# Macroeconomic and Financial Dynamics in Small Open Economies<sup>\*</sup>

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#### Abstract

The goal of this paper is to explore which financial frictions are more useful in explaining the joint dynamics of macroeconomic and credit-related variables in a small open economy. In particular, we set up a DSGE model of a small open economy featuring two types of financial frictions: one in the relationship between depositors and banks (as in Gertler and Karadi, 2011) and the other between banks and borrowers (along the lines of Bernanke et al., 1999). We use Chilean data to estimate the model, following a Bayesian approach. We emphasize several findings. First, the presence of both frictions is quite useful in matching the dynamics of observed macro and financial variables, as well as their co-movement. Second, we find that the friction related to borrowers risk is the relatively more relevant in explaining the behavior of both financial and non-financial variables, compared to the one associated with the banking problem. Finally, we assess how the presence of financial frictions change the propagation of the main driving forces of the economy.

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# 1 Introduction

The financial crisis of 2008 and the world recession that followed have renewed the interest for analyzing the joint behavior of macroeconomic variables (such as GDP, inflation, the exchange rate, among others) and financial series (like credit, different types of spreads, etc.). In principle, one can consider two alternative strategies to characterize these dynamics. One is to amend a DSGE model in the New Keynesian tradition (the prevailing tool for monetary policy analysis before 2008) by incorporating in an ad-hoc way "shocks" that can explain the departures from the frictionless financial market assumption typically embedded in these DSGE models. The other alternative is to extend the basic framework, introducing financial frictions to provide a more micro-founded explanation for the evolution of financial variables. Which of these approaches one should follow is a question that has to be answered by looking at the data, for it is not clear a priori which of these alternatives can better account for the joint dynamics of both macroeconomic and credit-related variables. In this paper we try to answer this question, focusing on small and open economies.

We develop a dynamic stochastic general equilibrium (DSGE) model of a small open economy featuring two types of domestic financial frictions. On one hand, there is a friction between depositors and banks that induces a spread between lending and deposit rates. We model this friction as a moral hazard problem following the work of Gertler and Karadi (2011) (GK).<sup>1</sup> On the other hand, there is a spread between the expected return for banks and the return to capital (known as the external finance premium), originated by a costly-state-verification problem following Bernanke *et al.* (1999) (BGG).<sup>2</sup> We also allow for anticipated shocks (news) to entrepreneurs risk, which Christiano et al (2014) find to be relevant driving force for the U.S.. The model also features loans to finance working capital, although we assume no informational asymmetries in this lending relationship.

We contrast this model with alternative versions that exclude one or both frictions but that, instead, include exogenous ad-hoc shocks to the lending spread or to the external finance premium.<sup>3</sup> We estimate these alternative setups using quarterly Chilean data from 2001 to 2012, including both macro and financial variables, following a Bayesian approach.

The estimation results are used to assess several relevant issues. First, we explore whether a model with financial frictions can help to improve the goodness-of-fit of the model in terms of non-financial variables. We find that a the model that includes both frictions can help to improve the fit of the model in many dimensions. In contrast, model that features just one frictions, or none at all, do not seem to be as useful in terms of replicating the dynamics of observed macro variables.

Second, we investigate which of the financial frictions we considered (between depositors and banks, or between banks and firms) is most useful in accounting for the dynamics of financial variables, and the co-movement between these and GDP. Our results show that the BGG friction appears to be relatively more relevant in accounting for those dynamics, although the model with both frictions replicate the observed movements in these variables better than other version of the model.

Third, we also find that shocks associated with the financial frictions, particularly the anticipated disturbances to borrowers risk emphasized by Christiano et al (2014), are more useful in accounting for the observed dynamics than simply assuming the existence of ad-hoc shocks that affect different spreads. Finally, we also analyze how

<sup>&</sup>lt;sup>1</sup>The Gertler and Karadi framework has become quite popular in the recent macroeconomic literature, particularly for the analysis of unconventional monetary policies (see, for instance, Gertler and Kiyotaki, 2011; Gertler and Karadi, 2013; Dedola *et al.*; 2013; Kirchner and van Wijnbergen, 2012; Rannenberg, 2012).

<sup>&</sup>lt;sup>2</sup>Some examples using this setup with a focus on emerging countries are Cespedes *et al.* (2004), Devereux *et al.* (2006), Gertler *et al.* (2007), Tovar 2006, Fernandez and Gulan (2012), among others.

 $<sup>^{3}</sup>$ The inclusion of ad-hoc shocks is to make the comparison with models that feature financial frictions more fair. Otherwise, trivially a frictionless model could not generate, for instance, a banking spread.

the propagation of structural shocks changes in the presence of financial frictions.

Our study makes several contributions to the related literature. First, to the best of our knowledge, we are the first to set up a model combining banks as in Gertler and Karadi (2011) with entrepreneurs as in Bernanke *et al.* (1999) in a small open economy framework.<sup>4</sup> In addition, we are the first to estimate a model featuring banks as in Gertler and Karadi (2011) for a small open economy.<sup>5</sup> Additionally, while several studies use estimated DSGE models to assess the role of financial frictions between domestic and foreign agents in propagating external shocks,<sup>6</sup> we are among the few that focus on the role of domestic financial frictions.<sup>7</sup> We also contribute to extend the work of Christiano et al (2014), by analyzing the importance of anticipated risk shocks in small open economies. Finally, while many papers compare the performance of models with and without financial frictions, studies comparing the relative relevance of alternative frictions to fit the data, like we do, are less frequent.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the parametrization and estimation strategy. The goodness-of-fit analysis is presented in Section 4. Section 5 describes impulse responses obtained with the different versions of the model. Section 6 concludes.

# 2 The Model

Our model shares many features with those in the literature of DSGE models for small open economies, many of them used at central banks.<sup>8</sup> The non-financial part of our framework is one of a small open economy with nominal and real rigidities. Domestic goods are produced with capital and labor, there is habit formation in consumption, adjustment costs in investment and variable capital utilization. Firms face a Calvo-pricing problem with partial indexation, and there is imperfect exchange rate pass-through to prices of imported goods in the short run due to local-currency price stickiness. In addition, households face a Calvo-type problem in setting wages, assuming also partial indexation to past inflation. We also assume that firms need to pay a fraction of their operating costs (working capital) in advance, which they finance with loans from banks. The economy also exports an exogenous endowment of a commodity good; a feature that is introduced to account for the importance of commodity exports (mainly copper) in Chile.

On top of that setup, we add two kinds of domestic financial frictions. On one hand, there are banks that intermediate credit from households to entrepreneurs (to finance capital accumulation) and to firms (for working capital), that are subject to a moral hazard problem along the lines of Gertler and Karadi (2011). On the other hand, capital accumulation by entrepreneurs is risky and subject to a costly-state-verification problem as in Bernanke *et al.* (1999), making the return on the loans obtained by banks state-contingent, as every period a fraction of the entrepreneurs will default on their loans.

The model features several exogenous sources of fluctuations: shocks to preferences, technology (neutral and investment-specific), commodity production, government expenditures, monetary policy, foreign demand, foreign inflation, foreign interest rates, the international price of the commodity good, and two financial shocks.

<sup>&</sup>lt;sup>4</sup>Rannenberg (2013) combines these two features but in a closed economy setup, using a calibrated model.

 $<sup>^{5}</sup>$ Some examples of estimations in closed-economy frameworks with these types of banks are Villa (2013), Villa and Yang (2013), and Areosa and Coelho (2013).

<sup>&</sup>lt;sup>6</sup>For instance, Tovar (2006) and Fernandez and Gulan (2012)

 $<sup>^{7}</sup>$ One example of a model featuring frictions between domestic agents, following the BGG approach, is the work by Christiano et al (2011).

<sup>&</sup>lt;sup>8</sup>Our base model (without financial frictions) is a simplified version of the model by Medina and Soto (2007), which is the DSGE model used for policy analysis and forecasting at the Central Bank of Chile. Given the simplifications that we make, the model is closer to that in Adolfson *at al.* (2007).

In the main part of the paper, we describe and set up the problems faced by each agent, leaving for the appendix the list of the relevant equilibrium conditions and the computation of the steady state.

## 2.1 Households

There is a continuum of infinitely lived households of mass one that have identical asset endowments and identical preferences that depend on consumption of a final good  $(C_t)$  and hours worked  $(h_t)$  in each period (t = 0, 1, 2, ...).<sup>9</sup> Households save and borrow by purchasing domestic currency denominated government bonds  $(B_t)$  and by trading foreign currency bonds  $(B_t^*)$  with foreign agents, both being non-state-contingent assets. They can also deposit resources at banks  $(D_t)$ . Expected discounted utility of a representative household is given by

$$E_t \sum_{s=0}^{\infty} \beta^s v_{t+s} \left[ \log \left( C_{t+s} - \varsigma C_{t+s-1} \right) - \kappa \frac{h_{t+s}^{1+\phi}}{1+\phi} \right], \tag{1}$$

where  $v_t$  is an exogenous preference shock.

Following Schmitt-Grohé and Uribe (2006), labor decisions are made by a central authority, a union, which supplies labor monopolistically to a continuum of labor markets indexed by  $i \in [0, 1]$ . Households are indifferent between working in any of these markets. In each market, the union faces a demand for labor given by  $h_t(i) = [W_t^n(i)/W_t^n]^{-\epsilon_W} h_t^d$ , where  $W_t^n(i)$  denotes the nominal wage charged by the union in market  $i, W_t^n$  is an aggregate hourly wage index that satisfies  $(W_t^n)^{1-\epsilon_W} = \int_0^1 W_t^n(i)^{1-\epsilon_W} di$ , and  $h_t^d$  denotes aggregate labor demand by firms. The union takes  $W_t^n$  and  $h_t^d$  as given and, once wages are set, it satisfies all labor demand. Wage setting is subject to a Calvo-type problem, whereby each period the household (or union) can set its nominal wage optimally in a fraction  $1 - \theta_W$  of randomly chosen labor markets, and in the remaining markets, the past wage rate is indexed to a weighted product of past and steady state CPI inflation with weights  $\vartheta_W \in [0, 1]$  and  $1 - \vartheta_W$ , respectively.

Let  $r_t$  and  $r_t^*$  denote the gross real returns on  $B_{t-1}$  and  $B_{t-1}^*$ , respectively. The real interest rate on deposits, by a non-arbitrage condition, will also equal  $r_t$ . Further, let  $W_t$  denote the real hourly wage rate, let  $rer_t$  be the real exchange rate (i.e. the price of foreign consumption goods in terms of domestic consumption goods), let  $T_t$  denote real lump-sum tax payments to the government and let  $\Sigma_t$  collect real dividend income from the ownership of firms. The period-by-period budget constraint of the household is then given by

$$C_t + B_t + rer_t B_t^* + D_t + T_t = \int_0^1 W_t(i)h_t(i)di + r_t B_{t-1} + rer_t r_t^* B_{t-1}^* + r_t D_{t-1} + \Sigma_t.$$
 (2)

The household chooses  $C_t$ ,  $h_t$ ,  $W_t^n(i)$ ,  $B_t$ ,  $B_t^*$  and  $D_t$  to maximize (1) subject to (2) and labor demand by firms, taking prices, interest rates and aggregate variables as given. The nominal interest rates are implicitly defined as

$$r_t = R_{t-1}\pi_t^{-1}, \ r_t^* = R_{t-1}^*\xi_{t-1}(\pi_t^*)^{-1},$$

where  $\pi_t$  and  $\pi_t^*$  denote the gross inflation rates of the domestic and foreign consumption-based price indices  $P_t$ 

<sup>&</sup>lt;sup>9</sup>Throughout, uppercase letters denote variables containing a unit root in equilibrium (either due to technology or due to long-run inflation) while lowercase letters indicate variables with no unit root. Real variables are constructed using the domestic consumption good as the numeraire. In the appendix we describe how each variable is transformed to achieve stationarity in equilibrium. Variables without time subscript denote non-stochastic steady state values in the stationary model.

and  $P_t^*$ , respectively. The variable  $\xi_t$  denotes a country premium given by<sup>10</sup>

$$\xi_t = \bar{\xi} \exp\left[-\psi \frac{rer_t B_t^* / A_{t-1} - rer \times \bar{b}^*}{rer \times \bar{b}^*} + \frac{\zeta_t^1 - \zeta^1}{\zeta^1} + \frac{\zeta_t^2 - \zeta^2}{\zeta^2}\right],$$

where  $\zeta_t^1$  and  $\zeta_t^2$  are exogenous shocks.<sup>11</sup> We consider these two shocks because the first one will be associated with an observable proxy for the country premium (namely, the EMBI spread), while the other is an unobservable shock that will be useful in accounting for deviations from the UIP. Finally, The foreign nominal interest rate  $R_t^*$ evolves exogenously, and the domestic central bank sets  $R_t$ .

#### 2.2 Production and Pricing

The supply side of the economy is composed by a set of monopolistically competitive firms producing different varieties of a home good with labor and capital services as inputs, a set of monopolistically competitive importing firms, and three groups of perfectly competitive aggregators: one packing different varieties of the home good into a composite home good, one packing imported varieties into a composite foreign good, and a final group that bundles (with different combinations) the composite home and foreign goods to create a final goods that will be purchased by household consumption  $(Y_t^C)$ , capital goods producers  $(I_t)$  and the government  $(G_t)$ . All of these firms are owned by domestic households. In addition, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad. A proportion of those commodity-exporting firms is owned by the government and the remaining proportion is owned by foreign agents. The total mass of firms in each sector is normalized to one. We denote productions/supply with the letter y and inputs/demand with x.

#### 2.2.1 Final Goods

The final consumption good that generates utility for households, the final investment good that is used to increase the stock of capital, and expenditures by the government are produced with different technologies combining composite home and foreign goods. The three productions function are, respectively,

$$Y_t^C = \left[ (1 - o_C)^{\frac{1}{\eta_C}} (X_t^{C,H})^{\frac{\eta_C - 1}{\eta_C}} + o_C^{\frac{1}{\eta_C}} (X_t^{C,F})^{\frac{\eta_C - 1}{\eta_C}} \right]^{\frac{\eta_C}{\eta_C - 1}},$$
(3)

$$I_t = \left[ (1 - o_I)^{\frac{1}{\eta_I}} (X_t^{I,H})^{\frac{\eta_I - 1}{\eta_I}} + o_I^{\frac{1}{\eta_I}} (X_t^{I,F})^{\frac{\eta_I - 1}{\eta_I}} \right]^{\frac{\eta_I}{\eta_I - 1}}, \tag{4}$$

$$G_t = \left[ (1 - o_G)^{\frac{1}{\eta_G}} (X_t^{G,H})^{\frac{\eta_G - 1}{\eta_G}} + o_G^{\frac{1}{\eta_G}} (X_t^{G,F})^{\frac{\eta_G - 1}{\eta_G}} \right]^{\frac{\eta_G}{\eta_G - 1}},$$
(5)

where  $X_t^{C,H}$ ,  $X_t^{I,H}$  and  $X_t^{G,H}$  denote the demands of home composite goods by each representative firm, while  $X_t^{C,F}$ ,  $X_t^{I,F}$  and  $X_t^{G,F}$  are the demands of foreign composite goods.<sup>12</sup> Each representative firm is competitive and takes input prices ( $p_t^H$  and  $p_t^F$ , measured in terms of the final consumption good) as well as selling prices (respectively, 1,  $p_t^I$  and  $p_t^G$ , in terms of the final consumption good) as given.

<sup>&</sup>lt;sup>10</sup>See, for instance, Schmitt-Grohé and Uribe (2003) and Adolfson *et al.* (2007).

<sup>&</sup>lt;sup>11</sup>The variable  $A_t$  (with  $a_t \equiv A_t/A_{t-1}$ ) is a non-stationary technology disturbance, see below.

 $<sup>{}^{12}</sup>Y_t^C$  will generally differ from  $C_t$  as we assume that utilization and monitoring costs are paid in final consumption units.

#### 2.2.2 Home Composite Goods

A representative home composite goods firm demands home goods of all varieties indexed by  $j \in [0, 1]$  in amounts  $X_t^H(j)$  and combines them according to the technology

$$Y_t^H = \left[\int_0^1 X_t^H(j)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}}.$$

Let  $p_t^H(j)$  denote the price of the good of variety j in terms of the home composite good. The profit maximization problem yields the following demand for the variety j:

$$X_t^H(j) = p_t^H(j)^{-\epsilon_H} Y_t^H.$$
(6)

#### 2.2.3 Home Goods of Variety j

Each home variety j is produced according to the technology

$$Y_t^H(j) = z_t K_t^d(j)^{\alpha} [A_t h_t^d(j)]^{1-\alpha},$$
(7)

where  $z_t$  is an exogenous stationary technology shock, while  $A_t$  (with  $a_t \equiv A_t/A_{t-1}$ ) is a non-stationary technology disturbance, both common to all varieties.  $K_t^d(j)$  denotes the demand for capital services by firm j while  $h_t^d(j)$ denotes this firm's demand for labor. Additionally, we assume that a fraction  $\alpha_L^{WC}$  of the operating costs needs to be financed with an intra-temporal loan (i.e.  $L_t^{WC} = \alpha_L^{WC}[W_t h_t(j) + r_t^K K_t^d(j)])$ , with a non-state contingent nominal rate of  $R_t^{L,WC}$  (with  $r_t^{L,WC} \equiv R_{t-1}^{L,WC}/\pi_t$ ).<sup>13</sup> The firm producing variety j has monopoly power but produces to satisfy the demand constraint given by (6). As the price setting decision is independent of the optimal choice of the factor inputs, the problem of firm j can also be represented in two stages. In the first stage, the firm hires labor and rents capital to minimize production costs subject to the technology constraint (7). Thus, the firm's real marginal costs in units of the final domestic good is given by

$$mc_t^H(j) = \frac{1}{\alpha^{\alpha} \left(1 - \alpha\right)^{1 - \alpha}} \frac{(r_t^K)^{\alpha} W_t^{1 - \alpha} [1 + \alpha_L^{WC} (R_t^{L, WC} - 1)]}{p_t^H z_t (A_t)^{1 - \alpha}},\tag{8}$$

which, given the assumptions, is the same for all varieties j.

In the second stage of firm j's problem, given nominal marginal costs, the firm chooses its price  $P_t^H(j)$  to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability  $1 - \theta_H$ , and if it cannot change its price, it indexes its previous price according to a weighted product of past inflation of home composite goods prices and steady state CPI inflation with weights  $\vartheta_H \in [0, 1]$  and  $1 - \vartheta_H$ .<sup>14</sup>

<sup>&</sup>lt;sup>13</sup>The assumption of working-capital loans being non-contingent is the usual choice in the related literature.

<sup>&</sup>lt;sup>14</sup>This indexation scheme eliminates the distortion generated by price dispersion up to a first-order expansion.

