Optimal Monetary Policy with Counter-Cyclical Credit Spreads

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Abstract

We study optimal monetary policy in a New Keynesian-DSGE model where the combination of a credit channel and customer-market features in banking gives rise to counter-cyclical credit spreads. In our setting, monopolistically competitive banks set lending rates in a forward-looking fashion as they internalize the fact that, due to borrowers’ bank-specific (hence deep) habits, current interest rates also affect the future demand for loans by financially constrained firms. In particular, during a phase of economic expansion, banks might find it optimal to lower current lending rates to build up a larger customer base, which will be locked into a long-term relationship. The resulting counter-cyclicity of credit spreads makes optimal monetary policy depart substantially from the efficient allocation (and hence from price stability), under both discretion and commitment. Our analysis shows that the welfare costs of setting monetary policy under discretion (with respect to the optimal Ramsey plan) and of using simpler sub-optimal policy rules are strictly increasing in the magnitude of deep habits in credit markets and market power in banking.

JEL Classification: E32, E44, E50.

Keywords: Optimal monetary policy; Cost Channel; New-Keynesian model; Credit frictions; Deep habits; Credit spreads

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1 Introduction

In several historical episodes, central banks around the world have reacted sharply to financial shocks and distress in credit markets. The most important examples are the policy developments in the United States in the late ’80s (when the interest rate was cut by more than suggested by the Taylor rule arguably in response to the credit crunch originated in the adoption of increased capital requirements by the Basel Accords) and, more recently after 2008, when many central banks started more aggressive bond-buying programs in response to the sub-prime mortgage crisis. This seems to suggest that central banks react to developments in financial markets. However, there is still no consensus on whether and how the interest rate should respond to financial variables in estimated versions of the Taylor rule, and up to the second half of last decade, the workhorse models used for optimal monetary policy analysis did not assign any role to external finance.

Only in 2006 Ravenna and Walsh (RW, hereafter) provide the first attempt to endogenize the credit/cost channel of monetary policy by introducing an endogenous cost-push shock to the Phillips curve. Since firms need to borrow to pay for working capital, then the interest rate itself enters the Phillips curve. This cost channel counteracts the standard interest rate or aggregate demand channel by mitigating the reaction of prices and amplifying the response of output to monetary policy shocks. The contribution of RW to the monetary policy literature was extremely relevant since they were the first to introduce a meaningful trade-off between stabilizing inflation and stabilizing the output gap. In RW agents can borrow freely at a riskless interest rate so that financial agents are non-existent or, at the most, nothing more than intermediaries acting as a veil. Just a couple of years after RW was published, the global financial crisis started calling for more emphasis on a model where the interaction between financial markets and the real sector is more meaningful.\footnote{Christiano et al (2011) express this need in the conclusions to their chapter in the Handbook of Monetary Economics. They state that the standard workhorse New-Keynesian DSGE model used for the study of monetary policy needs a richer financial sector to be able to study these policy questions.}

The main goal of our paper is to enhance our understanding of how credit market imperfections undermine the effectiveness of conventional monetary policy actions, and how the latter should be adjusted to account for their existence. We propose a channel which is capable of generating a positive spread between the lending and the policy rate without relying on the costly-state-verification/imperfect monitoring machinery used in the Bernanke-Gertler-Gilchrist-type financial accelerator models, or on \textit{ad hoc} costly-loan-origination technologies. This alternative (or complementary) channel has to do with the well-documented existence of imperfect competition and customer-market features (i.e.,
long-term relationships) in credit markets.\footnote{There is abundant evidence on product differentiation as the source of market power in banking. Differentiation makes the financial services from different banks imperfectly substitutable from the point of view of borrowers. See Kim et al. (2005), Northcott (2004), and Cohen and Mazzeo (2007), among others. Another empirically documented source of market power in banking is customer switching costs. See Yuan (2009) and Olivero and Yuan (2009) for evidence on the magnitude of these switching costs for a cross-section of countries.} \footnote{Fama (1985), Sharpe (1990), Rajan (1992), Petersen and Rajan (1994), Hart (1995), von Thadden (1995), Bolton and Scharfstein (1996), Detragiache et al. (1999), Neuberger and Schacht (2005), Ongena and Smith (2000) and Vulpes (2005) all provide evidence on the existence of banking relationships and on these being with multiple banks at the same time.} \footnote{The borrower “hold-up” problem can be rationalized in a context of asymmetric information between lenders and borrowers on borrowers creditworthiness. In this context incumbent banks gradually accumulate this information over time as they lend repeatedly to their customers, eventually earning an informational monopoly over these borrowers. This creates switching costs since it is costly for them to switch lenders and to start signalling private information on their creditworthiness to a new bank. Deep habits in credit markets provide a way to model the existence of these switching costs in a still tractable way, without the need to explicitly model informational asymmetries which is beyond our scope and cumbersome in a DSGE model. Also, as in Ravn et al (2006) with deep habits there is no actual switching in equilibrium when a bank raises its interest rate, but rather a gradual loss of customers for that bank. This is important because it makes the DSGE model computationally tractable.} More specifically, we study the conduct of optimal monetary policy in a small-scale NK-DSGE framework characterized by monopolistic competition and deep habits in banking, along the lines of Aliaga-Díaz and Olivero (2010a). Deep habits can be interpreted as representing the existence of switching costs for borrowers in the economy, and therefore to capture the documented borrower “hold-up” problem in a parsimonious way. Under deep habits, monopolistically competitive banks set lending rates in a forward-looking fashion: they internalize the fact that, due to habits in banking (which are meant to capture forward-looking customer markets for loans), current interest rates affect also the future demand for loans by financially constrained firms.

In particular and consistent with the empirical evidence, deep habits generate endogenously counter-cyclical spreads between the interest rates on loans (charged to credit-constrained firms) and the rate on deposits (which, in our model, corresponds to the policy rate). Here is the intuition why deep habits give rise to counter-cyclical spreads: In response to a positive shock that raises the demand for credit banks face the following trade-off. On the one hand, lowering spreads can mean lower current profits but higher future profits from a locked-in customer base (the “investment effect” in the language of the Industrial Organization literature). On the other, raising spreads means higher current profits (the “harvesting effect”). In the presence of highly autocorrelated aggregate shocks, the “investment effect” dominates, so that spreads fall in good times.\footnote{See, for instance, the customer-market model of Phelps and Winter (1970).} \footnote{Notice that spreads are also typically countercyclical in the canonical Bernanke-Gertler-Gilchrist model,}
Within this novel set-up, we study the conduct of optimal monetary policy, under both discretion and commitment. Our analysis shows that the introduction of deep-habits exacerbates the trade-off between stabilizing inflation and the output gap in the face of shocks to total factor productivity (TFP) and the spreads: the stronger the degree of deep habits, the larger the departure of the economy from full price stability. To grasp the underlying mechanism, consider the case of a cost channel with constant or no spreads (i.e. no deep habits), as in RW. Due to its impact on the credit-related component of marginal costs in the NK Phillips curve, the nominal interest rate cannot be used to insulate the economy from the technology shock. Under the optimal policy, the economy then displays a positive output gap, a deflation, and a larger drop in the policy rate than in the benchmark New-Keynesian model without the cost channel. Deep habits amplify this transmission mechanism via counter-cyclical credit spreads. By creating a positive output gap, the expansionary monetary policy lowers the credit spread, which in turn hampers the decline in inflation through the cost channel. The central bank counteracts this effect by lowering the nominal interest rate even further, with the objective of creating an even larger output gap to increase the wage component of marginal costs (via a standard aggregate demand channel) and hence contain deflation. The outcome is an equilibrium with a larger decrease in inflation and the nominal interest rate, and a larger output gap, compared to an economy with constant credit spreads (i.e. an economy where deep habits in banking are absent).

The amplifying effect of deep habits holds for the case of a policy-maker acting under both discretion and commitment, although in the latter case aggregate fluctuations are significantly contained. The welfare gains from committing to a Ramsey plan appear to be quite sizable and to be a strictly increasing function of the degree of market power and deep habits in banking. This result highlights the increased importance of optimal monetary policy commitment when there are imperfections in financial intermediation.

Our analysis also highlights the perils of trying to implement the optimal monetary policy plan through sub-optimal but simpler interest rate rules since we obtain sizable welfare costs of these rules.

The paper that is most closely related to ours is Aksoy et al (2013) who also study monetary policy and lending relationships. They do so by introducing the deep habits in credit markets model of Aliaga-Díaz and Olivero (2010) into an otherwise standard DSGE model of staggered pricing and a cost channel of monetary policy. They show that spread movements are crucial for policy even when a standard Taylor rule is employed, and that strong credit relationships may lead to indeterminacy of equilibrium which forces the central bank to react to changes in credit conditions. Aksoy et al explore several alternative

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so we do not see our model as competing with the canonical, but as a complementary explanation for the existence and countercyclicality of spreads.
interest rate rules. However, since they do not conduct an analysis of optimal policy, they are unable to assess the welfare properties of price stabilization and the welfare gains of commitment (relative to discretion) in the implementation of monetary policy. Thus, their setup and methodology are unable to shed light on how deep habits affect the stabilization trade-offs facing central banks.

Most of the work that followed RW on the macro-financial linkages and their implications for the conduct of monetary policy can be classified in three main strands.

The first line of work focuses on the implications for monetary policy of informational asymmetries in credit markets and the implied need for loan monitoring. Aikman and Paustian (2006), Carlstrom et al. (2010), Christiano et al. (2010), Williamson (2012), Agenor et al. (2014) and De Fiore and Tristani (2012) all model firms that need to externally finance their working capital, and due to asymmetric information, they need to pledge their net worth as collateral for these loans. This friction leads banks to optimally charge an external finance premium above the policy rate, which in turn enters as an endogenous cost-push shock in the Phillips curve. With this environment they obtain results qualitatively similar to ours: optimal monetary policy deviates from perfect price stability. Aikman and Paustian (2006) also conclude that responding to asset prices or credit growth as additional targets through the interest rate rule is detrimental to welfare relative to a policy of strict price stability. Also, differently from our case, in some of this work the central bank’s objective function includes as an additional target a measure of frictions in credit markets. Then, the target rule that characterizes optimal policy includes a reaction

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7While in De Fiore and Tristani (2012) what firms can use as collateral is an exogenous endowment, in Carlstrom et al (2009) it is endogenous, so that they introduce some additional feedback between endogenous net worth and asset prices. In Carlstrom et al (2009) a monetary policy response to supply-side shocks affects share prices and the market value of net worth and, through the collateral constraint, interest rates and the cost of labor. Under some conditions, optimal monetary policy still consists of strict inflation targeting.

8De Fiore and Tristani (2012) can reproduce countercyclical premia only when allowing for a set of additional shocks.
to credit spreads which is not necessarily the case in our model.\textsuperscript{9} \textsuperscript{10} \textsuperscript{11} Faia and Monacelli (2007) also introduce the financial accelerator into a monetary DSGE model. They conclude that a policy of strict inflation targeting is still welfare maximizing.\textsuperscript{12} However, they do not study optimal policy.

In the second strand banks are required to operate a costly function for the production or management of loans as in Goodfriend and McCallum (2007) and Cúrdia and Woodford (2009). With the marginal cost of loan production being pro-cyclical, this literature introduces some degree of procyclicality to the margin between loan and interbank rates. In particular, the increase in aggregate demand stimulated by expansionary monetary policy is accommodated by an expansion in hours worked at the banking sector and an elevated real wage. This in turn causes the capital-labor ratio in banking to fall and the cost of collateral to increase. Then, the cost of loans rises and introduces a financial “attenuator” effect which partially offsets the standard BGG financial “accelerator”.\textsuperscript{13} \textsuperscript{14}

\textsuperscript{9} Christiano et al. (2010) introduce BGG-type agency problems in financial contracts, Fisherian deflation of banks’ debt obligations to households, liquidity constraints facing banks and shocks that alter the perception of market risk into an otherwise standard monetary DSGE model. They study the endogenous response of the economy to a large set of shocks to financial wealth, risk, the marginal efficiency of investment, productivity and policy shocks, which allows them to interpret the consistently expansionary monetary policy after 2008-09 as compensating for the negative effects of bank liquidity shocks. However, they do not study optimal policy.

\textsuperscript{10} In Williamson (2012) banks enjoy economies of scale in the costly state verification technology and they insure private agents against liquidity shocks. In this environment the central bank will typically choose an inflation rate greater than the Friedman-rule rate, trading-off the benefits of taxing currency transactions with the costs of reducing trading efficiency.

\textsuperscript{11} Agenor et al. (2014) study a model in which firms are required to pledge output as collateral which gives rise to countercyclical spreads and significant amplification of the response to structural shocks.

