This paper studies optimal discretionary reduction of government debt in a New Keynesian model. The reduction of government debt by means of a distortionary tax instrument creates a trade-off between inflation and output. The optimal way to conduct monetary policy in this environment depends on the maturity of government debt. In the economy with one-period bonds, an incentive to reduce the tax rate by selling newly issued bonds at a higher price renders the stance of monetary policy loose. On the contrary, monetary policy is tight when government bonds are long-term because the dominant incentive is to mitigate inflation by restraining private demand. Introducing long-term bonds also slows down the speed of government debt reduction up to a level consistent with the existing empirical evidence on the persistence of government debt. Finally, lengthening maturity of government bonds brings down the welfare cost of debt reduction.
1 Introduction

The global financial and economic downturn of 2008-2009 has left a legacy of unprecedented government debt levels in many advanced economies. Not surprisingly, these fiscal developments have resumed and intensified a discussion among policymakers and economists alike regarding issues associated with government debt and a reduction thereof. Related to these issues is the debate on whether monetary policy is to play an important role if one is to reduce government debt or it is primarily a matter of fiscal policy. The conventional wisdom has it that it is optimal for fiscal policy to maintain control over government debt dynamics and smooth the associated distortions over time—see Kirsanova et al. (2009). Renewed concerns of monetary policy taking part in fiscal financing of highly indebted governments are often linked to the maturity of public debt.

In this paper, I study implications of long-term maturity of government debt for the joint conduct of monetary and fiscal policy. To this end, I use a baseline New Keynesian model where the benevolent government issues bonds and always repays debt but cannot commit to a specific path of policy instruments in the future. The monetary policy instrument is a one-period nominal interest rate that affects real economy due to price stickiness. The set of fiscal policy instruments consists of government spending on public goods and a linear labor income tax. The government faces a non-trivial problem of fiscal financing because the available instruments are distortionary. The government acts discretionary, each period it determines the amount of distortions it imposes today and the amount it postpones into the future by rolling debt over. Without commitment, government debt is required to be reduced over time towards a certain long-run level.

The novelty of this paper is to demonstrate that optimal monetary and fiscal policy along the transition to the steady state can be markedly different depending on whether the government issues one-period bonds or bonds of longer average maturity. I show this using a linear-quadratic representation of the policy problem and provide an analytical characterization of the equilibrium. The government that starts off with an excessive amount of debt is likely to set a high tax rate, which has a by-product of cost-push inflation. If lump-sum taxes were available, as in Clarida et al. (1999), a cost-push inflationary pressure would unambiguously call for monetary tightening to trade off a decline in output for a reduction in inflation. Here, however, a loose monetary policy could: (1) expand the tax base, (2) inflate away part of the outstanding nominal debt, and (3) improve the terms of new borrowing. A reduction in the real value of outstanding bonds due to inflation in a given period is independent of the maturity of these bonds. Differently, budget gains from an increase in the price of newly issued bonds are offset by capital losses on outstanding bonds more the longer is maturity of these bonds. It is through this effect maturity becomes relevant for the optimal stance of monetary policy in particular and the equilibrium outcome in general.

The main contribution of this paper is on the optimal stance of monetary policy during the transition period of government debt reduction. Calibrating the model at a quarterly frequency, I find that in the economy with one-period government bonds, monetary policy is driven primarily by the incentive to increase the price of newly issued bonds. Thus, optimal stance of monetary policy in this case is loose. In equilibrium, loose monetary policy results in low real interest rates that accommodate reduction of government debt. Importantly, this result is not robust to lengthening maturity of government debt. When the government issues bonds of longer maturity, monetary policy remains tight and real interest rates stay high during the transition. The driving incentive in this case is to restrain private demand and mitigate inflationary pressure.

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1Here monetary policy is referred to as tight (loose) if the one-period nominal interest rate reacts more (less) than one-to-one in response to expected inflation.
A relative simplicity of the baseline New Keynesian model can be seen as a drawback. Sharp characterization of the equilibrium in this model comes at a cost of leaving out a number of margins that are otherwise required to make dynamic effects of monetary policy on key macroeconomic variables in the model quantitatively consistent with the data. In order to provide a more rigorous quantitative assessment of the effect of government debt maturity on the stance of monetary policy, I extend the analysis into a workhorse medium-scale model. This model builds on the baseline model by adding working capital and wage-setting frictions, habit formation in consumption, and accumulation of capital with the costs of utilization and investment adjustment. Using posterior mean parameter estimates of the U.S. economy by Christiano et al. (2010), I show that the presence of government debt in excess of the steady-state level warrants loose monetary policy in the case of one-quarter bonds but lengthening average maturity just beyond three quarters reverses the optimal stance and makes it tight.

Effects of government debt maturity on the stance of monetary policy have important implications for debt dynamics during the transition period. A reduction of debt towards the steady state during this period is driven by strategic incentives to manipulate future selves of the government. Consider the case of one-period bonds. While loose monetary policy stance reduces the interest rate cost of issuing government debt in a given period, expectations of the government following the same policy in the future increase an implicit cost of issuing government debt. In particular, maintaining elevated level of government debt increases expectations of future inflation that push current inflation up. Thus, it is optimal for the government in a given period to leave a lower debt to its successor. As maturity of government debt lengthens and monetary policy weighs more on inflation, the implicit cost of government debt falls and debt reduction becomes more gradual.

Using a numerical method to solve the nonlinear version of the baseline model, I show that in the case of one-period bonds, it is optimal to reduce government debt by more than a half within a single quarter. Such a fast speed of reduction is at odds with existing empirical evidence on the persistence of government debt. Friedman (2005), for instance, uses the postwar U.S. data and finds a half-life equal to 85 quarters for the debt-to-GDP ratio response to its own shock in a univariate autoregressive setting. Accounting for a plausible average maturity makes debt reduction under optimal discretionary policy more in line with the empirical evidence. I show that in the case when average maturity of government debt is equal to 4 years, the half-life of debt reduction is equal to 92 quarters. Moreover, a more gradual transition with long-term bonds is associated with a higher welfare.

This paper also contributes to a discussion of a connection between maturity of government debt and inflation. As maturity of debt affects the stance of monetary policy and the speed of government debt reduction it consequently affects the magnitude and persistence of inflation. In particular, tight monetary policy and the slowdown of debt reduction, brought by the lengthening of maturity, increase the persistence but reduce the magnitude of inflation. This result is consistent with an empirical finding of Rose (2014), who shows that the existence of long-term government bonds markets may help to keep inflation low and stable.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 provides intuition for the key mechanism in a simplified version of the model with inflation-indexed debt and exogenous government spending. Section 4 describes the baseline model, and discusses parameterization and solution methods. Optimal policy and welfare in the baseline model is analyzed in Section 5. Section 6 discusses a medium-scale extension of the model. Section 7 concludes.

Marcet and Scott (2009) provide an extensive evidence of high persistence of the market value of debt.
2 Related Literature

A large strand of literature studies optimal monetary and fiscal policy. Faraglia et al. (2013) and Leeper and Zhou (2013) study impact of government debt maturity on inflation under the assumption of the joint monetary and fiscal commitment.\(^3\) They provide a rationale for a longer average maturity to lead to a stronger and more persistent inflation. In these studies, however, there is no intrinsic incentive to bring the level of debt towards a certain long-run level because government debt optimally follows a near random walk behavior. In this paper, on the other hand, under lack of commitment debt is required to be reduced until reaching the steady state.\(^4\)

This paper is mostly related to economies where the risk of default can be abstracted from so that the lack of commitment applies only to a path of policy instruments and not to a decision of debt repayment.\(^5\) A number of paper studied optimal discretionary policy under the assumption of one-period bonds. Debortoli and Nunes (2012) study effects of the lack of fiscal commitment on the steady-state level of debt in a real economy and show that government debt has to be reduced to zero. Gnocchi and Lambertini (2016) consider a nominal economy with sticky prices and show that commitment of monetary authority to an inflation target can lead to a positive steady-state level of debt despite the lack of fiscal commitment.\(^6\) In a similar environment, Leith and Wren-Lewis (2013) focus on optimal policy under joint monetary and fiscal discretion and take positive steady-state level of debt as given. They show that a reduction of debt towards the steady-state can be fast and accompanied by low real interest rates because the stance of monetary policy is loose. The current paper demonstrates that the stance of monetary policy in such an environment is sensitive to maturity of government debt and becomes tight if one introduces long-term bonds.

In a related work, Leeper et al. (2015) analyze the no-commitment solution with nominal bonds of mixed maturity when dynamic policy is determined jointly with the steady state. Differently, I study the effects on transition dynamics in isolation by assuming away steady-state distortions as if the government commits in the long run. This assumption leads to a very tractable equilibrium characterization that highlights comparative dynamic effects of the maturity of government debt. Bhattarai et al. (2015) show how shortening maturity can improve discretionary stabilization of the economy in response to demand shocks if the nominal interest rate is at the zero lower bound. I abstract from any uncertainty and analyze the model with the labor income tax as opposed to the lump-sum tax in their work. The latter does not have a by-product of cost-push inflation that generates the incentive to keep the stance of monetary policy tight during the reduction of government debt. Finally, I complement these studies by extending the analysis of optimal discretionary fiscal-monetary policy mix to a medium-scale version of the New Keynesian model.\(^7\) In a recent

\(^3\)Optimal fiscal and monetary policy with commitment has been extensively studied under the assumption of one-period bonds. Seminal contributions include Lucas and Stokey (1983) and Aiyagari et al. (2002) in real economies with complete and incomplete markets correspondingly as well as Chari et al. (1991) and Schmitt-Grohé and Uribe (2004) in nominal economies with flexible and sticky prices correspondingly.

\(^4\)A number of studies under commitment do find incentives that make it optimal to reduce government debt. Adam (2011) and Bhandari et al. (2016) show that budget risk due to incomplete markets makes it optimal to reduce government debt over time, albeit very slowly. Horvath (2011) shows that assuming unconditional welfare objective also implies gradual adjustment of government debt towards a certain mean value. With complete markets, Ferrière and Karantounias (2016) show that ambiguity aversion makes it optimal to reduce government debt to zero if intertemporal elasticity of substitution is sufficiently low.

\(^5\)A review of the issues related to sovereign default and further references can be found in Aguier et al. (2016).

\(^6\)Ellison and Rankin (2007), Díaz-Giménez et al. (2008), and Martin (2009) discuss implications of the lack of monetary and fiscal commitment for the steady state level of debt in a classical monetary economy with cash-in-advance constraint.

\(^7\)A number of contributions, including Burgert and Schmidt (2014), Eggertsson (2006), and Niemann et al. (2013), use baseline small-scale New Keynesian models to analyze jointly optimal discretionary monetary and fiscal policy with government debt in the form of one-period bonds.
study, Cantore et al. (2015) employ a medium-scale New Keynesian model with sovereign risk premium to analyze optimal fiscal and monetary policy, including that under discretion, in times of a debt crisis. As was previously mentioned, the focus of the current paper is on the times with a negligible sovereign risk.

This paper is also related to the literature studying optimal maturity structure of government debt. A number of studies incorporate portfolio problem into the optimal policy problem of the government under the assumption of commitment—see, e.g., Angeletos (2002), Buera and Nicolini (2004), Faraglia et al. (2014), and Lustig et al. (2008). More closely related are the studies that introduce lack of commitment and solve for optimal maturity structure in the real economy, see Debortoli et al. (2016), and in the nominal economy with cash-in-advance constraint, see Arellano et al. (2013). Differently, the current paper treats maturity structure as determined by a separate debt management authority that is not explicitly modeled. Optimal fiscal and monetary policy is then chosen given average maturity of government debt that does not change over time, which is not much at odds with the stability of portfolio shares documented in Faraglia et al. (2014) using postwar U.S. data.

An alternative approach to address questions related to government debt reduction is by abstracting from structural reasons that induce debt reduction and imposing it exogenously. Using this approach, Romei (2014) and Scheer (2015) analyze fiscal instruments for reduction of government debt in models of closed economies with nominal frictions. Both papers introduce monetary policy in the form of Taylor-type rules. Similarly, Andrés et al. (2016) study exogenous fiscal consolidation using various fiscal instruments amidst private deleveraging in a model of small open economy within a monetary union. Krause and Moyen (2013) examine the extent to which an exogenous change of the inflation target in a closed economy model may help to reduce the government debt burden. Thus, these studies abstract from issues related to the lack of policy commitment and the optimal role of monetary policy in reduction of government debt, which are the focus of the current paper.

3 A Primer

The analysis in this paper builds on a standard New Keynesian dynamic general equilibrium model, along the lines of Galí (2015) and Woodford (2003), of a closed economy with monopolistically competitive intermediate goods market and quadratic cost of price adjustment. The model is augmented with a fiscal sector à la Lucas and Stokey (1983) where the government issues bonds and levies distortionary labor income tax to finance government spending. There is no uncertainty, thus financial markets are complete.

This section develops intuition for the key mechanism in a stripped down version of the model where government spending level is assumed to be constant and bonds issued by the government are indexed to inflation. The analysis starts right away by postulating a linear-quadratic policy problem of the government subject to aggregate equilibrium conditions. Next section provides a detailed derivation and analysis starting from the first principles in a nonlinear version of the model with nominal government bonds and endogenously chosen government spending.

The model economy is assumed to have a deterministic steady-state with an efficient level of output $\bar{y}$, zero inflation $\bar{\pi} = 1$, and positive quantity of government bonds $\bar{b} > 0$. In the first period, the government

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8Empirical estimates of the impact of inflation on the real value of government debt in the U.S. can be found in Hall and Sargent (2011) and Hilscher et al. (2014) that use backward and forward looking approaches respectively.

9In the steady state, lump-sum tax is used to finance a constant employment subsidy that eliminates distortions arising from...
starts off with an excessive outstanding quantity of bonds (in real terms), $\hat{b}_{t-1} > 0$, where a hat is used to denote percentage deviation of the corresponding variable from its steady-state value. The government can credibly promise to repay its debt but cannot commit to a specific transition path of public debt reduction and, therefore, optimizes sequentially. In each period $t$ the government solves the following constrained social planning problem:

$$
U(\hat{b}_{t-1}) = \max_{\{q_t, \pi_t, y_t, h_t, r_t\}} -\frac{1}{2} (\theta \hat{y}_{t}^{2} + \hat{\pi}_{t}^{2}) + \beta U(\hat{b}_{t})
$$

subject to

$$
\begin{align*}
\hat{\pi}_t &= \kappa \hat{y}_t + \lambda \hat{\pi}_t + \beta \hat{\pi}_{t+1}, \\
\hat{y}_t &= \hat{y}_{t+1} - \gamma_c^{-1} (\rho \beta \hat{q}_{t+1} - \hat{q}_{t}), \\
\bar{y} \hat{b}_t &= \beta^{-1} \bar{y} \hat{b}_{t-1} - (1 - \rho) \bar{y} \hat{q}_t - \bar{\tau} \bar{w} y (1 + \bar{\tau} \bar{w}) \hat{r}_t + (1 + \gamma_c + \gamma_h) \hat{y}_t,
\end{align*}
$$

Equation (3.2) is the log-linearized flow budget constraint of the government written in real terms, where $\theta > 1$ is the relative price elasticity of demand for the intermediate goods, and $\beta \in (0, 1)$ is the time discount factor. Equation (3.2) is a log-linearized forward-looking Phillips curve that aggregates pricing decisions of individual firms, where $\lambda, \kappa > 0$ are given by

$$
\begin{align*}
\kappa &= \frac{(\theta - 1)}{\varphi} (\gamma_c + \gamma_h), \\
\lambda &= \frac{(\theta - 1)}{\varphi} \bar{\tau} \bar{w},
\end{align*}
$$

where $\varphi > 0$ measures the degree of nominal price rigidity, $\gamma_c \equiv \gamma_c (\bar{y} / \bar{c})$ and $\gamma_c > 0$ is the inverse of the household’s intertemporal elasticity of substitution, $\gamma_h > 0$ is the inverse of the Frisch elasticity of labor supply, and bar variables denote steady-state values of private consumption, $\bar{c}$, output, $\bar{y}$, the labor income tax rate, $\bar{\tau}$, and the real wage, $\bar{w}$. Equation (3.3) is a dynamic investment-savings equation that describes optimal, up to first order of approximation, intertemporal behavior of the representative household, where $\rho \beta \hat{q}_{t+1} - \hat{q}_{t}$ is the (real) holding period return on the portfolio of inflation-indexed government bonds with a structure of payoffs that decay exponentially at the rate $\rho \in [0, 1]$. The parameter $\rho$ determines the average maturity of government bonds: one can show that the steady-state duration of the portfolio is equal to $(1 - \beta \rho)^{-1}$. Let $\hat{\tau}_t$ be defined as a one-period nominal interest rate controlled by the government. Then, the real one-period interest rate is given by $\hat{r}_t = \hat{\tau}_t - \hat{\pi}_{t+1}$. By no arbitrage condition, the holding period return on the portfolio of inflation-indexed government bonds has to be equal to the one-period real interest rate

$$
\hat{r}_t = (\rho \beta \hat{q}_{t+1} - \hat{q}_{t}).
$$

Equation (3.4) is the log-linearized flow budget constraint of the government written in real terms, where $\bar{\Gamma} \equiv \bar{q} \bar{b}$ monoplastic competition and taxation of labor income. Next section provides a more detailed discussion.
is the steady-state market value of government bonds. The last term on the right-hand-side is a primary surplus written in terms of output and the tax rate using an implicit optimal labor supply decision of the households.