#### 2.2.4 Foreign Composite Goods

A representative foreign composite goods firm demands foreign goods of all varieties  $j \in [0, 1]$  in amounts  $X_t^F(j)$ and combines them according to the technology

$$Y_t^F = \left[\int_0^1 X_t^F(j)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}}.$$

Let  $p_t^F(j)$  denote the price of the good of variety j in terms of the foreign composite good. Thus, the input demand functions are

$$X_t^F(j) = p_t^F(j)^{-\epsilon_F} Y_t^F.$$
(9)

#### 2.2.5 Foreign Goods of Variety j

Importers buy an amount  $M_t$  of a homogenous foreign good at the price  $P_t^{F*}$  in the world market and convert this good into varieties  $Y_t^F(j)$  that are sold domestically, where  $M_t = \int_0^1 Y_t^F(j) dj$ . The firm producing variety jhas monopoly power but satisfies the demand constraint given by (9). As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F m c_t^F(j) = P_t^F m c_t^F = S_t P_t^{F*}.$$
(10)

Given marginal costs, the firm producing variety j chooses its price  $P_t^F(j)$  to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability  $1 - \theta_F$ , and if it cannot change its price, it indexes its previous price according to a weighted product of past inflation of foreign composite goods prices and steady state CPI inflation with weights  $\vartheta_F \in [0, 1]$  and  $1 - \vartheta_F$ . In this way, the model features delayed pass-trough from international to domestic prices.

#### 2.2.6 Commodities

A representative commodity producing firm produces a quantity of a commodity good  $Y_t^{Co}$  in each period. Commodity production evolves according to an exogenous process, and it is co-integrated with the non-stationary TFP process. The entire production is sold abroad at a given international price  $P_t^{Co*}$ . The real foreign and domestic prices are denoted as  $p_t^{Co*}$  and  $p_t^{Co}$ , respectively, where  $p_t^{Co*}$  is assumed to evolve exogenously. The real domestic currency income generated in the commodity sector is therefore equal to  $p_t^{Co}Y_t^{Co}$ . The government receives a share  $\chi \in [0, 1]$  of this income and the remaining share goes to foreign agents.

#### 2.3 Capital Accumulation

#### 2.3.1 Entrepreneurs

Entrepreneurs manage the economy's stock of capital  $(K_t)$ . Following Bernanke *et al.* (1999) (BGG for short), entrepreneurs have two distinctive features in this setup. On the one hand, they have a technology available to transform new capital produced by capital-goods producers (described below) into productive capital that can be used by firms. In particular, if at *t* they buy  $K_t$  units of new capital, the amount of productive capital available to rent to firms in t + 1 is  $\omega_{t+1}^e K_t$ . The variable  $\omega_t^e > 0$  is the source of heterogeneity among entrepreneurs and it is distributed in the cross section with a c.d.f.  $F(\omega_t^e; \sigma_{\omega,t-1})$ , and p.d.f.  $f(\omega_t^e; \sigma_{\omega,t-1})$ , such that  $E(\omega_t^e) = 1$ . The variable  $\sigma_{\omega,t}$  denotes the time-varying cross-sectional standard deviation of entrepreneurs' productivity, which is known in advance,<sup>15</sup> and it is assumed to follow an exogenous process, as in, for instance, Christiano *et al.* (2010, 2014). On the other hand, entrepreneurs have finite lifetimes (we describe this in more detail below) and when they exit the market they transfer all their remaining wealth to households.

In each period, after the idiosyncratic productivity shock is realized, entrepreneurs rent capital services (which for each individual entrepreneur equal  $u_t \omega_t^e K_{t-1}$ , where  $u_t$  denotes capital utilization) to home goods producing firms, at a rental rate (in real terms)  $r_t^{K,16}$ . They face a utilization cost per unit of capital, which in real terms is given by

$$\phi(u_t) = \frac{r^K}{\phi_u} \{ \exp[\phi_u(u_t - 1)] - 1 \},\$$

where  $r^{K}$  is the steady state value of the rental rate of capital services, and  $\phi_{u}$  governs the importance of these utilization costs.<sup>17</sup> After non-depreciated capital is returned, they sell it to capital goods producers at a real price  $q_{t}$ . Afterwards, they buy new capital  $(q_{t}K_{t})$ .

We assume that purchases of new capital have to be financed by loans from intermediaries. However, due to an informational asymmetry (described below) entrepreneurs will not be able to obtain loans to cover for the whole operation. This will create the incentives for entrepreneurs to accumulate net worth  $(N_t^e)$  to finance part of the capital purchases. Thus, we have

$$q_t K_t = N_t^e + L_t^K,$$

where  $L_t^K$  is the loan obtained from banks in real terms. We assume that the loan contract signed at t is nominal and it specifies a non-contingent interest rate  $R_t^{L,e}$  (with  $r_t^{L,e} \equiv R_{t-1}^{L,e}/\pi_t$ ). The fact that entrepreneurs have finite lifetimes prevents them from accumulating net worth beyond a point at which they can self-finance the operation.

The informational asymmetry takes the form of a costly-state-verification problem, as in BGG. In particular, we assume that  $\omega_t^e$  is only revealed to the entrepreneur ex-post (i.e. after loan contracts have been signed) and can only be observed by a third party after paying a monitoring cost, equivalent to a fraction  $\mu^e$  of the total revenues generated by the project. Thus, at the time entrepreneurs have to repay the loan they can choose to either pay it (plus the specified interest) or to default, in which case the intermediary will pay the monitoring cost and seize all entrepreneurial assets.

Following BGG, the optimal debt contract specifies a cut-off value  $\bar{\omega}_{t+1}^e$  such that if  $\omega_{t+1}^e \geq \bar{\omega}_{t+1}^e$  the borrower pays  $\bar{\omega}_{t+1}^e [r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]K_t$  units of final consumption goods to the lender and keeps  $(\omega_{t+1}^e - \bar{\omega}_{t+1}^e)[r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]K_t$ , while if  $\omega_{t+1}^e < \bar{\omega}_{t+1}^e$  the borrower receives nothing (defaults) and the lender obtains  $(1-\mu^e)\omega_{t+1}^e[r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]K_t$ . Therefore, under the assumption of a competitive the lending market, the mapping between the cut-off value and the interest rate on the loan  $R_t^{Le}$  satisfies

$$R_t^{L,e} = \bar{\omega}_{t+1}^e [r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}] \frac{K_t}{L_t^K} \pi_{t+1}, \tag{11}$$

where the right-hand side is the return obtained by the bank for each unit of money lent from an entrepreneur that pays back the loan. As we assume that entrepreneurs bear all the risk (as in BGG), this condition is assumed to hold state by state.

 $<sup>^{15}</sup>$ That is, at the time the financial contract is signed, everybody knows the distribution from which individual productivity will be drawn next period.

<sup>&</sup>lt;sup>16</sup>We are abusing the notation here, as  $u_t$ ,  $\omega_t^e$  and  $K_{t-1}$  should have an index identifying the individual entrepreneur. However, as we assume that entrepreneurs are identical ex-ante, that technology is linear, and that  $E(\omega_t^e) = 1$ , in equilibrium aggregate capital services will be given by  $u_t K_{t-1}$ .

<sup>&</sup>lt;sup>17</sup>Note that the choice of  $u_t$  is intra-periodic, so it does not depend on financing conditions.

While  $R_t^{L,e}$  denotes the interest rate of a loan signed at t, the ex-post return for the intermediary for each unit lent at t (which we denote by  $R_{t+1}^{L,K}$ , with  $r_t^{L,K} \equiv R_t^{L,K}/\pi_t$ ) is not equal to  $R_t^{L,e}$  for two reasons: not all loans will be repaid and, from those entrepreneurs who default, the intermediary receives their assets net of monitoring costs. This in particular implies that, while the interest rate on the loan is known at the time the contract is signed, the return obtained by the intermediary is instead state-contingent, for it depends on the aggregate conditions that determine whether entrepreneurs default or not. Therefore, for the intermediary to be willing to lend it must be the case that

$$L_t^K r_{t+1}^{L,K} \le g(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) [r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}] K_t,$$
(12)

where the term in brackets on the right-hand side of (12) is the average (across entrepreneurs) revenue obtained at t + 1 if the amount of capital purchases at t was  $K_t$ , and with

$$g(\bar{\omega}_t^e; \sigma_{\omega, t-1}) \equiv \bar{\omega}_t^e [1 - F(\bar{\omega}_t^e; \sigma_{\omega, t-1})] + (1 - \mu^e) \int_0^{\bar{\omega}_t^e} \omega^e f(\omega^e; \sigma_{\omega, t-1}) \mathrm{d}\omega^e.$$

The first term on the right-hand side is the share of total revenues that the intermediary obtains from those who pay back the loan, while the second is the value of the assets seized from defaulting entrepreneurs, net of monitoring costs. As we will see below, the banks' problem defines a non-arbitrage condition that relates the *expected value* of  $R_{t+1}^{L,K}$  with other interest rates relevant for banks. Thus, (12) is the participation constraint for the banks to be willing to lend. As before, this condition holds state-by-state under the assumption that entrepreneurs bear all the risk.<sup>18</sup>

From the entrepreneurs' viewpoint, the expected profits for the project of purchasing  $K_t$  units of capital equals

$$E_t\left\{ [r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]K_t h(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) \right\},\tag{13}$$

where

$$h(\bar{\omega}_t^e; \sigma_{\omega, t-1}) \equiv \int_{\bar{\omega}_t^e}^{\infty} \omega^e f(\omega^e; \sigma_{\omega, t-1}) \mathrm{d}\omega^e - \bar{\omega}_t^e [1 - F(\bar{\omega}_t^e; \sigma_{\omega, t-1})].$$
(14)

The first term on the right-hand side of (14) is the expected share of average revenue that entrepreneurs obtain given their productivity. The second is the expected repayment. Both are conditional on not defaulting (i.e.  $\bar{\omega}_t^e \ge \omega_t^e$ ).

Defining  $lev_t^e \equiv \frac{q_t K_t}{N_t^e}$ , and given the revelation principle, the optimal debt contract specifies a value for  $lev_t^e$  and a state-contingent  $\bar{\omega}_{t+1}^e$  such that (13) is maximized subject to (12) being satisfied with equality for every possible aggregate state at t + 1. As shown in the appendix, the optimality condition for this contract can be written as follows:

$$E_{t}\left\{\frac{\left[r_{t+1}^{K}u_{t+1}-\phi(u_{t+1})+(1-\delta)q_{t+1}\right]}{q_{t}}\left[\frac{h'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})g(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}{g'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}-h(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})\right]\right\}=E_{t}\left\{r_{t+1}^{L,K}\frac{h'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}{g'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}\right\},\quad(15)$$

The ratio  $efp_t \equiv E_t \left\{ \frac{[r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]}{q_t} \right\} / E_t \left\{ r_{t+1}^{L,K} \right\}$  is known as the external finance premium which,

 $<sup>^{18}</sup>$ See the appendix for a discussion about why this constraint needs to hold state-by-state if one wants merge the BGG model within the Gertler and Karadi framework.

as shown by BGG, is (up to first order) an increasing function of entrepreneurs' leverage  $lev_t^e$ .

Finally, average entrepreneurs' net worth evolves over time as follows. The average return an entrepreneur gets after repaying its loan at t is given by  $[r_t^K u_t - \phi(u_t) + (1 - \delta)q_t]K_{t-1}h(\bar{\omega}_t^e;\sigma_{\omega,t-1})$ . We assume that only a fraction v of entrepreneurs survives every period, and an equivalent fraction enters the market with an initial capital injection from households equal to  $\frac{\iota^e}{1-\upsilon}n^eA_{t-1}$ , with  $\iota^e > 0$  (i.e. a fraction  $\frac{\iota^e}{1-\upsilon}$  of balanced-growth-path net worth).<sup>19</sup> Thus, we have

$$N_t^e = v \left\{ [r_t^K u_t - \phi(u_t) + (1 - \delta)q_t] K_{t-1} h(\bar{\omega}_t^e; \sigma_{\omega, t-1}) \right\} + \iota^e n^e A_{t-1}.$$

# 2.3.2 Capital Goods

Capital goods producers operate the technology that allows to increase the economy-wide stock of capital. In each period, they purchase the stock of depreciated capital from entrepreneurs and combine it with investment goods (which they buy at a price  $P_t^I$ ) to produce new productive capital. The newly produced capital is then sold back to the entrepreneurs and any profits are transferred to the households. A representative capital producer's technology is given by

$$K_t = (1 - \delta)K_{t-1} + [1 - \Gamma (I_t/I_{t-1})]\varpi_t I_t,$$

where  $I_t$  denotes investment expenditures in terms of the final good as a materials input and

$$\Gamma\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2} \left(\frac{I_t}{I_{t-1}} - \bar{a}\right)^2$$

are convex investment adjustment costs. The variable  $\varpi_t$  is an investment shock that captures changes in the efficiency of the investment process (see, for instance, Justiniano *et al.*, 2011).

#### 2.4 Banks

We assume the presence of competitive financial intermediaries (banks) that take deposits from households and combine them with their own net worth to produce loans to both firms and to entrepreneurs. Following Gertler and Karadi (2011), the relationship between households and banks is characterized by a moral hazard problem that gives rise to a premium between the lending and deposit rates, even in the absence of BGG frictions.

The balance sheet of a representative financial intermediary at the end of period t is given by

$$L_t^{WC} + L_t^K = D_t + N_t,$$

where  $D_t$  denote deposits by domestic households at this intermediary,  $L_t^{WC}$  and  $L_t^K$  denote the intermediary's stock of loans to, respectively, home goods producing firms and entrepreneurs, and  $N_t$  denotes the intermediary's net worth (all in real terms of domestic units). The latter evolves over time as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1} = r_{t+1}^{L,WC} L_t^{WC} + r_{t+1}^{L,K} L_t^K - r_{t+1} D_t = (r_{t+1}^{L,WC} - r_{t+1}) L_t^{WC} + (r_{t+1}^{L,K} - r_{t+1}) L_t^K + r_{t+1} N_t$$
(16)

<sup>&</sup>lt;sup>19</sup>Entrepreneurs that leave that market transfer their remaining resources to households.

where  $r_t^{L,WC}$  and  $r_t^{L,K}$  denote the real gross returns on both types of loans.<sup>20,21</sup>

Financial intermediaries have finite lifetimes. At the beginning of period t+1, after financial payouts have been made, the intermediary continues operating with probability  $\omega$  and exits the intermediary sector with probability  $1 - \omega$ , in which case it transfers its retained capital to the household which owns that intermediary. Thus, the intermediary's objective in period t is to maximize expected terminal wealth  $(V_t)$ , which is given by

$$V_t \equiv E_t \sum_{s=0}^{\infty} (1-\omega)\omega^s \beta^{s+1} \Xi_{t,t+s+1} N_{t+s+1},$$

where  $\beta^s \Xi_{t,t+s}$  is the households' stochastic discount factor for real payoffs.

Further, following Gertler and Karadi (2011), a costly enforcement problem constrains the ability of intermediaries to obtain funds from depositors. In particular, at the beginning of period t, before financial payouts are made, the intermediary can divert an exogenous fraction  $\mu_t$  of total assets  $(L_t)$ . The depositors can then force the intermediary into bankruptcy and recover the remaining assets, but it is too costly for the depositors to recover the funds that the intermediary diverted. Accordingly, for the depositors to be willing to supply funds to the intermediary, the incentive constraint

$$V_t \ge \mu_t (L_t^{WC} + L_t^K) \tag{17}$$

must be satisfied. That is, the opportunity cost to the intermediary of diverting assets (i.e. to continue operating and obtaining the value  $V_t$ ) cannot be smaller than the gain from diverting assets. As can be seen, shocks that increase  $\mu_t$  will make this constraint tighter, making the financial problem more severe.

Using the method of undetermined coefficients,  $V_t$  can be expressed as follows (see the appendix):

$$V_t = \varrho_t^{L,WC} L_t^{WC} + \varrho_t^{L,K} L_t^K + \varrho_t^N N_t,$$
(18)

where

$$\begin{split} \varrho_t^{L,WC} &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1-\omega)(r_{t+1}^{L,WC} - r_{t+1}) + \omega \frac{L_{t+1}^{WC}}{L_t^{WC}} \varrho_{t+1}^{L,WC} \right] \right\} \\ \varrho_t^{L,K} &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1-\omega)(r_{t+1}^{L,K} - r_{t+1}) + \omega \frac{L_{t+1}^{K}}{L_t^{K}} \varrho_{t+1}^{L,K} \right] \right\}, \\ \varrho_t^N &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1-\omega)r_{t+1} + \omega \frac{N_{t+1}}{N_t} \varrho_{t+1}^N \right] \right\} \end{split}$$

Holding the other variables constant,  $\varrho_t^{L,WC}$  and  $\varrho_t^{L,K}$  are the expected discounted marginal gain of an additional unit of each type of loan, while  $\varrho_t^N$  is the expected discounted marginal gain of and additional unit of net worth.

The intermediary maximizes (18) subject to (17) taking  $N_t$  as given. The first-order conditions to this problem are as follows:

$$\begin{split} L_t^{WC} &: \quad (1+\varkappa_t)\varrho_t^{L,WC} - \mu_t \varkappa_t = 0, \\ L_t^K &: \quad (1+\varkappa_t)\varrho_t^{L,K} - \mu_t \varkappa_t = 0, \\ \varkappa_t &: \quad \varrho_t^{L,WC} L_t^{WC} + \varrho_t^{L,K} L_t^K + \varrho_t^N N_t - \mu_t (L_t^{WC} + L_t^K) \ge 0, \end{split}$$

 $^{20}$ These real rates relate to their nominal counterparts in a similar way as the real domestic deposit rate  $r_t$  defined above.

 $<sup>^{21}</sup>$ We assume that, while loans to working capital are intra-periodic, firms repay loans after banks' choices in period t have been made, which is the same as assuming that the return from this loan is received in the next period as in (16). This is in line with the assumption of working capital loans in the related literature without banks (e.g. Christiano *et al.* 2014).

where  $\varkappa_t \geq 0$  is the multiplier associated with the incentive constraint. The second condition holds with equality if  $\varkappa_t > 0$ , otherwise it holds with strict inequality. Notice that the optimality conditions for each type of loans imply that  $\varrho_t^{L,WC} = \varrho_t^{L,K} \equiv \varrho_t^L$ . In other words, as the incentive constraint is symmetric for both types of loans, banks need to be indifferent ex-ante between lending one unit to firms or to entrepreneurs. However, the arbitrage condition is not simply that the expected return of both loans are ex-ante identical (not even up to first order), because the marginal value for each type of loan depends on the growth rate of each of these loans. In addition, either of the conditions for the choice of loans imply that

$$\varkappa_t = \frac{\varrho_t^L}{\mu_t - \varrho_t^L},$$

such that the constraint is strictly positive if  $\mu_t > \varrho_t^L$ . That is, the incentive constraint holds with equality if the marginal gain to the financial intermediary from diverting assets and going bankrupt  $(\mu_t)$  is larger than the marginal gain from expanding assets by one unit of deposits (i.e. holding net worth constant) and continuing to operate  $(\varrho_t^L)$ . We assume that this is the case in a local neighborhood of the non-stochastic steady state. The condition for  $\varkappa_t$  holding with equality implies that

$$L_t \equiv L_t^{WC} + L_t^K = lev_t N_t$$

where

$$lev_t \equiv \frac{\varrho_t^N}{\mu_t - \varrho_t^L} \tag{19}$$

denotes the intermediary's leverage ratio. As indicated by (19), higher marginal gains from increasing assets  $\varrho_t^L$  support a higher leverage ratio in the optimum, the same is true for the higher marginal gains of net worth  $\varrho_t^N$ , while a larger fraction of divertable funds  $\mu_t$  lowers the leverage ratio.