\textsuperscript{12} They argue that the marginal benefit of counteracting the price stickiness distortion still largely outweighs that of neutralizing the credit friction.

\textsuperscript{13} Goodfriend and McCallum (2007) include a banking sector into an otherwise standard DNK model. In their framework banks operate a production function that combines labor as monitoring effort and collateral posted by borrowers to produce loans. Thus, in response to a positive TFP shock consumption, employment, the external finance premium, wages and the price of capital all rise persistently, while the inflation rate is essentially stabilized by the central bank cutting its interbank rate by 0.6%. A central bank unaware of the effects of costly banking activities on spreads would move the interest rate relatively little in response to a highly persistent productivity shock, significantly misjudging the desired policy action. Also, in response to a moderately persistent negative 1% shock to collateral, the central bank needs to cut the interbank rate by almost 5%.

\textsuperscript{14} Cúrdia and Woodford (2009) show that the presence of interest rate spreads does not change optimal monetary policy qualitatively, but it might from a quantitative standpoint. Cúrdia and Woodford (2010) conclude that changes in financial conditions, specially in interest rate spreads, should optimally be taken into account in the formulation of monetary policy. They show that modifying the standard Taylor rule to specify a funds rate target equal to the standard target minus the current value of the LIBOR-OIS spread can improve an economy’s response to a shock to the supply of credit.
The third line of work introduces staggered loan contracts that determine heterogeneously sticky interest rates and an empirically plausible incomplete pass-through from policy rates to lending rates (see Teranishi (2008), Hulsewig et al. (2009) and Gerali et al. (2010)). This allows for an accelerator effect through which economic fluctuations become more persistent and of greater amplitude.15

Our contribution relative to this work is fourfold. First, we contribute to the second line of work by departing from their standard and already well known financial accelerator environment, and by modelling a novel channel for endogenously countercyclical spreads. Second, we model spreads or external finance premia that are consistently countercyclical, more in line with the empirical evidence.16 Third, we contribute to the third line of work by endogenizing the incomplete pass-through from policy to lending rates through a customer-market type of story. The properties of the credit spread in our model are consistent with the empirical evidence on the sluggish adjustment of loan rates in response to shocks to open-market rates presented by Slovin and Sushka (1983) and Berger and Udell (1992). Fourth, in contrast to several of these works that focus on monetary policy conducted through exogenous interest rate rules, we study optimal policy and the welfare costs of using sub-optimal rules.

Our work is also related to a series of recent papers on the aggregate consequences for monetary and fiscal policy of “deep habits” in consumption. Leith et al. (2012) and Givens (2016) focus on monetary policy and Zubairy (2013 and 2014), on fiscal. Leith et al. (2012) study optimal monetary policy for alternative specification of consumption habits (“superficial versus deep” and “internal versus external”). They show that external consumption habits create new trade-offs for optimal monetary policy. In particular, under deep habits, the policy-maker optimally allows for a larger output gap and the economy slips further away from the flexible price efficient allocation. They also find that deep habits exacerbate the welfare cost of deviating from the optimal policy under commitment, with respect to the case of superficial habits. Givens (2016) studies the welfare gains from commitment in monetary policy. He shows that deep habits in consumption weaken the stabilization trade-offs facing a discretionary planner, and that commitment policy is more effective than discretion at managing these adverse stabilization effects stemming from deep habits. Thus, his results echo ours, but with deep habits concentrated in the retail sector rather than the banking sector. Zubairy (2013) introduces deep habits in consumption of

15In Teranishi interest rate smoothing and monetary easing in response to shocks to the spreads are both optimal. In Gerali et al these staggered contracts interact with endogenous accumulation of bank capital and borrowing constraints, so that banking also provides an attenuator effect that is sizable but short-lived. Hulsewig et al can reproduce the inflation inertia present in the data. None of these authors study optimal policy though.

16In De Fiore and Tristani (2012) the external premium is procyclical which is at odds with the empirical evidence and makes technology shocks partly inefficient.
both private and public goods into a medium-scale DSGE model to better estimate the
size of fiscal multipliers. Zubairy (2014) shows that deep habits can induce equilibrium
determinacy even if the Taylor principle is satisfied (i.e., the central bank increases the
short-term nominal interest rate more than proportionally with respect to inflation). 17
Gilchrist et al (2015) develop a model with both deep habits in consumption and firms
facing frictions in external financing, and show that these two features working together
give rise to inflation becoming less sensitive to output fluctuations.

Following this introduction the paper is structured as follows: Section 2 presents the
model, and section 3 introduces the semi-symmetric equilibrium that we study. Sections 4
and 5 study the steady-state and the aggregate dynamics, respectively. Section 6 contains
the results for optimal monetary policy and Section 7 concludes. The analytics of the
optimal monetary policy problem and some details on the computation of welfare costs are
left for the technical appendix.

2 A Model with Deep Habits in Banking

We study a closed economy made of a household sector, a production sector comprised
of manufacturing and retail firms, a banking sector, and a government. Households take
consumption-saving and labor-leisure decisions to maximize their expected lifetime utility.
Manufacturing firms produce intermediate goods with labor as the only input. These firms
use a composite of heterogeneous bank loans to finance working capital needs (a fraction
of the wage bill has to be paid at the beginning of the period before sales revenues are real-
ized). 18 Banks use households’ savings to provide loans in a monopolistically competitive
market. Monopolistically competitive retail firms subject to Calvo-type nominal rigidi-
ties produce final consumption goods using intermediate goods. As they are all owned by
households, expected future profits made by manufacturing firms, retail firms and banks
will all be discounted using the households’ stochastic discount factor in their respective
maximization problem.

17Also, Ravn et al. (2010) introduce deep habits in consumption into an otherwise standard New-
Keynesian model. They show that, by generating counter-cyclical mark-ups, deep habits can account
for both the price and the inflation persistence puzzle without relying on unreasonable extents of nominal
rigidities. Deep habits make firms set prices in a forward-looking manner (even under flexible prices) and
are therefore complementary to standard ways of introducing nominal price rigidities.

18Banks can differentiate their loans by targeting the financial services that they provide together with
a loan (i.e. firm monitoring, valuation of collateral and investment project evaluation) towards particular
sectors of economic activity. Also, banks can choose various quality characteristics to build reputation
and differentiate from competitors, like equity ratios, size, loss avoidance, etc. Last, lenders use different
product packages and the extensiveness and location of their branches, personalized service, accessibility
to the institution’s executives, hours of operation and ATM and remote access availability to differentiate
their services from those of competitors.
The key element of differentiation of our model with respect to a benchmark New Keynesian model with a cost channel of monetary policy transmission lies in the assumption that banks provide differentiated loan services to firms and that those services also depend on bank-specific stocks of past loans. This feature - which we refer to as “deep habits” in banking as in Aliaga-Díaz and Olivero (2010a) - is meant to capture the documented existence of long-term lender-borrower relationships in credit markets.

2.1 Manufacturing

A continuum of mass one of perfectly competitive manufacturing firms - each indexed by \( j \in [0, 1] \) - produces an undifferentiated intermediate good using labor \( \tilde{H}_{j,t} \) as the only input, where tildes are used to denote demand. In each period \( t \), the \( j \)-th firm sells its output \( I_{j,t} = A_t \tilde{H}_{j,t} \) at the unit price \( Q_{j,t} \) to retailers which use it to produce differentiated final products. The factor \( A_t \) denotes aggregate TFP, which evolves according to the following log-stationary stochastic process:

\[
\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t},
\]

\[
\rho_a \in (0, 1), \quad \varepsilon_{a,t} \sim iid \left(0, \sigma_a^2\right)
\]

The firm is subject to a working capital requirement: a fraction \( \alpha \) of labor costs has to be paid before sales revenues are realized. To finance those, the \( j \)-th firm uses a composite \( x_{j,t} \) of imperfectly substitutable heterogeneous loans provided by a mass one continuum of banks. This assumption is consistent both with the abundant evidence on the existence of product differentiation in banking (that makes the financial services from different banks imperfectly substitutable from the point of view of borrowers) and with the increasing size of syndicated loans (which account for roughly 50% of originate corporate finance in the U.S.).\(^{19}\)

Similar to Ravenna and Walsh (2006) and Christiano et al. (2010), we assume that loans are intra-period, in the sense that they are obtained at the beginning of the period for the firm to meet the working capital requirement, and repaid at the end of the same period. All loans are repaid in full.

In this setup, firms engage in multiple banking relationships by borrowing from several banks in the economy. This is in line with the rich empirical evidence presented by Ongena and Smith (2000a, 2000b).\(^{20}\) Then, without loss of generality, we assume that each firms borrows from all banks.

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\(^{19}\)Sufi (2007) provides some discussion on the size of the syndicated loans market in the U.S. For instance, he documents that the average number of lenders in a syndicated loan, in the U.S., is 8.

\(^{20}\)By looking at a sample of more than one thousand large firms in 20 European countries, these authors find that 50% of firms maintains up to 7 bank relationships, while 20% has more than 7. In their sample, the
To model the existence of borrower “hold-up” effects and costs of switching banks, the loan composite $x_{j,t}$ is assumed to depend also on the accumulated stock of past bank-specific loans as defined by equations (2) and (3):

$$x_{j,t} = \left[ \int_0^1 (l_{jn,t} - \theta s_{n,t-1}) \frac{\xi_{t-1}}{\xi_t} dn \right]^{\xi_t/\xi_{t-1}}$$

(2)

$$s_{n,t-1} = \rho_s s_{n,t-2} + (1 - \rho_s) l_{n,t-1}$$

(3)

where $l_{jnt}$ is the $j$-th firm demand for credit from the $n$-th bank in period $t$. The loan composite $x_{j,t}$ is expressed in units of an aggregate consumption good, which will sell at the market price $P_t$.\(^{21}\) In order to study the dynamic effects of exogenous shocks to credit, we assume that $\xi_t$, the elasticity of substitution across loan varieties, is stochastic with mean $\xi > 1$, and is generated by the following log-stationary process:

$$\ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + \varepsilon_{\xi,t}$$

$$\rho_\xi \in (0, 1), \quad \varepsilon_{\xi,t} \sim iid (0, \sigma^2_\xi)$$

(4)

The term $\theta s_{n,t-1}$ in $x_{j,t}$ is intended to capture the borrower “hold-up” effect, with the parameter $\theta$ measuring its extent. We will refer to the case of $\theta > 0$ as “deep habits in banking”. The term $s_{n,t-1}$ in (2) is defined as $s_{n,t-1} \equiv \int_0^1 s_{jn,t-1} dj$, which corresponds to the beginning of period $t$ cross-sectional (across manufacturing firms) average stock of accumulated past loans obtained from the $n$-th bank. The fact that $s_{n,t-1}$ is the average (rather than the individual) stock of past borrowing implies that habits are external and are therefore taken as exogenous by each individual borrowing firm.\(^{22}\) This simplifying assumption can be rationalized through banks exhibiting economies of scale in the management of informational asymmetries. Thus, the more all firms bank with one bank, the larger the information monopoly for that bank. The stock of habits $s_{n,t-1}$ follows the law

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\(^{21}\) This corresponds to the Consumer Price Index (CPI) defined in Section 2.4.

\(^{22}\) Just as in Ravn et al (2006), this assumption makes the model analytically tractable, since it preserves the separation of the dynamic problem of choosing total borrowing over time from the static problem of choosing individual borrowing from each bank at any given point in time. If this was not the case, the current demand from each bank $n$ would depend both on its current relative interest rate and on all future expected rates. Therefore, each bank would face an incentive to renege from past interest rate promises, and the problem would no longer be time consistent.
motion in equation (3): it is a linear function of its value in the previous period and the average level of borrowing from the \(n\)-th bank in \(t - 1\), \(l_{n,t-1} \equiv \int_0^1 l_{jn,t-1} dj\).

In each period \(t\), the \(j\)-th firm chooses the level of employment \(\tilde{H}_{j,t}\), the loans composite \(x_{j,t}\) and borrowing \(l_{jn,t}\) for \(n \in [0, 1]\), to maximize the expected present discounted value of its lifetime profits. Its optimization problem is given by:

\[
\max_{\{H_{j,t}, x_{j,t}, l_{jn,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} F_{0,t} V_{j,t}^M
\]

subject to:

\[
V_{j,t}^M = Q_{j,t} A_t \tilde{H}_{j,t} + P_t x_{j,t} - (1 - \tau) W_t \tilde{H}_{j,t} - \int_0^1 R_{n,t}^L P_t l_{jn,t} dn
\]

\[
x_{j,t} = \left[ \int_0^1 (l_{jn,t} - \theta s_{n,t-1}) \frac{\xi_{t-1}}{\xi_t} dn \right] ^{\xi_t} / \xi_t
\]

\[
P_t x_{j,t} \geq \alpha (1 - \tau) W_t \tilde{H}_{j,t} \quad \alpha \leq 1
\]

where \(F_{0,t} \equiv \beta^t U_{C,t} \frac{P_t}{U_{C,0} P_0} \) is the representative household’s stochastic discount factor.