Optimal discretionary monetary policy under the assumption of lump-sum taxes in an otherwise similar environment is analyzed in Clarida et al. (1999). There the driving forces are the exogenous shocks including a cost-push disturbance in the Phillips curve that creates a trade-off between inflation and output. The model studied here abstracts from exogenous shocks. Instead, the cost-push impact in the Phillips curve (3.2) arises endogenously due to an increase of the distortional labor income tax that pushes up the real marginal cost of production by affecting labor supply decision of the households. Introducing distortional taxes also breaks the Ricardian equivalence, which makes the budget of the government an effective policy constraint and the stock of outstanding government bonds becomes an endogenous state variable. Having an additional policy constraint changes intratemporal inflation-output trade-off. Moreover, emergence of an endogenous state variable creates an incentive for the government in a given period to manipulate future selves, which results in an explicit intertemporal trade-off. These results are discussed in the remainder of this section.

The solution of the policy problem yields the following optimality conditions:

\[
\frac{\hat{y}_t}{\tau w_0(1 + \tau \hat{w})} = \frac{\Gamma \lambda}{\tau w_0(1 + \tau \hat{w})} \hat{\pi}_t = \frac{\Gamma \lambda}{\tau w_0(1 + \tau \hat{w})} \hat{\pi}_{t+1} + \beta \Pi_b \hat{\pi}_t + \left(\gamma_c \gamma_b - \rho \beta Q_b\right) \frac{(1 - \rho) \Gamma \lambda}{\tau w_0(1 + \tau \hat{w})} \hat{\pi}_t. \tag{3.5}
\]

Condition (3.5) establishes optimal intratemporal relation between the welfare-relevant deviations (gaps) of output and inflation. The first term on the right-hand-side alone is identical to a “lean against the wind” targeting rule emerging in a model with lump-sum taxes. It prescribes to contract demand directly proportional to the gain in reduced inflation, \( \kappa \), and inversely proportional to the relative weight placed on output deviations, \( \vartheta \). The two remaining terms reflect effects that the lack of lump-sum taxes has on inflation-output trade-off. They capture an incentive to reduce distortional tax rate by stimulating demand directly proportional to the gain in reduced inflation, \( \lambda \), and inversely proportional to the relative weight placed on output deviations, \( \vartheta \).

More specifically, the second term describes the optimal amount of demand stimulus as directly proportional to the tax revenue gain from the increased output, \( \tau w_0(1 + \gamma_c + \gamma_h) \), and inversely proportional to the tax revenue loss from the tax rate reduction, \( \tau w_0(1 + \tau \hat{w}) \). The underlying effect can be referred to as an implicit profit tax because an increase in the taxed labor income corresponding to higher demand is a mirror image of an increase in the total cost of production that reduces monopolistic profits of the firms. The third term describes the optimal amount of demand stimulus as directly proportional to the reduction in the real interest rate generating a unit increase of output, \( \gamma_c \), combined with the net budget gain due to the effect of this change on the price of government bonds, \( (1 - \rho) \Gamma \), and inversely proportional to the tax revenue loss from the tax rate reduction, \( \tau w_0(1 + \tau \hat{w}) \). The underlying effect can be referred to as an interest rate manipulation because a reduction of the real interest rate allows the government to increase the price of newly issued bonds.

To compute the net budget gain of this change one also has to take into account corresponding capital losses on outstanding bonds. The longer is the average maturity of government bonds, i.e. the larger is parameter
\( \rho \), the lower is the net budget gain from an increase of the price of government bonds. Thus, having bonds with longer maturity reduces the incentive to stimulate demand *ceteris paribus*.

Another optimality condition (3.6) establishes intertemporal relation that implicitly determines the amount of bonds issued by the government so as to equate the welfare benefit of government borrowing with its welfare cost, where the two are expressed in terms of inflation. The left-hand-side represents the benefit of issuing government bonds instead of raising the tax rate in the current period that is directly proportional to the gain in reduced current-period inflation, \( \lambda \), and inversely proportional to the tax revenue loss from the tax rate reduction, \( \bar{\tau} \bar{w} \bar{y}(1 + \bar{\tau} \bar{w}) \). Three terms on the right-hand-side represent three channels that generate the cost of issuing government bonds.

In particular, the first term simply captures the cost of raising the tax rate in the next period instead of doing it in the current period in terms of the next-period inflation. The second term captures the cost of anticipated inflation, brought by issuing government bonds, that is directly proportional to the corresponding loss in increased current-period inflation, \( \beta \Pi \).

The formulation of the problem above assumes that the monetary policy instrument, \( \hat{i}_t \), is determined implicitly so as to support optimal choices of other variables. The optimality condition (3.5) is crucial in determining the overall stance of monetary policy in this economy. Let \( \Phi_y \) be the coefficient that summarizes the inflation-output trade-off so that \( \hat{y}_t = \Phi_y \hat{\pi}_t \). Analogously, let \( \Phi_\pi \) summarize optimal intertemporal trade-off in (3.6) so that \( \hat{\pi}_{t+1} = \Phi_\pi \hat{\pi}_t \). One can then use dynamic investment-savings equation (3.3) to show that the nominal interest rate in equilibrium has to satisfy

\[
\hat{i}_t = \left( 1 - \gamma_c \Phi_y \frac{(1 - \Phi_\pi)}{\Phi_\pi} \right) \hat{\pi}_{t+1}.
\]

where

\[
\Phi_y \equiv \left[ -1 + \frac{1 + 1/(\gamma_c + \gamma_h)}{1 + 1/\bar{\tau} \bar{w}} + \frac{(1 - \rho) \bar{Y}}{(1 + \gamma_h/\gamma_c) \bar{y}(1 + \bar{\tau} \bar{w})} \right] [\theta - 1], \tag{3.7}
\]

\[
\Phi_\pi \equiv \left[ 1 - \frac{\bar{y}(1 + \tau \bar{w})(\gamma_c + \gamma_h)}{\kappa \bar{y}} \beta \Pi - (1 - \rho) \left( \gamma_c \bar{Y}_b - \rho \beta \bar{Q}_b \right) \right]. \tag{3.8}
\]

Furthermore, the analysis is restricted to nonexplosive and monotone equilibria, i.e., such that \( 0 < \Phi_\pi < 1 \).

The magnitude of the nominal interest rate compared to expected inflation depends crucially on the sign of coefficient \( \Phi_y \). If fiscal effects on inflation-output trade-off are relatively weak then \( \Phi_y < 0 \) as in the model with lump-sum taxes. In this case the nominal interest rate should be raised sufficiently in response to expected positive inflation so that the real interest rate exceeds the long-run level. In other words, monetary
policy accompanying the reduction of government debt is tight. When the opposite happens and fiscal effects dominate, changes in the nominal interest rate are less aggressive and the real interest rate stays below the long-run level because $\Phi_y > 0$. Hence, in this case monetary policy is loose during the period of debt reduction. As was previously discussed, increasing parameter $\rho$ reduces the relative importance of fiscal effects in shaping inflation-output trade-off. Thus, longer maturity of government bonds favors tight monetary policy during the period of debt reduction.

As can be seen from the analysis above, the question of the optimal stance of monetary policy during the period of debt reduction is a quantitative one and depends on the dominant incentive shaping the inflation-output trade-off. This is also the case for such questions as the speed of government debt adjustment and the welfare analysis. Next sections address these questions in details by developing a model that relaxes assumptions of bonds indexation and fixed government spending.

4 The Baseline Model

This section provides a detailed description, starting from the first principles, of the baseline model underlying the analysis in this paper. The baseline model described in this section consists of four types of economic agents: the representative household, the representative final-good producer, a continuum of intermediate-goods producers, and the government. Compared to the previous section, the model described here relaxes the assumptions of exogenous government spending and indexation of government bonds to inflation. The baseline model features endogenous provision of public good, which is financed either by taxing labor income of the household or by issuing nominal government bonds.

4.1 Households

The economy is populated by a continuum of ex-ante and ex-post identical infinitely-lived households. The (representative) household dislikes labor and enjoys private consumption as well as consumption of public goods. Formally, preferences of the household are represented by the following lifetime utility

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + g(G_t) - v(h_t)],$$

(4.1)

where $\beta \in (0, 1)$ is the time discount factor, $c_t$ is private consumption of the final (aggregate) good, $h_t$ is labor supply (time endowment is normalized to one), $G_t$ is (real) government spending on public provision of the final good, $u$ and $g$ are period utility functions of private consumption and consumption of public goods, and $v$ is the period disutility function of labor. In what follows, functions $u$ and $g$ are assumed to be increasing and concave, and function $v$ is assumed to be increasing and convex.

The household enters period $t$ holding assets in the form of maturing nominal one-period (discount) government bonds $B_{t-1}$ and a long-term portfolio of government bonds $B_{t-1}$. Supply of newly-issued bonds in period $t$ is determined by government policies, which are discussed later. One-period bonds issued in period $t$ are purchased at a price $R_t^{-1}$, where $R_t$ is a one-period nominal interest rate. As in Woodford (2001), the portfolio of long-term bonds is defined as a set of perpetual bonds with nominal payoffs that start from
one and decay over time geometrically at the rate $\rho \in [0, 1]$. The outstanding long-term portfolio $B_{t-1}$ with remaining flow of payoffs is exchanged in period $t$ for a new long-term portfolio $B_t$ at market prices. One can use no-arbitrage argument to show that the price $q_t$ of the former is equal to the price $q_t$ of the latter scaled by the factor $\rho$, see Appendix A.1. Under the described market arrangement, flow budget constraint of the household takes the following form:

$$P_t c_t + R_t^{-1} B_t^s + q_t B_t = (1 - \tau_t) W_t h_t + B_{t-1}^s + (1 + \rho q_t) B_{t-1} + \int_0^1 \Pi_{i,t} di - T_t,$$

where $P_t$ is the unit price of the final good, $W_t$ is the nominal wage, $\tau_t$ is the linear tax rate on labor income, $\Pi_{i,t}$ is the share of profits from sales of intermediate good of type $i$ distributed in a lump-sum way, and $T_t$ is the lump-sum tax collected by the government. To have a well-defined intertemporal budget constraint an additional condition that rules out “Ponzi schemes” is implicitly imposed.

The household maximizes (4.1) by choosing consumption, labor and bond purchases $\{c_t, h_t, B_S^t, B_t\}_{t=0}^\infty$ subject to the budget constraint (4.2) and a no-Ponzi condition, taking as given prices, policies and firms’ profits $\{P_t, W_t, R_t, \tau_t, G_t, T_t, \Pi_t(i)\}_{t=0}^\infty$, as well as initial bond holdings $B_S^0$ and $B_{-1}$. The optimal plan of the household has to satisfy (4.2) and a standard transversality condition, as well as the following first-order conditions:

$$\lambda_t = \beta_t \frac{u'(c_t)}{P_t},$$

$$R_t^{-1} = \frac{\beta}{\pi_{t+1}} \frac{u'(c_{t+1})}{u'(c_t)},$$

$$q_t = \frac{\beta}{\pi_{t+1}} \frac{u'(c_{t+1})}{u'(c_t)},$$

$$w_t = \frac{1}{1 - \tau_t} \frac{u'(h_t)}{u'(c_t)},$$

where $\lambda_t$ is the Lagrange multiplier of the budget constraint (4.2), $\pi_{t+1} = P_{t+1}/P_t$ is the gross one-period inflation rate, and $w_t = W_t/P_t$ is the real wage. Equation (4.6) describes intratemporal trade-off between consumption and leisure. Equations (4.4) and (4.5) are Euler equations describing intertemporal allocation of consumption and savings. Combining Euler equations (4.4) and (4.5) yields the no-arbitrage condition between the one-period nominal interest rate and the price of long-term government bonds

$$R_t = \frac{1 + \rho q_{t+1}}{q_t}.$$  

4.2 Firms

Production of the final consumption good consists of two stages. On the upper stage of production process there is a (representative) perfectly competitive firm that assembles the final good $y_t$ from a bundle of imperfectly substitutable intermediate goods $y_{i,t}$ indexed by $i \in [0, 1]$ using the constant-returns-to-scale
technology

\[ y_t = \left( \int_0^1 \frac{\theta - 1}{\theta} \, dy_{i,t} \right)^{1/\theta}, \]

where \( \theta > 1 \) is the intratemporal elasticity of substitution across different varieties of intermediate goods. Solving profit maximization problem of the final-good producer results in the following demand schedule for every intermediate good \( i \)

\[ y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} y_t, \quad (4.8) \]

where \( P_{i,t} \) is the price of intermediate good \( i \). From the zero-profit condition—brought by the absence of an entry cost—it follows that \( P_t \) can be written as a price index:

\[ P_t = \left( \int_0^1 P_{i,t}^{1-\theta} \, di \right)^{1/\theta}. \]

On the lower stage of production process there is a continuum of firms of unit mass each producing an intermediate good with a technology that is linear in labor

\[ y_{i,t} = h_{i,t}, \]

where \( h_{i,t} \) is the labor input of firm \( i \). Imperfect price-elasticity of final-good producer’s demand (4.8) endows intermediate firms with market power to set prices, which in general distorts the economy. Another distortion in this economy is due to a nominal rigidity that makes price adjustment costly for a firm. It is modeled, following Rotemberg (1982), by introducing quadratic cost of adjusting nominal prices (measured in terms of the final good) given by

\[ \kappa_{i,t} = \frac{\varphi}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 y_t, \]

where \( \varphi \geq 0 \) measures the degree of nominal price rigidity. Higher values of \( \varphi \) indicate greater price stickiness, while \( \varphi = 0 \) corresponds to the case of perfectly flexible prices. The firm producing good \( i \) sets the price \( P_{i,t} \) and hires, in a perfectly competitive labor market, the quantity of labor that is necessary to satisfy realized demand. Present discounted real value of profits received by this firm is given by

\[ \sum_{t=0}^{\infty} \lambda_t \left[ P_{i,t} y_{i,t} - (1 - s) W_t y_{i,t} - P_t \kappa_{i,t} \right], \quad (4.9) \]

where \( s \) is the time-invariant rate of labor (employment) subsidy provided by the government to eliminate steady-state distortions created by monopolistic competition and taxation of labor income.\(^{11}\)

The pricing problem of the firm producing intermediate good \( i \) is dynamic due to the presence of the price adjustment cost. The firm chooses sequence of prices \( \{P_{i,t}\}_{t=0}^{\infty} \) so as to maximize its profits (4.9) subject to demand function (4.8), taking as given nominal wage, aggregate index of prices, and aggregate demand \( \{W_t, P_t, y_t\}_{t=0}^{\infty} \). In equilibrium all intermediate-goods producers behave symmetrically and charge identical prices \( P_{i,t} = P_t \) for all \( i \in [0, 1] \). Then, the optimizing behavior of intermediate-goods producers is

\(^{11}\)The specification follows Burgert and Schmidt (2014) and Leith and Wren-Lewis (2013). One can alternatively design a labor income subsidy, which works equivalently when the labor market is competitive.
characterized by the first-order condition of the pricing problem written as follows

\[(1 - s)w_t - \theta (\tau_t - s) w_t = \frac{\varphi}{\theta} (\pi_t - 1) \pi_t - \beta u_{t+1}\frac{w_{t+1}}{w_t} (\pi_{t+1} - 1) \pi_{t+1} + \psi \pi_t + (\pi_{t+1} - 1) \pi_{t+1}). \tag{4.10}\]

Equation (4.10) is a non-linear version of the New Keynesian Phillips curve that describes evolution of inflation over time as driven by the real marginal cost of production, which is proportional to the real wage, $w_t$.