The aggregate evolution of net worth follows from the assumption that a fraction  $1 - \omega$  of intermediaries exits the sector in every period and an equal number enters. Each intermediary exiting the sector at the end of period t-1 transfers their remaining net worth  $(\tilde{N}_{e,t} \equiv (r_t^{L,WC} - r_t)L_{t-1}^{WC} + (r_t^{L,K} - r_t)L_{t-1}^K + r_tN_{t-1})$  to households. At the same time, households transfer starting capital equal to  $\tilde{N}_{n,t} \equiv \frac{\iota}{1-\omega}nA_{t-1}$  to each new intermediary; with  $\iota > 0$  (i.e. the transfer equals a fraction  $\frac{\iota}{1-\omega}$  of balanced-growth-path net worth). Aggregate net worth then evolves as follows:

$$N_t = \omega \tilde{N}_{e,t} + (1-\omega)\tilde{N}_{n,t} = \omega \left[ (r_t^{L,WC} - r_t) L_{t-1}^{WC} + (r_t^{L,K} - r_t) L_{t-1}^K + r_t N_{t-1} \right] + \iota n A_{t-1}.$$

We define the average lending-deposit spread as

$$spr_{t} = \frac{(R_{t}^{L,WC}L_{t}^{WC} + R_{t}^{L,e}L_{t}^{K})}{L_{t}}\frac{1}{R_{t}}.$$
(20)

i.e. the average of both contractual loan rates, weighted by the size of each loan on total loans, relative to the deposit rate. This measure would be the data counterpart of the spread we will use for the estimation.

Finally, it is relevant to highlight that our treatment of working capital loans in the bank's problem is different than in the related literature. In the original paper by Gertler and Karadi, this type of loans were not considered. Other papers that do consider these loans in models featuring GK banks (e.g. Rannenberg, 2013; Villa, 2013; Villa and Yang, 2013; and Areosa and Coelho, 2013) assume that working capital loans are not subject to the moral hazard problem assumed by GK. However, it is not clear why this is an appropriate assumption, for it implies that somehow the ability of diverting funds by banks is different depending on the type of loans, or that depositors can easily distinguish between the several uses banks give to their funds. We believe that our setup, in which there is an indifference condition for both types of loans that is affected by the frictions faced by banks, is more realistic. Moreover, as we will latter describe, this assumption generates an additional propagation channel in the model.

# 2.5 Fiscal and Monetary Policy

The government consumes an exogenous stream of final goods  $(G_t)$ , levies lump-sum taxes, issues one-period bonds and receives a share  $\chi$  of the income generated in the commodity sector. We assume for simplicity that the public asset position is completely denominated in domestic currency. Hence, the government satisfies the following period-by-period constraint:

$$p_t^G G_t + r_t B_{t-1} = T_t + B_t + \chi p_t^{Co} Y_t^{Co}.$$

Monetary policy is carried out according to a Taylor rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[ \left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{Y_t/Y_{t-1}}{\bar{a}}\right)^{\alpha_y} \right]^{1-\rho_R} \exp(\varepsilon_t^R), \tag{21}$$

where  $\bar{\pi}$  is target inflation,  $Y_t$  is real GDP (defined below),  $\bar{a}$  is the long-run growth rate of TFP, and  $\varepsilon_t^R$  is an i.i.d. Gaussian shock that captures deviations from the rule.

# 2.6 The Rest of the World

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level  $P_t^{F*}$  is identical to the foreign consumption-based price index  $P_t^*$ . Further, let  $P_t^{H*}$  denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e.  $P_t^H = S_t P_t^{H*}$  and  $P_t^{Co} = S_t P_t^{Co*}$ . That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods according to (10). Therefore, the real exchange rate  $rer_t$  satisfies

$$rer_t = \frac{S_t P_t^*}{P_t} = \frac{S_t P_t^{F*}}{P_t} = \frac{P_t^F m c_t^F}{P_t} = p_t^F m c_t^F,$$

and the commodity price in terms of domestic consumption goods is given by

$$p_t^{Co} = \frac{P_t^{Co}}{P_t} = \frac{S_t P_t^{Co*}}{P_t} = \frac{S_t P_t^*}{P_t} p_t^{Co*} = rer_t p_t^{Co*}.$$

We also have the relation  $rer_t/rer_{t-1} = \pi_t^S \pi_t^*/\pi_t$ , where  $\pi_t^*$  denotes foreign inflation and  $\pi_t^S = S_t/S_{t-1}$ . Further, foreign demand for the home composite good  $X_t^{H*}$  is given by the schedule

$$X_t^{H*} = o^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\eta^*} Y_t^*,$$

where  $Y_t^*$  denotes foreign aggregate demand. Both  $Y_t^*$  and  $\pi_t^*$  evolve exogenously.

## 2.7 Aggregation and Market Clearing

Taking into account the market clearing conditions for all the different markets, we can define the trade balance in units of final goods as

$$TB_t = p_t^H X_t^{H*} + rer_t p_t^{Co*} Y_t^{Co} - rer_t M_t.$$

$$\tag{22}$$

Further, we define real GDP as follows:

$$Y_t \equiv C_t + I_t + G_t + X_t^{H*} + Y_t^{Co} - M_t.$$

Then, the GDP deflator  $(p_t^Y)$ , expressed as a relative price in terms of the final consumption good) is implicitly defined as

$$p_t^Y Y_t = C_t + p_t^I I_t + p_t^G G_t + T B_t.$$

Finally, we can show that the net foreign asset position evolves according to

$$rer_t B_t^* = rer_t r_t^* B_{t-1}^* + TB_t - (1-\chi)rer_t p_t^{Co*} Y_t^{Co}.$$

## 2.8 Driving Forces

The exogenous processes in the model are  $v_t, \varpi_t, z_t, a_t, \zeta_t, R_t^*, \pi_t^*, p_t^{Co*}, y_t^{Co}, y_t^*, g_t, \mu_t$  and  $\sigma_{\omega,t}$ . For each of them, we assume a process of the form

$$\log\left(x_t/\bar{x}\right) = \rho_x \log\left(x_{t-1}/\bar{x}\right) + \varepsilon_t^x, \qquad \rho_x \in [0, 1), \qquad \bar{x} > 0,$$

for  $x = \{v, \varpi, z, a, \zeta, R^*, \pi^*, p^{Co*}, y^{Co}, y^*, g, \mu, \sigma_\omega\}$ , where the  $\varepsilon_t^x$  are i.i.d. Gaussian shocks. In the case of  $\sigma_{\omega,t}$  we consider two alternatives for  $\varepsilon_t^{\sigma_\omega}$ . One is that it is just i.i.d. Gaussian. The other is a news structure as in Christiano et al (2014), i.e.

$$\varepsilon_t^{\sigma_\omega} = \tau_{0,t} + \tau_{1,t-1} + \dots + \tau_{p,t-p},$$

where the  $\tau_{j,t}$ 's are Guassian. It is assumed that at period t agents observe  $\tau_{j,t}$  for j = 0, ..., p. In that way,  $\tau_{0,t}$  is the unanticipated shock, while each of the  $\tau_{j,t}$  for j > 0 are the news shocks. Furthermore, we assume that the  $\tau_{j,t}$ 's have the following correlation structure

$$(\rho_{\sigma_{\omega},n})^{|i-j|} = \frac{E\{\tau_{i,t}\tau_{j,t}\}}{\sqrt{E\{\tau_{i,t}^2\}E\{\tau_{j,t}^2\}}} \quad \text{for } j, i = 0, ..., p,$$

with  $E\{\tau_{0,t}^2\} = \sigma_{\sigma_\omega}^2$  and  $E\{\tau_{j,t}^2\} = \sigma_{\sigma_\omega,n}^2$  for j > 0.<sup>22</sup> In the estimation, we set p = 4.

# 2.9 Alternative Versions of the Model

To empirically assess the relative importance of different financial channels and shocks we will consider alternative versions of the model. The complete model just presented considering only unanticipated shock to entrepreneurs risk will be labeled as GK+BGG, while the version that does include new shocks to this variable will be called as GK+BGG+CMR.

The Base model is one in which there are no financial frictions (i.e. with no banks and where entrepreneurs do not face idiosyncratic shocks), and where households lend directly to both firms for working capital (at a rate  $R_t^{L,WC}$ ) and to entrepreneurs (at a rate  $R_t^{L,e}$ ). Moreover, we include in such a model exogenous shocks to the spread between the lending rate and the policy and between the expected return on capital and the lending rate. In particular, the calibration implies that in steady state the difference between the policy rate and the return on capital is the same as in the GK+BGG model. In addition, it is assumed that only a share  $\alpha_L^K$  of total capital purchases is financed by loans. Let,  $\tilde{R}_t^e$  be the return on entrepreneurs operations. The above assumptions imply that,

$$R_t^{L,WC} = R_t^{L,e} = spr_t^{exo}R_t, \quad E_t\{\tilde{R}_{t+1}^e\} = efp_t^{exo}R_t^{L,e},$$
$$L_t^K = \alpha_L^K q_t K_t, \quad N_t^e = q_t K_t - L_t^K, \quad L_t = L_t^K + L_t^{WC}.$$

where  $spr_t^{exo}$  and  $efp_t^{exo}$  are exogenous AR(1) processes. In other words, this model features the same steady state as in the GK+BGG model, but the dynamics of the spread and the external-finance premium are driven by exogenous shocks instead of being micro-funded and endogenous.

The version called GK model features banks following Gertler and Karadi (2011) as we have described, but entrepreneurs face no financial frictions. However, as in the Base model, we assume the presence of an exogenous gap between that loan rate and the return on entrepreneurs operations. In this version of the model, we force the share of total capital purchases financed by loans to be the same as in the model with BGG entrepreneurs. Moreover, following Gertler and Karadi (2011), we assume in this version of the model that loans to entrepreneurs are state-contingent (with a rate  $\tilde{R}_{t+1}^{L,e}$ ). Overall, in the GK model we have

$$spr_{t} = \frac{(R_{t}^{L,WC}L_{t}^{WC} + E_{t}\{\tilde{R}_{t+1}^{L,e}\}L_{t}^{K})}{L_{t}}\frac{1}{R_{t}},$$
$$\tilde{R}_{t}^{e} = efp_{t-1}^{exo}\tilde{R}_{t}^{L,e}, \quad L_{t}^{K} = \alpha_{L}^{K}q_{t}K_{t}, \quad N_{t}^{e} = q_{t}K_{t} - L_{t}^{K}, \quad L_{t} = L_{t}^{K} + L_{t}^{WC},$$

and the relationship between  $R_t^{L,WC}$  and  $E_t\{\tilde{R}_{t+1}^{L,e}\}$  is determined by the bank's problem as in the full model. Thus, while the GK model makes the spread between lending rates and the policy rate endogenous, it assumes an exogenous external finance premium.

Finally, we also consider a model with entrepreneurs facing the costly-state-verification problem we have detailed above, but where they obtain funds directly from households. For this alternative, we set up two versions (BGG and BGG+CMR) that differ on the presence of news shocks. While this framework is able to endogenously generate positive spreads and external finance premium, to make a fair comparison with the other alternative models, we assume that the interest rate that represents the opportunity cost for households to lend is  $R_t spr_t^{exo}$ ,

 $<sup>^{22}</sup>$ In principle, one can include such a news structure for other driving forces of the model as well. Here we follow the results by Christiano et al (2014) that justify using this structure only for risk shocks on the grounds that they provide the best fit to the data.

which is also the interest rate on working-capital loans. Overall, we have six alternative versions of the model.

# **3** Parametrization Strategy

Our empirical strategy combines both calibrated and estimated parameters. The calibrated parameters and targeted steady state values are presented in Table 1. For most of the parameters not related with financial frictions we draw from related studies using Chilean data, as indicated in the table, while others are endogenously determined in steady state to target some first moments ( $\beta$ ,  $\pi^*$ ,  $\kappa$ ,  $o^*$ ,  $\bar{g}$ ,  $\bar{b}^*$  and  $\bar{y}^{Co}$ ). The parameters that deserve additional explanation are those related with financial frictions:  $\bar{\mu}$  (the steady state value of the fraction of divertible funds),  $\omega$  (the fraction of surviving banks),  $\iota$  (the capital injection for new banks),  $\mu^e$  (bankruptcy costs), v (the fraction of surviving entrepreneurs),  $\iota^e$  (the capital injection for new entrepreneurs), and  $\sigma_{\omega}$  (the steady state value of entrepreneurs' dispersion).

We target the following averages for financial variables. We set the spread between the interest rate on entrepreneurs  $(R^{L,e})$  and the deposit rate (R) to 380 basis points, which corresponds to the average spread between 90-days loans and the monetary policy rate.<sup>23,24</sup> We further set the bank leverage ratio to 9. This statistic is not easy to calibrate, for banks' balance sheets are more complicated in the data than in the model. Consolidated data from the banking system in Chile implies an average leverage ratio of around 13 between 2001 and 2012, but on the asset's side of the balance sheet there are other types of assets that are not loans. To pick the value that we use, we compute an average ratio of the stock of loans to total consolidated assets of the banking system of 66% and adjusted the observed average leverage of the banking system by this percentage (i.e.  $9 \approx 13 \times 0.66$ ). For the entrepreneurs' problem, we choose a steady state leverage of 2.05, which corresponds to the average leverage between 2001 and 2012 for the largest Chilean firms.<sup>25</sup> In addition, we also calibrate the external finance premium in steady state (efp), for which we choose a value of 120 basis points, which corresponds to the average between the A vs. AAA corporate-bond spread and the BBA vs. AAA spread, for the sample from 2001 to 2012.<sup>26</sup> Finally, as the steady state for both financial problems impose less restriction than parameters, we normalize  $\iota = 0.002$  (as in Gertler and Karadi, 2011),  $\upsilon = 0.97$  (the value used by BGG) and  $\mu^e = 0.12$  (in the range used by Christiano *et al.*, 2010, for the US and the EU). Thus, the parameters  $\bar{\mu}$ ,  $\omega$ ,  $\iota^e$  and  $\sigma_{\omega}$  are endogenously set in steady state to match these targets.

We also calibrate the parameters characterizing those exogenous processes for which we have a data counterpart or a good proxy. In particular, for g we use linearly-detrended real government expenditures, for  $y^{Co}$  we use linearly-detrended real mining production, for  $R^*$  we use the LIBOR rate, for  $y^*$  we use linearly-detrended real GDP of commercial partners, for  $\pi^*$  we use CPI inflation (in dollars) for commercial partners, and for  $p^{Co*}$  we use international copper price deflated by the same price index used to construct  $\pi^*$ .<sup>27</sup>

The other parameters of the model were estimated using Bayesian techniques, solving the model with a loglinear approximation around the non-stochastic steady state. The list of these parameters and the priors are described in columns one to four of Table  $4.^{28}$  The sample is from 2001Q3 to 2012Q4. We first include a set of

 $<sup>^{23}</sup>$ All rates and spread figures are presented here in annualized terms, although in the model they are included on a quarterly basis.  $^{24}$ We match this spread instead of the one defined in (20) because the computation of the steady state simplifies significantly with this choice. At the posterior mode, the difference between these two spreads is less than 3 annualized basis points.

 $<sup>^{25}</sup>$ This average is computed by consolidating balance sheet data compiled by the SVS (the stock market authority in Chile). On average, this includes the largest 300 firms in the country.

 $<sup>^{26}</sup>$ Here we follow Christiano *et al.* (2010) who use the spread on corporate bonds of different credit ratings as a proxy for the premium paid by riskier firms.

<sup>&</sup>lt;sup>27</sup>The data source for all Chilean-related data is the Central Bank of Chile, while the other variables are obtained from Bloomberg. <sup>28</sup>The prior means were set to represent (when available) the estimates of related papers for the Chilean economy (e.g. Medina

Param.	Description	Value	Source
σ	Risk aversion	1	Medina and Soto (2007)
$\phi$	Inverse Frish elasticity	5	Kirchner and Tranamil (2016)
$\alpha$	Capital share in production	0.33	Medina and Soto (2007)
δ	Capital depreciation	0.06/4	Medina and Soto (2007)
$\epsilon_H$	E.o.S. domestic aggregate	11	Medina and Soto (2007)
$\epsilon_F$	E.o.S. imported aggregate	11	Medina and Soto (2007)
$o_C$	Share of $F$ in $Y^C$	0.26	Input-ouput matrix (2008-2012)
$o_I$	Share of $F$ in $I$	0.36	Input-ouput matrix (2008-2012)
$O_G$	Share of $F$ in $G$	0	Normalization
	Government share in commodity sector	0.61	Average (1987-2012)
$\overset{\chi}{s^{tb}}$	Trade balance to GDP in SS	4%	Average (1987-2012)
$s^g$	Gov. exp. to GDP in SS	11%	Average (1987-2012)
$s^{Co}$	Commodity prod. to GDP in SS	10%	Average (1987-2012)
$\bar{\pi}$	Inflation in SS	3%	Inflation Target in Chile
$p^H$	Relative price of $H$ in SS	1	Normalization
h	Hours in SS	0.3	Normalization
$\bar{a}$	Long-run growth	2.50%	4.5% GDP - $2%$ labor force grth. (avg. 01-12)
R	MPR in SS.	5.80%	Fuentes and Gredig (2008)
$R^*$	Foreign rate in SS	4.50%	Fuentes and Gredig (2008)
GK bank			
ξ	Country premium in SS	$140 \mathrm{bp}$	EMBI Chile (avg. 01-12)
lev	Leverage financial sector	9	Own calculation (see text)
spread	90 days lending-borrowing spread	$380\mathrm{bp}$	Loan rate vs. MP rate (avg. 01-12)
ι	Injection for new bankers	0.002	Gertler and Karadi (2011)
$BGG \ en$	trepreneurs		
$\mu^e$	Bankruptcy cost	0.12	Christiano $et al.$ (2010)
v	Survival rate of entrepreneurs	0.97	Bernanke $et al.$ (1999)
efp	Entrepreneurs' external finance premium	$120 \mathrm{bp}$	Spread A vs. AAA, corp. bonds (avg. 01-12)
$lev^e$	Entrepreneurs' leverage	2.05	For the non-financial corp. sector (avg. 01-12)
Exogeno	us processes		
$ ho_{y^{Co}}$	Auto corr. $y^{Co}$	0.4794	Own estimation
$ ho_g$	Auto corr. $g$	0.6973	Own estimation
$\rho_{R^*}$	Auto corr. $R^*$	0.9614	Own estimation
$ ho_{y^*}$	Auto corr. $y^*$	0.8665	Own estimation
$\rho_{\pi^*}$	Auto corr. $\pi^*$	0.3643	Own estimation
$\rho_{p^{Co*}}$	Auto corr. $p^{Co*}$	0.962	Own estimation
$\sigma_{y^{Co}}$	St. dev. shock to $y^{Co}$	0.0293	Own estimation
$\sigma_g$	St. dev. shock to $g$	0.0145	Own estimation
$\sigma_{R^*}$	St. dev. shock to $R^*$	0.0011	Own estimation
$\sigma_{y^*}$	St. dev. shock to $y^*$	0.0062	Own estimation
$\sigma_{\pi^*}$	St. dev. shock to $\pi^*$	0.0273	Own estimation
$\sigma_{p^{Co*}}$	St. dev. shock to $p^{Co*}$	0.1413	Own estimation

Table 1: Calibrated Parameters.

