplier. This is what we study in the rest of the paper focusing on the case of government spending shocks.

5 Simulation results under state-dependent pricing

Price setting and the spending multiplier under a Taylor rule We start by looking at the effects of a government spending shock under a standard interest rate rule. The expenditure shock propagates through the usual channel in New Keynesian models: it raises inflation and inflation expectations. Given an active Taylor rule this increases the (long-run) real interest rate faced by households, so that consumption drops.

The results are presented in Figure 2. The figure plots the responses to an AR(1) government spending shock with $\rho = 0.9$, under a rule that only responds to current inflation with $\phi_\pi = 2$, of the following nine variables: the government spending process, consumption, GDP, the price level, the nominal interest rate, the short-term real interest rate (both annualized), the fraction of adjusting

firms, the “selection effect”, computed as in Costain and Nakov (2011), and the fiscal multiplier. The red lines with dots plot the responses with Calvo pricing, the green lines with circles plot those under fixed menu costs, and the blue lines with squares plot those under the SSDP model. The shock to government spending is scaled so that on impact, government expenditure increases by 0.1% of GDP — as we report below, a larger shock would result in all firms changing their price in the fixed menu cost model under a constant interest rate.

In all three models the government spending shock raises inflation and the real interest rate, and depresses consumption. However, some quantitative differences emerge when looking across the three models. The stronger rise of the real interest rate means that consumption falls by more under fixed menu costs (FMC) than it does under Calvo pricing. As a result, the impact multiplier in the menu cost model is closest to the flexible-price benchmark of $\Gamma = 2/3$. Under Calvo, although it is still less than one (as anticipated in the previous section for the case without idiosyncratic shocks), the fiscal multiplier exceeds the flexible price counterpart.

The price level rises by a cumulative 0.8% in the fixed menu cost model, while it rises only by 0.5% in the Calvo model. Under the SSDP model the rise in the price level is in-between the Calvo and the FMC models. The difference in the behavior of the price level is just the “selection effect” emphasized by Golosov-Lucas (2007): in the menu cost model the firms that adjust are those for which adjustment is most valuable, they tend to make large price changes and the aggregate price level is more flexible than in the Calvo model. Importantly, this effect operates even when the average fraction of adjusting firms remains virtually constant across models. To isolate the “selection effect” in the sense of Golosov and Lucas (2007), we decompose the inflation response as follows. Define the conditionally optimal price level $p_t^{*k} \equiv \operatorname{argmax}_p v_t(p, a^k)$, and also $x_t^{*jk} \equiv \log(p_t^{*k}/p^j)$, the desired log price adjustment of a firm at time t with productivity a^k and real price p^j . We can then write the average desired adjustment as $\bar{x}_t^* = \sum_{j,k} x_t^{*jk} \tilde{\Psi}_t^{jk}$, and write the fraction of firms adjusting as $\bar{\lambda}_t = \sum_{j,k} \lambda_t^{jk} \tilde{\Psi}_t^{jk}$.

Then inflation can be written as

$$\Pi_t = \sum_{j,k} x_t^{*jk} \lambda_t^{jk} \tilde{\Psi}_t^{jk}. \quad (51)$$

To a first-order approximation, we can decompose the deviation in inflation at time t as

$$\Delta\Pi_t = \bar{\lambda}\Delta\bar{x}_t^* + \bar{x}^*\Delta\bar{\lambda}_t + \Delta\sum_{j,k}x_t^{jk}(\lambda_t^{jk} - \bar{\lambda}_t)\tilde{\Psi}_t^{jk}, \quad (52)$$

where terms without time subscripts represent steady states, and Δ represents a change relative to steady state.²⁰

The “intensive margin”, $\mathcal{I}_t \equiv \bar{\lambda}\Delta\bar{x}_t^*$, is the part of inflation due to changes in the average desired adjustment, holding fixed the fraction of firms adjusting. The “extensive margin”, $\mathcal{E}_t \equiv \bar{x}^*\Delta\bar{\lambda}_t$, is the part due to changes in the fraction adjusting, assuming the average desired change among those who adjust equals the steady-state average in the whole population. The “selection effect”, $\mathcal{S}_t \equiv \Delta\sum_{j,k}x_t^{jk}(\lambda_t^{jk} - \bar{\lambda}_t)\tilde{\Psi}_t^{jk}$, is the inflation caused by redistributing adjustment opportunities from firms desiring small (or negative) price adjustments to firms desiring large (positive) adjustments, while fixing the total number adjusting. The middle panel in the last row of the figure reports the selection effect, showing that it contributes much to inflation in the menu cost model, while the fraction of adjusting firms barely moves.²¹