### 4.3 The Government

The government consists of a central bank and a treasury. The central bank controls the short-term nominal interest rate, $R_t$. The treasury chooses the amount of spending on public good provision to the household, $G_t$. To finance government spending, the treasury levies a labor income tax at the rate $\tau_t$ and participates in the bond market.

Assuming that simple one-period bonds are in zero net supply, the treasury can borrow on the bond market by issuing portfolio of long-term government bonds in the form of perpetuities with decaying nominal payoffs. This modeling approach emulates maturity structure of outstanding debt with shares that decay with remaining maturity. The payoff decay factor, $\rho$, parametrizes average maturity of the portfolio of government bonds. Setting $\rho = 0$ makes the model identical to the case of one-period bonds. Another extreme is the case of “consol” bonds obtained when $\rho = 1$. For a generic value of $\rho$, in a steady-state with zero inflation, average maturity of the portfolio as measured by duration is equal to $(1 - \beta \rho)^{-1}$ model periods, see Appendix A.2. The payoff decay factor is not a policy instrument of either the central bank or the treasury; it is assumed to be exogenously given and time-invariant. The assumption of exogeneity captures the fact that management of the maturity structure of government debt is not part of conventional monetary and fiscal policy.\(^{12}\)

Consolidated flow budget constraint of the government is given by

\[q_t B_t = (1 + \rho q_t) B_{t-1} + P_t (G_t - (\tau_t - s) w_t h_t) - T_t.\]

Lump sum tax $T_t$ is restricted to be used for the sole purpose of transferring resources corresponding to the labor subsidy. Furthermore, since the goal of the labor subsidy is to correct only long-run distortions in the economy, the value of the lump-sum tax is set to be constant over time and equal to the steady-state value of the subsidy.\(^{13}\) The flow budget constraint of the government in real terms is then given by

\[q_t b_t = (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + \left( G_t + \varsigma_t - \tau_t w_t h_t \right), \tag{4.11}\]

where $b_t \equiv B_t / \pi_t$ is the quantity of long-term government bonds in real terms, and $\varsigma_t \equiv s w_t h_t - s \bar{w} \bar{h}$ is the deviation of the labor subsidy from its steady-state level in real terms.

Note that iterating the no-arbitrage condition (4.7) forward and imposing a terminal condition results in

\[q_t \equiv \sum_{k=0}^{\infty} R_t R_{t+1} \cdots R_{t+k}.\]

\(^{12}\)See Bhattarai et al. (2015) for a model with time-varying and endogenously determined payoff decay factor that captures unconventional monetary policy of Quantitative Easing.

\(^{13}\)This assumption is different from the model in Section 3, where it is implicitly assumed that the subsidy is financed by lump-sum transfers both in the steady state and away from it.
which shows that the government sells the portfolio of long-term bonds issued in period $t$ at a market price that depends on the current as well as future monetary policy reflected by the path of one-period nominal interest rate, $R_t$. One can therefore think of $\{G_t, \tau_t, R_t\}_{t=0}^{\infty}$ as government policy instruments that pin down $\{b_t\}_{t=0}^{\infty}$ as satisfying government budget.

4.4 Implementable Equilibria

Symmetric pricing behavior in equilibrium leads to all the firms producing the same amount of output and hiring the same amount of labor, hence $y_{i,t} = y_t$ and $h_{i,t} = h_t$ for all $i \in [0, 1]$. Therefore, one can write down the aggregate production function of the form

$$y_t = h_t, \quad (4.12)$$

and the aggregate resource constraint resulting from the clearing of the goods market as follows

$$h_t = c_t + G_t + \frac{\varphi}{2} (\pi_t - 1)^2 h_t, \quad (4.13)$$

The aggregate resource constraint shows that nominal rigidity creates a wedge between the output and the aggregate consumption because a fraction of output is allocated to paying the price adjustment cost.

The set of implementable equilibria is restricted by the following definition.

**Definition 1.** The private-sector equilibrium is a sequence $\{c_t, y_t, h_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t, R_t\}_{t=0}^{\infty}$ satisfying equations (4.5)–(4.7), (4.10)–(4.13) and the transversality condition, for $t \geq 0$, given initial outstanding government debt $b_{-1}$.

The definition of private-sector equilibrium describes a (dynamic) system of 7 equations in 10 unknowns. The option to choose policy instruments creates three degrees of freedom that allow the government to implement certain equilibria from the set satisfying this system.

4.5 The First-Best Allocation

Before introducing the government’s problem of policy choice it is instructive to characterize an efficient allocation of private consumption, consumption of public good, and labor. This first-best allocation is meant to serve as a benchmark for the allocation that arises in the private-sector equilibrium under optimal government policy.

The efficient allocation is defined as a solution of the fictitious Social planner’s problem. The planner does not allow monopoly power in the production of intermediate goods and allocates resources effectively across varieties. The planner maximizes the household’s lifetime utility subject to the sequence of aggregate resource constraints of the form

$$h_t = c_t + G_t, \quad (4.14)$$

The first-order conditions imply, see Appendix A.3, that the period marginal utilities of private and public consumption be set equal to the marginal disutility of labor:
Efficiency conditions (4.14)–(4.16) are static, thus efficient allocation \( \{c_t, h_t, G_t\} \) is constant over time. In other words, it is optimal to allocate a fixed amount of labor to production of output and then allocate fixed shares of output to private and public consumption.

### 4.6 The Policy Problem

For the remainder of the paper it is assumed that preferences of the households for private consumption and consumption of public goods are described by 

\[
u(c_t) \equiv c_t^{1-\gamma} - \gamma c_t^{1-\gamma}
\]

and 

\[
g(G_t) \equiv \nu g G_t^{1-\gamma} - \gamma g_t^{1-\gamma}
\]

respectively, and the disutility from work is described by 

\[
v(h_t) \equiv \nu h_t^{1+\gamma} - \gamma h_t^{1+\gamma}
\]

Also, in what follows it is assumed that there is full cooperation between the central bank and the treasury and that such a consolidated government acts benevolently with the objective of maximizing the lifetime utility (4.1) of the representative household. The government controls policy instruments at its disposal in order to achieve optimal allocation as a part of the private-sector equilibrium. The government is modeled as not being able to commit to its future policy choices and instead acting discretionary in every period of time. The government, however, can still credibly commit to repay its debt in the future. The analysis in this paper abstracts from reputation mechanisms and focuses on optimal policy as an outcome of a dynamic game between successive selves of the government as if these were separate policymakers in every period of time. A stationary Markov-perfect equilibrium of this game is defined along the lines of Klein et al. (2008).

In a Markov-perfect equilibrium, strategies of the government depend on the minimal payoff-relevant state of the economy, which in every period \( t \) is entirely described by the quantity of outstanding government bonds, \( b_{t-1} \). In each period \( t \), the government maximizes utility of the representative household starting from its incumbent period onwards. When making the announcement of policy for the current period \( t \), the government takes into account how the private sector reacts, given anticipated future policy.

Formally, the Markov optimization problem of the discretionary government in any period \( t \) can be written as choosing \( \{c_t, y_t, h_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t, R_t\} \) that maximize

\[
\frac{c_t^{1-\gamma}}{1-\gamma} + \nu g \frac{G_t^{1-\gamma}}{1-\gamma} - \nu h_t^{1+\gamma} + \beta V(b_t)
\]

subject to

\[
0 = \Upsilon(b_{t-1}; c_t, y_t, h_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t, R_t; C(b_t), Y(b_t), \Pi(b_t), Q(b_t)),
\]

given outstanding quantity of bonds, \( b_{t-1} \), and anticipated future policy together with implied allocation and prices in the private-sector equilibrium as described by functions \( \{C, Y, H, \Pi, W, Q, B, G, T, R\} \) that provide continuation utility \( V \). For brevity, the vector-function \( \Upsilon \) is used to summarize the set of private-sector equilibrium constraints from Definition 1.
For optimal policy to be time-consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov-perfect equilibrium.

**Definition 2.** The Markov-perfect equilibrium is a function \( V(b_{t-1}) \) and a tuple of decision rules \( \{C, Y, H, \Pi, W, G, T, R\} \), each being a function of \( b_{t-1} \), such that for all \( b_{t-1} \):

1. Given \( V \), the tuple of rules solves the Markov problem of the government,
2. \( V \) is the value function of the government

\[
V(b_{t-1}) = \frac{C(b_{t-1})^{1-\gamma_c}}{1-\gamma_c} + \nu g \frac{G(b_{t-1})^{1-\gamma_g}}{1-\gamma_g} - \nu h \frac{H(b_{t-1})^{1+\gamma_h}}{1+\gamma_h} + \beta V(B(b_{t-1})).
\]

The analysis is restricted to equilibria with differentiable value function and equilibrium decision rules. Assuming that such an equilibrium exists, it can be characterized by the first-order conditions of the policy problem.\(^{14}\) A detailed formulation of the policy problem and derivation of the corresponding first-order conditions is delegated to Appendix A.4.

The first-order conditions imply that the three degrees of freedom provided by the ability to choose fiscal and monetary policy instruments are optimally controlled in accordance with the following three equations

\[
\Delta c_t \Omega_t \left( \nu g G_t^{\gamma_g} \right) = \nu g G_t^{\gamma_g} - c_t^{\gamma_c},
\]

\[
\left( \frac{\varphi}{2} (\pi_t - 1)^2 + \Delta h_t \Omega_t \right) \left( \nu g G_t^{\gamma_g} \right) = \nu g G_t^{\gamma_g} - \nu h y_t^{\gamma_h},
\]

\[
(1 + \Delta h_t) \Omega_t \left( \frac{\nu g G_t^{\gamma_g}}{c_t^{\gamma_c}} \right) = \Omega_{t+1} \left( \frac{\nu g G_{t+1}^{\gamma_g}}{c_{t+1}^{\gamma_c}} \right)
\]

where \( \Omega_t \) is an auxiliary variable defined as follows

\[
\Omega_t \equiv \frac{\varphi (\pi_t - 1) y_t}{\varphi (\pi_t - 1) y_t + \frac{\varphi}{2} (2\pi_t - 1) y_t + (1 + \rho q_t) \frac{b_{t-1}}{\pi_t}},
\]

with the numerator equal to the marginal resource cost of inflation and the denominator equal to the sum of the marginal resource cost and marginal benefits of inflation due to the implicit profit tax and the real liability effects. The real liability effect of inflation refers to a decline in the real value of outstanding government debt. The implicit profit tax effect of inflation refers to a decline of monopolistic markup charged by the firms due to the corresponding increase of the labor cost. The larger is the marginal resource cost of inflation compared to its marginal benefits the larger is \( \Omega_t \). Let \( \Omega_t \) be referred to as an inflation cost factor. The remaining auxiliary variables \( \Delta c_t, \Delta h_t \) and \( \Delta h_t \) are defined in Appendix A.4.

Equations (4.17) and (4.18) are the targeting rules that can be compared to the efficiency conditions (4.15) and (4.16). Both sets of equations establish an intratemporal relation between the marginal utility components. As has been previously discussed, efficiency dictates to equate marginal utilities of private consumption, government spending and disutility of work. In the case of optimal policy, however, there

\(^{14}\)The analysis in this paper refrains from a formal general proof of equilibrium existence and uniqueness. For a parameterized model, numeric results in the subsequent sections demonstrate existence and local uniqueness.
are nontrivial wedges between these marginal utility terms. These wedges arise due to the discretionary nature of government policy layered over the distortions present in the model economy. Equation (4.19) is the generalized Euler equation (GEE) that determines the optimal way to trade-off economic distortions intertemporally. This equation to a large extent determines the way how the wedges of the targeting rules evolve over time. The GEE is therefore crucial in determining dynamic properties of the allocation and underlying prices and policies.

Note that the inflation cost factor, $\Omega_t$, appears in the target rules, (4.17) and (4.18), and in the generalized Euler equation, (4.19). It allows to draw two intuitive conclusions. First, there is an interrelation between inflation and inefficiency. If Markov-perfect allocation in a given period of time differs from the efficient benchmark then corresponding inflation is different from zero. It also works in reverse, if inflation in the Markov-perfect equilibrium differs from zero in a given period of time then the corresponding allocation is different from the efficient benchmark. Second, the efficient allocation with zero inflation is a steady state of the Markov-perfect equilibrium. In other words, if the economy finds itself with efficient allocation (and zero inflation) in a given period of time it stays there forever. Approximated counterparts of the targeting rules and the GEE are analyzed in more details in the next section.

Following Leith and Wren-Lewis (2013), it is assumed that a labor subsidy is used by the government to offset permanent distortions in the economy. The labor subsidy rate, $s$, determines the amount of government debt at the efficient steady-state, see Appendix A.5. For practical purposes the subsidy rate is assumed to be such that the efficient steady state is supported by a positive amount of government debt. This assumption captures the idea of long-run commitment, while having discretionary policy along the transition path. One can solve for the efficient steady state independently of the optimal dynamic policy. Discretionary dynamics away from the steady state does not have a closed-form solution. Therefore, further equilibrium characterization and discussion of optimal government policy requires applying approximation methods and solving for the equilibrium numerically. The remainder of this section comments on solution methods and the describes baseline parameterization strategy.

4.7 Parameterization and Solution Methods

Solution for the Markov-perfect equilibrium as defined above can be computed numerically by means of a global nonlinear approximation method. The solution method employed in this paper is based on a projection method described in Debortoli and Nunes (2012). In a nutshell, the method is based on approximating equilibrium decision rules with cubic splines and solving a system of the first-order conditions of the policy problem by looking for a fixed-point of equilibrium decision rules. While accurate, results computed with this method lack analytical tractability. This is why, in addition to the nonlinear method, this paper relies on a local approximation of the policy problem in the vicinity of the steady state in order to derive a number of analytical and clear-cut numerical results.