Note: All rates and spreads are annualized figures.

macro-related variables: the growth rates of real GDP, private consumption and investment, the trade-balanceto-output ratio, the CPI inflation rate, the monetary policy rate, the multilateral real exchange rate, the growth rate of real wages, the EMBI Chile (a proxy for  $\xi_t$ ). We also include as observables the variables used to estimate the exogenous processes previously described.<sup>29</sup> In addition, we consider the following credit-related variables: the spread between the 90-days loans rate and the monetary policy rate (as a counterpart of  $spr_t$ ), the growth rate of total loans in the banking system,<sup>30</sup> the difference between the A vs. AAA corporate-bond as a proxy for  $efp_t$ , and the growth rate of the stock market index for Chile (IPSA), deflated by the CPI, as a proxy for the growth rate of  $N_t^e$ .<sup>31</sup>

Overall, the alternative versions of the model are estimated with 19 variables. Our estimation strategy also includes i.i.d. measurement errors whose variance is calibrated to represent 10% of the variance of the corresponding observable.<sup>32</sup>

# 4 Inference and Goodness of Fit

In this section, we first compare the models in terms of goodness-of-fit, in order to understand if and how the presence fo financial frictions helps to improve the ability of the model to account for the dynamics observed in the data. We then compare the inferred parameters and variance decomposition for each of the estimated models, with a special focus on the versions of the model that provides better match to the data.

## 4.1 Goodness of Fit

We begin with an overall measure of goodness of fit, computing the marginal data density of the dataset used for estimation in each alternative version of the model, as reported in the first line of Table 2. We first observe that, compared with the Base model, the GK version provides a worst overall fit to the data, while both versions of the BGG model (with and without anticipated risk shocks) improves this statistic more than 40 log points. Moreover, the difference is larger when including anticipated risk shocks. The GK+BGG+CMR also improves over the Base model and it is closer to the BGG model, although the model with just contemporaneous shocks (GK+BGG) provides an overall fit similar to the Base model. Thus, according to this measure, the BGG friction seems relatively more important, as well as the presence of anticipated risk shocks.

In lines two and three of Table 2, we use the posterior mode of each model to compute the marginal data density of different subsets of variables.<sup>33</sup> We can see that the Base and GK models do a better job in accounting for the dynamics of Macro variables, while both the BGG and BGG+CMR are the worst alternatives. But in terms of financial variables, the ranking of models is the opposite. In both cases, the GK+BGG and GK+BGG+CMR alternatives fall in the middle of the ranking. Thus, at least from this overall measure of fit, it seems that if we want to describe only macro variables, either the Base or the GK model are the best choices, while if we are interested in credit-related variables the BGG framework is a better alternative. Note however, that this analysis is mute about the joint dynamics of macro and credit-related variables, which we study below.

and Soto, 2007).

 $<sup>^{29}</sup>$ While the parameters of these exogenous processes were calibrated, including these variables in the data set is informative for the inference of the innovations associated with these exogenous processes.

<sup>&</sup>lt;sup>30</sup>Results do not significantly change if the growth rate of commercial loans only is used instead.

 $<sup>^{31}</sup>$ This set of financial variables is similar to that used by Christiano et al (2014) for the US.

<sup>&</sup>lt;sup>32</sup>We do this to for all the variables except for those that are observable driving forces  $(g, y^{Co}, R^*, y^*, \pi^*, p^{Co*})$ , for which we assume no measurement error.

 $<sup>^{33}</sup>$ See the note in Table for 2 the description of each group of variables.

Table 2: Log Marginal Data Density

			0	0	v	
Variables	Base	GK	BGG	BGG+CMR	GK+BGG	GK+BGG+CMR
All	-1497.8	-1537.0	-1451.4	-1431.4	-1506.8	-1466.9
Macro	-628.9	-631.1	-709.5	-704.1	-644.1	-650.8
Credit	-386.1	-475.0	-329.9	-325.4	-365.6	-352.5

Note: The first line represent, for each model, the marginal data density for the set of all variables used for estimation. The second is the marginal data density for the set of the following variables: the growth rates of real GDP, private consumption and investment, the trade-balance-to-output ratio, the CPI inflation rate, the monetary policy rate, the multilateral real exchange rate, and the growth rate of real wages. The third lines corresponds to a data set that includes: the growth rates of loans and entrepreneurs net worth, the spread and the external finance premium. All these were computed using a Laplace approximation at the posterior mode obtained with the estimation sample.

Another way to explore which dimensions of the data is better matched by each model is to study second moments of interest. Table 3 reports the standard deviation and first-order autocorrelation of a number of observed variables, both in the data and at the posterior mode for each model. We can see that the Base model tends to over-estimate the variance of most variables,<sup>34</sup> displaying more persistence as well. The GK model reduces the variances and persistence relative to the Base model, but is still far from the data-moments. For instance, the GK model generates a consumption growth that is less volatile than output (while in the data we observe the opposite) and the variance of credit growth and the spread are still overestimated.

The BGG model also overestimates the variance of most of the observables, particularly GDP and consumption growth, inflation, the policy rate, and the real exchange rate. On the other hand, the fit is much better for financial variables than with the GK, although the external finance premium seems to be more volatile and persistent than in the data. In addition, comparing both versions of the BGG model, the one that includes CMR-style shocks generate moments that are somehow closer to the data, but they are still significantly different.

The best overall fit for these second moments seems to be provided by the alternatives that include both frictions (GK+BGG and GK+BGG+CMR). These models seem to generate volatilities for both real macro aggregates as well as for nominal variables that are closer to those in the data, and the same pattern appears if we focus on autocorrelations. In terms of financial variables, these models approximate fairly well the moments for credit growth and the spread. On the down side, they generate a volatility and persistence of entrepreneurs net worth that is smaller than in the data, while the external finance premium is more volatile and persistent under these models.

So far, non of the goodness-of-fit measure that we have explore allow us to understand which is the best model in accounting for the joint dynamics of these variables (in particular, how financial series move relative to the business cycle). To that end, Figure 1 displays the leads-and-lags correlation for the four credit-related variables that we observe and GDP growth, both in the data and for the different models. In terms of the correlation with real credit growth, future values of this variables seems to be positively correlated with present values of GDP growth in the data, while the contemporaneous and past correlation does not appear to be significantly different from zero. Of the four models considered, the one that better captures this pattern is the GK+BGG setup.<sup>35</sup> All other models counter-factually produce negative lead correlations. Moreover, the GK model produces a positive contemporaneous correlation between these variables that is much larger than in the data.

 $<sup>^{34}</sup>$ The exceptions are the spread and the external finance premium, which happens trivially as they are, in this model, simple AR(1) processes.

 $<sup>^{35}</sup>$ In the figure, only the BGG+CMR and the GK+BGG+CMR are shown, as their counterparts without CMR shocks generate similar correlations.

								GK+
						BGG+	GK+	BGG+
Variable	Data		Base	$\operatorname{GK}$	BGG	CMR	BGG	CMR
		А.	Standa	ard Devi				
$\Delta GDP$	1.01	(0.13)	1.43	1.22	1.71	1.41	1.02	1.03
$\Delta C$	1.09	(0.15)	1.19	1.00	1.27	1.30	1.01	1.05
$\Delta I$	3.71	(0.45)	3.39	4.87	7.14	6.42	3.14	3.48
TB/GDP	5.26	(0.54)	4.70	4.07	6.89	6.89	4.01	4.20
$\Delta W$	0.61	(0.07)	0.65	0.61	1.44	1.23	0.57	0.58
R	0.46	(0.04)	0.72	0.61	3.03	3.07	0.59	0.71
$\pi$	0.73	(0.09)	0.78	0.70	2.86	2.75	0.66	0.77
rer	5.35	(0.49)	7.97	9.21	17.73	15.17	8.67	8.58
$\Delta L$	1.39	(0.16)	0.64	2.26	1.84	1.77	1.40	1.59
$\Delta N^e$	9.75	(0.91)	0.96	2.38	8.89	8.66	3.60	5.49
spr	0.26	(0.05)	0.23	1.39	0.32	0.36	0.35	0.43
efp	0.07	(0.01)	0.07	0.06	0.50	0.66	0.24	0.69
		В.	Autoco	orrelatio	n Order	1		
$\Delta GDP$	0.25	(0.22)	0.46	0.12	0.67	0.63	0.17	0.15
$\Delta C$	0.63	(0.21)	0.69	0.50	0.86	0.85	0.64	0.66
$\Delta I$	0.40	(0.22)	0.49	0.37	0.83	0.83	0.23	0.42
TB/GDP	0.73	(0.16)	0.95	0.93	0.97	0.97	0.94	0.94
$\Delta W$	0.40	(0.17)	0.49	0.21	0.77	0.74	0.45	0.45
R	0.88	(0.16)	0.95	0.90	1.00	1.00	0.93	0.96
$\pi$	0.63	(0.21)	0.70	0.57	0.98	0.98	0.72	0.77
rer	0.73	(0.16)	0.86	0.86	0.95	0.95	0.83	0.83
$\Delta L$	0.56	(0.18)	0.49	-0.16	0.34	0.29	0.28	0.46
$\Delta N^e$	0.20	(0.12)	0.09	-0.17	-0.04	-0.03	-0.03	-0.01
spr	0.68	(0.27)	0.74	0.12	0.86	0.90	0.52	0.55
efp	0.84	(0.13)	0.85	0.84	0.94	0.97	0.91	0.97

Table 3: Selected Second Moments

Note: The variables are, respectively, the growth rates of real GDP, Consumption, Investment, the trade-balance to output ratio, the growth rate of real wages, the policy rate, inflation the real exchange rate, the growth rates of loans and entrepreneurs net worth, the spread and the external finance premium. For the data moments, in parenthesis we report GMM standard errors. For those base on the models, they are computed as the unconditional moments using the posterior mode.

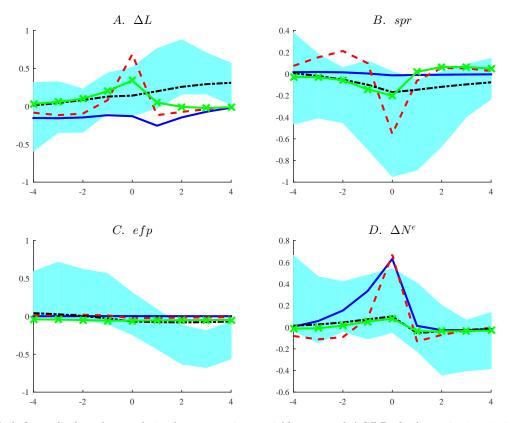


Figure 1: Leads and Lags Correlation with GDP Growth.

Note: Each figure displays the correlation between a given variable  $x_{t+h}$  and  $\Delta GDP_t$ , for h = -4, -3, ..., 3, 4, for the growth rate of credit (A), the banking spread (B), the external finance premium (C) and the growth rate of entrepreneurs net-worth (D). Gray areas represent a two-standard-error-band around the data moment (computed by GMM), blue-solid lines are from the baseline model, dashed-red lines corresponds to the GK model, dash-dotted-black lines are generated by the BGG+CMR, and crossed-solid-green lines are from the GK+BGG+CMR model.

In terms of the correlation of the spread with GDP growth, again the GK+BGG seems to better approximate the pattern observed in the data, where we observe a negative short-run correlation between these two variables. Regarding the external finance premium, none of the models can replicate the correlation observed in the data (where the external finance premium seems to lead credit growth), all of them generating values for these correlations that are much smaller relative to what seems plausible in the data. Still, in relative terms the GK+BGG model seems to better approximate this relationship. Finally, our proxy for the growth rate of entrepreneurs net-worth also seems to lead GDP growth in the data. This pattern seems to be better captured by both the BGG and the GK+BGG models, while both the Base and the GK model generate an excessive positive contemporaneous correlation between these variables.

Overall, we can draw several conclusion from this goodness of fit exercise. First, while the marginal data density favors the BGG+CMR model, the GK+BGG+CMR seems to provide a better fit for the moments we have analyzed. Second, the variance and persistence of financial variables seems to be better matched with models that include both frictions. Finally, including news shocks to entrepreneurs' risk improves the fit of the data

## 4.2 Estimated Parameters

We next describe the posterior mode for the different models, with a special focus on the alternatives that better fit the data, and how they compare with the Base model. These are presented in Tables 4 and 5. A first difference is the habits parameter  $\varsigma$ . As can be seen both BGG alternatives requires a value relatively high for this parameter, while the other models estimate a value between 0.7 and 0.8, closer to usual values in the related literature. Another difference is in the elasticities of substitution between home and foreign goods (both for consumption and investment) and also in terms of the elasticity of exports. In particular, we see that the BGG alternatives require a smaller values for these than the other setup. As these parameters are related to the expenditure switching channel, it seems that the model that includes only the BGG friction requires larger real exchange rate movements to affect the relative demand between domestic and foreign goods. This can be an explanation for the larger variance in the real exchange rate that the BGG models generate, as previously described.

A parameter that changes significantly across models is the one governing investment adjustment costs ( $\gamma$ ). In the Base, GK+BGG and GK+BGG+CMR models the posterior mode is around 10, in the GK model is much lower (close to 0.3) and both BGG alternatives this parameter is around 4.5. There are also difference in the inference for the parameter that determines the cost for capital utilization. It is expected that the presence of financial frictions changes the inference about these parameter. In fact, in one of the earliest contributions to the financial accelerator literature, Carlstrom and Fuerst (1997) present a financial accelerator similar to BGG as an alternative to capital adjustment costs in order to account for the hump-shaped dynamics of investment. From that perspective, one might expect the presence of financial frictions to diminish the need for capital adjustment costs to fit the data (i.e. a lower value for  $\gamma$ ). However, it is also true that when both capital adjustment costs and financial frictions are considered, one can also expect an interaction between both channels. In particular, it might be the case that with a lower  $\gamma$  the price of capital (Tobin's q) is too volatile (particularly after investment shocks), generating an excessive volatility in spreads that will get amplified by the presence of financial frictions. In our case, those models that only include one friction require a relatively lower cost for adjusting capital in order to match the volatility of investment and of financial variables, while with both frictions combined this parameter is closer to that in the Base model.

Another parameter that changes across models is the share working capital that is financed by loans ( $\alpha_L^{WC}$ ). In both GK+BGG alternatives, this parameter is relatively low (close to 0.1), while in the other alternatives seem

			Prior		D			terior	D	aa
Para.	Description	Dist.	Mean	St.Dev.	B Mode	ase St.Dev.			B Mode	GG St.Dev.
ς	Habits	β	0.7	0.1	0.82	0.04	0.70	0.05	0.90	0.02
$\psi$	Count. Prem. Elast.	$\Gamma^{-1}$	0.01	$\infty$	0.003	0.001	0.004	0.001	0.002	0.000
$\eta^{'C}$	E.o.S. $X^{C,H}$ , $X^{C,F}$	$\Gamma^{-1}$	1.4	0.4	2.39	0.27	2.41	0.31	2.01	0.30
$\eta^{I}$	E.o.S. $X^{I,H}, X^{I,F}$	$\Gamma^{-1}$	1.4	0.4	1.11	0.40	1.59	0.39	1.36	0.39
$\eta^*$	Elast. exports	$\Gamma^{-1}$	0.3	0.2	0.45	0.27	0.60	0.13	0.16	0.06
$\gamma$	Inv. Adj. Cost	$\mathcal{N}$	4	1.5	10.08	1.12	0.31	0.06	4.52	0.96
$ heta_W$	Calvo prob. $W$	$\beta$	0.75	0.1	0.97	0.00	0.95	0.01	0.96	0.00
$\vartheta_W$	Index. past infl. $W$	$\beta$	0.5	0.2	0.18	0.07	0.40	0.11	0.73	0.11
$ heta_{H}$	Calvo prob. $H$	$\beta$	0.75	0.1	0.41	0.07	0.72	0.03	0.97	0.00
$\vartheta_H$	Index. past infl. $H$	$\beta$	0.5	0.2	0.45	0.16	0.12	0.05	0.44	0.08
$ heta_F$	Calvo prob. $F$	$\beta$	0.75	0.1	0.70	0.06	0.87	0.03	0.64	0.04
$\vartheta_F$	Index. past infl. $F$	$\beta$	0.5	0.2	0.23	0.11	0.32	0.13	0.19	0.08
$ ho_R$	MPR Rule $R_{t-1}$	$\beta$	0.75	0.1	0.82	0.02	0.74	0.03	0.75	0.03
$\alpha_{\pi}$	MPR Rule $\pi_t$	$\mathcal{N}$	1.5	0.1	1.47	0.09	1.40	0.08	1.09	0.04
$\alpha_y$	MPR Rule growth	$\mathcal{N}$	0.13	0.1	0.15	0.05	0.13	0.05	0.11	0.05
$\phi_u$	Utilization Cost	$\mathcal{N}$	1	0.5	0.84	0.33	1.01	0.29	1.39	0.27
$\alpha_L^{WC}$	Share work. cap. Auto Correl. Shocks	$\mathcal{N}$	0.7	0.3	0.71	0.17	0.77	0.15	1.49	0.20
$ ho_v$	Pref.	β	0.75	0.1	0.83	0.06	0.71	0.07	0.97	0.01
$\rho_u$	Inv.	$\beta$	0.75	0.1	0.43	0.07	0.99	0.00	0.98	0.01
$\rho_z$	Temp. TFP	β	0.75	0.1	0.74	0.08	0.81	0.07	0.73	0.05
$\rho_a$	Perm. TFP	$\beta$	0.38	0.1	0.32	0.10	0.44	0.09	0.55	0.07
$\rho_{\zeta^1}$	Country prem.	$\beta$	0.75	0.1	0.86	0.06	0.83	0.05	0.82	0.05
$\rho_{\zeta^2}$	Country prem.	$\beta$	0.75	0.1	0.75	0.07	0.76	0.06	0.80	0.04
$\rho_{\mu}$	$\mu_t$	$\beta$	0.75	0.1	0.1.0		0.53	0.05	0.00	0.0-
$\rho_{\sigma_{\omega}}$	$\sigma_{\omega,t}$	β	0.75	0.1					0.76	0.07
$\rho_{spr_{exo}}$	$spr^{exo}$	β	0.75	0.1					0.81	0.08
$ ho_{efp_{exo}}$	$efp^{exo}$	$\beta$	0.75	0.1			0.84	0.05	0.0-	0.00
r cj pexo	St.Dev. Shocks	1-		-						
$\sigma_v$	Pref.	$\Gamma^{-1}$	0.01	$\infty$	0.04	0.01	0.03	0.01	0.10	0.02
$\sigma_u$	Inv.	$\Gamma^{-1}$	0.01	$\infty$	0.21	0.04	0.01	0.00	0.05	0.01
$\sigma_z$	Temp. TFP	$\Gamma^{-1}$	0.01	$\infty$	0.01	0.00	0.01	0.00	0.06	0.01
$\sigma_a * 10$	Perm. TFP	$\Gamma^{-1}$	0.10	$\infty$	0.03	0.01	0.03	0.01	0.07	0.01
$\sigma_{\zeta^1} * 10$	Country prem.	$\Gamma^{-1}$	0.03	$\infty$	0.008	0.001	0.008	0.001	0.008	0.001
$\sigma_{\zeta^2}^{*10}$	Country prem.	$\Gamma^{-1}$	0.03	$\infty$	0.07	0.02	0.08	0.02	0.07	0.02
$\sigma_R^*10$	MPR	$\Gamma^{-1}$	0.03	$\infty$	0.01	0.00	0.01	0.00	0.01	0.00
$\sigma_{\mu}$	$\mu_t$	$\Gamma^{-1}$	0.01	$\infty$	-		0.02	0.00	-	
$\sigma_{\sigma_\omega}$	Unanticipated risk	$\Gamma^{-1}$	0.01	$\infty$			-		0.05	0.01
$\sigma_{\sigma_{\omega,n}}$	Anticipated Risk	$\Gamma^{-1}$	0.01	$\infty$						
$\rho_{\sigma_{\omega},n}$	Anticipated Risk	$\mathcal{N}$	0	0.5						
$\sigma_{spr_{exo}}^{pow,n}$ *10	$spr^{exo}$	$\Gamma^{-1}$	0.10	$\infty$					0.17	0.03
$\sigma_{efp_{exo}} *10$	$efp^{exo}$	$\Gamma^{-1}$	0.10	$\infty$			0.03	0.00		0.00

