The Natural vs. Neutral Rate of Interest: Measurement and Policy Implications*

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Abstract
This paper measures and studies the policy implications of two closely related concepts of equilibrium rates of interest. We estimate the natural and the neutral rate of interest within a large-scale DSGE model, where the former rate is defined as the real interest rate that would prevail in the absence of nominal rigidities and inefficient variation in mark-ups while the latter rate is the real interest rate that sets output equal to its potential and hence closes the output gap. To assess the drivers behind the variation of these rates, we decompose latent variables into the contributions from the structural shocks. Furthermore, we propose a novel approach to decompose latent variables that accounts for shock contributions in terms of movements in input data. We show that fluctuations in both latent variables are driven by risk premium shocks at low frequencies and productivity at higher frequencies. We conclude that the labor market input data provide most information for inference about the neutral and natural interest rates in our model.

Keywords: Natural rate of interest; Neutral rate of interest; Latent variables; Bayesian estimation; Monetary Policy

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1. Introduction

Recently, the concept of a natural rate of interest, loosely following Wicksell (1898), has received renewed attention as a way of framing discussions about effective monetary policy as well as for understanding the extraordinarily long spell of interest rates at the effective lower bound in many countries. Formally, in Dynamic Stochastic General Equilibrium (DSGE) models, the natural rate of interest can be characterized as the real interest rate that would prevail in the absence of nominal rigidities and inefficient variation in mark-ups. In simple canonical New-Keynesian models, with only one source of purely nominal inefficiency, setting the monetary policy rate—adjusted for inflation—to the natural rate achieves both price stability and full employment (the divine coincidence). In such simple models, therefore, targeting the natural rate of interest is optimal. Moreover, the natural rate of interest coincides with the what we call the neutral rate of interest, aka the interest rate that closes the output gap by setting actual output equal to potential. Thus, the neutral rate of interest is neither expansionary nor contractionary with respect to aggregate economic activity.

In more complex models, featuring several sources of nominal rigidity, the natural and neutral rate of interest do not coincide. Moreover, targeting neither the natural nor the neutral rate of interest is fully optimal as measured by a model consistent welfare criterion such as lifetime utility of the representative household. The relative performance of these two measures of the monetary policy stance has not been assessed in the literature. We document that both lead to reasonably good welfare outcomes, with the neutral interest rate presenting lower volatility for the policymakers instrument in our model.

The natural rate of interest differs from the neutral rate of interest (as defined above) whenever there are multiple independent nominal frictions in the economy. For instance, in multi-sector models one can typically not set aggregate output equal to the level that would prevail in the absence of nominal rigidities by setting the real interest rate equal to the real natural rate of interest at all times. The reason is simple: with only one instrument policymakers can at most undo one distortion. In general, the real interest rate that eliminates the real effects of nominal rigidities in one sector is not the same as the real interest required to undo the real effects of nominal rigidities in another sector. A similar argument can be made for models with sticky prices and sticky wages. This then implies that by setting the real interest rate equal to the real natural rate, the output gap is typically not fully closed in more elaborate models. Furthermore, the rate of inflation is typically not fully at target for similar reasons. Of course, there always exists a state-contingent path of the real interest rate such that the output gap is closed, and we refer to it as the path for the neutral real rate. Both the natural and the neutral rate are key benchmarks for monetary policy deliberations,
because they are simple. A key contribution of this paper is to assess empirically whether the natural and the neutral rate differ in quantitatively important ways.

Literature has proposed various ways to infer the information regarding the natural and the neutral rate of interest, see, e.g., Laubach and Williams (2001). The DSGE approach to natural and neutral rate of interest has several advantages. First, the connection between the natural and neutral rate of interest and the movement in observable variables established by the models provides a framework for the measurement of this otherwise unobservable object. Second, the models can identify the fundamental sources of changes in the natural rate and relate them to the factors underlying macroeconomic fluctuations. In addition, we show how to account for shock contributions in terms of movements in input data. Third, it allows us to study possible policy implications of these two concepts.