Overall, these experiments confirm that the endogenous response of inflation is larger in the state-dependent model under a standard Taylor rule, leading to smaller real effects of shocks. As a result, the fiscal multiplier is less than one. We now turn to the case of a constant interest rate, for which the paradox of flexibility has been established under time-dependent pricing.

The multiplier under constant interest rates We report results under a constant interest rate, which we assume is anticipated not to react to the shock for 24 months. This is the standard approach in models with many state variables like ours. In Figure 3, the blue lines with squares plot the responses under SSDP, and the red lines with dots plot those under Calvo pricing. Figure 4 plots the responses under fixed menu costs in green lines with circles. The shock to government spending is again scaled so that on impact, government expenditure increases by 0.1% of GDP.

²⁰See Costain and Nakov (2011b) for further discussion of this decomposition.

²¹Note that the selection effect is absent by construction in the Calvo model. Therefore, while in the Calvo model with idiosyncratic shocks the “extensive margin” channel (namely the change in the size of price changes) is very close to that of the fixed menu cost model, the response of inflation is virtually identical to that in the textbook Calvo version without idiosyncratic shocks

Across all models, the fact that monetary policy does not react produces a persistent expansionary effect. This is consistent with the nominal interest rate not increasing with the rise in expected future inflation, so that the *long-run* ex-ante real interest rate drops in response to the shock. The result is that consumption rises and thus the multiplier exceeds unity in all three models. Strikingly, the fixed menu cost model now delivers a much larger multiplier than the Calvo model, an order of magnitude larger. Obviously this is reflected in a very large response of inflation and in a dramatic fall in the short term real rate. Intuitively, under a constant interest rate, more flexible prices lead to a larger response of the expected price level upon exit. As we have shown in the previous sections, contrary to the stochastic ZLB case analyzed e.g. by Woodford (2011), when the interest rate is kept constant as in our experiment for a period of deterministic duration, an equilibrium is defined for any admissible flexibility of prices. Under fixed menu costs, the effects of this policy are much bigger than under Calvo pricing with the same average frequency of price adjustment. The “paradox of flexibility” thus holds under state-dependent pricing and accounts for these very large responses. From the middle panel in the last row of Figure 3 and 4 we see that, indeed, the spike of inflation on impact in the FMC and SSDP models is due to the selection effect.

In fact, a very large multiplier is obtained also under the “intermediate” smoothly state-dependent pricing model (SSDP). Notice that in this model the response of inflation is larger than under Calvo pricing, and more persistent than in the fixed menu cost model, in which there is a large inflation spike on impact, that, however, dies out very quickly. This is because the SSDP model, due to its relatively smoother nature (see Costain and Nakov, 2011), exhibits inflation persistence similar to the Calvo model. This can be seen from the fact that the fraction of adjusting firms rises by much less than in the fixed menu cost (FMC) model. At the same time the SSDP model does feature an important selection effect, similar to the FMC model. So the fall in the long-term real interest rate is quite strong in the SSDP model and the resulting fiscal multiplier is also large.

We also look at the case in which the interest rate is kept constant for 36 months, reported in Figure 5. From our analytical results in the previous section, in the simple Calvo framework the multiplier is increasing in the number of periods T over which monetary policy is unresponsive to the shock. This expectation is confirmed in the Calvo model with idiosyncratic shocks and in the SSDP

model, but not in the FMC. In the latter the multiplier although still larger than in the Calvo model, is now smaller than its counterpart in Figure 4, and even smaller than that resulting from the SSDP model. This result stems from the fact that, as argued above, in the FMC model inflation is much less persistent than in the SSDP model. Therefore, the expected price level quickly stops increasing, and over a sufficiently long horizon, it can result in a lower expected inflation than in the SSDP model.²²

Finally, it is important to stress the result that even for the very small aggregate shock we consider, virtually all firms in the fixed menu-cost version of the model find it optimal to change their prices. A similar result would arise in the SSDP model for a larger shock, but still falling well short of the size usually assumed in ZLB literature. Therefore, when the degree of flexibility of the aggregate price level is allowed to be endogenous, its response may quickly converge to a situation close to full flexibility under a passive monetary policy. This result thus casts some doubt on the very large propagation mechanism of business cycle sized shocks found in the literature assuming an exogenous degree of flexibility. An investigation of this important issue is however beyond the scope of this paper.