From the firm’s perspective, the effective cash flow provided by loans is given by the loan composite \(x_{j,t}\) and not by the simple integral of loans across banks. Due to imperfect substitutability, each loan provides differentiated liquidity services to the borrowing firm. This is captured by the habit-adjusted Dixit-Stiglitz loan aggregator in (7).\(^{23}\) Letting \(W_t\) and \(R_{n,t}^L\) denote, respectively, the nominal wage rate and the gross interest rate contracted with the \(n\)-th bank, equation (6) defines then the \(j\)-th firm’s cash flow \(V_{j,t}^M\) in period \(t\) as sales revenues plus what the firm obtains from borrowing minus the sum of labor and borrowing costs. As standard in the New Keynesian literature, we assume that a fiscal authority subsidizes labor costs at a rate \(\tau\) in order to eliminate all distortions in the steady state equilibrium. The liquidity in advance constraint (8) states that the liquidity services provided by the differentiated loans should be at least equal to a fraction \(\alpha\) of working capital needs (in this case, labor costs).\(^{24}\)

We solve the problem in two steps. First, we find the \(j\)-th firm optimal relative demand for loans issued by the \(n\)-th bank. This is obtained by minimizing total borrowing costs, \(\int_0^1 R_{n,t}^L l_{jn,t} dn\), subject to (7). The solution gives an expression for \(l_{jn,t}\) - the \(j\)-th firm optimal demand for loans issued by the \(n\)-th bank - as a function of the relative loan rate charged by the \(n\)-th bank and the stock of borrowing habits related to the same loan.

\(^{23}\)Notice that under perfect substitutability (which, in the non-stochastic case, occurs in the limit case of \(\xi \to \infty\)) and without deep habits, the loan aggregator is simply \(x_{j,t} = \int_0^1 l_{jn,t} dn\).

\(^{24}\)For a similar specification of a liquidity in advance constraint with imperfect substitutable assets see Marimon et al. (2012), who study the competition among different forms of payments. Vég (2013) adopts a similar specification in a simple small open economy model with currency substitution.
variety:

\[ l_{jn,t} = \left( \frac{R_{n,t}^L}{R_t^L} \right)^{-\xi_t} x_{jt} + \theta s_{n,t-1} \]  \hspace{1cm} (9)

where \( R_t^L \equiv \int_0^1 (R_{n,t}^L)^{1-\xi_t} \, dn \) defines the aggregate loan rate index. As equation (9) states, \( l_{jn,t} \) is higher the cheaper is borrowing from the \( n \)-th bank (i.e. lower \( R_{n,t}^L \)) and/or the stronger the lender-borrower relationship established with that bank (i.e. larger \( \theta \) and/or \( s_{n,t-1} \)).

By simple calculus, we can derive an expression for the interest rate elasticity of the demand for loans as:

\[ -\frac{\partial l_{jn,t}}{\partial R_{n,t}^L} \frac{R_{n,t}^L}{l_{jn,t}} = \xi_t \left( 1 - \frac{\theta}{\gamma_{jn,t}} \right) \]  \hspace{1cm} (10)

where \( \gamma_{jn,t} \equiv \frac{l_{jn,t}}{s_{n,t-1}} \) can be interpreted as the growth rate of loans obtained from the \( n \)-th bank with respect to the initial stock of loan habits. By setting \( \theta = 0 \) in (9) and (10), we can show that, without deep habits in banking, the model boils down to a benchmark version where the demand for a specific loan variety depends only on the relative interest rate, and the interest-rate elasticity of the demand for credit is entirely pinned down by the elasticity of substitution across varieties \( \xi_t \). On the contrary, since the first derivative of the right hand side of (10) with respect to \( \gamma_{jn,t} \) is positive for \( \theta > 0 \), the interest rate elasticity of the demand for loans appears to be pro-cyclical. This last feature will play a key role in helping our model generate the empirically documented counter-cyclicality of credit spreads (see Aliaga-Díaz and Olivero (2010b and 2011) and Gilchrist and Zakrajsek (2012)).

Using (9) and the definition of the loan rate index \( R_t^L \), total borrowing costs entering \( V_{j,t}^M \) in equation (6) can be rewritten as \( \int_0^1 R_{n,t}^L l_{jn,t} \, dn = R_t^L x_{jt} + \Delta_t^L \), where \( \Delta_t^L \equiv \theta \int_0^1 R_{n,t}^L l_{n,t-1} \, dn \) is a term equal across all manufacturing firms. We can then write problem (6) in a simpler form:

\[
\max_{\tilde{H}_{j,t},x_{j,t}} V_{j,t}^M = Q_{j,t} A_t \tilde{H}_{j,t} + P_t x_{j,t} - (1 - \tau) W_t \tilde{H}_{j,t} - P_t \left( R_t^L x_{j,t} + \Delta_t^L \right) \\
\text{s.t.} \\
P_t x_{j,t} \geq \alpha (1 - \tau) W_t \tilde{H}_{j,t}
\]

\(^{25}\)These two expressions are isomorphic to those found in Ravn et al. (2006), with the demand for a specific loan variety, \( l_{jn,t} \), and the interest rate index, \( R_t^L \), replacing, respectively, the demand for a specific consumption variety and the price index. Habits emerge when the demand for a particular loan is increasing in the stock of habit associated with that variety which requires \( \theta > 0 \).
Taking first order conditions with respect to $\tilde{H}_{j,t}$ and $x_{j,t}$ and rearranging, we obtain an expression for the optimal pricing of the intermediate good sold by the $j$-th manufacturer:

$$Q_{j,t} = (1 - \tau) \frac{W_t}{A_t} \left[ 1 + \alpha \left( R_t^j - 1 \right) \right] = Q_t$$  \hspace{1cm} (11)

As in Ravenna and Walsh (2006), the working capital requirement implies that prices now also reflect firms’ borrowing costs.

### 2.2 Retailers

A mass one continuum of retail firms, indexed by $i \in [0, 1]$ engage in simple and costless activities such as packaging. They buy the homogeneous intermediate good from manufacturers to transform it into differentiated final consumption goods for the households. The $i$-th retail firm operates in a monopolistically competitive market and produces output $Y_{i,t}$ using the following technology:

$$Y_{i,t} = K \tilde{I}_{i,t}$$  \hspace{1cm} (12)

where $\tilde{I}_{i,t}$ is the demand for the intermediate good by the $i$-th firm and $K$ is a constant factor of transformation. Notice that $I_{i,t} = I_{j,t}$ since each retailer $i$ is randomly matched to one manufacturer $j$. The $i$-th retail firm sells its output at a price $P_{i,t}$ per unit, facing a standard downward-sloping demand: i.e., $Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$ where $P_t$ is the economy-wide price index (to be defined in a later section), $Y_t$ is aggregate demand and $\epsilon > 1$ is the (constant) elasticity of substitution across differentiated final consumption goods. Retail firms are subject to Calvo-type nominal price rigidities: in each period they face a constant probability $\vartheta$ of not being able to reset their price optimally. Their profit maximization problem is standard:

$$\max_{P_{i,t}^*, 0} E_0 \sum_{t=0}^{\infty} \vartheta^t F_{0,t} V_{i,t}^R$$  \hspace{1cm} (13)

s.t.

$$V_{i,t}^R = \left[ \left( P_{i,t}^* - MC_t \right) Y_{i,t} \right]$$  \hspace{1cm} (15)

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$$  \hspace{1cm} (16)

where $MC_t \equiv K^{-1}Q_t$ are nominal marginal costs. After taking first order conditions and rearranging terms, we obtain the optimal price setting rule for the $i$-th firm:

$$P_{i,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \vartheta^k \beta k \frac{U_{C,t+k}}{U_{C,t}} \frac{MC_{t+k}}{P_{t+k}} \left( \frac{P_{t+k}}{P_t} \right)^{\epsilon-1} Y_{t+k}}{E_t \sum_{k=0}^{\infty} \vartheta^k \beta k \frac{U_{C,t+k}}{U_{C,t}} \left( \frac{P_{t+k}}{P_t} \right)^{\epsilon-1} Y_{t+k}}$$  \hspace{1cm} (17)
2.3 The Banking Sector

There is a continuum of size one of banks indexed by \( n \in [0, 1] \). Each variety of loans/financial services is produced by a bank operating in a monopolistically competitive loan market. Banks are competitive in the market for deposits.

In every period \( t \), the \( n \)-th bank chooses its demand for deposits \( (D_{n,t}) \) and the interest rate on its loans \( (R_{n,t}^L) \) to maximize the expected present discounted value of lifetime profits. Its optimization problem is given by:

\[
\max_{\{D_{n,t}, L_{n,t}, R_{n,t}^L\}} \sum_{t=0}^{\infty} F_{0,t} V_{n,t}^B
\]

s.t.

\[
V_{n,t}^B = R_{n,t}^L L_{n,t} - R_t P_t D_{n,t} \tag{18}
\]

\[
L_{n,t} = D_{n,t} \tag{19}
\]

\[
L_{n,t} = l_{n,t} \equiv \int_0^1 l_{jn,t} dj = \int_0^1 \left( \frac{R_{n,t}^L}{R_t^L} \right)^{-\xi_t} x_{jt} + \theta s_{n,t-1} \right) dj \tag{20}
\]

where \( x_t \equiv \int_0^1 x_{jt} dj \) is the cross-sectional (across manufacturers) average of the demand for the loan composite. The terms \( L_{n,t} \) and \( D_{n,t} \) are measured in units of the aggregate consumption good, and are therefore multiplied by \( P_t \) in (18). Equation (18) defines the bank’s cash flow, where \( R_t \) is the common risk-free gross interest rate on deposits paid by all banks. Equation (19) defines the bank’s balance sheet, equating loans to deposits (there is no reserve requirement), while equation (20) defines total loans issued by the bank as the sum of the relative demands by all manufacturing firms. Due to loan differentiation and monopolistic competition, the relative loan demand faced by the \( n \)-th bank is downward-sloping with respect to \( R_{n,t}^L \), and is internalized by the bank while setting \( R_{n,t}^L \).

The Lagrangian for the \( n \)-th bank profit maximization problem is given by:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} F_{0,t} \left\{ (R_{n,t}^L - R_t) P_t L_{n,t} + \nu_{n,t} \left[ \left( \frac{R_{n,t}^L}{R_t^L} \right)^{-\xi_t} x_t + \theta s_{n,t-1} - L_{n,t} \right] \right\}
\]

Taking first order conditions with respect to \( L_{n,t} \) and \( R_{n,t}^L \) gives, respectively, the following two expressions:

\[
P_t \nu_{n,t} = P_t (R_{n,t}^L - R_t) + \theta (1 - \rho_s) E_t (F_{t,t+1} \nu_{n,t+1}) \tag{21}
\]

\[
L_{n,t} = \xi_t \nu_{n,t} x_t \left( \frac{R_{n,t}^L}{R_t^L} \right)^{-\xi_{t-1}} \tag{22}
\]
Using (20) and (22) to find an expression for \( \nu_{n,t} \) - the shadow value of per-unit profits - and substituting the result into (21), after simple algebra, we obtain an expression for the credit spread \( R_{n,t}^L - R_t \):

\[
(R_{n,t}^L - R_t) = \frac{R_{n,t}^L}{\xi_t} \frac{\gamma_{n,t}}{\gamma_{n,t} - \theta} - \theta(1 - \rho_s) E_t F_{t+1} p_{t+1} \nu_{n,t+1}
\]  

(23)

where \( \gamma_{n,t} \equiv \frac{L_{n,t}}{\nu_{n,t+1}} \). Through a close inspection of equation (23) we can see how the introduction of a customer-market channel in banking is capable of generating countercyclical credit spreads. As banks face an increase in current demand conditions (i.e. larger \( \gamma_{n,t} \)) and/or the expected shadow value of future expected profits (i.e. larger \( \nu_{n,t+1} \)), they lower current loan rates to capture a larger customer base that will be locked-in in the future and will allow them to attain higher per unit profits. Another channel, although exogenous, leading to a lower credit spread is related to an increase in the elasticity of substitution across loans \( \xi_t \), which leads to reduced market power in banking.