Table 4: Estimated Parameters, Prior and Posterior Mode.

$ \begin{array}{c c c c c c c c c c c c c c c c c c c $				Prior				Posterior			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						BGG	+CMR			GK+B0	GG+CMR
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Para.	Description	Dist.	Mean	St.Dev.	Mode	St.Dev.	Mode	St.Dev.	Mode	St.Dev.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ς	Habits	β	0.7	0.1	0.90	0.02	0.77	0.02	0.78	0.05
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\psi$	Count. Prem. Elast.	$\Gamma^{-1}$	0.01	$\infty$	0.003	0.000	0.004	0.000	0.004	0.001
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\eta^C$	E.o.S. $X^{C,H}, X^{C,F}$		1.4	0.4	2.11	0.30	2.57	0.33	2.59	0.31
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\eta^{I}$	E.o.S. $X^{I,H}$ , $X^{I,F}$		1.4	0.4	1.40	0.39	1.55	0.39	1.55	0.39
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			$\Gamma^{-1}$	0.3	0.2	0.17	0.06	0.73	0.07	0.79	0.14
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Inv. Adj. Cost	$\mathcal{N}$	4	1.5	4.61	0.96	10.56	1.03	11.62	0.82
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Calvo prob. $W$	$\beta$	0.75	0.1	0.96	0.004	0.97	0.005	0.96	0.009
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Index. past infl. $W$	β		0.2	0.69	0.11	0.43	0.11	0.48	0.12
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Calvo prob. H	β	0.75	0.1	0.96	0.00	0.48	0.01	0.47	0.07
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			,			0.66					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Index. past infl. $F$	β	0.5	0.2	0.16	0.08	0.44	0.07	0.50	0.14
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\rho_R$			0.75		0.76	0.03	0.84	0.03	0.83	0.03
$\begin{array}{cccccccccccccccccccccccccccccccccccc$					0.1				0.05	1.33	0.09
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Utilization Cost	$\mathcal{N}$	1	0.5	1.24	0.27	1.00	0.28	0.95	0.29
Auto Correl. Shocks $\rho_v$ Pref. $\beta$ 0.75         0.1         0.97         0.01         0.84         0.01         0.85         0.06 $\rho_u$ Inv. $\beta$ 0.75         0.1         0.98         0.006         0.23         0.01         0.26         0.04 $\rho_z$ Temp. TFP $\beta$ 0.75         0.1         0.62         0.05         0.78         0.07         0.80         0.06 $\rho_a$ Perm. TFP $\beta$ 0.75         0.1         0.81         0.05         0.83         0.01         0.26         0.08 $\rho_{\zeta^1}$ Country prem. $\beta$ 0.75         0.1         0.80         0.04         0.69         0.05         0.68         0.07 $\rho_{\mu}$ $\mu_t$ $\beta$ 0.75         0.1         0.80         0.04         0.02         0.98         0.07         0.84         0.02         0.98         0.05 $\rho_{sprexo}$ $sprexo$ $\beta$ 0.75         0.1         0.85         0.08         0.00         0.07         0.00 $\rho_{sprexo}$ $sprexo$	$\alpha_L^{WC}$	Share work. cap.	$\mathcal{N}$	0.7		1.06	0.20	0.07	0.23		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Ľ	Auto Correl. Shocks									
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\rho_v$		β	0.75	0.1	0.97	0.01	0.84	0.01	0.85	0.06
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	-	Temp. TFP	,								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	-		$\beta$	0.38	0.1	0.58	0.07	0.41	0.08	0.42	0.10
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Country prem.	$\beta$	0.75	0.1	0.81	0.05	0.83	0.05	0.83	0.04
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Country prem.	β	0.75	0.1	0.80	0.04	0.69	0.05	0.68	0.07
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	*		,								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\sigma_{\omega,t}$			0.1	0.98	0.07	0.84	0.02		0.05
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$spr^{exo}$		0.75	0.1	0.85	0.08	0.00	0.07	0.00	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$efp^{exo}$		0.75	0.1						
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1 - ) / CLO	St.Dev. Shocks	,								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\sigma_v$			0.01	$\infty$	0.11	0.02	0.03	0.02	0.04	0.01
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		Inv.	$\Gamma^{-1}$	0.01				0.30		0.30	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		Temp. TFP	$\Gamma^{-1}$		$\infty$		0.010		0.017	0.01	0.002
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			$\Gamma^{-1}$								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\sigma_{c^1}*10$		$\Gamma^{-1}$								0.001
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\sigma_{c^2}^*10$	° 1	$\Gamma^{-1}$								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\sigma_R^*10$		$\Gamma^{-1}$								0.003
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			$\Gamma^{-1}$		$\infty$			0.02		0.01	0.004
$σ_{\sigma\omega,n}$ Anticipated Risk $\Gamma^{-1}$ 0.01       ∞       0.03       0.00       0.03 $\rho_{\sigma\omega,n}$ Anticipated Risk $\mathcal{N}$ 0       0.5       0.22       0.14       0.31 $\sigma_{uvre}$ *10 $spr^{exo}$ $\Gamma^{-1}$ 0.10       ∞       0.16       0.03       0.02			$\Gamma^{-1}$			0.005		0.04			
$\rho_{\sigma_{\omega,n}}$ Anticipated Risk $\mathcal{N}$ 0 0.5 0.22 0.14 0.31 $\sigma_{\sigma_{w}} *10 \qquad spr^{e_{xo}} \qquad \Gamma^{-1}  0.10  \infty  0.16  0.03 \qquad 0.02$		÷							0.00		
$\sigma_{ave} *10 = spr^{exo} = \Gamma^{-1} = 0.10 = \infty = 0.16 = 0.03 = 0.02$		Anticipated Risk	$\mathcal{N}$								
1 CWO 1	$\sigma_{spr_{ero}}$ *10	$snr^{exo}$	$\Gamma^{-1}$				0.03				
$\sigma_{efp_{exo}}$ *10 $efp^{exo}$ $\Gamma^{-1}$ 0.10 $\infty$		$efp^{exo}$	$\Gamma^{-1}$								

Table 5: Estimated Parameters, Prior and Posterior Mode, Cont.

to require a larger value (greater than one for the BGG alternatives). As we will describe in the next section, the working capital channel plays a somehow different role when both GK and BGG frictions are included, which might help to explain why the estimated value for  $\alpha_L^{WC}$  is lower in the GK+BGG models.

In terms of nominal rigidities, the GK, BGG and BGG+CMR models seems to require a relatively higher degree of price stickiness for home goods. This happens despite the fact that, as previously described, the BGG alternatives greatly exacerbate the volatility of inflation. These differences and those previously described for expenditure-switching-related parameters might help to explain the better adjustment in the second moments for the real exchange rate in the models including both financial frictions. In addition, it is worth mentioning that all models require a relatively high degree of wage stickiness as well.

Finally, the parameters describing the exogenous driving forces also change between models. However, instead of comparing them directly, it is more instructive to see how these different values affect the way each shock explains aggregate fluctuations. To that end, Table 6 displays the variance decomposition obtained with the Base, the GK+BGG and the GK+BGG+CMR models, computed at their respective posterior modes.<sup>36</sup> In the Base model (Panel A), the most important driving force to explain GDP, consumption and net worth seems to be the preference shock v. The shock to the marginal efficiency of investment ( $\varpi$ ) plays the most relevant role in explaining investment and credit growth, while both productivity shocks (z and a) are also relevant to explain the growth rates of output, wages and inflation. In terms of international variables, the world interest rate shock ( $R^*$ ) is quite relevant to account for the volatility of nominal variables and the real exchange rate, and it also explains a no trivial fraction of output and financial quantities. Moreover, the shock to commodity prices ( $p^{Co*}$ ) has a relevant role in explaining the variance of the trade balance and the real exchange rate. The shock to the deviations in UIP ( $\zeta^2$ ) accounts for a non trivial part of output-growth volatility, as well as the real exchange rate and financial quantities. Finally, we can see that the exogenous shock to the spread and the external finance premium are relatively irrelevant in describing the variance of these selected observables.<sup>37</sup>

When both financial frictions are included but without news shocks (Panel B) the relative contribution of each shock is quite different. First, the importance of the preference shock is greatly diminished, being now relevant to explain mostly consumption volatility and, to a lesser extent, the variability of output, the policy rate and inflation. Investment shocks become less relevant to explain output growth, but they are quite important in describing the dynamics of financial variables. The role of both productivity shocks increases in this model, the importance of foreign shocks is similar compared to the base mode, and the relative importance of the monetary policy shock is somehow larger. In terms of financial shocks ( $\mu$  and  $\sigma_{\omega}$ ), they explain an important part of the spread and the external finance premium, but have little impact of other variables

If we also allow for the possibility of news to entrepreneurs' risk shock (Panel C), we see that they play a relatively larger role, not only to account for the variance of financial variables, but also in terms of investment, inflation and the policy rate. Moreover, in line with the results by Cristiano et al (2014), the increased importance of this driving force crowds-out the relevance of shocks to the marginal efficiency of investment, although not completely as can be seen. In particular, these news shocks explain a non trivial fraction of investment, the policy rate and inflation, and they are also an important determinant of financial variables. The presence of these shocks also reduce the relative relevance of productivity shocks.

Overall, this variance decomposition exercise allow us to reach some important conclusions. First, there is no

 $<sup>^{36}</sup>$ We omit some of the shocks because they do not have a significant impact to explain the variance of these variables. The only exception is the shock to the endowment of commodities that explains between 15 and 20% of the variance of real GDP growth but it has a negligible impact on other variables.

 $<sup>^{37}</sup>$ Notice that this does no imply that a either of these shocks produce a zero effect on these variables; but in relative terms all other shocks play a significantly larger role.

							Cet	-			077		
	v	$\varpi$	z	a	$R^*$	$\pi^*$	$p^{Co*}$	R	$\zeta^2$	$efp^{exo}$	$spr^{exo}$	$\mu$	$\sigma_{\omega}$
							Base						
$\Delta GDP$	40	13	9	7	2	4	1	3	10	0	0		
$\Delta C$	76	4	0	4	5	1	7	1	1	0	0		
$\Delta I$	6	89	0	3	0	0	1	0	0	0	0		
TB/GDP	1	14	1	2	11	5	59	0	5	0	0		
$\Delta W$	0	2	39	25	11	3	3	0	16	0	0		
R	6	12	9	2	33	4	14	7	12	0	0		
$\pi$	3	6	34	2	24	4	8	1	17	0	0		
rer	1	3	2	1	24	11	29	2	26	0	0		
$\Delta L$	6	83	1	4	2	0	1	1	2	0	0		
$\Delta N^e$	43	6	10	5	8	1	1	9	12	0	2		
spr	0	0	0	0	0	0	0	0	0	0	100		
efp	0	0	0	0	0	0	0	0	0	100	0		
					]		X + BGG	£					
$\Delta GDP$	16	9	17	8	3	6	0	8	16			0	0
$\Delta C$	69	1	2	4	6	2	9	2	3			0	0
$\Delta I$	0	91	1	2	2	0	1	3	0			1	0
TB/GDP	5	2	1	1	14	9	62	0	6			0	0
$\Delta W$	1	1	43	44	3	1	2	1	3			1	0
R	13	4	12	1	25	5	14	19	7			0	0
$\pi$	9	3	47	6	14	3	6	5	7			1	0
rer	1	1	2	0	23	12	23	4	33			0	0
$\Delta L$	1	74	12	2	2	0	1	4	1			2	0
$\Delta N^e$	1	37	1	3	0	0	0	43	0			11	4
spr	1	29	2	2	1	0	0	7	1			54	2
efp	2	43	0	0	2	1	2	11	1			2	34
					С. С	GK+I	BGG+0	CMR					
$\Delta GDP$	18	7	13	9	3	6	0	5	19			0	1
$\Delta C$	72	1	1	4	6	2	7	1	2			0	3
$\Delta I$	0	66	0	1	1	0	1	1	0			0	29
TB/GDP	6	2	1	1	14	8	57	0	5			0	5
$\Delta W$	3	3	34	44	3	2	2	1	5			1	3
R	16	4	7	1	22	4	13	9	7			0	16
$\pi$	13	4	29	7	14	3	6	3	8			1	12
rer	1	1	1	0	23	12	22	2	34			0	3
$\Delta L$	1	55	8	2	1	0	0	2	1			2	26
$\Delta N^e$	1	17	0	1	0	0	0	11	0			2	67
spr	1	21	1	2	1	0	0	3	1			33	37
efp	0	6	0	0	0	0	Ő	1	0			0	92
- J F	~	~	~	~	~	2	v	*	~			~	

 Table 6: Variance Decomposition

Note: The variance decomposition for each model is computed at their respective posterior mode. In Panel C, the column labeled  $\sigma_{\omega}$  includes the contribution of all risk shocks combined.

single shock that can explain a large fraction of all variables at the same time; a characteristic that will be further emphasize with the impulse response analysis in the next section. Second, with the exception of the investment shock (and to a lower degree the monetary shock as well), the typical macro-related driving forces (both domestic and external) seem to play negligible role in explaining the volatility of financial variables. Finally, financial shocks, particularly news shock to entrepreneurs' risk, are relevant mainly to explain financial variables, although they can help to describe the behavior of some macro variables as well, investment in particular.

# 5 Impulse Responses

The goal of this section is to understand how the presence of financial frictions changes the propagation of the main driving forces in the economy. While we will mainly focus on the comparison between the Base and the GK+BGG models we are also interested in understanding the dynamics under the other two alternatives to see if we can understand what type of dynamics are rejected by the data. To keep the comparison as clean as possible, we will fix the estimated parameters at the posterior mode obtained with the GK+BGG+CMR model, and then compute the responses for the other alternatives by shutting down the relevant channels.

We begin by analyzing the effects generated by a monetary policy shock, displayed in Figure 2. As it is usual in New-Keynesian models of small and open economies, in the Base model a positive shock to the Taylor rule leads to a fall in investment, consumption and GDP, while inflation decreases and the real exchange appreciates. In all models with financial frictions, the effect of the shock on investment gets amplified. As the shock generates a contraction in aggregate demand, financial conditions for entrepreneurs will, ceteris paribus, worsen; leading to a rise in the external finance premium and in the spread. This will further contract investment, activating the financial accelerator. As a result, investment falls much more than in the Base model.

On the contrary, consumption in the BGG and the GK+BGG models decreases by less than in the Base model after this shock. This can be related to the fact that, in equilibrium, the path for the policy rate is less contractionary in these models than in the Base alternative, leading to a smaller contraction on consumption. This does not happen when only the GK friction is considered. As can be seen, in such a model the rise in the spread is much larger than in the others. In turn, due to the working capital loans, this puts an upward pressure on inflation, as the spread is part of the marginal cost faced by firms. As a result, the path for the policy rate is relatively more contractionary in the GK model, which leads to a further drop in consumption compared to other models.

Overall, output decreases by less in the BGG and the GK+BGG models compared to the Base version, indicating that the smaller contraction on consumption and the rise on the trade balance compensates for the larger drop on investment.

An interesting (and a priori counter-intuitive) results is that in the BGG and the GK+BGG model the contractionary policy shock raises loans to finance entrepreneurs, while a simple intuition may lead us to think that loans should fall.<sup>38</sup> However, the key variable for the determination of the external finance premium is not just the loans obtained but entrepreneurs' leverage (which equals the value of capital over net worth), that indeed drops after the contractionary policy shock. In principle, for a given level of net worth, one would expect that the fall in the value of capital generated by the policy shock should lead to a drop in loans received by entrepreneurs. However, net worth does not remain fixed; it decreases after the shock. Thus, the result that we obtain is generated because, for the chosen parametrization, net worth falls by more than the contraction in the

<sup>&</sup>lt;sup>38</sup>Total loans still fall as working-capital loans drop by more than the rise in loans to entrepreneurs.