Both solution methods eventually call for assigning values to structural parameters of the model. Baseline parameter values used for computation are summarized in Table 1. Each time period in the model represents one quarter of a year. The time discount factor, $\beta$, is set equal to 0.99. Curvature parameters of the utility functions, $\gamma_c$, $\gamma_g$ and $\gamma_h$, are all set equal to 1. Values chosen for the utility weights $\nu_h$ and $\nu_g$ equal to 20 and 0.25 correspondingly so that in the steady state households spend one quarter of their unitary time endowment working and government spending amounts to 20 percent of the value added.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>$\gamma_c$</td>
<td>Intertemporal elasticity for $C$</td>
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<td>$\gamma_g$</td>
<td>Intertemporal elasticity for $G$</td>
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<td>$\gamma_h$</td>
<td>Inverse Frisch elasticity</td>
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<tr>
<td>$\nu_g$</td>
<td>Utility weight on gov.t spending</td>
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</tr>
<tr>
<td>$\nu_h$</td>
<td>Utility weight on labor</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Price adjustment cost</td>
<td>116.505</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution among goods</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 1 – Baseline parameterization of the baseline model

The monopoly power of the firms is described by the elasticity of substitution between intermediate goods, $\theta$, which is set equal to 11 in order to match desired markup of the price over the marginal cost of 10 percent. Given the value for $\theta$, the parameter of price adjustment cost, $\phi$, is set equal to 116.505, which makes the slope of the Phillips curve consistent, up to the first order of approximation, with a Calvo (1983) price-setting specification where one quarter of the firms reoptimize their prices every period or, equivalently, the average price duration is equal to one year.

With respect to government debt characteristics, the model is parameterized as follows. The baseline target for the market value of government debt in the steady state is set equal to 40 percent of annual GDP. This number is consistent with the pre-crisis U.S. data available from the Federal Reserve Bank of Dallas. Parameter $\rho$ is set to match certain values of the average maturity of government debt. Two baseline cases are considered: one-period bonds and bonds with the average maturity equal to four years. The latter is consistent with the pre-crisis duration of government debt in the U.S. as reported in Greenwood et al. (2014). Equilibrium characterization that relies on the local approximation method also looks into wider ranges of values for both the maturity and the market value of government debt. Simulations of government debt reduction that rely on the global nonlinear approximation method use an initial value such that the market value of government debt at the beginning of transition is 30% higher than in the steady state, which is consistent with the post-crisis U.S. data available from the Federal Reserve Bank of Dallas.

5 Optimal Policy

In order to facilitate understanding of the key mechanisms driving the results it is convenient to reformulate the policy problem in a linear-quadratic form. In order to obtain the linear-quadratic representation of the policy problem, the objective function of the government is approximated up to the second order, whereas equilibrium constraints are approximated linearly. The approximation is done in logarithmic deviations around the efficient steady state with zero inflation and a non-zero amount of government debt, see Appendix A.6 for derivations.

Using the first-order conditions of the linear-quadratic policy problem allows to provide a rich local analytical characterization of a (differentiable) Markov-perfect equilibrium around the steady state. This characterization parallels and extends discussion of a restricted model with exogenous government spending.

See Woodford (2003) for a detailed discussion of the linear-quadratic approach. One could alternatively derive a (log-)linear approximation of the first-order optimality conditions of the non-linear Markov-perfect equilibrium. It is possible to show that these two approaches would deliver identical results.
and inflation-indexed debt in Section 3. The Markov-Perfect equilibrium is approximated by a set of rules \( \{ \hat{C}, \hat{Y}, \hat{H}, \hat{H}, \hat{V}, \hat{Q}, \hat{B}, \hat{G}, \hat{T}, \hat{I} \} \) each being a linear function of \( \hat{b}_{t-1} \), such that for any \( \hat{b}_{t-1} \), the quantities, prices, and policies generated by these rules \( \{ \hat{c}_t = C_t \hat{b}_{t-1}, \hat{y}_t = Y_t \hat{b}_{t-1}, \ldots, \hat{i}_t = I_t \hat{b}_{t-1} \} \) satisfy the system of the first-order conditions that contains private-sector equilibrium conditions (4.5)–(4.7), (4.10)–(4.12) approximated up to the first order:

\[
\begin{align*}
\hat{y}_t &= \hat{h}_t, \quad (5.1) \\
\hat{\pi}_t &= \beta \rho \hat{q}_{t+1} - \hat{q}_t, \quad (5.2) \\
\hat{\omega}_t &= \bar{\pi} \hat{w}_t + \gamma_c \hat{c}_t + \gamma_h \hat{y}_t, \quad (5.3) \\
\hat{q}_t &= \gamma_c (\hat{c}_t - \hat{c}_{t+1}) + \beta \rho \hat{q}_{t+1} - \hat{\pi}_{t+1}, \quad (5.4) \\
\hat{b}_t &= \beta^{-1} \hat{b}_{t-1} - (1 - \rho) \hat{q}_t - \beta^{-1} \hat{\pi}_t + \hat{\Gamma}^{-1} \hat{G} \hat{C}_t - \hat{\Gamma}^{-1} \hat{w} \hat{y} ((\bar{\pi} - \hat{s}) (\hat{y}_t + \hat{w}_t) + \bar{\pi} \hat{t}_t), \quad (5.5)
\end{align*}
\]

where variables with a bar denote steady-state values, variables with a hat denote percentage deviations from the steady state. Additionally, \( \hat{\Gamma} = \hat{b} \hat{q} \) is the market value of government debt in the steady state, \( \hat{t}_t \) denotes the percentage deviation of the short-term nominal interest rate, \( R_t \). The remaining first-order conditions are

\[
\begin{align*}
\hat{c}_t &= \Phi_c \hat{\pi}_t, \quad (5.6) \\
\hat{y}_t &= \Phi_y \hat{\pi}_t, \quad (5.7) \\
\hat{G}_t &= \Phi_g \hat{\pi}_t, \quad (5.8) \\
0 &= \frac{\theta \hat{\Gamma}}{\varphi_y} [\hat{\pi}_{t+1} - \hat{\pi}_t] + \beta \Pi_b \hat{\pi}_t + (1 - \rho) (\gamma_c \hat{C}_b + \Pi_b - \beta \rho \hat{q}_b) \frac{\theta \hat{\Gamma}}{\varphi_y} \hat{\pi}_t, \quad (5.9)
\end{align*}
\]

where the coefficients \( \Phi_c, \Phi_y \) and \( \Phi_g \) depend, among other things, on parameter \( \rho \) governing average maturity of government bonds as well as on the market value of government debt in the steady state, \( \hat{\Gamma} \). Expressions defining coefficients \( \Phi_c, \Phi_y \) and \( \Phi_g \) are delegated to Appendix A.6.

Given the motivation of historically high levels of government debt in developed economies, the analysis below assumes that the model economy starts off away from the steady state with an initial condition of inherited debt in excess of the steady-state level. The main purpose of this section is to characterize equilibrium with the focus on comparative dynamic effects of the maturity of government bonds and to assess the robustness of these effects with respect to the steady-state market value of government debt. A plain \emph{ceteris paribus} change of parameter \( \rho \) would not only change average maturity of government debt but would also impact the amount of payments associated with every bond issued by the government. The latter effect would change the steady-state price of government bonds, \( \hat{q} \), which by definition would affect the market value of government debt in the steady state, \( \hat{\Gamma} \). Differently, the effects of changes in the maturity of government bonds discussed below are associated with variation of parameter \( \rho \) conditional on sterilizing the effect of portfolio composition on the steady-state market value of government debt.

The analysis that follows is restricted to locally non-explosive equilibria with government debt converging to the steady state monotonously or, formally, with \( 0 < \beta_b < 1 \). Despite tractability and linearity of decision
rules, discretionary equilibria in the linear-quadratic models in general are not immune to multiplicity, see Blake and Kirsanova (2012). The method of undetermined coefficients allows one to characterize equilibrium in the baseline model as a solution of two nonlinear equations in terms of the decision rule coefficients for government debt and inflation, $B_b$ and $Π_b$, see Appendix A.6 for details. Numerical checks show that under baseline parameter values the approximate model does not feature multiple equilibria.

Figure 1 plots coefficients in the decision rules for government debt, $B_b$, and inflation, $Π_b$, as functions of the market value of government debt in the steady state when government issues one-period bonds whilst other parameters are set equal to their values from the baseline parameterization. First, this graph shows that the monotone equilibrium exists only when the steady-state market value of government debt is below 60% of annual GDP. Furthermore, this graph shows that a positive deviation of government debt from the steady-state level leads to an upward pressure on the price level in the economy. Contemporaneous response of inflation increases with the steady-state market value of government debt. Also, a higher steady-state indebtedness makes government debt dynamics less persistent, which speeds-up transition towards the steady state.

[Figure 1 about here.]

Figure 2 plots the same coefficients in the case when average maturity of government debt is equal to four years. Comparing it with the previous graph shows that longer maturity corresponds to a weaker contemporaneous response of inflation to a deviation of government debt from its steady-state level. Furthermore, longer maturity makes government debt dynamics more persistent, which slows-down transition towards the steady state. Also note that with longer maturity there is a smaller effect of an increase in the market value of government debt in the steady state on coefficients of the decision rules for inflation and government debt.

[Figure 2 about here.]

When analyzing fiscal and monetary policy in separate models (without nominal rigidity and with lump-sum taxes correspondingly), looking at the dynamics of government debt or inflation tells a lot about the underlying policy. Analysis of the model in this paper is less straightforward because the two variables are intertwined and determined by the fiscal-monetary policy mix. The analysis below takes advantage of the linearity of the first-order conditions (5.1)–(5.9) so as to disentangle the underlying policy. The analysis is split in two parts. First part studies government debt dynamics and discusses incentives to reduce government debt when it exceeds the steady-state level. It explicitly shows that the path of government reduction towards the steady state depends both on fiscal and monetary policy. Second part of the analysis looks in more details at the policy mix that implements debt reduction in the equilibrium.

5.1 Dynamics of Government Debt Reduction

Equation (5.9) describes optimal way to balance the intertemporal trade-off faced by the government when issuing government debt. It can be referred to as a generalized Euler equation (GEE) due to the presence of derivatives of equilibrium decision rules. The GEE is written as a linear combination of the welfare-relevant
wedges expressed in terms of inflation in the two consecutive periods. This equation is crucial in determining
dynamic properties of the economy.

Using linear equilibrium decision rule for inflation, the GEE can be written in terms of government debt
as follows

\[ 0 = \frac{\theta \Gamma}{\varphi \gamma} [\hat{b}_t - \hat{b}_{t-1}] + \beta \Pi_b \hat{b}_{t-1} + (1 - \rho) \left( \gamma \sigma C_b + \Pi_b - \beta \rho Q_b \right) \frac{\theta \Gamma}{\varphi \gamma} \hat{b}_{t-1}. \]  

(5.10)

This representation of the GEE shows explicitly that if the government has outstanding debt different from
the steady-state level then optimal amount of newly issued bonds balances direct gains from smoothing debt
and corresponding distortions over time against indirect losses coming from anticipated effects of doing so on
decision of the government in the successive period.

Think of the government that starts off with an outstanding debt in excess of the steady-state level. As
discussed earlier in Section 3, having excessive government debt can be supported by raising the tax rate,
which is costly due to the cost-push inflationary effect. The first term in the GEE reflects direct gains from
smoothing adjustment of the tax rate intertemporally. This term introduces permanent component into the
dynamic behavior of the economy. The government in the successive period, however, does not internalize
effects of its policy choice on trade-offs faced by the government in the current period. As a result, sustaining
inefficient debt level in the long-run ceases to be optimal because it brings along indirect losses. It becomes
optimal for the government to reduce debt towards the steady state so as to mitigate these losses.

The second term in the GEE reflects indirect loss from leaving debt level unchanged that comes from
the marginal effect of having extra inflation in the successive period. Expectation of inflation works its way
through the Phillips curve, (5.3), and makes firms more willing to raise prices further up in the current
period.\(^{17}\) The third term in the GEE reflects indirect loss from leaving debt level unchanged that comes
from the marginal effects on the successive period consumption, inflation, and the price of government
bonds. Joint expectations regarding these variables affect consumption-savings decision of the household,
as described by the Euler equation, (5.4), and make agents less willing to save in the current period. Note
that inflation expectations enter this term because the household saves in nominal bonds, which is different
from the analysis in Section 3 where bonds are indexed to inflation. Such a real interest rate effect makes
it more expensive for the government to borrow today, which requires a stronger tax rate increase \textit{ceteris
paribus}.\(^{18}\) These two GEE terms introduce a transitory component and pin down the persistence of the
dynamic behavior of the economy.

In particular, it follows directly from (5.10), equilibrium dynamics of government debt follows an
autoregressive process with coefficient \((1 - \frac{1}{\varphi} \beta \gamma \gamma \Pi_b - (1 - \rho) (\gamma \sigma C_b + \Pi_b - \beta \rho Q_b))\). The larger are the indirect
losses the stronger is debt reduction intended to strategically manipulate future policy and the faster is
convergence to the steady state. Furthermore, linear structure of the decision rules implies that the remaining
economic variables inherit the persistence of government debt. Thus, one can see that economies with
different market value of government debt in the steady state, \(\Gamma\), and parameter \(\rho\) governing the maturity of
government bonds are going to have different dynamics. First, the difference comes through the change in the
sensitivity of government budget in a given period to marginal effects of newly issued debt on the equilibrium

\(^{17}\)This is a distinct loss that emerges due to the presence of nominal rigidity in the model. It is absent in a model with
competitive goods market and flexible prices, see Debortoli and Nunes (2012).

\(^{18}\)This loss is nil in the case of consol bonds, \(\rho = 1\), due to the flat payoff structure of such bonds as discussed by Debortoli
et al. (2016) in the context of a model with competitive goods market and flexible prices.
decision rules in a subsequent period. Second, the difference comes through changes in the equilibrium decision rules themselves. Equilibrium decision rules depend on the entire policy mix that includes the tax instrument as well as government spending and monetary policy interest rate. Next part of this section looks at the optimal policy mix in detail.

5.2 Policy Instruments

Equations (5.6)–(5.8) constitute a set of targeting rules. Targeting rules in general offer a convenient way to describe the outcome of solving optimal policy problems. The set of targeting rules in this model describes optimal intratemporal relations between deviations (gaps) of target variables, namely private consumption, \( \hat{c}_t \), output, \( \hat{y}_t \), government spending, \( \hat{G}_t \), and inflation, \( \hat{\pi}_t \), that the government seeks to maintain in every period \( t \). Comparing to the primer example in Section 3, the baseline model features two more target variables and, therefore, two more targeting rules. The targeting rules allow to characterize equilibrium paths of monetary and fiscal policy instruments during the transition of the economy with an excessive amount of government debt towards the steady state. The signs of targeting coefficients \( \Phi_g, \Phi_c, \) and \( \Phi_y \), are crucial in characterizing the qualitative nature of the underlying policy.

The targeting rule (5.8) for government spending has a negative coefficient \( \Phi_g \), hence the sign of the government spending gap is opposite of inflation. This result is robust to variations of the steady-state market value and the maturity of government debt.\(^{19}\) Recall numerical results in the beginning of this section that show how government debt in excess of the steady-state level leads to an upward pressure on prices, \( \Pi_b > 0 \). Targeting rule (5.8) then implies that a reduction of government debt is accompanied by keeping government spending low relative to its steady-state level. The optimal reduction of government spending balances the cost of providing inefficiently low amount of public goods against direct and indirect benefits that allow to reduce the adverse cost-push effect of raising the tax rate. The direct benefit stems from an increase in the primary surplus, whereas indirect benefit comes from an offsetting effect of a corresponding reduction in public demand on the willingness of firms to raise prices.