Notice that if \( \theta = 0 \) (no habits in banking), then equation (23) reduces to the following:

\[
(R_{n,t}^L - R_t) = \frac{1}{\xi_t - 1} R_t
\]  

(24)

If the liquidity services provided by the loan composite do not depend on the past stock of accumulated loans, then the \( n \)-th bank sets \( R_{n,t}^L \) as a time-varying mark-up on the deposit rate \( R_t \) (the marginal cost of issuing one unit of loans to firms). In this case, however the credit spread might vary only because of exogenous shocks to the elasticity \( \xi_t \).

2.4 Households

The representative household’s expected lifetime utility is given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{1-\sigma} - H_t^{1+\varphi}}{1-\sigma} \right]
\]

(25)

where \( H_t = \int_0^1 H_{j,t} dj \) are total hours supplied to the manufacturing sector, while \( C_t \) is a Dixit-Stiglitz consumption aggregator of a continuum of imperfect substitute final goods, indexed by \( i \):

\[
C_t = \left[ \int_0^1 C_{i,t} (\epsilon-1)i dt \right]^{\epsilon/(\epsilon-1)}
\]  

(26)

where \( \epsilon \) denotes the elasticity of substitution across varieties in the goods market.

Given the consumption aggregator (26), the relative consumption demand for the \( i \)-th good is given by:

\[
C_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\epsilon} C_t
\]  

(27)
where \( P_t = \left[ \int_0^1 P_{i,t}^{1-\epsilon} \, di \right]^{1/(1-\epsilon)} \) is the aggregate consumer price index, such that \( P_t C_t = \int_0^1 P_{i,t} C_{i,t} \, di \).

The household is allowed to save by accessing a competitive market for intra-period bank deposits and by holding money balances. Firms’ and banks’ profits are rebated to households in a lump-sum fashion. The household seeks to maximize (25) subject to the following constraints:

\[
M_t + P_t C_t = M_{t-1} + W_t H_t + P_t (R_t - 1) D_t + V_t - P_t T_t
\]

\[
P_t C_t \leq M_{t-1} + W_t H_t - P_t D_t
\]

where \( D_t = \int_0^1 D_{n,t} \, dn \) are total deposits and \( V_t = \int_0^1 V_{M,i} \, dj + \int_0^1 V_{R,i} \, di + \int_0^1 V_{B,n} \, dn \) are total profits rebated to the household in each period by manufacturers, retailers and banks. Equation (28) is a standard budget constraint. On its right hand side, the household’s resources come from previous period money balances, wage income, interest payments on intra-period deposits, and distributed profits net of lump sum taxes. Equation (29) is a cash-in-advance constraint: the household’s consumption expenditure cannot exceed money balances accumulated from the previous period, plus wage income, net of resources deposited at the banks. This constraint represents the implicit cost of holding intra-period deposits: money deposited at the bank yields interest, but cannot be used for transaction services.\(^\text{26}\) Taking first order conditions and rearranging, we obtain the following relationships:

\[
C_t^{-\sigma} = \beta R_t E_t \left[ \frac{C_{t+1}^{\sigma}}{\Pi_{t+1}} \right]
\]

\[
H_t^{\sigma} C_t^{\sigma} = W_t
\]

where \( \Pi_{t+1} = \frac{P_{t+1}}{P_t} \) is gross CPI inflation. Equation (30) is a standard Euler equation relating consumption growth to the \textit{ex ante} real interest rate. Equation (31) describes the household’s labor supply schedule.

\[2.5 \text{ The Government}\]

The government is made of a fiscal and a monetary authority. The fiscal authority sets lump sum taxes to finance the labor subsidy to firms in the manufacturing sector. It is subject to a balanced-budget rule:

\[\text{\textsuperscript{26}Without a cash-in-advance constraint the gross deposit rate (and hence the policy rate too) would be equal to one in every period.}\]}
\[ P_t T_t = \tau W_t \tilde{H}_t \tag{32} \]

where \( \tilde{H}_t = \int_0^1 \tilde{H}_{j,t} dj \) is total labor demand by manufacturers.

Our analysis will mainly focus on optimal monetary policy, whereby the monetary authority sets the short-term riskless interest rate on deposits \( R_t \) in order to maximize aggregate welfare. We will consider both the case of a monetary authority acting under commitment (Ramsey policy) and under discretion (time-consistent policy). We will also compute the welfare costs of adopting simpler but sub-optimal policy rules, such as instrumental Taylor rules and policies of both strict and flexible inflation targeting.

3 Equilibrium

We consider a semi-symmetric equilibrium in the following sense. On the one hand, all banks in the banking sector and all firms in the manufacturing sector will behave identically: that is, banks will set the same interest rate and supply the same amount of loans to all firms, while manufacturers will hire the same amount of labor, produce the same amount of homogeneous intermediate goods and take on the same amount of loans from banks. On the other hand, due to Calvo contracts, there will be price dispersion in the retail sector: in equilibrium a fraction \( \vartheta \) of firms will not be able to optimally reset its price, while a fraction \( 1 - \vartheta \) will. As a consequence, the pricing and production of final goods will differ across retail firms.

Symmetry in the banking sector implies that \( R_{n,t} = R_t \) and \( L_{n,t} = L_t \) for \( n \in [0,1] \), while symmetry in the manufacturing sector implies that \( l_{j,n,t} = l_{n,t} \), \( x_{j,t} = x_t \), \( \tilde{H}_{j,t} = \tilde{H}_t \) and \( I_{j,t} = I_t \) for \( j \in [0,1] \) (such that the last two equalities also imply that \( I_t = A_t \tilde{H}_t \)). Since market clearing requires that \( L_{n,t} = l_{n,t} \) for \( n \in [0,1] \), then \( l_{n,t} = l_t = L_t \) as well as \( s_{n,t} = s_t \) for \( n \in [0,1] \). Using these conditions, from (9), we obtain the following relationship:\(^{27}\)

\[ x_t = l_t - \theta s_{t-1} \tag{33} \]

Under symmetry, using equations (22) and (33), the credit spread equation (23) becomes:

\[ (R_t^L - R_t) = \frac{R_t^L}{\xi_t} \left( 1 - \frac{\theta}{\gamma_t} \right)^{-1} - \frac{\theta(1 - \rho_s)}{\xi_t} E_t \left[ \mathcal{F}_{t,t+1} \Pi_{t+1} R_{t+1}^L \left( 1 - \frac{\theta}{\gamma_{t+1}} \right)^{-1} \right] \tag{34} \]

where \( \gamma_t \equiv \frac{l_t}{s_{t-1}} \). Equation (34) further stresses the intertemporal dimension of optimal loan rate setting by banks due to deep habits in credit markets. By forward iteration, it immediately follows that the current rate on bank loans depends both on current as well

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\(^{27}\)Notice that this can be also obtained by imposing symmetry on (2).
as expected future market conditions, in particular on expected future policy rates and growth of loan demand.

Real marginal costs faced by retail firms are given by:

\[ mc_t \equiv \frac{MC_t}{P_t} = \frac{Q_t}{P_t K} \quad (35) \]

where \( Q_t \) has been defined in (11). By market clearing in the final goods market \( Y_{i,t} = C_{i,t} \) for \( i \in [0,1] \) such that \( Y_t = C_t \), the relative demand (27) and technology in the retail sector (12), we have that \( K \tilde{I}_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t \). After integrating both sides across the continuum of \( i \)-indexed retail firms, using the market clearing condition for the intermediate goods market \( \int_0^1 \tilde{I}_{i,t} di = I_t \), the technology in the manufacturing sector \( I_t = A_t H_t \), market clearing in the labor market \( \tilde{H}_t = H_t \), and defining the price dispersion index \( \Xi_t \equiv \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} di \), we obtain the following equilibrium condition:

\[ Y_t \Xi_t = K A_t H_t \]

Finally, under symmetry the working capital constraint (8) and the household’s labor supply schedule become, respectively:

\[ x_t = \alpha (1 - \tau) \frac{W_t}{P_t} H_t \quad (36) \]

\[ H_t^\sigma C_t^\sigma = \frac{W_t}{P_t} = A_t \quad (37) \]

## 4 Steady State

We focus on a zero inflation (\( \Pi = 1 \)) non-stochastic steady state equilibrium where \( A_t = 1 \), \( \xi_t = \xi \), and all remaining variables are constant. Without loss of generality, we set \( K = 1 \) in the retail sector. First, from the law of motion of real loans (33), we obtain \( s = l \) and \( x = (1 - \theta) s \). Since \( x \) has to be positive, the second equality restricts the deep habits parameter \( \theta \) to be smaller than unity.

From the households Euler equation (equation (30)) we obtain the steady state gross interest rate: \( R = \beta^{-1} \). From equation (34), we have that the lending rate \( R^L \) is equal to the riskless rate \( R \) times the constant gross credit mark-up \( \mu^R \):

\[ R^L = \mu^R R \quad (38) \]

where

\[ \mu^R \equiv \frac{m \xi}{m \xi - 1} \quad \text{and} \quad m \equiv \frac{(1 - \theta)}{[1 - \theta \beta (1 - \rho_s)]} \quad (39) \]
By imposing an upper bound \( \bar{\theta} < 1 \) on the deep habits parameter \( \theta \), the following assumption guarantees that \( \mu^R > 1 \) (hence, a positive credit spread) at the steady state.

**Assumption 1:** \( \theta < \frac{\xi - 1}{\xi - \beta (1 - \rho_s)} \equiv \bar{\theta} \).

Since \( \frac{\partial \mu^R}{\partial \theta} > 0 \), the steady state mark-up under deep habits, \( \frac{m \xi}{m \xi - 1} \), is always larger than what would obtain under monopolistic competition in banking alone, \( \frac{\xi}{\xi - 1} \), for any \( \theta \in (0, \bar{\theta}) \). Moreover, this difference appears to be more significant the larger the persistence of habits: i.e., \( \frac{\partial}{\partial \rho_s} \left( \frac{\partial \mu^R}{\partial \theta} \right) > 0 \).

From the optimal price setting by retailers, we have that, at the steady state, real marginal costs are equal to the inverse of the steady state gross mark-up in the final goods market, \( \mu \equiv \frac{\xi}{\xi - 1} \):

\[
MC = (1 - \tau) \frac{W}{P} \left[ 1 + \alpha (R_L - 1) \right] = \frac{1}{\mu}
\]

We assume that the fiscal authority sets the subsidy rate \( \tau \) to equate the marginal rate of substitution between consumption and labor to the marginal productivity of labor. This implies setting \( \tau \) to make \( \frac{W}{P} = 1 \), or, more specifically, \( \tau = \frac{\mu [1 + \alpha (R_L - 1)]^{-1}}{\mu [1 + \alpha (R_L - 1)]} \). The latter together with the aggregate technology, \( Y = H \), market clearing, \( Y = C \), and the labor supply equation (31) implies that \( Y = 1 \).

### 5 Log-Linearization and Aggregate Dynamics

We log-linearize the equilibrium conditions around the unique non-stochastic steady state.\(^{29}\)

Before doing that, we define \( \mu^R_t \equiv \frac{R^L_t}{R^R_t} \) as the gross mark-up in the loan market, such that \( \hat{\mu}_t = \hat{r}_t^L - \hat{r}_t \). Henceforth, we will refer to \( \hat{\mu}_t^R \) as the credit spread. From the households Euler equation (30) and market clearing in the goods market we obtain:

\[
\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} \left( \hat{r}_t - E_t \hat{\pi}_{t+1} \right)
\]  

(40)

From the optimal pricing rule of retail firms (17), we obtain the linearized New-Keynesian Phillips curve:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{mc}_t,
\]

(41)

\(^{28}\)For \( \theta > \bar{\theta} \), we have that \( \mu^R < 0 \). Assumption 1 easily holds for any realistic parameterization of the degree of imperfect competition in banking, as indexed by \( \xi \). For instance, if \( \rho_s = 0 \) (i.e., the stock of habits is equivalent to the level of past period loans), then \( \bar{\theta} = \frac{\xi - 1}{\xi - \beta} \), which is very close to unity for \( \beta \approx 1 \). If instead \( \rho_s \to 1 \), then \( \bar{\theta} \to \frac{\xi - 1}{\xi} \). The latter is also rather close to unity unless one assumes an unrealistically high degree of market power in banking (i.e. low \( \xi \)).