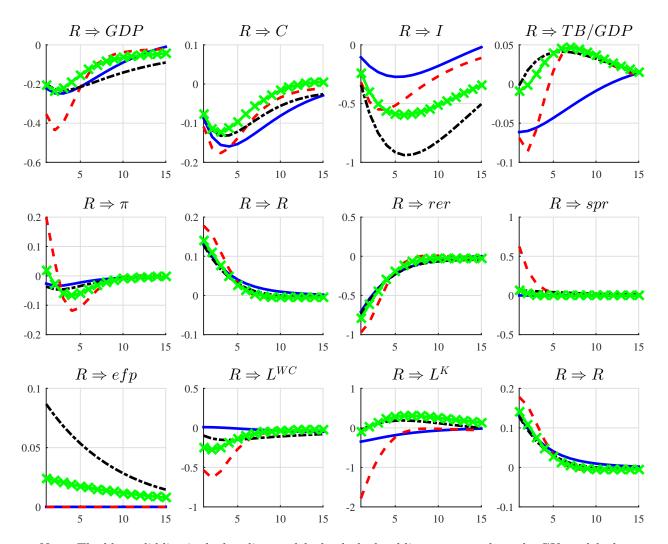


Figure 2: Impulse Responses to a Monetary Policy Shock.

Note: The blue solid line is the baseline model, the dashed red line corresponds to the GK model, the dash-dotted black line BGG, and the crossed-solid green line is the GK-BGG model. The variables are GDP, consumption, investment, the trade-balance-to-output ratio, inflation, the monetary policy rate, the real exchange rate, the spread, the external finance premium, loans to working capital, loans to entrepreneurs, and the variable being shocked. All variables are measured in percentage deviations which respect to the steady state, and all rates and spreads are expressed in a quarterly basis.

value of capital, and therefore loans to entrepreneurs actually rise. In other words, the observed average values for leverage and spreads generate a net worth that is more elastic to changes in financial conditions than the value of assets. We have experimented with alternative calibrations and there are some values for parameters describing the BGG friction that can generate the opposite response for loans, however these alternative parameterizations do not seem to be favored by the data.<sup>39</sup>

Figure 3 shows the responses to a commodity price shock. Qualitatively, this shock generates a positive wealth effect (which is quite large given the estimated persistence for this shock) that raises consumption. In turn, by increasing the demand for domestic goods, the rise in desired consumption raises the marginal product of capital, expanding also investment. This increase in absorption leads to a real appreciation. In the Base model with no financial frictions, inflation experiences a fall, led by a reduction in the domestic price of imported goods after the real appreciation. Consequently, the policy rate drops.

In models with financial frictions, the rise in investment is relatively milder than in the Base model. Here we have the two opposing effects. On one hand, the rise in aggregate demand tends to make investment more attractive, which should tend increase the price of capital, activating the financial accelerator. On the other hand, the real appreciation reduces the marginal product of capital for tradable firms; which in turn worsens the financial position entrepreneurs. Given the estimated parameters, the second effect seems to dominate, as can be seen by the increase in the external finance premium in the BGG and GK+BGG model. Similarly, in the GK model this is manifested as an increase in the spread. In equilibrium the real exchange rate appreciation seems to be more persistent in models with financial frictions, which further emphasize the interaction between financial conditions and the real exchange rate.

The effect on consumption is also milder in models with financial frictions, which is associated to the smaller drop in the policy rate in these models. Overall, the effect on output is also less expansionary than in the Base model.

Next we investigate the responses to a preference shock, which are displayed in Figure 4. In the Base model, the shock generates a rise in desired consumption, increasing the demand for all goods. In equilibrium, this increases production and investment of home goods, as well as prices, and the real exchange rate appreciates. Given this co-movement between these variables, it is easy to see why this shocks is one of the most important driver in the Base model.

However, when considering financial frictions, the propagation of this shock is significantly modified. In particular, the real appreciation combined with financial frictions will *ceteris paribus* worsen the financial situation of entrepreneurs (similar to what we described with the commodity price shock). As can be seen, the response of investment is significantly different than in the Base model, being even negative in the GK+BGG model. This is also an indication of why this shock is much less relevant in models with financial frictions than in the Base model, particularly to capture the dynamics of of investment and financial variables.

Another domestic disturbance that explains a significant fraction of the volatility in the data is the investment shock; the responses shown in Figure 5. In the Base model this shock increases the return on investment, leading to a rise in investment and the stock of capital. This shift in the supply of capital lowers its price. Inflation increases (and so does the monetary policy rate) due to pressure on aggregate demand for final goods from higher investment, which offsets the supply-side effect that the shocks generates.<sup>40</sup> At the same time consumption drops

<sup>&</sup>lt;sup>39</sup>Actually, we have experimented with the BGG setup in many alternative versions of the model (e.g. closed economies, real models, calibrations closer to more developed countries, among others) and realized that this increase in loans arises in many other cases as well.

 $<sup>^{40}</sup>$ By this we mean that the shock should, ceteris paribus, decrease the expected rental rate of capital as the stock of capital gets accumulated. Thus, ceteris paribus, the shock generates a reduction in expected marginal costs.

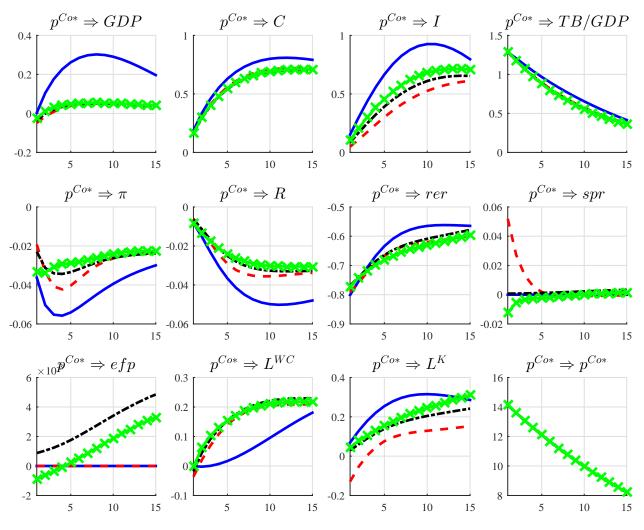


Figure 3: Impulse Responses to a Commodity Price Shock

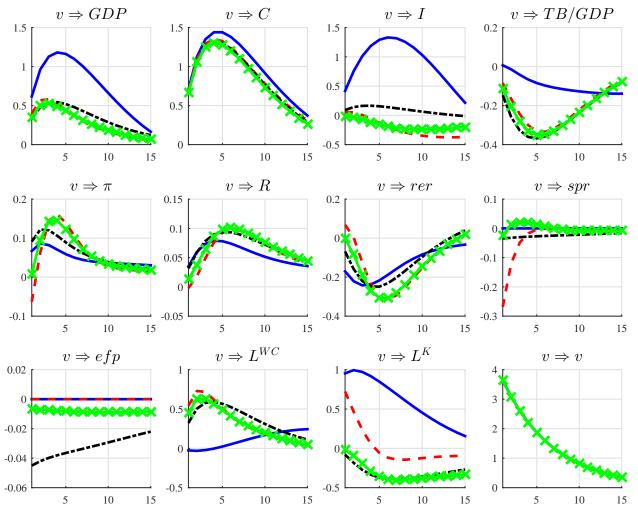


Figure 4: Impulse Responses to a Preference Shock

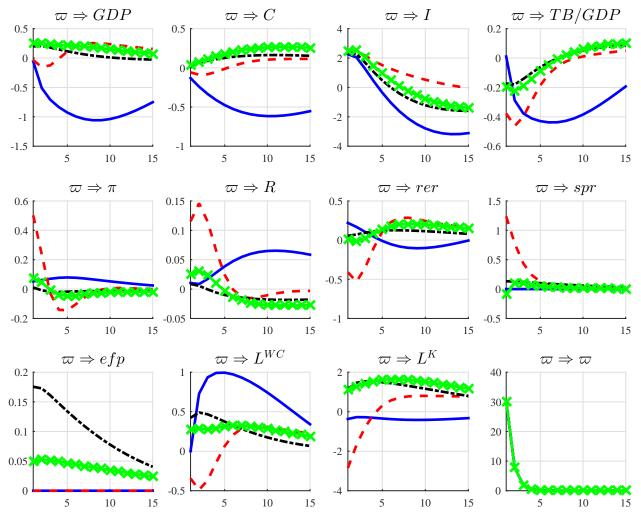


Figure 5: Impulse Responses to a Investment Shock

due to the more contractionary path for the policy rate, which compensates the positive wealth effect that the increases in production of home goods will generate. In fact, this drop in consumption is so large and persistent in the Base model than output actually drops after the shock. With these responses, the co-movement between the main macro variables is not in line with the typical business cycle movements, which explains the somehow limited role for this shock in explaining the volatility of domestic variables in the Base model.

When financial frictions are in place the responses are quite different. When the financial accelerator is present, *ceteris paribus* this shock should generate a larger and more persistent increase in desired investment. As a result, loans to capital accumulation increase. In addition, the larger stock of capital tends to reduce marginal cost, and therefore the effect on inflation is more limited, leading to less contractionary policy rate path. In turn, and given the larger wealth effect originated in this case, consumption increases, leading to an expansion in GDP as well. Therefore, we can see that the after we include financial frictions this shocks has a much better chance to be a relevant driving force, particularly to explain the co-movement between macro and financial variables.

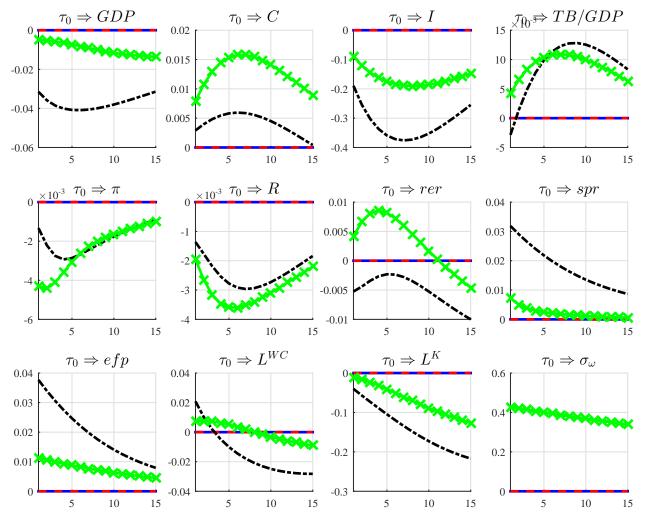
Finally, the other driving force that was particularly important to explain the evolution of financial variables is the shock to entrepreneurs' risk. Figure 6 shows the impulse responses to an unanticipated risk shock. The shock acts as an exogenous increases in the external finance premium, generating a drop in investment. Inflation is affected by two channels that generate opposing effects, similar to what we described for the investment shock. On one hand, the contraction on aggregate demand will tend to reduce inflation. On the other, the rental rate of capital and the lending rate will increase, putting an upward pressure on inflation trough the expected rise in marginal costs. In equilibrium, the first effect seems to dominate. As a response, the monetary policy rate drops, which rises consumption. We can also see that, in line with the analysis of second moments, the BGG model generates a more volatile spread and external finance premium, which explains why this model tends to over estimate the volatility of both macro and financial variables, relative to the GK+BGG.

Finally, Figure 7 displays the responses to a news shock to entrepreneurs' risk. In particular, the figure shows the case in which at period zero people realize about a rise in the risk of entrepreneur four periods in advance. As can be seen, with the exception of the spread and the external finance premium, the shock generates a qualitatively similar response compared to the unanticipated case.

# 6 Conclusions

In this paper we set up and estimate a DSGE model of a small open economy that includes two types of domestic financial frictions: one between domestic depositors and banks, and another between banks and domestic borrowers. We estimate several version of the model using Chilean data from 2001 to 2012. Our main goal was to see if such a model can do a better job in describing the evolution of both macroeconomic and credit-related variables observed in the data, relative to a model without frictions but augmented by ad-hoc financial shocks.

Our results highlight several relevant lessons. First, we find that the presence of financial frictions can help to improve the fit of the model in several relevant dimensions of the data. In particular, the friction between banks and borrowers (the BGG setup) seems to be more relevant than the friction embedded in the GK framework alone; although the combined effect of both frictions yield a better fit to the data in many dimensions of interest. Second, we highlight that the presence of financial frictions significantly alter the propagation of structural shocks, both domestic and foreign disturbances. Moreover, the relative importance of alternative driving forces is significantly modified in the presence of financial frictions. Finally, we have also identified that allowing for the possibility of news shocks to risk significantly improve the goodness of fit of the data.



# Figure 6: Impulse Responses to an unanticipated Risk Shock

Note: See figure 2.

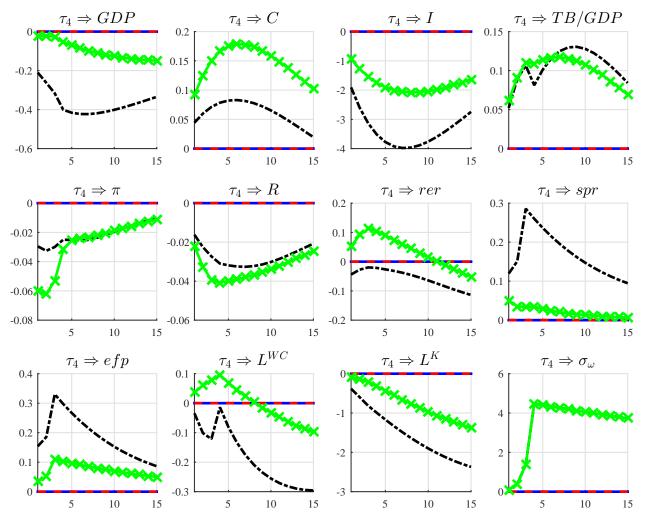


Figure 7: Impulse Responses to an unanticipated Risk Shock

To conclude, there are several aspect of our framework that deserve to be discussed, for they can point to future improvements in the analysis. First, in our model financial frictions are always binding. In contrast, part of the literature has emphasized financial frictions that are only occasionally binding, particularly in the lending relationship between domestic and foreign agents.<sup>41</sup> Assuming that frictions are always binding is convenient from a computational point of view (for it allows to solve the model using perturbation methods),<sup>42</sup> but of course we can be missing important dynamics. For instance, while the EMBI spread for Chile has been relatively small, it still experienced a spike during the 2008 world financial crisis. A similar sudden increase can also be observed in domestic spreads in the same period. This might reflect that financial conditions became suddenly more restrictive than in normal times. Thus it might be of interest to extend our analysis by considering a model in which financial frictions bind occasionally. Nonetheless, while of course this might be relevant from a quantitative point of view, qualitatively the analysis in this paper is still useful to understand the relevant channels that might be part of the propagation of both domestic and foreign shocks.

In addition, given the highlighted relevance of the real exchange rate, it would be of interest to consider a multi-sector model, with tradables and non-tradables. Arguably, some of the channels that we have emphasized arise because all goods are tradable and thus, for instance, a real depreciation may improve the financial position of these firms. But if firms in the non-traded sector are also subject to financial constraints, a real depreciation will deteriorate their financial conditions, making less clear what the final effect would be.

Another relevant issue that we did not tackle in this paper is the relative importance of domestic vis-a-vis foreign financial frictions in propagating external shocks.<sup>43</sup> To perform such a comparison, one would need to set up a model with both types of frictions at the same time. For instance, one could consider that banks obtain funds also from abroad, subject to the same type of frictions that we assume between banks and domestic depositors. In such a setup, movements in the real exchange rate will also alter the banks' balance sheet, leading to an additional amplification channel.

Alternatively, one could consider that firms (at least part of them) obtain funds not only in the domestic market but also abroad (for instance through corporate debt or equity markets). In particular, this can lead to additional relevant dynamics; as emphasized, for instance, by Caballero (2002) to explain how the Asian crisis propagated to the Chilean economy. If large firms can obtain funds both domestically and abroad while smaller firms (particularly in the non-traded sector) have only access to domestic financing, a sudden stop in capital inflows will lead these large firms to turn to the domestic market for financing, crowding-out the credit available for smaller firms. Thus, in such a setup both domestic and foreign financial frictions would be relevant channels to describe the dynamics of the economy.

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<sup>&</sup>lt;sup>41</sup>Some examples are Mendoza (2010), Benigno *et al.* (2013), and Bianchi (2011).

 $<sup>^{42}</sup>$ Linearization not only allows to estimate the model with a likelihood approach more easily, but it also allows to consider many other potentially relevant features of the economy in the model. Global solution methods, required to solve models with occasionally binding constraints, generally require to limit significantly the size of the model (for instance, it would be quite costly to compute the model of Mendoza (2010) assuming also sticky prices and wages, as well as indexation).

 $<sup>^{43}</sup>$ Many studies simply assume firms only borrow from abroad, for instance Cespedes *et al.* (2004), Devereux *et al.* (2006), Gertler *et al.* (2007), among others.

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# A Model Appendix

## A.1 Intermediary Objective

This section shows that the objective of financial intermediaries, given by

$$V_t = E_t \sum_{s=0}^{\infty} (1-\omega)\omega^s \beta^{s+1} \Xi_{t,t+s+1} \left[ (r_{t+1+s}^{L,WC} - r_{t+1+s}) L_{t+s}^{WC} + (r_{t+1+s}^{L,K} - r_{t+1+s}) L_{t+s}^{K} + r_{t+1+s} N_{t+s} \right]$$

can be expressed as

$$V_t = \varrho_t^{L,WC} L_t^{WC} + \varrho_t^{L,K} L_t^K + \varrho_t^N N_t.$$

First, notice that

$$V_t = E_t \sum_{s=0}^{\infty} (1-\omega)\omega^s \beta^{s+1} \Xi_{t,t+s+1} \left[ (r_{t+1+s}^{L,WC} - r_{t+1+s}) \frac{L_{t+s}^{WC}}{L_t^{WC}} L_t^{WC} + (r_{t+1+s}^{L,K} - r_{t+1+s}) \frac{L_{t+s}^K}{L_t^K} L_t^K + r_{t+1+s} \frac{N_{t+s}}{N_t} N_t \right].$$

Thus,

$$\varrho_t^{L,WC} \equiv E_t \sum_{s=0}^{\infty} (1-\omega) \omega^s \beta^{s+1} \Xi_{t,t+s+1} (r_{t+1+s}^{L,WC} - r_{t+1+s}) \frac{L_{t+s}^{WC}}{L_t^{WC}},$$
$$\varrho_t^{L,K} \equiv E_t \sum_{s=0}^{\infty} (1-\omega) \omega^s \beta^{s+1} \Xi_{t,t+s+1} (r_{t+1+s}^{L,K} - r_{t+1+s}) \frac{L_{t+s}^K}{L_t^K},$$

and

$$\varrho_t^N \equiv E_t \sum_{s=0}^{\infty} (1-\omega) \omega^s \beta^{s+1} \Xi_{t,t+s+1} r_{t+1+s} \frac{N_{t+s}}{N_t}.$$

In terms of  $\varrho_t^N$ ,

$$\varrho_t^N = E_t \left\{ (1-\omega)\beta \Xi_{t,t+1} r_{t+1} + \sum_{s=1}^{\infty} (1-\omega)\omega^s \beta^{s+1} \Xi_{t,t+s+1} r_{t+1+s} \frac{N_{t+s}}{N_t} \right\},\,$$

or,

$$\varrho_t^N = E_t \left\{ (1-\omega)\beta \Xi_{t,t+1} r_{t+1} + \beta \omega \Xi_{t,t+1} \frac{N_{t+1}}{N_t} \sum_{s=1}^\infty (1-\omega)\omega^{s-1}\beta^s \Xi_{t+1,t+s+1} r_{t+1+s} \frac{N_{t+s}}{N_{t+1}} \right\}.$$

Finally, changing the index in the sum by j = s - 1, we obtain

$$\varrho_t^N = E_t \left\{ (1-\omega)\beta \Xi_{t,t+1} r_{t+1} + \beta \omega \Xi_{t,t+1} \frac{N_{t+1}}{N_t} \sum_{j=0}^\infty (1-\omega)\omega^j \beta^{j+1} \Xi_{t+1,t+1+j+1} r_{t+1+j+1} \frac{N_{t+j+1}}{N_{t+1}} \right\},$$

or, using the definition of  $\varrho_t^N$  evaluated at t+1,

$$\varrho_t^N = E_t \left\{ (1-\omega)\beta \Xi_{t,t+1} r_{t+1} + \beta \omega \Xi_{t,t+1} \frac{N_{t+1}}{N_t} \varrho_{t+1}^N \right\} = \beta E_t \left\{ \Xi_{t,t+1} \left[ (1-\omega)r_{t+1} + \omega \frac{N_{t+1}}{N_t} \varrho_{t+1}^N \right] \right\}.$$

With an analogous procedure we can obtain the expression for  $\varrho_t^{L,WC}$  and  $\varrho_t^{L,K}$ .