6 Conclusion

We have studied the effects of state-dependent pricing on the amplification of shocks under a constant interest rate, focusing on the government spending multiplier. Since the type of price setting by firms is crucial for the determination of the long-term real interest rate in the workhorse DSGE model, it is a primary determinant of the size of the spending multiplier. To have a multiplier substantially bigger than unity under a standard interest rate rule, both monetary policy has to be accommodative, and the aggregate price level has to be sufficiently sticky. Conversely, when the nominal interest is constant or at the zero lower bound, if the monetary policy is such that the price level has a positive drift, a paradox of flexibility arises: the multiplier, and thus shock amplification, is increasing in the degree of price flexibility. We thus find much larger multipliers in state-dependent models than under Calvo pricing, when the nominal interest rate is constant for a deterministic time period, under the

²²In results available upon request we also verify that under a constant money growth rule, while the nominal interest rate is always constant, the multiplier is less than one across all models of price setting, and smaller under state-dependent pricing.

same frequency of price adjustment.

In concluding, it is however important to remember that our results concern the size of policy multipliers in the workhorse New Keynesian model, which is the core of DSGE models also used to inform policy debates, but do not consider the quantitative importance of the many additional factors in the latter. These other factors, such as distortionary taxes, credit and financial constraints for households and firms, wage rigidities, information frictions, and many others, are important for specific quantitative answers and for a full welfare evaluation. Their role in a model with state-dependent pricing is left to future research.