\(^{29}\)Hatted lower-case variables denote percentage deviation of variables from their respective steady state.
where, from the linearization of (11) and (35),

$$\hat{μ}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t + η\hat{ρ}_t$$

(42)

with $κ = \frac{(1-θ)θ(1-β)}{θ}$, $η = \frac{α\hat{μ}_R}{(1-α)β+α\hat{μ}_R}$ and $\hat{a}_t \equiv \ln A_t$ following the process in (1). The real wage $\hat{w}_t - \hat{p}_t$ is obtained from the consumption-leisure trade-off condition (37) combined with market clearing, $C_t = Y_t$, and the aggregate technology, $Y_t = A_tH_t$:

$$\hat{w}_t - \hat{p}_t = (σ + φ)\hat{y}_t - φ\hat{a}_t$$

(43)

It follows that the inflation dynamics in our economy are regulated by a Phillips curve augmented with a cost channel and a time-varying credit spread:

$$\hat{π}_t = βE_t\hat{π}_{t+1} + κ(σ + φ)\hat{y}_t + κη\hat{r}_t + κη\hat{μ}_R^R - κφ\hat{a}_t$$

(44)

The dynamics of $\hat{μ}_R^R$ are then determined by the log-linearized version of (34), describing the optimal interest rate setting in the banking sector. After simple manipulation we obtain:

$$\left[1 - \frac{ω}{1 - βθ(1 - ρ_s)}\right]\hat{μ}_R^R = \frac{(1-θ)^{-1}θω}{1 - βθ(1 - ρ_s)} [θβ(1 - ρ_s)E_t\hat{γ}_{t+1} - \hat{γ}_t] - \frac{θβω(1 - ρ_s)}{1 - βθ(1 - ρ_s)}(E_t\hat{r}_{t+1} - \hat{r}_t) - \frac{θβω(1 - ρ_s)}{1 - βθ(1 - ρ_s)}E_t\hat{μ}_R^R_{t+1} + \frac{θβω(1 - ρ_s)}{1 - βθ(1 - ρ_s)}(\hat{r}_t - E_t\hat{π}_{t+1}) - ω\dot{ξ}_t$$

(45)

where $ω \equiv \frac{1-βθ(1-ρ_s)}{ξ(1-θ)}$, $\hat{γ}_t = \hat{l}_t - \hat{s}_{t-1}$ and $\dot{ξ}_t \equiv \ln (ξ_t/ξ)$ evolves according to (4). Using the definition of $ω$, simple algebra shows that the term within squared brackets multiplying $\hat{μ}_R^R$ on the left hand side of (45) is strictly positive if and only if $θ < \frac{ξ-1}{ξ}$. Since $ω > 0$ always, then, as long as $θ < \frac{ξ-1}{ξ}$, a negative shock to $\dot{ξ}_t$ (higher market power in banking) is equivalent to a positive shock to the credit spread. Notice that without deep habits, $θ = 0$, equation (45) reduces to $\hat{μ}_R^R = -\frac{\dot{ξ}_t}{ξ-1}$. In this case, the credit spread $\hat{μ}_R^R$ is equivalent to an exogenous cost push shock entering the Phillips curve (44).

A close inspection of (45) shows how the credit spread reacts to different market forces. First of all, since in our model firms borrow more during upturns, by the first term in squared bracket on the right hand side, the credit spread decreases during current booms (higher $\hat{γ}_t$), but increases in response to expected future ones (higher $E_t\hat{γ}_{t+1}$). Second, it decreases with respect to the expected future change in the deposit rate, $E_t\hat{r}_{t+1} - \hat{r}_t$: banks would rather wait to charge higher rates on loans if they foresee a higher cost on deposits in

30The inequality $θ < \frac{ξ-1}{ξ}$ is stricter than what required by Assumption 1. However, it easily holds for any calibration of $ξ$ consistent with the average credit spreads observed in the data.
the near future. Third, it decreases with respect to expected future spreads, $E_t \hat{\mu}_{t+1}$: once a bank anticipates the possibility of charging higher spreads in the future, it would most likely lower current rates to expand its customer base. Fourth, it increases with respect to the real interest rate, $\hat{r}_t - E_t \hat{\pi}_{t+1}$: a higher real interest rate lowers the present discounted value of future bank’s profit, thus making the building of a future customer base (through lower current rates) less appealing.

From the linearization of (33) and (36), and the expression for the real wage in (43), we obtain:

$$\hat{L}_t = \theta \hat{s}_{t-1} + (1 - \theta) \hat{x}_t \quad (46)$$

where

$$\hat{x}_t = (1 + \sigma + \varphi) \hat{y}_t - (1 + \varphi) \hat{a}_t \quad (47)$$

After linearizing (3) and using (46), we obtain an expression for the law of motion of the stock of habits:

$$\dot{s}_t = \left[ \rho_s + (1 - \rho_s) \theta \right] \hat{s}_{t-1} + (1 - \rho_s) (1 - \theta) \hat{x}_t \quad (48)$$

The model is closed by a path for the nominal interest rate $\hat{r}_t$ chosen by the monetary authority.

The next section describes the optimal monetary policy plan.

### 6 Optimal Monetary Policy

We study the consequences of deep habits in banking for the design of optimal monetary policy under both discretion and commitment. Before performing such analysis, we define the efficient allocation in our economy. The latter is derived by solving a social planner’s optimization problem, which, given the absence of capital, is equivalent to maximizing the representative agent’s temporary utility subject to the aggregate technology $Y_t = A_t H_t$ and market clearing $Y_t = C_t$.

$$\max_{c_t} \frac{c_t^{1-\sigma} - H_t^{1+\varphi}}{1 - \sigma} \frac{1 + \varphi}{s.t. \quad Y_t = A_t H_t = C_t}$$

The solution to this problem gives us $Y_t^e$, the efficient level of output in the economy:\footnote{\textsuperscript{31}The social planner overcomes all frictions in the economy, namely, monopolistic competition, the need for cash in transactions, the nominal rigidities in the intermediate good sector, the working capital needs in manufacturing, and hence the need for financial intermediation.}

\footnote{\textsuperscript{32}As in Ravenna and Walsh (2006), in our model the efficient level of output $Y_t^e$ is not equal to the level of output that would occur under flexible prices, $Y_t^f$. Under flexible prices (i.e. $\vartheta = 0$ in (17)) real
Taking logs of both sides, we obtain that (log) efficient output is proportional to (log) TFP:

$$\hat{y}_t^e = 1 + \frac{\varphi}{\sigma + \varphi} \hat{a}_t$$

(51)

The central bank’s objective is obtained by taking a second order approximation to the representative agent’s welfare. Similar to a benchmark New-Keynesian model, the objective is expressed in terms of squared deviations of inflation and output from their respective efficient levels:

$$L_0 = -\frac{1}{2} \frac{\epsilon}{\kappa} \bar{H}^{1+\varphi} E_0 \sum_0^\infty \beta^t \left[ \pi_t^2 + \psi (\hat{y}_t^\theta)^2 \right]$$

(52)

where \( \bar{H} \) is steady state hours, \( \tilde{y}_t^\theta \equiv \hat{y}_t - \hat{y}_t^e \) is the welfare relevant output gap and \( \psi \equiv \frac{(\sigma + \varphi) \kappa}{\epsilon} \) is the central bank’s relative concern for output gap versus inflation stabilization in the micro-founded loss. The latter does not depend on structural parameters related to the credit constraint on firms (the share of the wage bill to be paid in advance, \( \alpha \)) and banking (the degree of imperfect competition, \( \xi \), or the measure of deep habits, \( \theta \)). Optimal monetary policy requires the maximization of (52) subject to a set of equilibrium constraints given by the IS equation (30) together with the NK Phillips curve (44), the spread equation (45), the equations for \( l \) and \( x \), respectively (46) and (47), and the law of motion for \( s \) in (48), where output \( \hat{y}_t \) is now expressed as \( \hat{y}_t = \tilde{y}_t^\theta + \tilde{y}_t^e \).

As an analytical characterization of the optimal plan is unattainable (under both discretion and commitment), we resort to numerical methods and study the model economy’s response to exogenous TFP and credit shocks under both regimes. For this purpose, we adopt the following quarterly calibration. We set the risk aversion parameter \( \sigma \) equal to 2 and labor disutility parameter \( \varphi \) equal to 1/4. The latter is consistent with a Frisch elasticity of labor supply equal to 4, as supported by the macro-based labor literature. The subjective discount factor \( \beta \) is set to 0.99, which gives a 4% annual real interest rate. For marginal costs \( \frac{MC_t}{\mu} \) are equal to the inverse gross mark-up \( \frac{1}{\mu} \). This, together with the households’ labor supply condition and market clearing, implies:

$$Y_t^e = \Psi_t^{-1} Y_t^\psi$$

(49)

where \( \Psi_t \equiv \mu (1 - \tau) \left[ 1 + \alpha \left( R_t^{L^f} - 1 \right) \right] \) and \( R_t^{L^f} \) denotes the lending rate under flexible prices. Clearly, \( Y_t^\psi = Y_t^f \) in a model without the cost channel (\( \alpha = 0 \)), given that, at the steady state, we would have \( \mu (1 - \tau) = 1 \). With a cost channel instead, the equivalence is broken, and the flexible price level of output deviates from efficiency because of the interest rate distortion, \( R_t^{L^f} \). Because of the assumption of a constant labor subsidy to manufacturing firms, the efficient equilibrium and the equilibrium of our decentralized economy share the same steady state, which is therefore undistorted.
the degree of imperfect competition in the goods market, we choose \( \epsilon = 6 \), corresponding to a net markup of 20\%. For what concerns the degree of price rigidity, we set the Calvo probability of no price change equal to 0.66. This gives an average duration of prices equal to three quarters, consistent with the empirical evidence provided by Nakamura and Steinsson (2010). We set both AR(1) coefficients \( \rho_a \) and \( \rho_\xi \) equal to 0.9.\(^{33}\)

To better highlight the quantitative role of deep habits in credit markets, we will consider two alternative parametrizations for the steady state gross mark up in credit markets, \( \mu^R \). In the first case, \( \mu^R \) is set to give a 2\% annual credit spread (we will refer to this case as the case of a “developed economy”, e.g. the U.S. or Western Europe). In the second case \( \mu^R \) is set to give a 4\% annual credit spread (we will refer to this as the case of an “emerging market economy”, e.g. a Latin American or Asian high growth economy).\(^{34}\)

For what concerns the degree of deep habits in credit markets, we will consider two alternative values: \( \theta = 0 \) (no deep habits, which will make our model isomorphic to the cost channel model studied by Ravenna and Walsh, 2006), and \( \theta = 0.5 \) (deep habits in credit markets, which is consistent with the empirical estimates by Aliaga-Diaz and Olivero, 2010). For given parameterization, using the expressions (38) and (39), we will then retrieve the elasticity of substitution across bank loans, indexing the degree of imperfect competition in the banking system which is consistent with each of the two calibrated values of \( \mu^R \).

6.1 The Case of Discretion

Under discretion, we compute the optimal time consistent monetary policy. For this purpose, we restrict the analysis to the concept of Markov-Perfect-Equilibrium, that is, an equilibrium where endogenous variables are functions only of relevant state variables, e.g. the outstanding stock of habits \( \hat{s}_{t-1} \), and the shocks \( \hat{a}_t \) and \( \hat{\xi}_t \). Although the lack of commitment implies that policy announcements are not credible, current policy choices can still affect future expectations via their impact on the stock of habits \( \hat{s}_t \), a state variable in the next period. Despite the fact that the policy-maker cannot strategically exploit this linkage (as it takes it as a given equilibrium relationship), the optimal monetary policy problem is dynamic also under discretion. The latter is an important element of differentiation with respect to the cost channel model of Ravenna and Walsh (2006) for which, because of the absence of endogenous states, the optimal time consistent policy can be found by solving (analytically) a simple static loss minimization problem. Since this is not possible in our case, we solve for the optimal monetary policy under discretion following the computational

\[^{33}\text{Our results are qualitatively robust to alternative parameterizations.}\]

\[^{34}\text{The average credit spreads for the U.S. and for emerging markets are computed using data from the FRED Dataset at the Federal Reserve Bank of St. Louis. Bleaney et al. (2016) provide evidence on credit spreads across major Western European countries.}\]
procedure proposed by Soderlind (1999).\textsuperscript{35}

Figure 1 displays the impulse responses to a one-percent shock to TFP for key endogenous variables, under three alternative parametrizations: namely, the cost channel model of Ravenna and Walsh (2006) (where $\alpha = 1$ but $\theta = 0$: there is a cost channel but deep habits in banking are absent); and two versions of our deep habits economy differing with respect to the size of the steady state credit spread, as discussed in the previous section.\textsuperscript{36}

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Impulse responses to TFP shock for optimal monetary policy under discretion}
\end{figure}

Consider first the case studied by Ravenna and Walsh (2006). Under the optimal policy, the economy displays a positive output gap, a deflation and a drop in the policy rate. These responses differ from the benchmark NK model without a cost channel where, by lowering the policy rate, the monetary authority is successful at stabilizing both inflation and the output gap at their efficient steady state level. Since with a cost channel movements in the nominal interest rate also affect the credit-related component of marginal costs in the NKPC, the nominal interest rate cannot be used to fully insulate the economy from the technology shock. It therefore appears optimal to generate a larger drop in the policy rate (with respect to the no cost channel case), while letting output and inflation deviate from full efficiency. To capture the underlying mechanism, consider a positive shock to TFP, which, without any policy intervention would determine a negative output gap. Lowering the nominal interest rate by as much as in the no cost channel New-Keynesian model might suffice to restore a zero output gap, but at the cost of a deflation as the policy rate (also the lending rate) pulls down the interest rate component of marginal costs in the Phillips curve. To counteract this, optimal policy requires the central bank to lower the nominal rate by a larger amount. By doing this, the central bank induces a positive output gap, which contains the deflation by raising the wage component of marginal costs.