The signs of the remaining targeting coefficients, \( \Phi_c \), and \( \Phi_y \), depend on the steady-state market value and average maturity of government debt. As a result, transition behavior of interest and tax rates also varies depending on these characteristics of government debt. One can combine the Euler equation, (5.4), decision rule for government debt, targeting rule (5.6) and no-arbitrage condition (5.1), to derive an equilibrium relation between the one-period nominal interest rate and expected inflation

\[
\hat{i}_t = \left(1 - \gamma_c \Phi_c \frac{(1-B_b)}{B_b}\right) \hat{\pi}_{t+1}.
\] (5.11)

Equation (5.11) provides a clear criterion of the equilibrium stance of monetary policy. Monetary policy raises the nominal interest rate less than one-to-one in response to expected inflation—a loose stance—if and only if targeting coefficient \( \Phi_c \) is positive. As in Section 3, the sign of the targeting coefficient \( \Phi_c \) is determined by opposing incentives to reduce private demand so as to mitigate the adverse cost-push effect of raising the tax rate and to stimulate private demand so as to reduce the need to raise the tax rate in the first place. Relaxing assumption of inflation-indexed government bonds in the baseline model creates an

\(^{19}\) Using analytical expression defining \( \Phi_g \) in Appendix A.6, one can see that its sign is unambiguously negative given previously made assumption of a positive amount of government debt in the steady state.
additional incentive to stimulate demand because it leads to a higher inflation that reduces the real value of outstanding government debt.

It is straightforward to see that the stance of monetary policy has direct implication for the path of real interest rate during the transition of the economy towards the steady state. One can characterize equilibrium dynamics of the real interest rates by rewriting (5.11) using decision rules for inflation and government debt:

$$\hat{r}_t = -\gamma_c \Phi_c \Pi_b (1 - B_b) \hat{b}_{t-1},$$  \hspace{1cm} (5.12)

where $\hat{r}_t = \hat{i}_t - \hat{\pi}_{t+1}$ is the one-period real interest rate. Thus, whenever monetary policy is loose a reduction of government debt is accompanied by real interest rates that are below the long-run level.

Analogously, the equilibrium tax rate can be characterized by using decision rule for inflation and government debt, targeting rules (5.6)–(5.7), and equation (5.2) that describes optimal intratemporal choice of the household between consumption and leisure in order to rewrite the Phillips curve, (5.3), as follows

$$\bar{T} \Pi_b (1 - \bar{T}) \hat{\tau}_t = \frac{\varphi}{\theta - 1} (1 - \beta B_b) \hat{b}_{t-1} - (\gamma_c \Phi_c + \gamma_h \Phi_y) \hat{b}_{t-1}.$$  \hspace{1cm} (5.13)

For convenience, let tax policy be referred to as tight when reduction of government debt in equilibrium is implemented with the labor tax rate above the long-tun level and be referred to as loose otherwise. A sufficient condition for tax policy to be tight is to have a linear combination of targeting coefficients $(\gamma_c \Phi_c + \gamma_h \Phi_y)$ with a negative sign. Having the same linear combination with a positive sign is a necessary condition for tax policy to be loose. It is not a sufficient condition because if equilibrium speed of government debt reduction is fast enough then tax policy implementing it has to be tight. Note that targeting coefficient $\Phi_y$ can be written as a linear combination of targeting coefficients $\Phi_c$ and $\Phi_g$ that summarizes effects of public and private demand stimuli provided by interest rate and government spending policies on aggregate demand.

[Figure 3 about here.]

Figure 3 depicts the signs of targeting coefficients $\Phi_c$ and $\Phi_y$ for various combinations of the steady-state market value and average maturity of government debt whilst other parameters are set equal to their baseline values. The signs of these two targeting coefficients coincide in most of the cases. Moreover, the sign of targeting coefficient $\Phi_y$ is always negative in the region where the sign of targeting coefficient $\Phi_c$ is negative. Hence, tight monetary policy always goes together with tight tax policy. Loose monetary policy, however, potentially can co-occur both with tight and with loose tax policy. It is natural to continue by drawing on the previous characterization and discuss comparative dynamic effects of government debt characteristics.

The mix of tight monetary and tight tax policy is observed in the region where the steady-state market value of government debt is low enough and/or where the average maturity of government debt is long enough. In economies with debt characteristics from this region, it is optimal to reduce excessive government debt with the policy that sets high tax rates, low government spending, and high real interest rates. Inflation observed during the transition in these economies is due to the cost-push effect of high tax rates. Tight stance of monetary policy that results in high real interest rates is primarily driven by the incentive to reduce private demand so as to mitigate cost-push inflation. In other words, incentives that call for loose monetary
policy to stimulate private demand so as to reduce the need to raise the tax rate are relatively weak in these economies.

Shortening average maturity of government debt increases relative strength of the incentives that call for loose stance of monetary policy. In particular, shorter maturity creates a stronger incentive to benefit from a higher price of newly issued debt by keeping real interest rate low because of a smaller corresponding capital loss on outstanding debt. Thus, short enough maturity may reverse optimal stance of monetary policy and make it loose in equilibrium. Changes in the optimal stance of monetary policy depending on the maturity of government bonds also require a high enough steady-state value of government debt. Figure 3 shows that in the economy with a steady-state market value of debt around 40 percent of annual GDP optimal stance of monetary policy turns loose with maturity of government bonds only as short as one quarter. Importantly, increasing steady-state market value of debt still requires a very short average maturity of government debt to make it optimal for monetary policy to take a loose stance. Looking at the economies with steady-state market value of government debt as high as 160 percent of annual GDP, monetary policy turns loose with average maturity of government debt shorter than one year.

5.3 Transition Simulations

The analysis above provided tractable equilibrium characterization by using linear-quadratic approximation of the policy problem. This part presents transition simulations that provide illustrative examples supporting that characterization and provides a quantitative assessment of the comparative dynamic effects of the maturity of government bonds. Reported simulations are produced from a fully nonlinear solution of the policy problem. Using nonlinear solution shows that the characterization provides valid insights into dynamics of the model even when moving away from vicinity of the steady-state.

Figure 4 shows transition dynamics of two economies towards the steady state. Red dotted lines correspond to the economy where the government issues one-period bonds. Solid black lines correspond to the economy where the government issues bonds with average maturity equal to 4 years. Both economies start off with an initial quantity of bonds that makes the market value of outstanding government debt 30 percent higher than in the steady state. The graphs report dynamics of private consumption, government spending, aggregate output, and real market value of outstanding government debt in percentage deviation from the steady state. Inflation, the (net) nominal and real interest rates are reported in annualized percentages. The labor tax rate is reported in percentages of labor income.

First, consider the case of one-period bonds. Debt reduction is performed fast with the half-life of roughly one quarter, which means that every period excess debt is cut by half. Tax rate is initially set almost twice as high as the long-run level and declines over time. Also, initially there is a large negative gap of more than 30 percent in government spending that declines during the transition. Such dynamics of government spending mitigates adverse cost-push effect of elevated tax rates and helps to avoid further increases of the tax rate. Low government spending pulls down aggregate demand and results in the negative output gap along the transition despite a relative boom in private consumption. The latter is due to a loose stance of monetary policy that manifests itself in the real interest rates below the long-run level. Monetary policy is driven primarily by the incentives to expand the tax base, increase the price of newly issued government
bonds, and inflate away outstanding government bonds. Naturally, observed monetary policy jointly with tax policy lead to a surge in inflation that goes up to 10 percent initially and then declines over time.

Economy where the government issues bonds with average maturity equal to four years exhibits notably different transition dynamics. While the behavior of fiscal policy instruments is qualitatively unchanged compared to the case of one-period bonds, optimal stance of monetary policy changes. Longer maturity reduces the incentive to use interest rate policy in order to manipulate the price of government bonds. The incentive to reduce private demand so as to mitigate adverse cost-push effect of elevated tax rates becomes relatively more important, which makes optimal stance of monetary policy tight. As a consequence, transition path features negative consumption gap and high real interest rates. Switch in the stance of monetary policy yields a decline in the indirect cost of smoothing reduction of government debt over time. Therefore debt reduction with long-term bonds is more gradual and has a half-life of approximately 23 years, which is a slowdown by the factor of 92 compared to one-period bonds. The slowdown comes with a sizable reduction in amplitudes of changes in other variables. Inflation, in particular, runs only as high as 15 basis points in the first period and then declines.

5.4 Welfare Analysis

Given that maturity of government debt affects dynamic properties of the economy during transition towards the steady state, it is of interest to analyze and compare corresponding changes in welfare. Using the baseline parameterization of the model, Figure 5 shows welfare losses incurred because of the reduction of government debt depending on its average maturity under optimal discretionary policy. Losses are measured using welfare-equivalent permanent reduction of consumption in the steady state. The computation is done by solving for nonlinear transitions towards the steady state with a length of 1000 years each starting from the quantity of bonds that makes the market value of outstanding government liabilities in real terms 30 percent higher than in the steady state. The absolute size of welfare losses depends on the initial condition that determines the amount of debt in excess of the steady-state level. Naturally, the further away from the steady-state is initial condition the larger is the accruing loss. Therefore, the reported losses are scaled relative to the welfare loss of the transition in the economy with consol bonds.

As shown on the left panel of Figure 5, under the standard assumption of one-period government bonds the consumption loss is larger by the factor of 80 than the analogous loss in the economy with consol bonds. As the maturity of government bonds lengthens to one year, the relative loss declines 60 times and is equal to 1.34. The right panel of Figure 5 shows that the decline of the relative loss continues as the maturity of government debt increases further, albeit at a slower speed.

These results show that welfare losses of government debt reduction decline notably if the maturity of government debt is longer than one period. An important reason is the underlying change of monetary policy stance from loose to tight. As was previously discussed, this change leads to a decline of the indirect cost of smoothing reduction of government debt over time and an effective slowdown of the speed of government debt reduction. A more smooth reduction of government debt then translates in to an improvement of welfare.
6 The Medium-Scale Model

Analysis of the baseline model has shown a difference of the optimal discretionary policy depending on the maturity of government debt. In particular, the key result shows that optimal stance of monetary policy reverses depending on the maturity of government debt. Quantitative assessment of the baseline model pointed out that given average maturity of government bonds consistent with the data it is optimal to have tight monetary policy in an economy that starts off with government debt in excess of the steady-state level. Under baseline parameterization, it would be optimal to switch the stance of monetary policy and make it loose only if the government were to issue one-period bonds.

The baseline New Keynesian model, however, is known for its failure to account for the inertial behavior of inflation and persistence of aggregate dynamics in response to variation of the monetary policy interest rate. The purpose of this section is to assess the extent to which the effect of government debt maturity on the stance of monetary policy holds in a model with empirically relevant dynamic effects of monetary policy. To this end, analysis in this section builds on a workhorse medium-scale New-Keynesian model as described, for instance, in Christiano et al. (2010). The model includes monopolistic competition not only in the goods market but also in the labor market. Following Erceg et al. (2000), both markets feature nominal rigidity in the form of sticky prices and wages. Real rigidities include working capital friction, habit formation in consumption, and accumulation of capital with the costs of utilization and investment adjustment.

The workhorse medium-scale model is extended by introducing fiscal sector in a way identical to the earlier analysis of the baseline model in this paper. This model is then used to solve for optimal discretionary monetary and fiscal policy under fixed parameter values. Most of the parameter values are set to be equal to the posterior mean parameter estimates of the U.S. economy by Christiano et al. (2010). The model is solved by deriving the linear-quadratic approximation and then using the method described by Debortoli et al. (2014). A more detailed description of the parameterization and the equilibrium conditions can be found in Appendix B.

The medium-scale model studied here has six endogenous state variables. It is capable to display rich and complicated dynamics studying which in detail is beyond the scope of this paper. The focus here is on the conduct of monetary policy in times when government debt exceeds its steady-state level. Figure 6 displays the spread of the real interest rate over its steady-state counterpart in a single period of time where the stock of government debt exceeds its steady state level by one percent while other state variables are equal to their steady-state values. The value of the spread is plotted against the average maturity of bonds issued by the government.

A negative value of the real interest rate spread implies that an excessive government debt makes it optimal for monetary policy to stimulate the economy. In other words, monetary policy is loose in such a case. A positive values of the spread implies the opposite: the stance of monetary policy is tight if government debt is above the steady-state value. Thus, one can see that optimal stance of monetary policy in the medium-scale model exhibits a switch from loose to tight when one lengthens maturity of government bonds.

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20 As in the baseline model, the real interest rate depends on the intratemporal and intertemporal optimality conditions. Qualitatively, the stance of monetary policy in the baseline model was shown to be determined exclusively by the intratemporal tradeoff. Complexity of the medium-scale model does not permit one to disentangle these two dimensions.
Moreover, similar to the baseline model, the switch occurs at a very short average maturity of around three quarters.

The real interest spread would differ in more general cases with different initial conditions of other state variables such as, for instance, capital. To put it differently, the overall optimal stance of monetary policy in equilibrium depends on all states of the economy. Nevertheless, linearity of the decision rules implies that the analysis presented here would be valid to the extent that it represents how an excessive amount of government debt affects the stance of monetary policy ceteris paribus.

7 Conclusion

In the aftermath of the recent global economic downturn, governments of many countries amassed public debt that exceeds historic averages. A pressing policy concern is whether there is a need to reduce the stock of this debt, and if yes, then how fast should the adjustment be performed and, equally important, which policies are to be used for this purpose. The topic that spurs a lot of controversies is a capacity of central banks to ease the burden of government debt. This paper looks into a relative importance of the need of government debt reduction in shaping optimal stance of monetary policy.

The analysis here focuses on the case where reduction of government debt occurs because of the lack of commitment by the government to its future policy choices. Analytical results demonstrate that the stance of monetary policy during the period of government debt reduction is driven by opposing incentives to reduce private demand so as to mitigate the adverse cost-push effect of raising the tax rate and to stimulate private demand so as to reduce the need to raise the tax rate in the first place. Quantitative results demonstrate a strong support for the dominance of the former incentive and the resulting tight stance of monetary policy. A crucial feature of the analysis is that it takes into account long-term nature of government debt. In can be optimal to switch the stance of monetary policy and make it loose only if the government were to issue bonds with the average maturity below one year. Government debt in the form of such a short-term bonds also has to be reduced at a fast speed. More generally, the optimal speed of government debt reduction is gradual.

Analysis of this paper is built under the assumption that the government can always adjust either monetary or fiscal policy instruments to maintain fiscal sustainability. Clearly, there are cases when keeping public debt on a sustainable path requires the government to perform debt adjustment via an outright default. Hence, abstracting from the default risk may be not without loss of generality and may be a relevant direction for future research.
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A Appendix: The Baseline Model

A.1 Recursive structure and pricing of government bonds

Consider two long-term portfolios of government bonds, $B_{t-1}$ and $B_t$, issued in the two consecutive periods $t-1$ and $t$ correspondingly. The former portfolio pays 1 in period $t$, $\rho$ in period $t+1$, $\rho^2$ in period $t+2$, etc. The latter portfolio pays 1 in period $t+1$, $\rho$ in period $t+2$, $\rho^2$ in period $t+3$, etc.

Let $q_{t-o}$ and $q_t$ denote period $t$ prices of $B_{t-1}$ and $B_t$ correspondingly. Taking into account the structure of payoffs, these two prices has to satisfy the following asset pricing equations

\[
q_{t-o} = \sum_{j=1}^{\infty} \frac{\prod_{i=1}^{j} \rho^i}{R_{t+i-1}},
\]

\[
q_t = \sum_{j=1}^{\infty} \frac{\rho^{j-1}}{R_{t+i-1}},
\]

where the one-period nominal interest rate is used for discounting the payoffs.\textsuperscript{21} It is then straightforward to see that market prices of the two portfolios satisfy

\[
q_{t-o} = \rho q_t.
\]

A.2 Duration of government bonds

Wighted average maturity of the portfolio of government bonds issued in period $t$ is measured as the Macaulay duration of perpetuities entering this portfolio.