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Appendix: figures

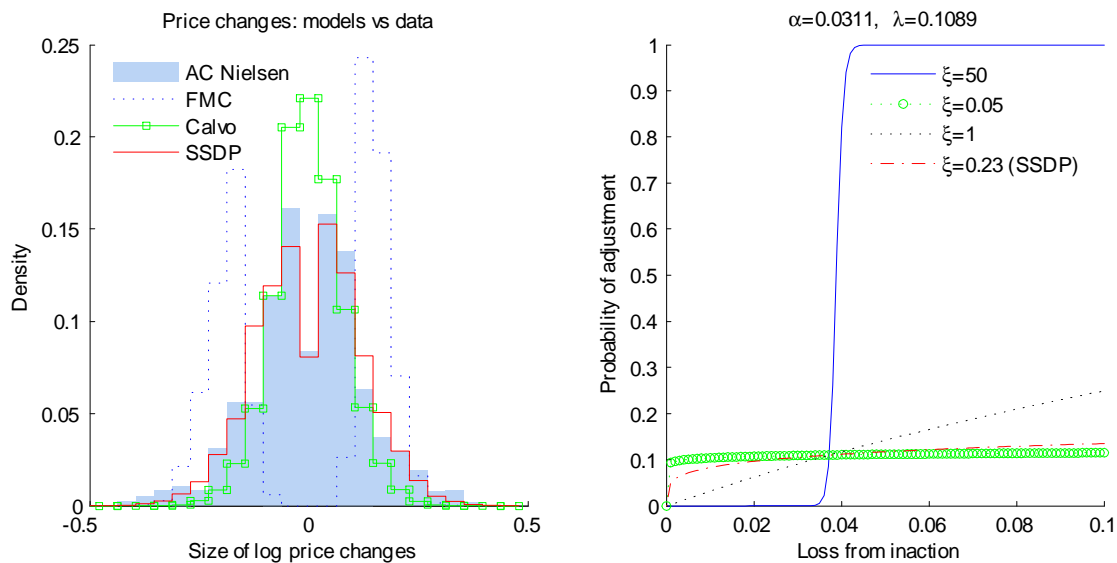


Fig.1: Adjustment probability functions and histograms of price changes

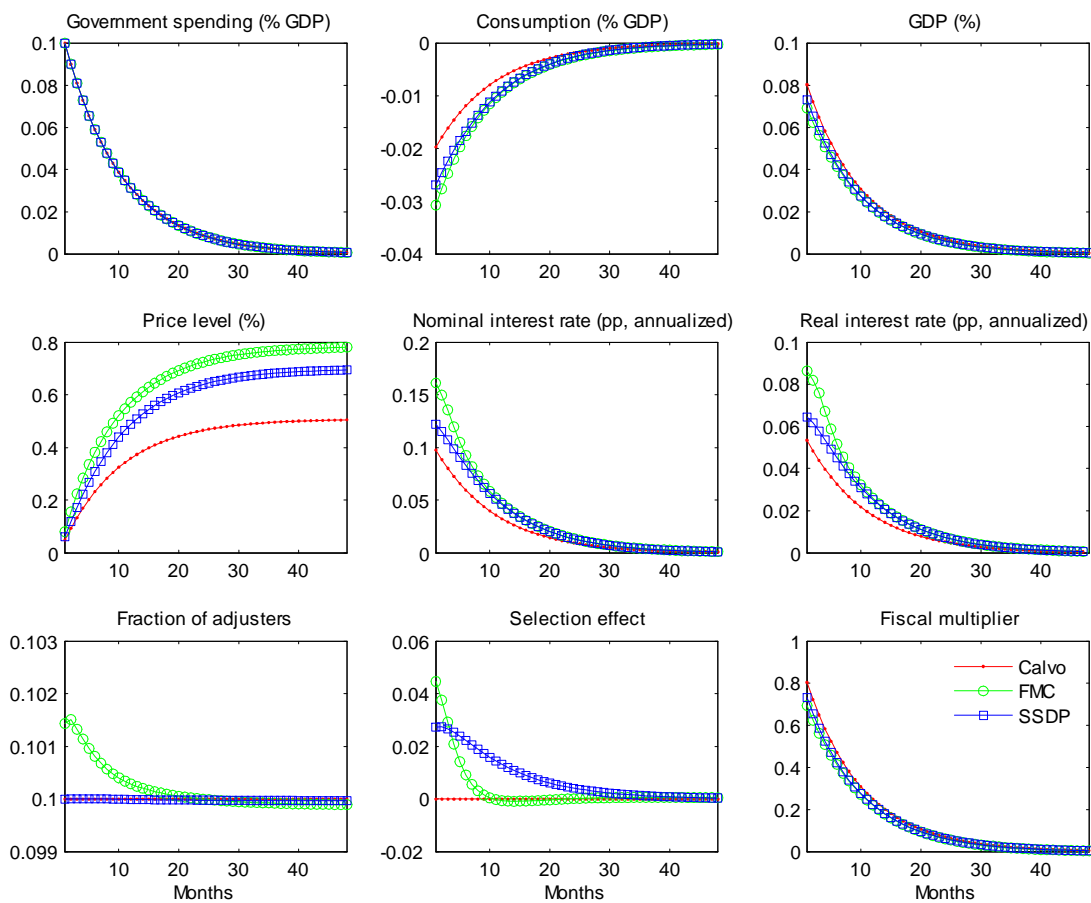