Deep habits amplify the transmission we have just described and determine an even stricter stabilization trade-off for the central bank. As a result, the optimal equilibrium features an even larger output gap, a stronger deflation and a bigger drop in the policy rate. Key to this mechanism is the counter-cyclicality of credit spreads generated by deep habits. By creating a positive output gap, the expansionary monetary policy lowers the credit spread, which in turn hampers the decline in inflation. The central bank counteracts this channel by lowering the nominal interest rate even further, with the objective of creating an

\textsuperscript{35}A similar approach is used in Steinsson (2003) and Leith et al. (2012).
\textsuperscript{36}The results for the case of a benchmark New-Keynesian model where the cost channel is completely absent are already well-known, and we do not show them here.
even larger output gap in the attempt to increase the wage component of marginal costs (via a standard aggregate demand channel) and hence contain deflation. As the figure shows, this channel appears to be stronger in an emerging market economy where the (average) credit spread is larger. From equation (44) and the definition of the composite parameter \( \eta \), it is easy to see that a larger \( \mu^R \) makes the credit spread dynamics quantitatively more relevant for inflation determination, leading to an amplification of the cost channel of policy transmission.\(^{37}\)

Another interesting consequence of deep habits is the hump-shaped response of loans to the TFP shock.\(^{38}\) Given that output, \( \hat{y}_t = \hat{y}_t^d + \hat{y}_t^c \) (not plotted) positively responds on impact to the TFP shock, this result implies that in our model loans lag output, a feature that is consistent with the empirical evidence provided by Demirel (2012). He finds that business loans are more positively correlated with past than with current output, both in the U.S. and the Euro area, a feature that a standard model of business and credit fluctuations with agency costs cannot generate. While Demirel (2013) shows that this could be amended with the introduction of costly financial intermediation, we are able to obtain a similar pattern through deep habits in banking.

\[\text{Figure 2 about here}\]

\[\text{Figure 2: Impulse responses to credit spread shock for optimal monetary policy under discretion}\]

Figure 2 considers instead the case of a 1% unanticipated shock to the credit spread.\(^{39}\) Without deep habits, a higher spread generates positive inflation (through a cost-push effect) and a negative output gap (through a negative impact on aggregate activity). To counteract higher prices, the central bank raises the policy rate, which, together with the exogenously-driven higher spread, leads to a larger lending rate and a subsequent further pressure on inflation. The positive response of inflation and the negative response of output are exacerbated by the introduction of deep habits: as output declines, the credit spread increases beyond the initial exogenous shock; this leads to a further cost-push increase in inflation and a harsher recession.\(^{40}\)

\(^{37}\)Other things being equal, the responses to a TFP shock can be amplified by increasing the size of \( \theta \).

\(^{38}\)This is not limited to the case of optimal policy under discretion, but equally holds under commitment (see below).

\(^{39}\)Since a period in the model corresponds to one quarter, the shock corresponds to a 4% increase in the credit spread at an annual frequency. Given the expression in (45), other things being equal, a one percent positive exogenous shock to the credit spread \( \tilde{\mu}_t^R \) is equivalent to a \( \frac{0.01}{\omega} \left[ 1 - \frac{\omega}{1 - \beta \theta (1 - \rho_s)} \right] \) percent negative shock to the elasticity \( \tilde{\xi}_t \).

\(^{40}\)These results contrast with those obtained in the earlier literature according to which aggressive monetary easing is optimal in response to financial shocks (see for instance Aikman and Paustian (2006), Carlstrom et al (2009) and De Fiori and Tristani (2012)).

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To better grasp the role played by deep habits in banking for optimal policy design, it is useful to compare the targeting rule implied by our model (DH) with those one would obtain in a benchmark New-Keynesian set-up (NK) and in the standard cost channel model (CC). As we show in full details in Appendix A.1, for given future expectations, the targeting rule can be written as follows:\footnote{For both the NK and the CC models, the targeting rule's expressions implied by (53) are exact. For the DH model instead, the targeting rule (53) also includes a term depending on expected future inflation. See the Appendix for the details, as well as for the definitions of the composite parameters entering $\Psi_{DH}$.}

\begin{equation}
\hat{\pi}_t = -\frac{\psi}{\Psi_k} \hat{y}_t^g, \text{ for } k = NK, CC, DH,
\end{equation}

for

\begin{align*}
\Psi_{NK} &\equiv \kappa (\sigma + \varphi) \\
\Psi_{CC} &\equiv \{\kappa (\sigma + \varphi) - \kappa \eta \sigma\} \\
\Psi_{DH} &\equiv \{\kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta)\},
\end{align*}

where $\Theta$ is a positive composite parameter defined in equation (A.13) in the appendix, and $\eta \equiv \frac{\alpha \bar{\mu}^\beta}{(1-\alpha)\beta + \alpha \bar{\mu}^\beta}$ as previously defined. These analytical expressions allow us to highlight the following results. First, by the definition of $\psi$ and $\Psi_{NK}$, we have that $\frac{\psi}{\Psi_{NK}} = \epsilon^{-1}$. As $\epsilon > 1$, the latter implies that, in benchmark New Keynesian setting, under the optimal policy, inflation is always less volatile than the output gap. Second, since $\eta \in (0, 1]$, we also have that $\Psi_{CC} < \Psi_{NK}$. This inequality means that, for given output gap volatility, the cost channel model features higher inflation volatility with respect to the benchmark New Keynesian model. As already pointed out by Ravenna and Walsh (2006), the credit channel makes it harder to stabilize prices as the central bank has to move the policy rate to counteract the (supply-side) shocks to TFP. In addition to that, inflation can be optimally more volatile than output - that is, $\frac{\psi}{\Psi_{CC}} > 1$ - if $\eta > \frac{1}{\epsilon}$, which occurs if $\alpha$ is sufficiently large.\footnote{Notice that this is always the case if $\alpha = 1$ (100% working capital requirement) since, in that case, $\eta = 1$.} Third, as shown in the Appendix, we also have that $\Psi_{DH} < \Psi_{CC} < \Psi_{NK}$. According to this last inequality, deep habits in credit market imply a harsher output gap-inflation stabilization trade-off under the optimal policy, which leads to a more significant departure from full price stability.\footnote{It is possible to show numerically that $\Psi_{DH}$ is strictly decreasing in $\theta$.}

6.2 The Case of Commitment

Under commitment the policy-maker announces and implements the optimal state-contingent Ramsey plan that maximizes aggregate welfare, taking into account its direct impact on
individual expectations. This allows the policy-maker to attain a better trade-off between stabilizing inflation and the output-gap in the face of any shock affecting price setting by individual firms. Figure 3 displays the results in response to a TFP shock for alternative parametrizations, as for the case of discretion displayed in Figure 1.

[Figure 3 about here]

[Figure 3: Impulse responses to TFP shock for optimal monetary policy under commitment]

As for the case of discretion, deep habits exacerbate the inflation-output gap stabilization trade-off faced by the central bank. The optimal response of the economy to the technology shock is a positive output gap, and an initial decline followed by an increase in inflation. The mechanism that leads to larger deviations from full efficiency for a positive $\theta$ is again related to the counter-cyclicality of credit spreads generated by deep habits. As the figure shows, increasing the steady state mark-up in credit markets amplifies the impact of a TFP shock, bringing the economy further away from full stabilization of inflation and the output gap. As expected, conducting monetary policy under commitment achieves a better stabilization trade-off with respect to the case of discretion, for all parametrization considered in Figures 1 and 3. This allows the economy - in particular inflation and the output gap - to remain closer to the efficient (steady state) allocation.

The impulse response functions in response to a shock to credit spreads under commitment are presented in Figure 4. In this case, larger spreads introduced through either deep habits and/or smaller elasticity of the demand for credit do not change the results significantly. Under commitment fluctuations are contained by roughly the same amount across all three parameterization.

[Figure 4 about here]

[Figure 4: Impulse responses to credit spread shock for optimal monetary policy under commitment]

6.3 Welfare Analysis

In this section we evaluate the quantitative importance of deep habits in banking for what concerns the welfare costs associated with 1) the policy-maker’s inability to commit to the optimal Ramsey plan; and 2) the adoption of simpler but sub-optimal policy rules. In both cases, we compute the welfare cost both in terms of consumption equivalent (CE) and inflation equivalent (IE) variations.
With respect to CE, we follow Schmitt-Grohé and Uribe (2007) and define the welfare cost of adopting an alternative policy \( A \) with respect to a reference policy \( R \) as the fraction \( \nu \) of the consumption path under policy \( R \) that must be given up to make the household as well off under policy \( A \) as under policy \( R \). In our case policy \( R \) is commitment and policy \( A \) is discretion. With respect to IE, we follow Dennis and Soderstrom (2006) by computing the permanent decrease in annual inflation needed to compensate the household for a switch from commitment to discretion.

### 6.3.1 Discretion versus Commitment

Since the seminal work of Kydland and Prescott (1977), it is well known that policy actions taken under commitment deliver higher welfare than those under discretion. As private agents’ decisions depend on future expectations, by announcing credible policy plans, a committed government can strategically manipulate expectations and attain a more favorable policy trade-off between stabilizing the output gap and inflation. The impulse response analysis presented in the previous section highlights the possibility of even higher welfare gains if deep habits in credit markets are at work.

<table>
<thead>
<tr>
<th>Table 1. Welfare Analysis: Discretion versus Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal Relative Volatility:</strong> ( SD(\pi) / SD(y^g) )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>A. Benchmark NK model</td>
</tr>
<tr>
<td>B. Credit Channel model</td>
</tr>
<tr>
<td>1. ( \theta = 0 )</td>
</tr>
<tr>
<td>2. ( \theta = 0.25 )</td>
</tr>
<tr>
<td>a. 2% spread</td>
</tr>
<tr>
<td>b. 4% spread</td>
</tr>
<tr>
<td>3. ( \theta = 0.5 )</td>
</tr>
<tr>
<td>a. 2% spread</td>
</tr>
<tr>
<td>b. 4% spread</td>
</tr>
</tbody>
</table>

Table 1 presents the results of our analysis for the benchmark New Keynesian model (where the credit channel is absent) and alternative parametrizations of the credit channel model (differing in the extent of deep habits and the steady state size of the credit spread). The second column reports the optimal relative volatility (ORV) - i.e., the standard deviation of inflation relative to that of the output gap occurring under the optimal discretionary policy - while the third and fourth columns report the CE and IE welfare cost of acting under discretion (with respect to commitment). As already stressed towards
the end of Section 6.1, in the benchmark setting, optimal monetary policy makes inflation less volatile than the output gap. For this simple case, the relative standard deviation is in fact equal to $\epsilon^{-1}$, which equals 0.16 under our calibration.

It also appears that the welfare costs of acting under discretion are quite negligible, both in CE and IE terms. This is clearly not the case once a credit channel and deep habits in banking are considered. In particular, the following results emerge. First, the credit channel makes inflation more volatile than output. For the case of $\theta = 0$, the ORV is equal to $\frac{\sigma + \varphi}{\sigma(1 - \eta) + \varphi}$, which reduces to $\frac{\sigma + \varphi}{\epsilon \varphi} = 1.5$ under our calibration. This pattern is reinforced by our banking friction: if deep habits and/or the credit spread are strengthened, the ORV of inflation to output increases. This finding is consistent with the analytical result presented of equation (53), where, for given output gap volatility, the departure from price stability is larger the stronger the banking friction.

Second, the welfare costs of acting under discretion (with respect to commitment) are strictly increasing in the degree of deep habits and the size of the credit spread. For instance, if $\theta = 0.5$ (B.3. in the table) the CE costs are fifty percent larger than those occurring without deep habits with a 2% spread, and twice as large with a 4% spread. This effect is also evident if measured in terms of permanent inflation. For $\theta = 0.5$ and a 2% spread, the household would require almost a 1.6% permanent decrease in yearly inflation in order to accept a switch from full commitment to discretion. This value goes up to almost 1.8% if the credit spread becomes 4%.