#### A.2 Entrepreneurs' Optimization Problem

Using the definition for  $lev_t^e$  and (13), the Lagrangian for the optimal-contract problem can be written as,

$$E_t \left\{ lev_t^e \frac{[r_{t+1}^K + (1-\delta)q_{t+1}]}{q_t} h(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) + \eta_{t+1} \left[ g(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) \frac{[r_{t+1}^K + (1-\delta)q_{t+1}]}{q_t} lev_t^e - (lev_t^e - 1)r_{t+1}^{L,K} \right] \right\},$$

where  $\eta_{t+1}$  is the Lagrange multiplier. The choice variables are  $lev_t^e$  and a state-contingent  $\bar{\omega}_{t+1}^e$ . The first order conditions are the constraint holding with equality and

$$E_t \left\{ \frac{[r_{t+1}^K + (1-\delta)q_{t+1}]}{q_t} h(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) + \eta_{t+1} \left[ g(\bar{\omega}_{t+1}^e; \sigma_{\omega,t}) \frac{[r_{t+1}^K + (1-\delta)q_{t+1}]}{q_t} - r_{t+1}^{L,K} \right] \right\} = 0,$$
  
$$h'(\bar{\omega}_{t+1}^e) + \eta_{t+1}g'(\bar{\omega}_{t+1}^e) = 0.$$

Combining these to eliminate  $\eta_{t+1}$  and rearranging we obtain (15) in the text.

Finally, we need a functional form for  $F(\omega_t^e; \sigma_{\omega,t-1})$ . We follow BGG and assume that  $\ln(\omega_t^e) \sim N(-.5\sigma_{\omega,t-1}^2, \sigma_{\omega,t-1}^2)$  (so that  $E(\omega_t^e) = 1$ ). Under this assumption, we can define

$$aux_t^1 \equiv \frac{ln(\bar{\omega}_t^e) + .5\sigma_{\omega,t-1}^2}{\sigma_{\omega,t-1}},$$

and, letting  $\Phi(\cdot)$  be the standard normal c.d.f. and  $\phi(\cdot)$  its p.d.f., we can write,<sup>44</sup>

$$\begin{split} g(\bar{\omega}_{t}^{e};\sigma_{\omega,t-1}) &= \bar{\omega}_{t}[1-\Phi(aux_{t}^{1})] + (1-\mu^{e})\Phi(aux_{t}^{1}-\sigma_{\omega,t-1}), \\ g'(\bar{\omega}_{t}^{e};\sigma_{\omega,t-1}) &= [1-\Phi(aux_{t}^{1})] - \bar{\omega}_{t}^{e}\phi(aux_{t}^{1})\frac{1}{\sigma_{\omega,t-1}}\frac{1}{\bar{\omega}_{t}^{e}} + (1-\mu^{e})\phi(aux_{t}^{1}-\sigma_{\omega,t-1})\frac{1}{\sigma_{\omega,t-1}}\frac{1}{\bar{\omega}_{t}^{e}} \\ &= [1-\Phi(aux_{t}^{1})] - \mu^{e}\phi(aux_{t}^{1}), \\ h(\bar{\omega}_{t}^{e};\sigma_{\omega,t-1}) &= 1-\Phi(aux_{t}^{1}-\sigma_{\omega,t-1}) - \bar{\omega}_{t}^{e}[1-\Phi(aux_{t}^{1})], \\ h'(\bar{\omega}_{t}^{e};\sigma_{\omega,t-1}) &= -\phi(aux_{t}^{1}-\sigma_{\omega,t-1})\frac{1}{\sigma_{\omega,t-1}}\frac{1}{\bar{\omega}_{t}^{e}} - [1-\Phi(aux_{t}^{1})] + \bar{\omega}_{t}^{e}\phi(aux_{t}^{1})\frac{1}{\sigma_{\omega,t-1}}\frac{1}{\bar{\omega}_{t}^{e}} \\ &= -[1-\Phi(aux_{t}^{1})]. \end{split}$$

Finally, a technical note is in order. As we have stated the model, it turns out that whether the participation constraint (11 in the text) holds state-by-state or in expectations (as in, for instance, Rannenberg, 2013) is (up to first order) irrelevant for the characterization of the optimal contract (in equilibrium it will hold without expectations anyway, as in Rannenberg, 2013). What is key to allow to merge the BGG model within the Gertler and Karadi framework is the assumption that the loan rate  $r_t^{L,e}$  is not contingent on the aggregate state, and if this is not the case the equilibrium is indeterminate. The intuition for this result is as follows. In the original BGG model, if the participation constraint for the lender holds state-by-state, the nature of  $r_t^{L,e}$  is irrelevant. This is so because, as the required return  $r_{t+1}^{L,K}$  is determined elsewhere, the participation constraint pins down the current value of  $\bar{\omega}_{t+1}^e$  and then the other optimality condition of the optimal contract ((15) in the text) pins down the external finance premium (in fact, given that such a setup is the usual way the BGG model is implemented, an equation like (11) is generally omitted as an equilibrium condition). However, if in the original BGG model the participation constraint for the lender holds in expectations, we do require  $r_t^{L,e}$  to be non-contingent. In such

<sup>&</sup>lt;sup>44</sup>See, for instance, the appendix of Devereux *et al.* (2006).

a case, it is precisely equation (11) that pins down  $\bar{\omega}_{t+1}^e$ , while the participation constraint alone just determines (up to first order)  $E_t\{\bar{\omega}_{t+1}^e\}$ .

In our setup the reason why we need  $r_t^{L,e}$  to be non-contingent is because  $r_{t+1}^{L,K}$  is not determined by any other equilibrium condition (the intermediary's problem just pins down  $E_t\{r_{t+1}^{L,K}\}$ ). Thus, in our framework, equation (11) pins down  $\bar{\omega}_{t+1}^e$  and, given that value, (12) determines  $r_{t+1}^{L,K}$ . Under the other alternative the equilibrium is indeterminate because only equation (12) displays both  $r_{t+1}^{L,K}$  and  $\bar{\omega}_{t+1}^e$ , and there is no other equation that determines one of these.

#### A.3 Equilibrium Conditions

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock  $A_t$ . We need to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by  $A_{t-1}$  (e.g.  $c_t \equiv \frac{C_t}{A_{t-1}}$ ). The only exception is the Lagrange multiplier  $\Lambda_t$  that is multiplied by  $A_{t-1}$  (i.e.  $\lambda_t \equiv \Lambda_t A_{t-1}$ ), for it decreases along the balanced growth path.

The rational expectations equilibrium of the stationary version of the model model is the set of sequences

$$\{\lambda_{t}, c_{t}, h_{t}, h_{t}^{d}, w_{t}, \tilde{w}_{t}, mc_{t}^{W}, f_{t}^{W}, \Delta_{t}^{W}, i_{t}, k_{t}, r_{t}^{K}, q_{t}, y_{t}, y_{t}^{F}, y_{t}^{H}, x_{t}^{C,F}, x_{t}^{C,H}, x_{t}^{I,F}, x_{t}^{I,H}, x_{t}^{I,F}, x_{t}^{I,H}, x_{t}^{H,*}, R_{t}, \\ \xi_{t}, \pi_{t}, \pi_{t}^{S}, rer_{t}, p_{t}^{H}, \tilde{p}_{t}^{H}, p_{t}^{F}, \tilde{p}_{t}^{F}, p_{t}^{Y}, p_{d}^{G}, p_{t}^{I}, mc_{t}^{H}, f_{t}^{H}, \Delta_{t}^{H}, mc_{t}^{F}, f_{t}^{F}, \Delta_{t}^{F}, b_{t}^{*}, m_{t}, tb_{t}, u_{t}, l_{t}, l_{t}^{WC}, l_{t}^{WC}, d_{t}, n_{t}, \\ \varrho_{t}^{L}, \varrho_{t}^{N}, lev_{t}, r_{t}^{L,K}, r_{t}^{L,e}, r_{t}^{L,WC}, R_{t}^{L,e}, R_{t}^{L,WC}, spr_{t}, \bar{\omega}_{t}, n_{t}^{e}, rp_{t}, lev_{t}^{e}\}_{t=0}^{\infty},$$

(64 variables) such that for given initial values and exogenous sequences

$$\{v_t, \varpi_t, z_t, a_t, \zeta_t, \varepsilon_t^R, R_t^*, \pi_t^*, p_t^{Co*}, y_t^{Co}, y_t^*, g_t, \mu_t, \sigma_{\omega, t}\}_{t=0}^{\infty},$$

the following conditions are satisfied. Households:

$$\lambda_t = \left(c_t - \varsigma \frac{c_{t-1}}{a_{t-1}}\right)^{-1} - \beta \varsigma E_t \left\{ \frac{v_{t+1}}{v_t} \left(c_{t+1}a_t - \varsigma c_t\right)^{-1} \right\},\tag{E.1}$$

$$w_t m c_t^W = \kappa \frac{h_t^{\phi}}{\lambda_t},\tag{E.2}$$

$$\lambda_t = \frac{\beta}{a_t} R_t E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\pi_{t+1}} \right\},\tag{E.3}$$

$$\lambda_t = \frac{\beta}{a_t} R_t^* \xi_t E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\pi_{t+1}^S \lambda_{t+1}}{\pi_{t+1}} \right\},\tag{E.4}$$

$$f_t^W = mc_t^W \tilde{w}_t^{-\epsilon_W} h_t^d + \beta \theta_W E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_t^{\vartheta_W} \pi^{1-\vartheta_W}}{\pi_{t+1}} \right)^{-\epsilon_W} \left( \frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\epsilon_W} \left( \frac{w_t}{w_{t+1}} \right)^{-1-\epsilon_W} f_{t+1}^W \right\}, \quad (E.5)$$

$$f_t^W = \tilde{w}_t^{1-\epsilon_W} h_t^d \left(\frac{\epsilon_W - 1}{\epsilon_W}\right) + \beta \theta_W E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\vartheta_W} \pi^{1-\vartheta_W}}{\pi_{t+1}}\right)^{1-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{1-\epsilon_W} \left(\frac{w_t}{w_{t+1}}\right)^{-\epsilon_W} f_t^W \right\}, \quad (E.6)$$

$$1 = (1 - \theta_W)\tilde{w}_t^{1 - \epsilon_W} + \theta_W \left(\frac{w_{t-1}}{w_t} \frac{\pi_{t-1}^{\vartheta_W} \pi^{1 - \vartheta_W}}{\pi_t}\right)^{1 - \epsilon_W},$$
(E.7)

$$\Delta_t^W = (1 - \theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_W \left(\frac{w_{t-1}}{w_t} \frac{\pi_{t-1}^{\vartheta_W} \pi^{1-\vartheta_W}}{\pi_t}\right)^{-\epsilon_W} \Delta_{t-1}^W, \tag{E.8}$$

$$h_t = h_t^d \Delta_t^W. \tag{E.9}$$

Composite final goods:

$$y_t^C = \left[ (1 - o_C)^{\frac{1}{\eta_C}} (x_t^{C,H})^{\frac{\eta_C - 1}{\eta_C}} + o_C^{\frac{1}{\eta_C}} (x_t^{C,F})^{\frac{\eta_C - 1}{\eta_C}} \right]^{\frac{\eta_C}{\eta_C - 1}},$$
(E.10)

$$i_t = \left[ (1 - o_I)^{\frac{1}{\eta_I}} (x_t^{I,H})^{\frac{\eta_I - 1}{\eta_I}} + o_I^{\frac{1}{\eta_I}} (x_t^{I,F})^{\frac{\eta_I - 1}{\eta_I}} \right]^{\frac{\eta_I}{\eta_I - 1}},$$
(E.11)

$$g_t = \left[ (1 - o_G)^{\frac{1}{\eta_G}} (x_t^{G,H})^{\frac{\eta_G - 1}{\eta_G}} + o_G^{\frac{1}{\eta_G}} (x_t^{G,F})^{\frac{\eta_G - 1}{\eta_G}} \right]^{\frac{\eta_G}{\eta_G - 1}},$$
(E.12)

$$x_t^{C,H} = (1 - o_C)(p_t^H)^{-\eta_C} y_t^C,$$
(E.13)

$$x_t^{C,F} = o_C(p_t^F)^{-\eta_C} y_t^C,$$
(E.14)

$$x_t^{I,H} = (1 - o_I) \left(\frac{p_t^H}{p_t^I}\right)^{-\eta_I} i_t,$$
(E.15)

$$x_t^{I,F} = o_I \left(\frac{p_t^F}{p_t^I}\right)^{-\eta_I} i_t, \qquad (E.16)$$

$$x_t^{G,H} = (1 - o_G) \left(\frac{p_t^H}{p_t^G}\right)^{-\eta_G} g_t, \qquad (E.17)$$

$$x_t^{G,F} = o_G \left(\frac{p_t^F}{p_t^G}\right)^{-\eta_G} g_t.$$
(E.18)

Home goods:

$$mc_t^H = \frac{1}{\alpha^{\alpha} \left(1 - \alpha\right)^{1 - \alpha}} \frac{(r_t^K)^{\alpha} w_t^{1 - \alpha} [1 + \alpha_L^{WC} (R_t^{L, WC} - 1)]}{p_t^H z_t a_t^{1 - \alpha}},$$
(E.19)

$$\frac{u_t k_{t-1}}{h_t^d} = a_{t-1} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^K},\tag{E.20}$$

$$l_t^{WC} = \alpha_L^{WC} \left( w_t h_t^d + r_t^K u_t \frac{k_{t-1}}{a_{t-1}} \right),$$
(E.21)

$$f_t^H = \left(\tilde{p}_t^H\right)^{-\epsilon_H} y_t^H m c_t^H + \beta \theta_H E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_t^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_{t+1}} \right)^{-\epsilon_H} \left( \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left( \frac{p_t^H}{p_{t+1}^H} \right)^{-1-\epsilon_H} f_{t+1}^H \right\}, \quad (E.22)$$

$$f_t^H = \left(\tilde{p}_t^H\right)^{1-\epsilon_H} y_t^H \left(\frac{\epsilon_H - 1}{\epsilon_H}\right) + \beta \theta_H E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_{t+1}}\right)^{1-\epsilon_H} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H}\right)^{-\epsilon_H} f_{t+1}^H \right\},$$
 (E.23)

$$y_t^H \Delta_t^H = z_t \left(\frac{u_t k_{t-1}}{a_{t-1}}\right)^{\alpha} (a_t h_t^d)^{1-\alpha},$$
(E.24)

$$1 = \theta_H \left( \frac{p_{t-1}^H}{p_t^H} \frac{\pi_{t-1}^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_t} \right)^{1-\epsilon_H} + (1-\theta_H) (\tilde{p}_t^H)^{1-\epsilon_H},$$
(E.25)

$$\Delta_t^H = (1 - \theta_H)(\tilde{p}_t^H)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{\pi_{t-1}^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_t}\right)^{-\epsilon_H} \Delta_{t-1}^H.$$
(E.26)

Capital accumulation:

$$k_t = (1 - \delta) \frac{k_{t-1}}{a_{t-1}} + \left[ 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right)^2 \right] \varpi_t i_t,$$
(E.27)

$$\frac{p_t^I}{q_t} = \left[ 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right)^2 - \gamma \left( \frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right) \frac{i_t}{i_{t-1}} a_{t-1} \right] \varpi_t \\
+ \frac{\beta}{a_t} \gamma E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t} \left( \frac{i_{t+1}}{i_t} a_t - \bar{a} \right) \left( \frac{i_{t+1}}{i_t} a_t \right)^2 \varpi_{t+1} \right\}.$$
(E.28)

Imported goods:

$$mc_t^F = rer_t / p_t^F, (E.29)$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} y_t^F m c_t^F + \beta \theta_F E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_t^{\vartheta_F} \pi^{1-\vartheta_F}}{\pi_{t+1}} \right)^{-\epsilon_F} \left( \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left( \frac{p_t^F}{p_{t+1}^F} \right)^{-1-\epsilon_F} f_{t+1}^F \right\}, \quad (E.30)$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{1-\epsilon_F} y_t^F \left(\frac{\epsilon_F - 1}{\epsilon_F}\right) + \beta \theta_F E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\vartheta_F} \pi^{1-\vartheta_F}}{\pi_{t+1}}\right)^{1-\epsilon_F} \left(\frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F}\right)^{1-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F}\right)^{-\epsilon_F} f_{t+1}^F \right\}, \quad (E.31)$$

$$1 = \theta_F \left( \frac{p_{t-1}^F}{p_t^F} \frac{\pi_{t-1}^{\vartheta_F} \pi^{1-\vartheta_F}}{\pi_t} \right)^{1-\epsilon_F} + (1-\theta_F) (\tilde{p}_t^F)^{1-\epsilon_F}.$$
 (E.32)

$$m_t = y_t^F \Delta_t^F, \tag{E.33}$$

$$\Delta_t^F = (1 - \theta_F)(\tilde{p}_t^F)^{-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{\pi_{t-1}^{\vartheta_F} \pi^{1-\vartheta_F}}{\pi_t}\right)^{-\epsilon_F} \Delta_{t-1}^F.$$
(E.34)