Fig.2: Responses to a government spending shock under a Taylor rule

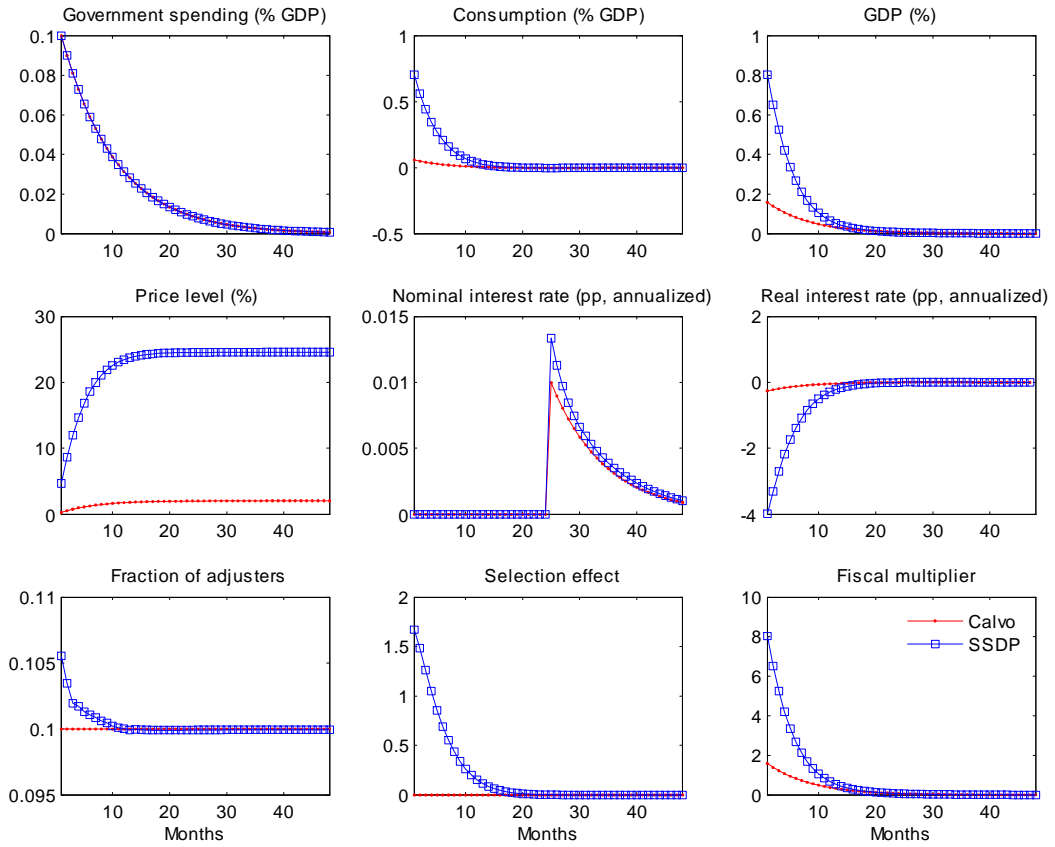


Fig.3: Responses to a government spending shock under a constant interest rate (Calvo, SSDP)