To compute the duration first define the yield-to-maturity $R_{t-o}^m$ on a perpetual bond as an implicit constant interest rate at which the discounted value of its payoffs equals its price

\[
q_t = \sum_{j=1}^{\infty} \frac{\rho^{j-1}}{(R_{t-o}^m)^j} = \frac{1}{R_{t-o}^m - \rho}.
\] (A.1)

The Macaulay duration is defined as the weighted average of the time until each payoff, with the weights determined by discounted payoffs as a fraction of the bond’s price

\[
d_t = \frac{1}{q_t R_{t-o}^m} \sum_{j=1}^{\infty} j \left( \frac{R_{t-o}^m}{R_{t}^m} \right)^{j-1} = \frac{1}{q_t R_{t-o}^m} R_{t-o}^m \left( \frac{R_{t-o}^m}{R_{t}^m} \right) \partial \left( \sum_{j=1}^{\infty} \frac{\rho^j}{R_{t}^m} \right)\bigg|_{\rho=R_{t-o}^m} = \frac{1}{q_t R_{t-o}^m} \partial \left( \frac{\rho}{R_{t-o}^m} \right) \partial \left( \frac{R_{t-o}^m}{R_{t}^m} \partial \left( \frac{1}{R_{t-o}^m} \right) \right)
\]

\[
= \frac{1}{q_t R_{t-o}^m} \left( R_{t-o}^m \right)^2 = \frac{R_{t-o}^m}{R_{t}^m - \rho},
\] (A.2)

\textsuperscript{21}In a model with uncertainty one would have to use the stochastic discount factor.
where the last step uses equation (A.1). The duration as defined above is measured in units of a time period of the model.

Note further that the Euler equation (4.5) at the steady state with zero inflation takes the form

\[ \bar{q} = \frac{1}{\beta - 1 - \rho}, \]  

where \( \bar{q} \) denotes the steady-state values of the variables. Comparison of (A.1) and (A.3) reveals the fact that the yield-to-maturity of any long-term portfolio of government bonds at the steady state is equal to the inverse of the time discount factor \( \bar{R}_m = \beta^{-1} \). It is then straightforward to see, using (A.2), that the corresponding steady-state duration is determined as follows

\[ \bar{d} = (1 - \beta \rho)^{-1}. \]

A.3 The First-Best Allocation

Lagrangian corresponding to the Planner’s problem is

\[ \mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + g(G_t) - v(h_t) + \gamma_t (h_t - c_t - G_t) \right]. \]

First-order conditions with respect to \( \{c_t, G_t, h_t\} \) are as follows

\[ u'(c_t) = \gamma_t, \]
\[ g'(G_t) = \gamma_t, \]
\[ v'(h_t) = \gamma_t. \]

Eliminating Lagrange multiplier \( \gamma_t \) leaves the system with two equations

\[ 0 = g'(G_t) - u'(c_t), \]
\[ 0 = g'(G_t) - v'(h_t), \]

that together with the resource constraint \( h_t = c_t + G_t \) characterize the first-best allocation as a solution of the Planner’s problem.
A.4 Markov-Perfect Equilibrium Characterization

The Markov-perfect equilibrium is associated with a solution to the following Bellman equation

\[ V(b_{t-1}) = \max_{(c_t, y_t, \pi_t, w_t, q_t, g_t, \tau_t)} \left\{ \left( \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu Y_t^{1-\gamma_y} - \nu h \frac{y_t^{1+\gamma}}{1+\gamma} \right) + \beta V(b_t) \right\} \]

subject to

\[ 0 = c_t + G_t - y_t \left( 1 - \frac{1}{2} \varphi (\pi_t - 1)^2 \right), \]  
(A.4) \[ 0 = (1 - \tau_t) w_t - \nu \theta c_t^{1-\gamma} y_t^{\gamma}, \]  
(A.5) \[ 0 = q_t - \beta \left( \frac{c_t}{C_b(b_t)} \right)^{\gamma_c} \left( \frac{1 + \rho Q(b_t)}{H(b_t)} \right), \]  
(A.6) \[ 0 = w_t - \frac{\theta}{(1-s)\theta} - \frac{\varphi}{\theta(1-s)} \left( \pi_t (1 - 1) - \beta \left( \frac{c_t}{C_b(b_t)} \right)^{\gamma_c} \right) \frac{\gamma_t}{y_t} I(b_t) (I(b_t) - 1), \]  
(A.7) \[ 0 = q_t b_t - \left( 1 + \rho q_t \right) b_{t-1} - G_t + \tau_t w_t y_t - s (w_t y_t - \bar{w} g_t), \]  
(A.8)

This is a dynamic functional problem whose solution consists of a value function \( V(b_{t-1}) \) and policy rules \( \{C, \gamma, I, W, Q, B, G, T\} \) that determine private-sector equilibrium in a given period of time as a function of outstanding debt, \( b_{t-1} \), as in \( c_t = C(b_{t-1}) \), \( y_t = \gamma(b_{t-1}) \), etc. To ease the exposition, two variables have been eliminated from the system of private-sector equilibrium constraints using two of its equations. These variables are the labor supply, \( h_t \), and the short-term nominal interest rate, \( R_t \), whereas the corresponding equations are the aggregate production function (4.12) and the no-arbitrage condition (4.7). Markov-perfect rules determining the former, \( \mathcal{H}(b_{t-1}) \) and \( \mathcal{R}(b_{t-1}) \), are recovered using the latter, given the solution of Bellman equation above.\(^{22}\)

This paper considers equilibria with differentiable functions, which makes it possible to characterize these equilibria with the first-order conditions. Let \( \{\lambda_{r,t}, \lambda_{h,t}, \lambda_{q,t}, \lambda_{\pi,t}, \lambda_{b,t}\} \) be Lagrange multipliers corresponding to private-sector equilibrium constraints (A.4)–(A.8). The first-order condition with respect to \( b_t \) reads as follows

\[ 0 = \beta V'(b_t) - q_t \lambda_{b,t} \]

\[ - \beta \left( \frac{c_t}{C(b_t)} \right)^{\gamma_c} \left[ \left( 1 + \rho Q(b_t) \right) \left( \frac{1}{H(b_t)} \right) - \rho \frac{Q'(b_t)}{H(b_t)} \right] \lambda_{q,t} + \beta \frac{\varphi}{(1-s)\theta y_t} \left( \frac{c_t}{C(b_t)} \right)^{\gamma_c} \left[ \left( I(b_t) (I(b_t) - 1) \left( \frac{1}{H(b_t)} \right) - \gamma_t \right) \gamma_t \right] \lambda_{\pi,t} - \beta \frac{\gamma_t}{y_t} I(b_t) (I(b_t) - 1) \gamma(b_t) \]

(A.9)

The first-order conditions with respect to \( \{c_t, y_t, \pi_t, w_t, q_t, G_t, \tau_t\} \) read

\(^{22}\)Treating the short-term nominal interest rate in such a “residual” way is innocuous as long as the Zero Lower Bound on nominal interest rates is ignored or is never binding.
\[ 0 = c_t^{-\gamma_c} - \lambda_{r,t} + \nu_h \gamma_c C_t^{\gamma_c-1} y_t^{\gamma_c} \lambda_{h,t} + \beta \frac{\gamma_c}{c_t} \left( \frac{C_t}{C(b_t)} \right)^{\gamma_c} \left( 1 + \rho Q(b_t) \right) \lambda_{q,t} + \frac{\beta}{c_t} \frac{\gamma_c}{C(b_t)} \frac{\gamma_c}{y_t} \frac{C(b_t)}{C(b_t)} \lambda_{\pi,t} \]

\[ 0 = -\nu_h y_t^\gamma + \left( 1 - \frac{1}{2} \varphi (\pi_t - 1)^2 \lambda_{r,t} + \nu_h \gamma_h y_t^{\gamma_h-1} c_t^{\gamma_h} \lambda_{h,t} \right) \]

\[ + \frac{\beta}{c_t} \frac{\gamma_c}{C(b_t)} \left( \frac{C(b_t)}{y_t^2} \right) \lambda_{\pi,t} - (\tau_t - s) w_t \lambda_{b,t}, \]  

\[ 0 = -\varphi (\pi_t - 1) y_t \lambda_{r,t} + \frac{\varphi}{\theta} \left( \frac{2 \pi_t - 1}{1 - s} \right) \lambda_{\pi,t} - \frac{(1 + \rho Q_t) b_t}{\pi_t^2} \lambda_{b,t}, \]  

\[ 0 = \lambda_{q,t} + \left( b_t - \frac{\rho b_t}{\pi_t} \right) \lambda_{b,t}, \]

\[ 0 = \nu_y G_t^{-\gamma_y} - \lambda_{r,t} + \lambda_{b,t}, \]

\[ 0 = w_t \lambda_{h,t} - y_t w_t \lambda_{b,t}, \]  

One can use equations (A.12)–(A.16) to solve for Lagrange multipliers

\[ \lambda_{r,t} = (1 - \Omega_t) \left( \nu_y G_t^{-\gamma_y} \right), \]

\[ \lambda_{h,t} = -y_t \Omega_t \left( \nu_y G_t^{-\gamma_y} \right), \]

\[ \lambda_{q,t} = \left( b_t - \frac{\rho b_t}{\pi_t} b_t \right) \Omega_t \left( \nu_y G_t^{-\gamma_y} \right), \]

\[ \lambda_{\pi,t} = (1 - s) y_t \Omega_t \left( \nu_y G_t^{-\gamma_y} \right), \]

\[ \lambda_{b,t} = -\Omega_t \left( \nu_y G_t^{-\gamma_y} \right), \]

where \( \Omega_t \) is an auxiliary variable referred to as an inflation cost factor and defined as follows

\[ \Omega_t \equiv \frac{\varphi (\pi_t - 1) y_t}{\varphi (\pi_t - 1) y_t + \frac{\varphi}{\theta} \left( 2 \pi_t - 1 \right) y_t + (1 + \rho Q_t) b_t \frac{\rho b_t}{\pi_t^2}}. \]

It is straightforward to see that \( \Omega_t < 1 \). Moreover, \( \Omega_t \) is unambiguously positive when debt level and inflation are positive. If there were to be no inflation, then \( \Omega_t \) would be equal to zero and (A.4) would be the only binding constraint thus making the problem isomorphic to that of the Social Planner. It is inability to fully contain inflation at all states that makes the policy problem nondegenerate. The larger is the marginal resource cost of inflation compared to its marginal benefits the larger is \( \Omega_t \) and the tighter are the constraints (A.5)–(A.8).

The Envelope condition

\[ \lambda^{'}(b_t) = \left( \frac{1 + \rho Q(b_t)}{H(b_t)} \right) \lambda_{b,t+1} \]
allows to substitute for the derivative of the value function in equation (A.9). Consequently, equations (A.17)–(A.21) allow to substitute for Lagrange multipliers in equations (A.9)–(A.11) that become

\[(1 + \Delta_{b,t}) \Omega_t \left( \frac{\nu_y G_t^{-\gamma_y}}{c_t^{\gamma_c}} \right) = \Omega_{t+1} \left( \frac{\nu_y G(b_t)^{-\gamma_y}}{C(b_t)^{-\gamma_c}} \right) \quad (A.22)\]

\[\Delta_{c,t} \Omega_t \left( \nu_y G_t^{-\gamma_y} \right) = \nu_y G_t^{-\gamma_y} - c_t^{-\gamma_c} \quad (A.23)\]

\[\left( \frac{\varphi}{2} (\pi_t - 1)^2 + \Delta_{h,t} \Omega_t \right) \left( \nu_y G_t^{-\gamma_y} \right) = \nu_y G_t^{-\gamma_y} - \nu_h y_t^{\gamma_h} \quad (A.24)\]

where auxiliary variables \(\Delta_{b,t}, \Delta_{c,t}\) and \(\Delta_{h,t}\) are defined as follows

\[\Delta_{b,t} \equiv \frac{\varphi}{\theta} \left( \frac{\Pi(b_t)}{1 + \rho Q(b_t)} \right) \left[ \frac{\Pi(b_t) \left( \Pi(b_t) - 1 \right)}{\gamma_c C(b_t)^{1 + \gamma_c}} \right] \left( \gamma_c \frac{C'(b_t)}{C(b_t)} \gamma C(b_t) - \gamma_c \nu_y \Pi'(b_t) \left( \Pi(b_t) - 1 \right) \gamma C(b_t) \right) - \left( \frac{\nu_y G(b_t)}{C(b_t)^{-\gamma_c}} \right) \]

\[\Delta_{c,t} \equiv 1 + \frac{\varphi}{\theta} \left( \frac{c_t}{C(b_t)} \right)^{\gamma_c} \left[ \frac{1 + \rho Q(b_t)}{\Pi(b_t)} \left( \Pi(b_t) - 1 \right) - \frac{\varphi}{\theta} \gamma C(b_t) \gamma \left( \Pi(b_t) - 1 \right) \right] - \gamma_c \nu_h y_t^{1+\gamma_h} c_t^{\gamma_c} \]

\[\Delta_{h,t} \equiv \left( 1 - \frac{\varphi}{2} (\pi_t - 1)^2 \right) + \gamma_h \nu_h y_t^{\gamma_h} c_t^{\gamma_c} - \beta \frac{\varphi}{\theta} \left( \frac{c_t}{C(b_t)} \right)^{\gamma_c} \gamma C(b_t) \gamma \left( \Pi(b_t) - 1 \right) - (\tau_t - s) \omega_t \]

Equations (A.4)–(A.8), (A.22)–(A.24) constitute a dynamic system of 8 functional equations in 8 unknowns \(\{C, Y, \Pi, W, Q, B, G, T\}\). Presence of derivatives of the policy functions in (A.9) complicates the use of this system for the purposes of characterizing and solving for the Markov-perfect equilibrium because solving for unknown policy functions requires taking into account how these functions react to variation of the state of the economy.

### A.5 Steady-State Analysis

Let bars denote steady-state values. Then equation (A.22) at a steady-state becomes

\[\bar{\Delta}_b \bar{\Omega} = 0, \quad (A.25)\]

where

\[\bar{\Omega} = \frac{\varphi (\pi - 1) \bar{y}}{\varphi (\pi - 1) \bar{y} + \frac{\varphi}{\theta} \left( \frac{2\bar{\pi} - 1}{\bar{\pi} + 1 + \rho q} \right) \bar{\gamma}^{\gamma_c}}, \]

\[\bar{\Delta}_b = \varphi \left( \frac{\bar{\pi}}{1 + \rho q} \right) \left[ \bar{\pi} \left( \gamma_c \frac{c_t^{\gamma_c} \bar{y} - \bar{y}^{\gamma_c}}{c_t^{\gamma_c}} - \pi_t (2\bar{\pi} - 1) \bar{y} \right) - \left( \gamma_c \frac{c_t^{\gamma_c} \bar{y}^{\gamma_c}}{\bar{y}^{\gamma_c}} - \frac{\rho q}{1 + \rho q} \right) \left( \bar{\pi} - \rho \right) \bar{b}, \right] \]
It implies that there are two types of steady states. In the first steady state, where $\bar{\Omega} = 0$, inflation is zero and the efficient allocation is implemented. In the second steady state, where $\bar{\Delta}_b = 0$, marginal change of debt does not provide any gains from affecting future period policy and allocation.