Figure 5 summarizes the main findings of the welfare analysis. The left-hand panel displays the ORV (under discretion) as a strictly increasing function of $\theta$. While the same pattern remains, a higher average credit spread significantly increases the ORV for any given $\theta$. The right-hand panel displays instead the CE welfare costs of setting policy under discretion. Similarly to the ORV, welfare costs appear to be strictly increasing in $\theta$, with the size of the average credit spread acting as a upward shifter.

[Figure 5 about here]

[Figure 5: Optimal relative volatility (under discretion) and welfare costs: the role of deep-habits in credit markets]

6.3.2 Sub-Optimal Policy Rules

Optimal monetary policy requires the monetary authority to have full knowledge of the economy’s underlying structural relationships, otherwise it would not be able to attain the desired inflation-output stabilization. It is therefore interesting to consider the welfare consequences of adopting sub-optimal policy rules which do not require full information on behalf of the monetary authority. More specifically, we compute the CE variation welfare
costs (with respect to the truly optimal policy under commitment) for the following alternative policy rules: strict inflation targeting, SIT (i.e. the nominal interest rate is moved in order to keep inflation at its target at all times, $\hat{\pi}_t = 0$); flexible inflation targeting, FIT (i.e. the inflation-output gap targeting rule that the monetary authority would implement in the benchmark New-Keynesian model without credit frictions, $\hat{\pi}_t = -\frac{\psi}{\kappa(\sigma+\varphi)}\Delta\hat{y}^g_t$); a standard Taylor rule, TR$_1$ (i.e. the nominal interest rate $\hat{r}_t$ is set according to the rule $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t$, with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, as proposed by Taylor (1993)); a spread-augmented Taylor rule, TR$_2$ (i.e. the policy rate responds negatively to the credit spread, $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \phi_\mu \hat{\mu}_t^R$, with $\phi_\mu = 0.25$); and a credit-augmented Taylor rule, TR$_3$ (i.e. the policy rate responds positively to loans, $\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_l \hat{l}_t$, with $\phi_l = 0.25$). Notice that both Taylor rules are expressed only in terms of observables. Namely, it is assumed that the monetary authority responds to the output level and not the output gap, as the latter is hardly observed with precision by the policy-maker. The negative and positive sign restrictions imposed, respectively, on $\phi_\mu$ and $\phi_l$ follow the common wisdom in the related literature.\footnote{See, for instance, Curdia and Woodford (2010) and Christiano et al. (2010).}

The results are reported in Table 2.

<table>
<thead>
<tr>
<th>Table 2. Welfare Analysis: Sub-Optimal Policy Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare Costs (CE Variations) w.r.t. Commitment</td>
</tr>
<tr>
<td>SIT</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>A. Benchmark NK Model</td>
</tr>
<tr>
<td>2.22E-0.14</td>
</tr>
<tr>
<td>B. Credit Channel Model</td>
</tr>
<tr>
<td>1. $\theta = 0$</td>
</tr>
<tr>
<td>IND 1.13E-05</td>
</tr>
<tr>
<td>2. $\theta = 0.5$</td>
</tr>
<tr>
<td>a. 2% spread</td>
</tr>
<tr>
<td>IND 2.25E-05</td>
</tr>
<tr>
<td>b. 4% spread</td>
</tr>
<tr>
<td>IND 3.53E-05</td>
</tr>
</tbody>
</table>

IND: indeterminacy; IT: strict inflation targeting ($\hat{\pi}_t = 0$ for every $t$);
FIT: flexible inflation targeting ($\hat{\pi}_t = -\frac{\psi}{\kappa(\sigma+\varphi)}\Delta\hat{y}^g_t$, as in benchmark NK model);
TR$_1$: standard Taylor rule ($\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t$, for $\phi_\pi = 1.5$, $\phi_y = 0.5/4$);
TR$_2$: spread-augmented Taylor rule ($\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t - \phi_\mu \hat{\mu}_t^R$, for $\phi_\mu = 0.25$);
TR$_3$: credit-augmented Taylor rule ($\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_l \hat{l}_t$, for $\phi_l = 0.25$).

While it provides a good approximation to the truly optimal policy in the benchmark New Keynesian model, strict inflation targeting (SIT) always leads to equilibrium indeterminacy (hence, sunspot-driven fluctuations) in a model with a credit channel, independently from the extent of deep habits. While its quantitative performance is clearly superior to
SIT and all Taylor rule specifications considered, flexible inflation targeting (FIT) displays welfare costs which are strictly increasing in deep habits. For instance, they become about three times larger if we move from the simple zero-spread cost channel model (line B.1.) to a deep habits model with $\theta = 0.5$ and a 4% yearly spread (line B.3.).

Credit frictions also make less desirable to follow a standard Taylor rule (TR$_1$). Its welfare costs for the case of $\theta = 0.5$ and a 4% yearly spread (line B.3.) are more than thirty times larger than for the benchmark NK model (line A.). Nevertheless, the analysis shows that there can be some welfare gains from letting the policy rate respond to movements in credit demand when counter-cyclical spreads are present (compare TR$_1$ with TR$_3$ for the case of $\theta = 0.5$, under both a 2% and 4% spread). On the contrary, there appear to be no welfare gains (actually, even losses) from responding to the observed credit spread (compare TR$_1$ with TR$_2$).

A possible interpretation of the apparently conflicting results given by rules TR$_2$ and TR$_3$ is the following. Consider the spread-augmented rule TR$_2$, $\hat{r}_t = \phi_{\pi} \hat{\pi}_t + \phi_y \hat{y}_t - \phi_{\mu} \hat{\mu}_t^R$. Using the definition $\hat{\mu}_t^R = \hat{r}_t^L - \hat{r}_t$, such rule is equivalent to $\hat{r}_t = \frac{\phi_{\pi}}{1+\phi_{\mu}} \hat{\pi}_t + \frac{\phi_y}{1+\phi_{\mu}} \hat{y}_t - \frac{\phi_{\mu}}{1+\phi_{\mu}} \hat{\mu}_t^L$. An increase in $\phi_{\mu}$ has then two opposite effects. On the one hand, it is beneficial: following an increase in the loan rate, it leads to a lower policy rate, and hence a lower marginal cost for banks, which, in turn, provides a lower incentive to raise loan rates. On the other hand, it is detrimental as it implicitly determines a milder response to both inflation and output, which, as well known, could potentially lead to larger aggregate fluctuations. It appears that these two effects cancel out each other in our quantitative analysis. Consider now the credit-augmented rule TR$_3$, $\hat{r}_t = \phi_{\pi} \hat{\pi}_t + \phi_y \hat{y}_t + \phi_l \hat{l}_t$. Because of the counter-cyclical behavior generated by deep habits, a positive $\phi_l$ is equivalent to an implicit negative response of the policy rate to the credit spread (as the latter decreases when loan demand is higher). However, differently from the rule TR$_2$, the rule TR$_3$ does so without inducing a milder response to inflation and/or output.

7 Conclusions

We have augmented a small-scale New Keynesian DSGE framework with two frictions in the banking sector: monopolistic competition and features of a customer-market type of model. We have modeled the latter as in the deep-habits in credit markets model of Aliaga-Díaz and Olivero (2010), by assuming that the liquidity services to borrowing constrained firms arise from a loan composite which depends on the amount of past bank-specific loans. By making the interest rate elasticity of the demand for loans pro-cyclical, this feature (which, in reduced form, captures the documented borrower “hold-up” effect and the existence of switching costs in banking relationships) implies that during a phase of economic expansion banks might find optimal to lower current lending rates to greatly expand their customer
base, which will then be locked into a long-term relationship. As a result, our model is capable of generating endogenously countercyclical spreads between the interest rates on loans and deposits, a well-documented feature in the data. We have then used this framework to study the conduct of optimal monetary policy, as well as the welfare costs of the lack of commitment and of alternative sub-optimal policy rules.

Our analysis shows that the combination of monopolistic competition and deep habits in credit markets exacerbate the trade-off between stabilizing inflation and the output gap in optimal monetary policy making, under both discretion and commitment. In particular, because of its impact on the credit-related component of marginal costs, the nominal interest rate cannot be used to fully insulate the economy from shocks to aggregate productivity. After a positive technological shock, optimal policy prescribes a positive output gap, a deflation, and a drop in the policy rate that are all larger than in the credit channel New-Keynesian model without deep habits. This result hinges on the countercyclicality of credit spreads generated by deep habits: as the monetary authority induces a positive output gap (via a cut in the policy rate) to contain deflation, the credit spread falls, putting further downward pressure on marginal costs, which in turn leads to a more severe deflation.

From a quantitative perspective, the departure from price stability under the optimal policy is substantially larger than that implied by the simple cost channel model of Ravenna and Walsh (2006). For our benchmark calibration of deep habits, the deviation of inflation from its target under the discretionary regime can be 40-60 percent larger, depending on the degree of market power in banking. The welfare costs of deviating from the optimal Ramsey plan are higher than in the standard cost channel model by 100 percent in terms of the consumption-equivalent variation and 40 percent in terms of the inflation-equivalent variation.

The welfare costs of committing to simpler but sub-optimal policy rules also appear to be quite sizable and to be strictly increasing in the degree of deep habits in banking. Taking into account that the welfare gains from commitment in a benchmark New-Keynesian model without credit market imperfections are typically very small, this result highlights the quantitative importance of optimal monetary policy commitment when there are imperfections in financial intermediation.

Overall, the propagation mechanism identified by our structural model can be seen as alternative or complementary to the New Keynesian DSGE financial accelerator models hinging on agency costs and default risk. Most importantly, it overcomes one of their major drawbacks, i.e., the fact that in some cases they counterfactually predict credit spreads to be pro-cyclical. Our structural model can in fact lead to countercyclical spreads without imposing any requirement on the underlying structural shocks, as is done, for instance, in
Working on this paper we were able to uncover the fact that countercyclical credit spreads can become a source of equilibrium indeterminacy in the sense that even when satisfying the Taylor principle, simple instrumental interest rate rules can induce non-fundamental sunspot-driven fluctuations. This result casts some doubts on the desirability of feedback rules to implement the optimal policy plan. Studying this indeterminacy feature in depth is left for future work.

To conclude, in our framework banks are not subject to any type of macro-prudential regulations. Understanding how the introduction of these regulations interacts with optimal monetary policy making is worth of further efforts in the literature.

Faia and Monacelli (2007) assume that the mean of the idiosyncratic productivity shock (on the realization of which firms decide whether or not to default on debt obligations) is strictly increasing in aggregate TFP, which makes the default risk decrease (increase) during upturns (downturns). De Fiore and Tristani (2012) argue that their model would be able to generate the observed counter-cyclicality if shocks other than TFP were allowed to drive business cycle fluctuations.