Entrepreneurs:

$$r_t^K = r^K \exp[\phi_u(u_t - 1)],$$
 (E.35)

$$l_{t-1}^{K} r_{t}^{L,K} = g(\bar{\omega}_{t}; \sigma_{\omega,t-1}) [r_{t}^{K} u_{t} - \phi(u_{t}) + (1-\delta)q_{t}] k_{t-1},$$
(E.36)

$$E_{t}\left\{\frac{\left[r_{t+1}^{K}u_{t+1}-\phi(u_{t+1})+(1-\delta)q_{t+1}\right]}{q_{t}}\left[\frac{h'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})g(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}{g'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}-h(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})\right]\right\}=E_{t}\left\{r_{t+1}^{L,K}\frac{h'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}{g'(\bar{\omega}_{t+1}^{e};\sigma_{\omega,t})}\right\},\quad (E.37)$$

$$r_{t-1}^{L,e} = \bar{\omega}_t [r_t^K u_t - \phi(u_t) + (1-\delta)q_t] \frac{k_{t-1}}{l_{t-1}^K},$$
(E.38)

$$n_t^e = \frac{\upsilon}{a_{t-1}} \left\{ [r_t^K u_t - \phi(u_t) + (1-\delta)q_t] k_{t-1} h(\bar{\omega}_t^e; \sigma_{\omega,t-1}) \right\} + \iota^e n^e,$$
(E.39)

$$l_t^K = q_t k_t - n_t^e, \tag{E.40}$$

$$rp_{t} = \frac{E_{t} \left\{ \frac{[r_{t+1}^{K}u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}]}{q_{t}} \right\}}{E_{t} \left\{ r_{t+1}^{L,K} \right\}},$$
(E.41)

$$lev_t^e = \frac{q_t k_t}{n_t^e},\tag{E.42}$$

$$r_{t-1}^{L,e} = \frac{R_{t-1}^{L,e}}{\pi_t}.$$
(E.43)

Banks:

$$\varrho_t^L = \frac{\beta}{a_t} E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left[ (1-\omega)(r_{t+1}^{L,WC} - r_{t+1}) + \omega \frac{l_{t+1}^{WC}}{l_t^{WC}} a_t \varrho_{t+1}^L \right] \right\},\tag{E.44}$$

$$\varrho_t^L = \frac{\beta}{a_t} E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left[ (1-\omega)(r_{t+1}^{L,K} - r_{t+1}) + \omega \frac{l_{t+1}^K}{l_t^K} a_t \varrho_{t+1}^L \right] \right\},\tag{E.45}$$

$$\varrho_t^N = \frac{\beta}{a_t} E_t \left\{ \frac{v_{t+1}}{v_t} \frac{\lambda_{t+1}}{\lambda_t} \left[ (1-\omega)r_{t+1} + \omega \frac{n_{t+1}}{n_t} a_t \varrho_{t+1}^N \right] \right\},\tag{E.46}$$

$$lev_t = \frac{\varrho_t^N}{\mu_t - \varrho_t^L},\tag{E.47}$$

$$l_t = lev_t n_t, \tag{E.48}$$

$$l_t = l_t^K + l_t^{WC}, (E.49)$$

$$d_t = l_t - n_t, \tag{E.50}$$

$$n_t = \frac{\omega}{a_{t-1}} \left[ (r_t^{L,WC} - r_t) l_{t-1}^{WC} + (r_t^{L,K} - r_t) l_{t-1}^K + r_t n_{t-1} \right] + \iota n,$$
(E.51)

$$spr_t = \frac{(R_t^{L,WC} l_t^{WC} + R_t^{L,e} l_t^K)}{l_t} \frac{1}{R_t},$$
 (E.52)

$$r_t^{L,WC} = \frac{R_{t-1}^{L,WC}}{\pi_t}.$$
(E.53)

Rest of the world:

$$x_t^{H*} = o^* \left(\frac{p_t^H}{rer_t}\right)^{-\eta^*} y_t^*,$$
(E.54)

$$\xi_t = \bar{\xi} \exp\left[-\psi \frac{rer_t b_t^* - rer \times \bar{b}^*}{rer \times \bar{b}^*} + \frac{\zeta_t - \zeta}{\zeta}\right].$$
(E.55)

$$\xi_t = \bar{\xi} \exp\left[-\psi \frac{rer_t b_t^* - rer \times b^*}{rer \times b^*} - \psi_2 \frac{E_t \pi_{t+1}^S \pi_t^S - (\pi^S)^2}{(\pi^S)^2} + \frac{\zeta_t - \zeta}{\zeta}\right],\tag{E.56}$$

Monetary policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[ \left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{y_t a_{t-1}}{y_{t-1}\bar{a}}\right)^{\alpha_y} \right]^{1-\rho_R} \exp(\varepsilon_t^R).$$
(E.57)

Market clearing and definitions:

$$y_t^H = x_t^{C,H} + x_t^{G,H} + x_t^{G,H} + x_t^{H*},$$
(E.58)

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t},\tag{E.59}$$

$$y_t = c_t + i_t + g_t + x_t^{H*} + y_t^{Co} - m_t,$$
(E.60)

$$tb_t = p_t^H x_t^{H*} + rer_t p_t^{Co*} y_t^{Co} - rer_t m_t,$$
(E.61)

$$rer_t b_t^* = rer_t \frac{b_{t-1}^*}{a_{t-1}\pi_t^*} R_{t-1}^* \xi_{t-1} + tb_t - (1-\chi)rer_t p_t^{Co*} y_t^{Co},$$
(E.62)

$$p_t^Y y_t = c_t + p_t^I i_t + p_t^G g_t + tb_t.$$
(E.63)

$$y_t^F = x_t^{C,F} + x_t^{G,F} + x_t^{G,F}, (E.64)$$

$$y_t^C = c_t + \frac{r^K}{\phi_u} \{ \exp[\phi_u(u_t - 1)] - 1 \} \frac{k_{t-1}}{a_{t-1}} + \mu^e [r_t^K u_t - \phi(u_t) + (1 - \delta)q_t] \frac{k_{t-1}}{a_{t-1}} \Phi(aux_t^1 - \sigma_{\omega, t-1}).$$
(E.65)

The exogenous processes are

$$\log\left(x_t/\bar{x}\right) = \rho_x \log\left(x_{t-1}/\bar{x}\right) + \varepsilon_t^x, \qquad \rho_x \in [0,1), \qquad \bar{x} > 0,$$

for  $x = \{v, \varpi, z, a, \zeta, R^*, \pi^*, p^{Co*}, y^{Co}, y^*, g, \mu, \sigma_\omega\}$ , where the  $\varepsilon_t^x$  are n.i.d. shocks (including also  $\varepsilon_t^R$ ).

### A.4 Steady State

We show how to compute the steady state for given values of R, h,  $p^H$ ,  $s^{tb} = tb/(p^Y y)$ ,  $s^g = p^G g/(p^Y y)$ ,  $s^{Co} = rer \times p^{Co*} y^{Co}/(p^Y y)$ ,  $\Gamma \equiv r^{L,e}/r$ , lev,  $\iota$ , efp, v,  $lev^e$  and  $\mu^e$ . The parameters  $\beta$ ,  $\bar{\pi}^*$ ,  $\kappa$ ,  $o^*$ ,  $\bar{g}$ ,  $\bar{y}^{Co}$ ,  $\bar{\mu}$ ,  $\omega$ ,  $\bar{\sigma}_{\omega}$  and  $\iota^e$  are determined endogenously while the values of the remaining parameters are taken as given.

From the exogenous processes for  $v_t$ ,  $u_t$ ,  $z_t$ ,  $a_t$ ,  $y_t^{Co}$ ,  $R_t^*$ ,  $y_t^*$  and  $p_t^{Co*}$ ,

$$v = \bar{v}, \ u = \bar{u}, \ z = \bar{z}, \ a = \bar{a}, \ y^{Co} = \bar{y}^{Co}, \ \zeta = \bar{\zeta}, \ R^* = \bar{R}^*, \ y^* = \bar{y}^*, \ p^{Co*} = \bar{p}^{Co*},$$

 $\xi = \overline{\xi}.$ 

From (E.56),

From (E.57),

$$\pi = \bar{\pi}.$$

From (E.3),

$$\beta = a\pi/R.$$

From (E.35),

$$u = 1,$$

which implies that monitoring costs  $\phi(u)$  are zero in steady state. From (E.4),

$$\pi^S = a\pi/(\beta R^*\xi).$$

From (E.59) and the exogenous process for  $\pi_t^*$ ,

$$\pi^* = \bar{\pi}^* = \pi/\pi^S.$$

Also, from (E.36), (E.37), (E.41) and (E.42),

$$efp\left[h'(\bar{\omega}^e;\sigma_{\omega})g(\bar{\omega}^e;\sigma_{\omega}) - h(\bar{\omega}^e;\sigma_{\omega})g'(\bar{\omega}^e;\sigma_{\omega})\right] = h'(\bar{\omega}^e;\sigma_{\omega}),$$
$$\frac{lev^e - 1}{lev^e} = g(\bar{\omega}^e;\sigma_{\omega})efp.$$

These two equation can be solved numerically to obtain  $\bar{\omega}^e$  and  $\sigma_{\omega}$ . Then, from the definition of  $\Gamma$ ,

$$R^{L,e} = \Gamma R,$$

and combining (E.36) and (E.38),

$$R^{L,K} = \frac{R^{L,e}}{efp} \frac{lev^e - 1}{lev^e \bar{\omega}^e}, \quad r^{L,K} = R^{L,K} / \pi.$$

Thus, from (E.44) and (E.45)

$$R^{L,WC} = R^{L,K}, \ r^{L,WC} = r^{L,K}.$$

From (E.10)-(E.18),

$$p^{F} = \left[\frac{1}{o_{C}} - \frac{1 - o_{C}}{o_{C}}(p^{H})^{1 - \eta_{C}}\right]^{\frac{1}{1 - \eta_{I}}},$$
$$p^{I} = \left[(1 - o_{I})(p^{H})^{1 - \eta_{I}} + o_{I}(p^{F})^{1 - \eta_{I}}\right]^{\frac{1}{1 - \eta_{I}}},$$
$$p^{G} = \left[(1 - o_{G})(p^{H})^{1 - \eta_{G}} + o_{G}(p^{F})^{1 - \eta_{G}}\right]^{\frac{1}{1 - \eta_{G}}}.$$

From (E.25), (E.32) and (E.7),

$$\tilde{p}^H = 1, \ \tilde{p}^F = 1, \ \tilde{w} = 1.$$

From (E.26), (E.34) and (E.8),

$$\Delta^H = (\tilde{p}^H)^{-\epsilon_H}, \ \Delta^F = (\tilde{p}^H)^{-\epsilon_F}, \ \Delta^W = \tilde{w}^{-\epsilon_W}.$$

From (E.22)-(E.23), (E.30)-(E.31) and (E.5)-(E.6),

$$mc^{H} = \frac{\epsilon_{H} - 1}{\epsilon_{H}} \tilde{p}^{H}, \ mc^{F} = \frac{\epsilon_{F} - 1}{\epsilon_{F}} \tilde{p}^{F}, \ mc^{W} = \left(\frac{\epsilon_{W} - 1}{\epsilon_{W}}\right) \tilde{w}.$$

From (E.9),

$$h^d = h/\Delta^W.$$

From (E.28),

$$q = \frac{p^I}{\varpi}.$$

From (E.41),

$$r^{K} = q \left[ efp \times r^{L,K} - (1-\delta)q \right].$$

From (E.19),

$$w = \left\{ \frac{\alpha^{\alpha} \left(1 - \alpha\right)^{1 - \alpha} p^{H} m c^{H} z a^{1 - \alpha}}{(r^{K})^{\alpha} [1 + \alpha_{L}^{WC} (R^{L, WC} - 1)]} \right\}^{\frac{1}{1 - \alpha}}.$$

From (E.5),

$$f^W = \tilde{w}^{-\epsilon_W} h^d m c^W / (1 - \beta \theta_W).$$

From (E.20),

$$k = \frac{\alpha a w h^d}{\left(1 - \alpha\right) r^K}.$$

From (E.24),

$$y^H = z \left( k/a \right)^{\alpha} (ah^d)^{1-\alpha} / \Delta^H.$$

From (E.22),

$$f^H = mc^H (\tilde{p}^H)^{-\epsilon_H} y^H / (1 - \beta \theta_H).$$

From (E.27),

$$i = k \left( \frac{1 - (1 - \delta)/a}{\varpi} \right).$$

 $rer = mc^F p^F.$ 

From (E.29),

From (E.15)-(E.16),

$$x^{I,H} = (1 - o_I) \left(\frac{p^H}{p^I}\right)^{-\eta_I} i,$$
  
$$x^{I,F} = o_I \left(\frac{p^F}{p^I}\right)^{-\eta_I} i.$$

Let  $mon \equiv \mu^e [r^K + (1 - \delta)q] \frac{k}{a} \Phi(aux^1 - \sigma_{\omega})$  be the monitoring costs payed in steady state. From GDP equal to value added, equivalent to (E.63), and (E.33),

$$p^{Y}y = p^{H}y^{H} + p^{Y}ys^{Co} + p^{F}(1 - mc^{F}\Delta^{F})y^{F} - mon.$$

Using (E.64) and (E.14),

$$p^{Y}y = p^{H}y^{H} + p^{Y}ys^{Co} + p^{F}(1 - mc^{F}\Delta^{F})\left[o_{C}(p^{F})^{-\eta_{C}}y^{C} + x^{IF} + o_{G}\left(\frac{p^{F}}{p^{G}}\right)^{-\eta_{G}}g\right] - mon.$$

Using (E.65),

$$p^{Y}y = p^{H}y^{H} + p^{Y}ys^{Co} + p^{F}(1 - mc^{F}\Delta^{F})\left[o_{C}(p^{F})^{-\eta_{C}}(c + mon) + x^{IF} + o_{G}\left(\frac{p^{F}}{p^{G}}\right)^{-\eta_{G}}g\right] - mon.$$

Using (E.63),

$$p^{Y}y = p^{H}y^{H} + p^{Y}ys^{Co} + p^{F}(1 - mc^{F}\Delta^{F})\left[o_{C}(p^{F})^{-\eta_{C}}(p^{Y}y(1 - s^{tb} - s^{g}) - p^{I}i + mon) + x^{IF} + o_{G}\left(\frac{p^{F}}{p^{G}}\right)^{-\eta_{G}}\frac{p^{Y}ys^{g}}{p^{G}}\right] - mon.$$

Thus,

$$p^{Y}y = \frac{p^{H}y^{H} + p^{F}(1 - mc^{F}\Delta^{F})[-o_{C}(p^{F})^{-\eta_{C}}(p^{I}i - mon) + x^{IF}] - mon}{1 - s^{Co} - p^{F}(1 - mc^{F}\Delta^{F})\left[o_{C}(p^{F})^{-\eta_{C}}(1 - s^{tb} - s^{g}) + o_{G}\left(\frac{p^{F}}{p^{G}}\right)^{-\eta_{G}}\frac{s^{g}}{p^{G}}\right]}$$

From  $s^{tb} = tb/(p^Y y)$ ,  $s^g = p^G g/(p^Y y)$ ,  $s^{Co} = rer \times p^{Co*} y^{Co}/(p^Y y)$  and the exogenous process for  $g_t$ ,

$$tb = s^{tb}p^{Y}y, \ g = \bar{g} = \frac{s^{g}p^{Y}y}{p^{G}}, \ y^{Co} = \bar{y}^{Co} = s^{Co}p^{Y}y/(rer \times p^{Co*}).$$

From (E.63),

$$c = p^Y y - p^I i - p^G g - tb.$$

From (E.65),

$$y^C = c + mon.$$

From (E.13)-(E.14) and (E.17)-(E.18),

$$x^{C,H} = (1 - o_C)(p^H)^{-\eta_C} y^C, \quad x^{C,F} = o_C(p^F)^{-\eta_C} y^C,$$
$$x^{G,H} = (1 - o_G) \left(\frac{p^H}{p^G}\right)^{-\eta_G} g, \quad x^{G,F} = o_G \left(\frac{p^F}{p^G}\right)^{-\eta_G} g.$$

From (E.58),

$$x^{H*} = y^H - x^{C,H} - x^{I,H} - x^{G,H}.$$

From (E.64),

$$y^F = x^{C,F} + x^{I,F} + x^{G,F}.$$

From (E.30),

$$f^F = mc^F (\tilde{p}^F)^{-\epsilon_F} y^F / (1 - \beta \theta_F).$$

From (E.33),

$$m = y^F \Delta^F.$$

From (E.60),

$$y = c + i + g + x^{H*} + y^{Co} - m.$$

From (E.63),

$$p^Y = (c + p^I i + p^G g + tb)/y.$$

From (E.1),

$$\lambda = \left(c - \varsigma \frac{c}{a}\right)^{-1} - \beta \varsigma \left\{ \left(ca - \varsigma c\right)^{-1} \right\}.$$

From (E.2).

From (E.2),  

$$\kappa = me^{W} \lambda w/h^{\phi}.$$
From (E.54),  

$$\sigma^{*} = (x^{H*}/y^{*})(p^{H}/rer)^{y^{*}}.$$
From (E.62),  

$$b^{*} = \overline{b}^{*} = \frac{tb - (1 - \chi)rer \times p^{Co*}y^{Co}}{rer(1 - (R^{*} + \xi)/(\pi^{*}a)]}.$$
From (E.38),  

$$r^{Le} = \overline{\omega}^{e} \frac{[r^{K} + (1 - \delta)g]k}{l}.$$
From (E.40),  

$$r^{Le} = \frac{gk}{lev^{e}}.$$
From (E.51),  

$$\omega = a(1 - \iota) / [(r^{L,K} - r)lev + r].$$
From (E.51),  

$$\omega = a(1 - \iota) / [(r^{L,K} - r)lev + r].$$
From (E.40),  

$$e^{N} = \frac{\beta}{a} \frac{1 - \omega}{1 - \beta \omega} r.$$
From (E.41),  

$$e^{L} = \frac{\beta}{a} \frac{1 - \omega}{1 - \beta \omega} (r^{L,K} - r).$$
From (E.41),  

$$\mu = \varrho^{L} + \frac{\varrho^{N}}{lev}.$$
From (E.42),  

$$l^{WC} = \alpha_{L}^{WC} \left(wh + r^{K}\frac{k}{a}\right).$$
From (E.49),  
From (E.50),  

$$n = \frac{l}{lev}.$$
From (E.50),  

$$n = \frac{l}{lev}.$$
From (E.52),  

$$spr = \frac{(R^{L,WC}l^{WC} + R^{L,e}l^{K})}{l} \frac{1}{R}.$$