Consider steady-state of the first type. The remaining Markov-perfect equilibrium conditions, (A.4)–(A.8), (A.23) and (A.24), at the steady-state with zero inflation read as

$$0 = \bar{c} + \bar{G} - \bar{\gamma}, \quad (A.26)$$
$$0 = \left(1 - \bar{\tau}\right) \bar{\omega} - \nu h \bar{y}^{\gamma_h} \bar{e}^{\gamma_c}, \quad (A.27)$$
$$0 = \bar{q} - \beta (1 + \rho \bar{q}), \quad (A.28)$$
$$0 = \bar{w} - \frac{\theta - 1}{(1 - s) \theta}, \quad (A.29)$$
$$0 = \bar{q} \bar{b} - (1 + \rho \bar{q}) \bar{b} - \bar{\tau} \bar{\omega} \bar{y}, \quad (A.30)$$
$$0 = \nu g \bar{G}^{-\gamma_g} - \bar{e}^{-\gamma_c}, \quad (A.31)$$
$$0 = \nu g \bar{G}^{-\gamma_g} - \nu h \bar{y}^{\gamma_h}. \quad (A.32)$$

Equations (A.25)–(A.32) implicitly determine the Markov-perfect steady-state $\{\bar{c}, \bar{y}, \bar{\pi}, \bar{w}, \bar{q}, \bar{b}, \bar{G}, \bar{\tau}\}$. Furthermore, the steady-state values of hours worked and the short-term real interest rate are easily obtained from the aggregate production function (4.12) and the no-arbitrage condition (4.7):

$$\bar{h} = \bar{y}, \quad \bar{R} = \beta^{-1}. \quad (A.33)$$

Equations (A.26), (A.31) and (A.32) coincide with the first-order conditions of the Planner’s problem, which confirms that the efficient allocation is implemented at this steady-state. Furthermore, combining (A.31) and (A.32) results in

$$0 = 1 - \nu h \bar{y}^{\gamma_h} \bar{e}^{\gamma_c},$$

which, together with (A.27), yields the constraint $1 = (1 - \bar{\tau}) \bar{\omega}$, where the tax rate and the real wage can be further substituted for using equations (A.28)–(A.30) so as to get

$$1 = \frac{\theta - 1}{(1 - s) \theta} - \frac{\bar{G}}{\bar{y}} - \left(\frac{1 - \beta}{1 - \beta \rho}\right) \frac{\bar{b}}{\bar{y}}. \quad (A.33)$$

Note that the steady-state levels of government spending and output, $\bar{G}$ and $\bar{y}$, are determined independently of equation (A.33). Therefore, equation (A.33) implicitly determines the steady-state level of government debt, $\bar{b}$, as a function of the labor subsidy rate, $s$. A direct application of the Implicit function theorem

$$\frac{\partial \bar{b}}{\partial s} = \bar{y} \left(\frac{\theta - 1}{\theta} \right) \left(\frac{1 - \beta \rho}{1 - \beta}\right) \frac{1}{(s - 1)^2} > 0,$$

shows that the steady-state level of debt is an increasing function of the labor subsidy rate. This result is intuitive because accumulating assets and increasing subsidy rate are alternative means to offset, in the long-run, distortions introduced by monopolistic competition and labor taxation.
A.6 Linear-Quadratic Approximation

The backbone of the Linear-Quadratic approach is the use of the second-order approximation to the household utility in the vicinity of the steady-state. Quadratic functional form of the approximated utility preserves the ranking of government policy alternatives when maximizing subject to the (log-)linearized private-sector equilibrium constraints.

A.6.1 Objective Function

Household utility in period \( t \) is

\[
U_t(c_t, G_t, h_t) \equiv \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h}.
\]

In order for the second-order approximation of the utility function to be an accurate welfare criterion in the neighborhood of the efficient steady-state, one can substitute for consumption using the resource constraint (A.4). The resulting function reads as

\[
U_t(\hat{\pi}_t, G_t, h_t) = \frac{(\bar{h}_t (1 - \frac{\gamma}{2} (\pi_t - 1)^2) - G_t)^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h}.
\]  

(A.34)

Prior to approximating, I make a variable change by substituting the original variables with their log-deviations from the efficient steady-state, where I use hats to denote log-deviations. Formally, I use the following identity for a variable \( X_t \)

\[
X_t = \bar{X} e^{\hat{X}_t}, \quad \text{where } \hat{X}_t \equiv \ln X_t - \ln \bar{X}.
\]  

(A.35)

Utility (A.34), after the change of variables, approximated to the second-order using Taylor expansion is

\[
U_t(\hat{\pi}_t, \hat{G}_t, \hat{h}_t) \simeq -\frac{1}{2} \left[ \bar{h} \gamma_c (\bar{h} - \bar{G})^{-\gamma_c-1} - (\bar{h} - \bar{G})^{-\gamma_c} + (\gamma_h + 1) \nu_h \bar{h}^{\gamma_h} \right] \bar{h} \hat{h}_t^2
\]

\[
- \frac{1}{2} \left[ (\bar{h} - \bar{G})^{-\gamma_c} (\bar{G} (\gamma_c - \gamma_g) + \bar{h} \gamma_h) \right] \bar{G} \hat{G}_t^2
\]

\[
+ \left[ \gamma_c (\bar{h} - \bar{G})^{-\gamma_c-1} \right] \bar{G} \bar{h}_t \hat{h}_t - \frac{1}{2} \left[ \varphi \bar{h} (\bar{h} - \bar{G})^{-\gamma_c} \right] \bar{h}_t^2 + \text{t.i.p.},
\]  

(A.36)

where t.i.p. is equal to the steady-state value of the period utility that does not depend on policy, and linear terms have been eliminated using steady-state equations (A.26), (A.31) and (A.32).

To simplify the expression and to make the ultimate linear-quadratic policy problem resemble the nonlinear counterpart in Appendix A.4, I rewrite (A.36) after reintroducing consumption, \( \hat{c}_t \), using the resource constraint (A.4) and substituting labor supply, \( \hat{h}_t \), with output, \( \hat{y}_t \), using the aggregate production function (4.12) so as to get the following quadratic approximation to the household utility in period \( t \) in the vicinity of the efficient steady-state

\[
U_t(\hat{c}_t, \hat{\pi}_t, \hat{G}_t, \hat{y}_t) \simeq -\frac{1}{2} \bar{c}^{-\gamma_c} \left( \gamma_c \bar{c}_t^2 + \gamma_g \bar{G} \hat{G}_t^2 + \gamma_h \bar{y}_t^2 + \varphi \bar{y}_t^2 \right) + \text{t.i.p.}
\]  

(A.37)

\(^{23}\) Alternatively, one can approximate the original function up to the second order and then use second-order approximation of the resource constraint, which would eliminate the linear terms and introduce the remaining quadratic terms.
A.6.2 Constraints

The remaining step required to specify the Linear-Quadratic policy problem is to describe the set of optimization constraints. The set of constraints consists of the private-sector equilibrium constraints log-linearized around the steady-state with a nonzero amount of government debt:

\[ 0 = \ddot{c}_t - \ddot{y}_t + \ddot{G}\dot{c}_t, \tag{A.38} \]
\[ 0 = \dot{w}_t - \dot{\pi}_t - \gamma_c \dot{c}_t - \gamma_h \dot{y}_t, \tag{A.39} \]
\[ 0 = \dot{q}_t + \dot{\pi}_{t+1} - \beta \rho \dot{q}_{t+1} - \gamma_c (\dot{c}_t - \dot{c}_{t+1}), \tag{A.40} \]
\[ 0 = \ddot{w}_t - \frac{\varphi}{\theta - 1} \ddot{\pi}_t + \beta \frac{\varphi}{\theta - 1} \ddot{\pi}_{t+1}, \tag{A.41} \]
\[ 0 = b\dot{q}_t - bq \frac{\ddot{b}_{t-1}}{\beta} + (1 - \rho) b\dot{q}_t + bq \frac{\ddot{\pi}_t}{\beta} - G\dot{G}_t + \ddot{w}_t ((\ddot{\pi} - s) (\ddot{y}_t + \ddot{w}_t) + \ddot{\pi}_t), \tag{A.42} \]
\[ 0 = \dot{h}_t - \ddot{y}_t, \tag{A.43} \]
\[ 0 = \dot{i}_t - \beta \rho \dot{q}_{t+1} + \dot{q}_t. \tag{A.44} \]

where \( \dot{i}_t \) conventionally denotes the log-deviation of the short-term nominal interest rate, \( R_t \), equation (A.38) is the log-linearized version of the aggregate resource constraint (4.13), equation (A.39) is the log-linearized household leisure optimality condition (4.6), equation (A.40) is the log-linearized household Euler equation (4.5), equation (A.41) is the log-linearized Phillips curve (4.10), equation (A.42) is the log-linearized government budget constraint (4.11), equation (A.43) is the log-linearized aggregate production function (4.12), and equation (A.44) is the log-linearized no-arbitrage condition (4.7).

A.6.3 Equilibrium Characterization

The approximated Markov-Perfect equilibrium is associated with a recursive optimization problem where the benevolent government maximizes approximated lifetime utility defined as a discounted sum of period utilities (A.37) by choosing \( \{\ddot{c}_t, \ddot{y}_t, \ddot{\pi}_t, \dot{w}_t, \dot{q}_t, \dot{b}_t, \dot{\pi}_t, \dot{G}_t\} \) subject to the approximated private-sector equilibrium constraints (A.38)–(A.42) taking as given policy in the future periods. Formally, the Bellman equation reads as

\[ U(\dot{b}_{t-1}) = \max_{\{w_t, q_t, \pi_t, c_t, r_t, G_t\}} \left[ \frac{-1}{2} e^{-\gamma_c \left( \gamma_c c_t^2 + \gamma_y \ddot{G}\dot{c}_t^2 + \gamma_y \ddot{y}_t^2 + \varphi \ddot{q}_t^2 \right) + \beta U(\dot{b}_t) \right] \]

subject to

\[ 0 = \ddot{c}_t - \ddot{y}_t + \ddot{G}\dot{c}_t, \tag{A.45} \]
\[ 0 = \dot{w}_t - \dot{\pi}_t - \gamma_c \dot{c}_t - \gamma_h \dot{y}_t, \tag{A.46} \]
\[ 0 = \dot{q}_t + \dot{\pi}_{t+1} - \beta \rho \dot{q}_{t+1} - \gamma_c (\dot{c}_t - \dot{c}_{t+1}), \tag{A.47} \]
\[ 0 = \ddot{w}_t - \frac{\varphi}{\theta - 1} \ddot{\pi}_t + \beta \frac{\varphi}{\theta - 1} \ddot{\pi}_{t+1}, \tag{A.48} \]
\[ 0 = b\dot{q}_t - bq \frac{\ddot{b}_{t-1}}{\beta} + (1 - \rho) b\dot{q}_t + bq \frac{\ddot{\pi}_t}{\beta} - G\dot{G}_t + \ddot{w}_t ((\ddot{\pi} - s) (\ddot{y}_t + \ddot{w}_t) + \ddot{\pi}_t), \tag{A.49} \]
where $U(\hat{b}_{t-1})$ is the value function, and functions $\hat{H}(\hat{b}_t), \hat{Q}(\hat{b}_t), \hat{C}(\hat{b}_t)$ determine inflation, government bond price and consumption in period $t + 1$ as functions of the debt level outstanding at the beginning of period $t + 1$, $\hat{b}_t$. As in the original nonlinear policy problem, labor supply, $\hat{h}_t$, and the short-term nominal interest rate, $\hat{i}_t$, have been eliminated from the optimization problem but can be easily recovered using the aggregate production function (A.43) and the no-arbitrage condition (A.44).

The approximated Markov-perfect equilibrium can be characterized with the first-order conditions. Let $\{\alpha_{r,t}, \alpha_{h,t}, \alpha_{q,t}, \alpha_{\pi,t}, \alpha_{b,t}\}$ be Lagrange multipliers corresponding to the constraints (A.45)–(A.49). The first-order conditions with respect to $\{\hat{\pi}_t, \hat{w}_t, \hat{q}_t, \hat{G}_t, \hat{\tau}_t\}$ can be used to solve for Lagrange multipliers

\[
\alpha_{r,t} = -\tilde{c}^{-\gamma} \left( \gamma \hat{G}_t + \Psi \hat{\pi}_t \right),
\]
\[
\alpha_{h,t} = -\tilde{c}^{-\gamma} \tilde{y} \Psi \hat{\pi}_t,
\]
\[
\alpha_{q,t} = \tilde{c}^{-\gamma} (1 - \rho) \tilde{b} \Psi \hat{\pi}_t,
\]
\[
\alpha_{\pi,t} = \tilde{c}^{-\gamma} (1 - s) \tilde{y} \Psi \hat{\pi}_t,
\]
\[
\alpha_{b,t} = -\tilde{c}^{-\gamma} \Psi \hat{\pi}_t,
\]

where the coefficient $\Psi$ scales the effect that inflation has on tightness of the constraints and is defined as follows

\[
\Psi = \frac{\varphi \tilde{y}}{\frac{\tilde{y}}{\bar{y}} + \beta^{-1} \bar{\Gamma}},
\]

where $\bar{\Gamma} \equiv \tilde{b} \tilde{q}$ is the market value of government debt in the steady-state. It is straightforward to see that $\Psi$ is unambiguously positive when the steady-state market value of debt is positive. The coefficient $\Psi$ has an interpretation similar to the inflation cost factor, $\Omega_t$, in the nonlinear analysis: the larger is the marginal resource cost of inflation compared to its marginal benefits the larger is $\Psi$. Also note that if an outstanding amount of debt in excess of its level in the steady state, $\hat{b}_{t-1} > 0$, makes the government budget constraint tighter, $\alpha_{b,t} < 0$, then condition (A.54) implies that the corresponding inflation has to be positive, $\hat{\pi}_t > 0$.

The first-order conditions with respect to $\hat{c}_t$ and $\hat{y}_t$ are as follows

\[
0 = \tilde{c}^{-\gamma} c \hat{c}_t + \tilde{c} \alpha_{r,t} - \gamma_c \alpha_{h,t} - \gamma_c \frac{\beta}{1 - \beta \rho} \alpha_{q,t},
\]
\[
0 = \tilde{c}^{-\gamma} \gamma_h \hat{y}_t - \tilde{y} \alpha_{r,t} - \gamma_h \alpha_{h,t} + \tilde{y} \tilde{w} (\hat{\tau} - s) \alpha_{b,t}
\]

Lagrange multipliers in these two equations can be eliminated with the help of (A.50)–(A.54). Combining the resulting equations with the aggregate resource constraint results in the three targeting rules:

\[
\hat{c}_t = \Phi_c \hat{\pi}_t, \quad \hat{y}_t = \Phi_y \hat{\pi}_t, \quad \hat{G}_t = \Phi_g \hat{\pi}_t,
\]

where
The first-order condition with respect to \( \hat{b}_t \) reads

\[
0 = \beta U'(\hat{b}_t) - \bar{\Pi}_b\hat{b}_t - q\left(\gamma_c\hat{C}'(\hat{b}_t) + \hat{H}'(\hat{b}_t) - \beta \rho \hat{Q}'(\hat{b}_t)\right)\alpha_{q,t} - \frac{\varphi \beta}{\theta (1 - s)} \hat{H}'(\hat{b}_t)\alpha_{\pi,t}, \tag{A.57}
\]

The derivative of the value function, \( U'(\hat{b}_t) \), can be expressed in terms of Lagrange multiplier \( \alpha_{b,t+1} \) using the Envelope condition

\[
U'(\hat{b}_t) = \beta^{-1}\bar{\Pi}_b\alpha_{b,t+1}.
\]

Furthermore, Lagrange multipliers \( \alpha_{q,t} \), \( \alpha_{\pi,t} \) and \( \alpha_{b,t} \) have been previously solved for in terms of inflation as described by (A.52)–(A.54). It allows to rewrite (A.57) as follows

\[
0 = \ddot{\pi}_{t+1} - \ddot{\pi}_t + (1 - \rho) \left(\gamma_c\hat{C}'(\hat{b}_t) + \hat{H}'(\hat{b}_t) - \beta \rho \hat{Q}'(\hat{b}_t)\right)\ddot{\pi}_t + \frac{\varphi \beta}{\theta (1 - s)} \hat{H}'(\hat{b}_t)\ddot{\pi}_t. \tag{A.58}
\]