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A Appendix

A.1 Optimal Monetary Policy: Some Analytics

The optimal monetary policy problem can be expressed as the minimization of the following
Lagrangian expression:

$$
\min \mathcal{L}_0 = \min E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[ \pi_t^2 + \psi (\hat{y}_t^g)^2 \right] + \lambda_{1,t} \left[ \beta \pi_{t+1} + \kappa (\sigma + \varphi) \hat{y}_t^g + \kappa \hat{r}_t^R + \kappa \hat{\mu}_t^R - \hat{\pi}_t \right] \\
+ \lambda_{2,t} \left[ \hat{y}_{t+1}^g - \frac{1}{\sigma} \hat{r}_t + \frac{1}{\sigma} \mathbb{E}_t \hat{\pi}_{t+1} + \hat{\mu}_t - \hat{y}_t^g \right] \\
+ \lambda_{3,t} \left[ A_{r} \hat{r}_t - A_{s} \pi_{t+1} - A'_{r} \hat{r}_{t+1} - A'_{s} \hat{\mu}_{t+1} \\
+ A'_y \hat{y}_{t+1}^g + A'_s \hat{r}_t - A_y \hat{y}_{t+1}^g + A_s \hat{r}_{t+1} \\
+ (1 - \theta) \chi A_{a} \hat{a}_t - \omega \hat{\xi}_t - A_{s} \hat{\mu}_t^R \right] \\
+ \lambda_{4,t} \left[ \tilde{\rho}_s \hat{s}_{t-1} + (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \hat{y}_t^g \right] \\
+ (1 - \rho_s) (1 - \theta) \chi \hat{a}_t - \hat{s}_t \right\}
$$

where $\hat{u}_t \equiv \frac{(1+\varphi)}{(\sigma+\varphi)} (E_t \hat{a}_{t+1} - \hat{a}_t)$, $\tilde{\rho}_s \equiv \rho_s + (1 - \rho_s) \theta$, and, for notational purposes, we have defined the following composite structural parameters which multiply the variables in equation (45):

- $A_{\pi} = A'_r = A'_s \equiv \frac{\theta \beta \omega (1 - \rho_s)}{1 - \beta \theta (1 - \rho_s)}$, $A_r = 2A_{\pi}$
- $A'_y \equiv \theta (1 + \sigma + \varphi) A_{\pi}$, $A'_s = -\theta A_{\pi}$
- $A_s \equiv \frac{\theta \omega}{[1 - \beta \theta (1 - \rho_s)]}$, $A_y \equiv (1 + \sigma + \varphi) A_s$
- $A_a \equiv \frac{\rho \theta^2 \beta \omega (1 - \rho_s) - \theta \omega}{(1 - \theta) [1 - \beta \theta (1 - \rho_s)]}$, $A_{s} \equiv 1 \frac{\omega}{1 - \beta \theta (1 - \rho_s)}$

First order conditions with respect to $\hat{\pi}_t, \hat{y}_t^g, \hat{r}_t, \hat{\mu}_t^R$ and $\hat{s}_t$ give the following equations:
• for $t = 0$ :

$$
\hat{\pi}_t = \lambda_{1,t} \tag{A.1}
$$

$$
\psi \hat{y}_t^g = -\kappa (\sigma + \varphi) \lambda_{1,t} + \lambda_{2,t} + A_y \lambda_{3,t} + (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \lambda_{4,t} \tag{A.2}
$$

$$
\frac{1}{\sigma} \lambda_{2,t} = \kappa \eta \lambda_{1,t} + A_r \lambda_{3,t} \tag{A.3}
$$

$$
\kappa \eta \lambda_{1,t} = A_M \lambda_{3,t} \tag{A.4}
$$

$$
\lambda_{4,t} = \tilde{\rho}_s \beta E_t \lambda_{4,t+1} + A_s' \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1} \tag{A.5}
$$

• for $t \geq 1$ :

$$
\hat{\pi}_t = \lambda_{1,t} - \lambda_{1,t-1} - \frac{1}{\sigma \beta} \lambda_{2,t-1} + \frac{A_\pi}{\beta} \lambda_{3,t-1} \tag{A.6}
$$

$$
\psi \hat{y}_t^g = -\kappa (\sigma + \varphi) \lambda_{1,t} + \lambda_{2,t} + A_y \lambda_{3,t} - \frac{\lambda_{2,t-1}}{\beta} - \frac{A_y'}{\beta} \lambda_{3,t-1}
+ (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \lambda_{4,t} \tag{A.7}
$$

$$
\frac{1}{\sigma} \lambda_{2,t} = \kappa \eta \lambda_{1,t} + A_r \lambda_{3,t} - \frac{A_r'}{\beta} \lambda_{3,t-1} \tag{A.8}
$$

$$
\kappa \eta \lambda_{1,t} = A_M \lambda_{3,t} + \frac{A_M'}{\beta} \lambda_{3,t-1} \tag{A.9}
$$

$$
\lambda_{4,t} = \tilde{\rho}_s \beta E_t \lambda_{4,t+1} + A_s' \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1} \tag{A.10}
$$

As pointed out by McCallum and Nelson (2004), one could conceive the optimal solution under discretion in the following manner. The policy-maker implements (A.1)-(A.5) in period 0 - where $\lambda_{j,-1} = 0$ for $j = 1, 2, 3$ since there are no past promises - and plans to implement (A.6)-(A.10) in each subsequent period. However, when period 1 arrives, the discretionary government re-solves the optimal policy problem and implements (A.1)-(A.5) for $t = 1, 2, \ldots$.  

---

46This concept of optimal monetary policy under discretion is different from the Markov Perfect Equilibrium concept we have used in Section 6.1. However, it proves useful to derive (to a good approximation) the
After substituting out the Lagrange multipliers in the system (A.1)-(A.5), simple algebra allows us to get the following relationship:

\[ \psi_t^g = -\left\{ \kappa (\sigma + \varphi) - \kappa \eta \left[ \sigma \left( 1 + \frac{A_r}{A_\mu} \right) + \frac{A_y}{A_\mu} \right] \right\} - \delta \lambda_{4,t} \]  

(A.11)

where \( \delta \equiv (1 - \rho_s) (1 - \theta) (1 + \sigma + \varphi) \). Next, consider equation (A.5), which regulates the equilibrium dynamics of the multiplier \( \lambda_{4,t} \) and let \( \zeta_t \equiv A'_s \lambda_{3,t} + \beta A_s E_t \lambda_{3,t+1} \). Since we are restricting to a stationary solution, assume that \( \zeta_t \) follows a simpler AR(1) process with persistence parameter \( \rho_\zeta \in (0, 1) \) and a mean-zero iid disturbance. Given that \( \bar{\rho}_s \beta \in (0, 1) \), equation (A.5) can be solved forward to give the following solution: \( \lambda_{4,t} = (1 - \rho_\zeta \bar{\rho}_s \beta)^{-1} \zeta_t \).

From the system (A.1)-(A.5), we also have that \( \lambda_{3,t} = \frac{\kappa \eta}{A_\mu} \pi_t \), such that:

\[ \lambda_{4,t} = \kappa \eta A'_s \frac{A_\mu}{A_\mu (1 - \rho_\zeta \bar{\rho}_s \beta)} \hat{\pi}_t + \frac{\kappa \eta \beta A_s}{A_\mu (1 - \rho_\zeta \bar{\rho}_s \beta)} E_t \hat{\pi}_{t+1} \]  

(A.12)

This expression for \( \lambda_{4,t} \) can then be substituted into (A.11) to give the following relationship:

\[ \psi_t^g = -\left\{ \kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta) \right\} \hat{\pi}_t - \delta \frac{\kappa \eta \beta A_s}{A_\mu (1 - \rho_\zeta \bar{\rho}_s \beta)} E_t \hat{\pi}_{t+1} \]  

(A.13)

where \( \Theta \equiv \sigma A'_r + A_y + \delta \frac{\theta A_s}{A_\mu (1 - \rho_\zeta \bar{\rho}_s \beta)} \). Equation (A.13) corresponds to what Svensson and Woodford (2005) refer to as a targeting rule: it defines the optimal output-gap inflation volatility trade-off faced by the policy-maker. To grasp how deep habits in banking affect this trade-off, we define the following composite coefficients:

...
\[
\Psi_{NK} \equiv \kappa (\sigma + \varphi) \\
\Psi_{CC} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta \sigma\} \\
\Psi_{DH} \equiv \{\kappa (\sigma + \varphi) - \kappa \eta (\sigma + \Theta)\}
\]

They correspond to the coefficient multiplying inflation in (A.13) in, respectively, the benchmark New-Keynesian model, the cost channel model of Ravenna and Walsh (2006) and our model with deep habits in banking. Equation (A.13) implies that, for given \(E_t \hat{\pi}_{t+1}\) and given volatility for the output gap, stabilizing inflation is more costly (i.e. inflation volatility would be larger in equilibrium) the smaller is \(\Psi\). For instance, since \(\Psi_{CC} < \Psi_{NK}\), it immediately follows that the cost channel model of Ravenna and Walsh features more inflation volatility with respect to the benchmark New-Keynesian model.

Consider now the case of deep habits, for which \(\Psi_{DH}\) is the relevant coefficient. Notice that \(A_r, A_y\) and \(A_\pi\) are all strictly positive, and so are \(\delta\) and \((1 - \rho_s \tilde{p}_s \beta)\), while simple (but tedious) algebra shows that, under Assumption 1, \(A_\mu > 0\) as well. It then follows that \(\Theta > 0\) and therefore \(\Psi_{DH} < \Psi_{CC}\): that is, for given volatility of the output gap, equilibrium inflation is more volatile in a model with deep habits in credit markets.

A.2 Welfare Cost Computation

A.2.1 Consumption Equivalent Variation

We follow Schmitt-Grohé and Uribe (2007) and define the welfare cost of adopting an alternative policy \(A\) with respect to a reference policy \(R\) as the fraction \(\upsilon\) of the consumption path under policy \(R\) that must be given up to make the household as well off under policy \(A\) as under policy \(R\). More specifically, \(\upsilon\) is computed as the unique solution to the following equality:
The computational procedure involves following steps. Let $R$ strand for “optimal policy under commitment” and $A$ for “optimal policy under discretion”.

**Step 1.** We compute the value of aggregate welfare under policy $J$, for $J=A, R$, using the second-order-approximation procedure described in Schmitt-Grohe and Uribe (2002) for the computation of unconditional welfare:

$$U^J \equiv \bar{U} - \epsilon \frac{C^{1-\sigma}}{1-\beta} \left[ \text{Var} \left( \hat{x}_{J,t} \right) + \psi \text{Var} \left( \hat{y}_{J,t}^g \right) \right]$$

(A.14)

where $\bar{U} \equiv \frac{1}{1-\beta} \left[ \bar{C}^{1-\sigma} (1-\beta) - \bar{H}^{1+\varphi} \right]$.

**Step 2.** For $J=A, R$, we compute the percentage reduction in steady state consumption, $\nu_J$, that would make the representative agent indifferent between the efficient steady state allocation and the allocation occurring under policy regime $J$. Given the value of $U^J$ computed in Step 1, we solve the following equation with respect to $\nu_J$:

$$U^J = \frac{1}{1-\beta} \left[ \bar{C} (1-\nu_J)^{1-\sigma} - \bar{H}^{1+\varphi} \right]$$

(A.15)

Letting $\Upsilon_J \equiv (1-\beta)U^J + \bar{H}^{1+\varphi}$ and $\kappa_J \equiv (1-\sigma)\Upsilon_J^{1-\sigma}$, simple algebra gives that

$$\nu_J = 1 - \frac{\kappa_J}{C}.$$

**Step 3.** The consumption-based welfare cost associated with adopting the alternative policy $A$ with respect to the reference policy $R$ is then $\nu \equiv \nu_A - \nu_R$.

### A.2.2 Inflation Equivalent Variation

We follow Dennis and Soderstrom (2006), and a more recent application by Demirel (2013), by computing the permanent decrease in yearly inflation needed to compensate the household for a switch from commitment to discretion. For given arbitrary paths of inflation
and the output gap, \( \{ \hat{\pi}_t, \hat{y}^g_t \}_{t=0}^{\infty} \), we have that (a second order approximation to) aggregate welfare, conditional on information available at \( t = 0 \), is equal to:

\[
W_0 = \frac{\bar{U}}{1 - \beta} - \frac{1}{2} \epsilon \bar{C}^{1-\sigma} E_0 \sum_{0}^{\infty} \beta^t \left[ \hat{\pi}_t^2 + \psi(y^g_t)^2 \right]
\]  

(A.16)

Given (A.16), the welfare gain from a \( x\% \) permanent decrease in quarterly inflation is then equal to \( \frac{\epsilon \bar{C}^{1-\sigma} (\frac{x}{100})^2}{2 (1 - \beta) \kappa} \).

For the computation of \( x \) we use Monte-Carlo simulation methods. Let \( N \) be the number of simulations (we set \( N = 10000 \)). Then, for \( n = 1 : N \), we proceed as follows.

**Step 1.** We simulate time series for inflation and the output gap under both discretion and commitment. We then compute the values of (A.16), under both regimes, by truncating the infinite summation at a very large \( T \). We denote the respective values by \( W_{0,n}^{\text{disc}} \) and \( W_{0,n}^{\text{com}} \), where the subscript \( n \) denotes the specific simulation.

**Step 2.** Since \( W_{0,n}^{\text{com}} > W_{0,n}^{\text{disc}} \), we compute the percentage decrease in inflation that is required to increase welfare by \( \Delta_n \equiv W_{0,n}^{\text{com}} - W_{0,n}^{\text{disc}} \) by solving the following equation:

\[
\frac{\epsilon \bar{C}^{1-\sigma}}{2 (1 - \beta) \kappa} \left( \frac{x_n}{100} \right)^2 = \Delta_n
\]

**Step 3.** Given the series \( x_n \), for \( n = 1 : N \), we compute its sample mean and multiply it by four to obtain annual values: \( x = 4 \times \frac{\sum_{1}^{N} x_n}{N} \).

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References


Figure 1
Figure 2
Figure 3

- Inflation
- Output Gap
- Nominal Interest Rate
- Lending Rate
- Credit Spread
- Loans

Legend: 
- $\theta = 0$
- $\theta = 0.5$ (Developed Economy)
- $\theta = 0.5$ (Emerging Market Economy)
Figure 4