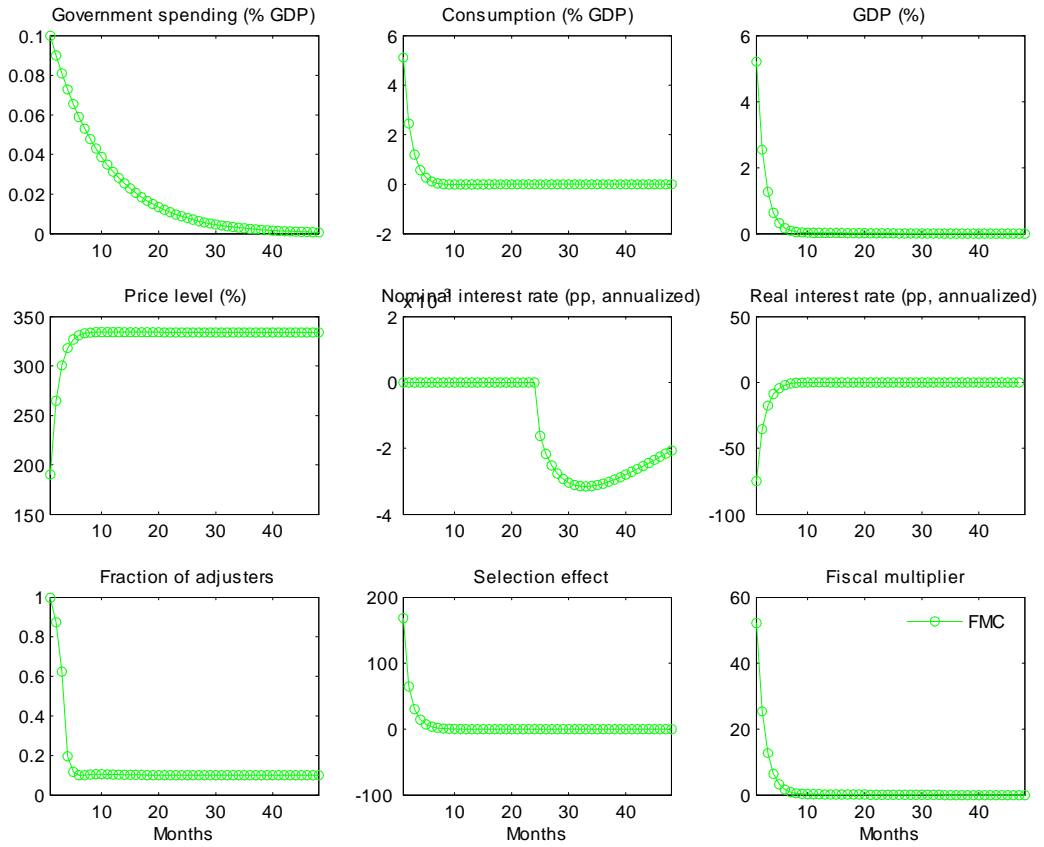


Fig.4: Responses to a government spending shock under a constant interest rate (FMC)

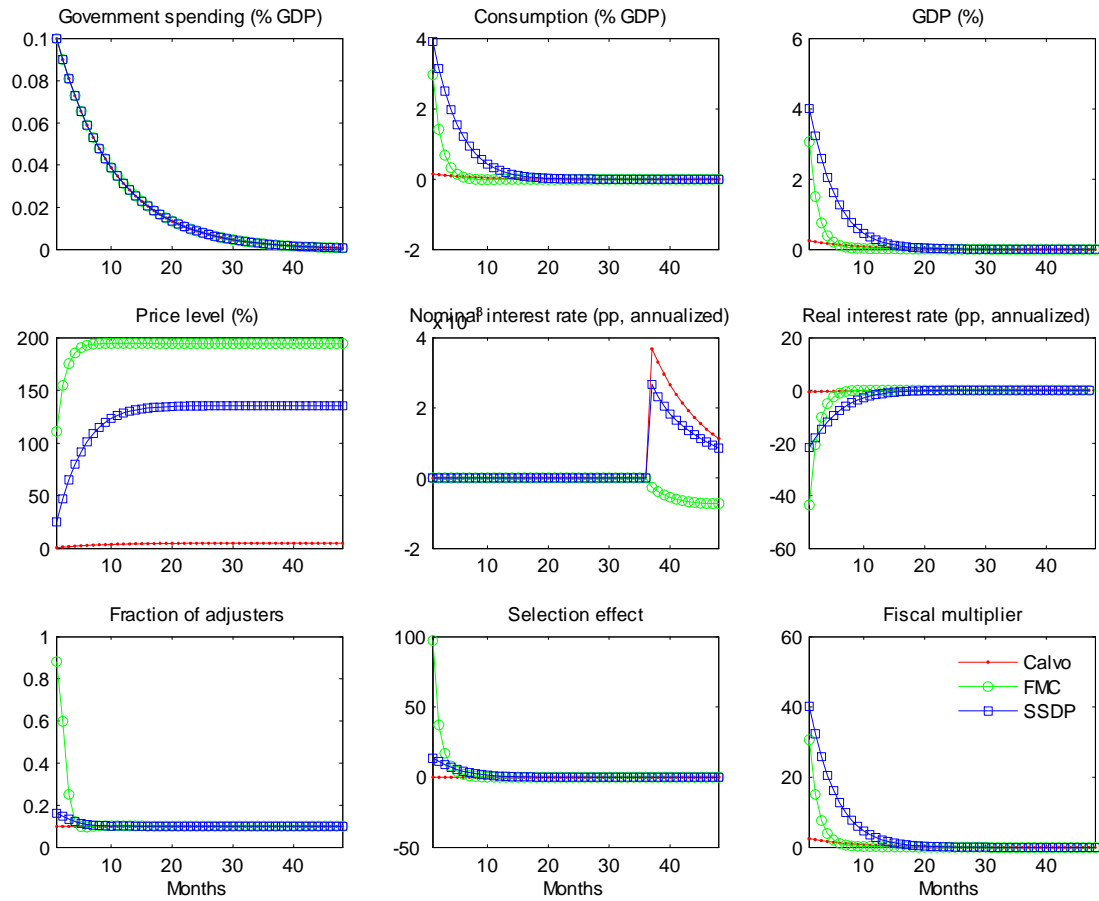


Fig.5: Responses to a government spending shock under a constant interest rate for 3 years