Linear-quadratic structure of the optimization problem underlying the approximated Markov-perfect equilibrium is known for its nice property that solutions belong to the class of linear functions of state variable. In other words, the approximated Markov-perfect equilibrium is completely represented by a set constant coefficients \( \{C_b, \gamma_b, \Pi_b, \beta \Pi_b, Q_b, B_b, G_b, T_b, I_b\} \) that determine economic outcome in a given period as proportional to the outstanding level of government debt:

\[
\begin{align*}
\hat{c}_t &= C_b \hat{b}_{t-1}, & \quad \hat{y}_t = \gamma_b \hat{b}_{t-1}, & \quad \hat{h}_t = \Pi_b\hat{b}_{t-1}, & \quad \hat{\pi}_t = \bar{\Pi}_b\hat{b}_{t-1}, & \quad \hat{w}_t = \bar{W}_b\hat{b}_{t-1}, \\
\hat{q}_t &= Q_b \hat{b}_{t-1}, & \quad \hat{b}_t = B_b\hat{b}_{t-1}, & \quad \hat{G}_t = G_b\hat{b}_{t-1}, & \quad \hat{\tau}_t = \bar{T}_b\hat{b}_{t-1}, & \quad \hat{\eta}_t = \bar{I}_b\hat{b}_{t-1}.
\end{align*}
\]

Imposing linear solution structure on to the system of the first-order equation allows to use the method of undetermined coefficients so as to solve for the equilibrium. One can start with rewriting the GEE (A.58) and the government budget constraint (A.49) as follows

\[
0 = (B_b - 1) + (1 - \rho) (\gamma_c C_b + \Pi_b - \beta \rho Q_b) + \frac{\varphi \beta}{\theta (1 - s)} \beta \Pi_b, \tag{A.59}
\]

\[
0 = (B_b - \beta^{-1}) + (1 - \rho) Q_b + \frac{\Pi_b}{\beta} - \frac{\bar{G} \gamma_b}{\bar{\Gamma}} G_b + \frac{\bar{w}_t}{\bar{\Gamma}} ((\bar{\pi} - s) (\gamma_b + W_b) + \bar{\tau} T_b). \tag{A.60}
\]

The remaining first-order conditions, namely three targeting rules, and equations (A.39)–(A.41), (A.43) and (A.44) can be rearranged to deliver the following equilibrium restrictions:
\[ C_b = \Phi_c I_b, \]
\[ \mathcal{Y}_b = \Phi_y I_b, \]
\[ G_b = \Phi_y I_b, \]
\[ H_b = \Phi_y I_b, \]
\[ W_b = \frac{\varphi(1 - \beta B_b)}{\theta - 1} I_b, \]
\[ Q_b = \frac{\gamma_c \Phi_c (1 - B_b) - B_b}{1 - \beta \rho B_b} I_b, \]
\[ I_b = -\left(\gamma_c \Phi_c (1 - B_b) - B_b\right) I_b, \]
\[ T_b = \frac{1 - \bar{\tau}}{\bar{\tau}} \left( \frac{\varphi}{\theta - 1} (1 - \beta B_b) - (\gamma_c \Phi_c + \gamma_h \Phi_y) \right) I_b, \]

It is then straightforward to use the last set of equations to substitute for all the coefficients but \( B_b \) and \( I_b \) in (A.59)–(A.60). The last step is to solve the resulting system of two nonlinear equations for the pair of coefficients \((B_b, I_b)\). The analysis of this paper is restricted to non-explosive and monotone solutions with \( 0 < B_b < 1 \). Checking existence and uniqueness of \((B_b, I_b)\) solutions is equivalent to checking existence and uniqueness of approximated Markov-Perfect equilibria. It is not possible to derive analytical criteria of either existence or uniqueness for generic values of structural parameters. When checking numerically, the model with baseline parameter values does not display multiplicity of equilibria for \( \rho \in [0, 1] \).
B Appendix: The Medium-Scale Model

B.1 List of variables

Allocation and related variables

- $X_t$ - aggregate output
- $L_t$ - aggregate labor
- $C_t$ - aggregate private consumption
- $I_t$ - investment
- $K_t$ - physical capital stock
- $Z_t$ - level of capital utilization
- $K_t$ - capital services
- $v_t$ - real marginal cost

Prices and related variables

- $r^k_t$ - real rental price of capital services
- $i_t$ - net nominal interest rate (monetary policy rate)
- $q_t$ - price of government bonds portfolio
- $\pi_t$ - gross inflation rate
- $p_t^*$ - reset price
- $s_t$ - price dispersion
- $\omega_t$ - real wage
- $\omega_t^*$ - reset real wage
- $s_t^{\omega}$ - real wage dispersion

Fiscal variables

- $\tau_t$ - labor income tax rate
- $G_t$ - government spending
- $b_t$ - portfolio of government bonds

Auxiliary variables $\lambda_t, \mu_t, f_{1,t}, f_{2,t}, x_{1,t}, x_{2,t}$. 
B.2 Summary of the Private-Sector Equilibrium Conditions

Utility of the representative household

\[
\sum_{t=0}^{\infty} \beta^t \left[ \log c_t + \nu_g \log G_t - \frac{\eta h_t^{1+\chi}}{1+\chi} \right] \tag{B.1}
\]

Private sector equilibrium conditions

\[
\lambda_t = (C_t - bC_{t-1})^{-1} - b\beta(C_{t+1} - bC_t)^{-1}, \tag{B.2}
\]

\[
\mu_t = \beta(1 - \delta)\mu_{t+1} + \beta\lambda_{t+1} \left( r_{t+1}^k Z_{t+1} - \gamma_1(Z_{t+1} - 1) - \frac{\gamma_2}{2}(Z_{t+1} - 1)^2 \right), \tag{B.3}
\]

\[
\lambda_t = \mu_t \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) + \beta \kappa \mu_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right), \tag{B.4}
\]

\[
r_{t}^k = \gamma_1 + \gamma_2(Z_t - 1), \tag{B.5}
\]

\[
\lambda_t = \beta(1 + i_t)\lambda_{t+1} \pi_{t+1}^{-1}, \tag{B.6}
\]

\[
\eta_t = \beta(1 + \rho\eta_{t+1})\frac{1}{\pi_t} \lambda_{t+1} \pi_{t+1}^{-1}, \tag{B.7}
\]

\[
K_t = (1 - \delta)K_{t-1} + \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 I_t, \tag{B.8}
\]

\[
K_t = Z_t K_{t-1}, \tag{B.9}
\]

\[
\omega_t^i = \sigma f_{1,t}(\sigma - 1)f_{2,t})^{-1}, \tag{B.10}
\]

\[
f_{1,t} = \eta(\omega_t^i \omega_t^1)^{-\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi \omega(\pi_t^{*\omega} \pi_{t+1}^{*\omega})^{-\sigma(1+\chi)} (\omega_{t+1}^* \sigma(1+\chi) f_{1,t+1}, \tag{B.11}
\]

\[
f_{2,t} = \lambda_t(1 - \tau_t)(\omega_t^i \omega_t^1)^{-\sigma} L_t + \beta \xi \omega(\pi_t^{*\omega} \pi_{t+1}^{*\omega})^{-\sigma} (\omega_t^* \sigma)_{f_{2,t+1}}, \tag{B.12}
\]

\[
1 = (1 - \xi)_{\omega_t}^{(\omega_t^i \omega_t^1)^{-1}} + \xi (\pi_t^{*\omega} \pi_{t+1}^{*\omega} \omega_t^{1+\omega_t^1})^{1-\sigma}, \tag{B.13}
\]

\[
s_t = (1 - \xi)_{\omega_t}^{(\omega_t^i \omega_t^1)^{-\sigma(1+\chi)}} + \xi (\pi_t^{*\omega} \pi_{t+1}^{*\omega} \omega_t^{1+\omega_t^1})^{-\sigma(1+\chi)} s_{t-1}^\omega, \tag{B.14}
\]

\[
K_t = \alpha \frac{\nu_t}{(1 - s_k) r_t^s} \left( s_t X_t + F \right), \tag{B.15}
\]

\[
L_t = (1 - \alpha) \frac{\nu_t}{(1 - s_k)(1 + i_t)\omega_t} \left( s_t X_t + F \right), \tag{B.16}
\]

\[
s_t X_t = K_t^{\alpha} L_t^{1-\alpha} - F, \tag{B.17}
\]

\[
p_t^s = \theta x_{1,t}((\theta - 1)x_{2,t})^{-1}, \tag{B.18}
\]

\[
x_{1,t} = \lambda_t X_t \nu_t + \beta \xi (\pi_t^{*\omega} \pi_{t+1}^{*\omega} - x_{1,t+1,} \tag{B.19}
\]

\[
x_{2,t} = \lambda_t X_t + \beta \xi (\pi_t^{*\omega} \pi_{t+1}^{*\omega} - x_{2,t+1}, \tag{B.20}
\]

\[
1 = (1 - \xi)_{\pi_t}^{(\pi_t^{*\omega} \pi_{t+1}^{*\omega})^{-\sigma}} + \xi (\pi_t^{*\omega} \pi_{t+1}^{*\omega})^{-\sigma} s_{t-1}, \tag{B.21}
\]

\[
X_t = C_t + G_t + I_t + \left( \gamma_1(Z_t - 1) + \frac{\gamma_2}{2}(Z_t - 1)^2 \right) K_{t-1}, \tag{B.22}
\]

\[
q_t b_t = (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + (G_t + \varsigma_t - \pi_t w_t L_t), \tag{B.24}
\]
B.3 The Policy Problem

As in the baseline model, the policy problem of the government consists of maximizing utility function of the representative household (B.1) subject to private-sector equilibrium constraints (B.2)–(B.24) while lacking commitment to its own future choices. The policy problem can be solved using the Linear-Quadratic approach. To do so, one can follow the steps described earlier in relation with the baseline model. This section makes a number of remarks specific to the medium-scale version without repeating these steps in detail.

First, note that the approximation of the medium-scale model is done around the efficient steady state supported by a positive government debt. As in the baseline model, all the static distortions are eliminated by designing appropriate production subsidies. In the medium-scale model one can do so using two subsidies, $s_K$ and $s_L$, one for each factor of production.

Second, the approximation requires one to derive a quadratic approximation of the utility-based objective function. Let $U_t$ be a per-period utility component in (B.1). One can then write down its quadratic approximation in the vicinity of the steady state as follows

$$U_t \simeq -\frac{1}{2} \tilde{\mu} \left( \bar{C} \hat{C}_t^2 + \frac{b \bar{C}}{(1 - b)(1 - b\beta)} \hat{C}_t^2 + G \hat{G}_t^2 + \kappa \hat{I}_t^2 + \gamma_1 \bar{K} \hat{K}_t^2 
+ \gamma_2 \bar{\hat{I}}_t^2 + (1 - \alpha)(1 + \chi)(1 - \phi)(\bar{X} + F) \hat{\tilde{L}}_t^2 - \frac{\bar{X}^2}{\bar{X} + F} \hat{\tilde{X}}_t^2 
+ \frac{\sigma \xi_\omega (1 - \alpha)(1 - \phi)(\sigma \chi + 1)(\bar{X} + F)}{(1 - \beta \xi_\omega)(1 - \xi_\omega)} \hat{\tilde{\omega}}_t^2 + \frac{\theta \xi_p \bar{X}}{(1 - \beta \xi_p)(1 - \xi_p)} \hat{\tilde{\pi}}_t^2 \right) + \text{t.i.p.,}$$

where

$$\hat{C}_t \equiv \hat{C}_t - \hat{C}_{t-1},$$
$$\hat{I}_t \equiv \hat{I}_t - \hat{I}_{t-1},$$
$$\hat{\pi}_t \equiv \hat{\pi}_t - \xi_p \hat{\pi}_{t-1},$$
$$\hat{\omega}_t \equiv \hat{\omega}_t - \hat{\omega}_{t-1} + \hat{\pi}_t - \xi_\omega \hat{\pi}_{t-1},$$

and a bar denotes the steady-state value of the corresponding variable whereas a hat is used to denote the percentage deviation from this steady-state value.
### Table 2 – Baseline parameterization of the Medium-scale Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<tr>
<td>$b$</td>
<td>Internal habit formation</td>
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<td>$\nu_g$</td>
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<tr>
<td>$\eta$</td>
<td>Labor disutility weight</td>
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<td>$\chi$</td>
<td>Inverse labor supply elasticity</td>
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<td>$\kappa$</td>
<td>Investment adjustment cost</td>
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<td>$\delta$</td>
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<td>Utilization adjustment cost linear term</td>
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<tr>
<td>$\gamma_2$</td>
<td>Utilization adjustment cost squared term</td>
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<tr>
<td>$\alpha$</td>
<td>Capital share</td>
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<tr>
<td>$F$</td>
<td>Fixed cost of production</td>
<td>$\bar{\Pi} = 0$</td>
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</tbody>
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#### B.4 Parameterization

Most of parameter settings, displayed in Table 2, are based on fixed parameters and posterior mean estimates of the U.S. economy by Christiano et al. (2010). Parametric differences with respect to the model estimates in Christiano et al. (2010) are as follows. The medium-scale model in this paper abstracts from the drift in productivity terms that generate trend growth in output. Moreover, the model abstracts from all stationary disturbances. The model has optimal zero inflation in the steady state.

The fiscal sector of the model, which is absent in Christiano et al. (2010), is parameterized following the logic of the baseline model. The value for the weight on the public consumption utility component is chosen so that in the steady state government spending amounts to 20 percent of the value added. The target for the market value of government debt in the steady state is set equal to 40 percent of annual GDP.
Figure 1 – Linear Equilibrium Decision Rules for Government Debt and Inflation: one-period bonds

Notes: Parametric plot of coefficients in the linear equilibrium decision rules for government debt (vertical axis) and inflation (horizontal axis) as functions of the market value of government debt as a fraction of (annual) GDP in the steady state. Government debt in the form of one-period bonds. Data is plotted with an increment of 5% of GDP.
Figure 2 – Decision Rules for Government Debt and Inflation: bonds with 4 years average maturity

Notes: Parametric plot of coefficients in the linear equilibrium decision rules for government debt (vertical axis) and inflation (horizontal axis) as functions of the market value of government debt as a fraction of (annual) GDP in the steady state. The duration of government debt is 4 years. Data is plotted with an increment of 5% of GDP.
**Figure 3** – Targeting Rules Coefficients

Notes: Contour plots of the coefficients in the targeting rules for consumption and output as functions of government debt characteristics. *Light gray area*: negative values of coefficients. *Dark gray area*: positive values of coefficients. *White area*: positive values of coefficients but a non-monotone equilibrium.
Notes: The figure plots dynamics of the equilibrium variables over time, given an initial quantity of government bonds such that the market value of outstanding government debt in real terms is 30% higher than the steady-state level. The steady-state market value of debt is 40% of annual GDP. **Solid black lines and black axes**: bonds with average maturity equal to four years. **Dotted red lines and red axes**: one-period bonds.
Figure 5 – Consumption Equivalent Losses of Debt Reduction

Notes: The steady-state market value of debt is 40% of annual GDP. Transition starts from an initial quantity of government bonds such that the market value of outstanding government debt in real terms is 30% higher than the steady-state level. Welfare losses are computed in terms of equivalent permanent steady-state consumption reduction and normalized by the welfare loss in the economy with consol bonds.
Notes: On the vertical axes is the difference, measured in annualized percentage points, between the real interest rate in the period $t$ where the stock of outstanding government debt, $b_{t-1}$, is the only state variable that is different from the steady-state value and the real interest rate in the steady state. The outstanding stock of government debt exceeds the steady-state value by one percent. The real interest rate is given by $\hat{r}_t = \hat{i}_t - \hat{\pi}_{t+1}$, where $\hat{i}_t$ is the nominal interest rate and $\hat{\pi}_{t+1}$ is expected inflation.