In this paper, we use a large scale DSGE model to provide model consistent estimates of the natural rate of interest for the United States and forecasts at medium-term horizons. The model under analysis is the EDO model by Chung et al. (2010), which generalizes the basic Smets and Wouters (2007) model to account for the more rapid growth of the productivity in the production of capital goods, compared to the production of consumption goods. Core labor market dynamics are driven by monopolistically competitive labor supply by households, as in the Smets and Wouters (2007) model. Following Gali, Smets and Wouters (2011), the labor force is defined as the fraction of households who would choose to work, given wages and consumption, but in the absence of this monopoly power. The EDO model reproduces several empirical regularities of the U.S. economy and is used for policy analysis.

The **natural rate of interest** is a latent variable and, hence, model dependent. Furthermore, substantial uncertainty surrounding real-time estimates of the natural rate could diminish the value of using such estimates as indicators for policy. We employ our model to gauge the usefulness of the natural rate of interest as an indicator of the stance of monetary policy. In particular, we compare the effectiveness of targeting both short-term and long-term natural rate, against a number of plausible benchmarks, such as nominal income targeting.

The business cycle variation in the natural rate of interest is largely driven by shocks to aggregate risk premium, while shocks to the capital-specific risk premium and aggregate technology drive most of the lower-frequency variation. Given the aggregate risk premium shock appears in the model as a wedge only in the Fisher equation, so it passes through one-for-one into the natural rate.

Estimates of “natural” variables are, of necessity, highly indirect and the relation between these variables and movements in data may not be readily apparent. To gain additional insight to the mapping between data and the natural variables, we exploit the linear-Gaussian structure of the model to decompose movements in latent variables into variations in the data. This decomposition reveals, among other insights, that data on employment are strong
indicators of the natural rate, largely because these data are highly informative about the aggregate risk premium.

2. Latent variable decomposition

As is well known, any linear DSGE model can be represented by the system $X_t = GX_{t-1} + H\epsilon_t$, where $X_t$ is the state vector, related to the data by an observation equation of the form $Y_t = HX_t$. In many models, moreover, the innovations $\epsilon_t$ are assumed to be independently and identically distributed normal random variables. From these assumptions, it follows that the paths of $X_t$ and $Y_t$ are multivariate normally distributed as well and, as a generic property of such random variables, there exist data-independent matrices $J_t(u)$ such that

$$E(X_t|Y_{1:T}) = \sum_{u=1}^{T} J_t(u)Y_u$$  \hspace{1cm} (1)

Using the decomposition in equation 1, we can thus directly characterize the data signature of movements in the natural rate.

We can gain even more insight into the model’s inference about the natural rate by combining equation 1 with the standard historical shock decomposition, obtained by iterating on the law of motion

$$X_s = G^sX_0 + \sum_{u=1}^{s} G^{s-u}H\epsilon_u$$  \hspace{1cm} (2)

Substituting a decomposition of the shocks into the historical decomposition, of the form $E(\epsilon_t|Y_{1:T}) = \sum_{u=1}^{T} J_{\epsilon,t}(u)Y_u$, we arrive at a double decomposition of the latent variables

$$E(X_t|Y_{1:T}) = \sum_{s=1}^{S} \sum_{u=1}^{T} J_t(u,s)Y_u$$  \hspace{1cm} (3)

, where the index $s$ runs over the structural shocks (including, for this purpose, the initial condition).

The decomposition 3 shows how variation in the data maps into contributions of specific structural shocks to the latent variable of interest. Specifically, an element of the matrix $J_t(u,s)$ might represent the effect of data at time $u$ on latent variables at time $t$ (holding all other data fixed), via movements in shocks of type $s$. One advantage of this decomposition is that the relation between the data and the model’s estimates of structural shocks may be more intuitive than the mapping between the data and the latent variable of interest. In that case, the tracing the model’s inference about the shock contributions may be more insightful than proceeding directly to the level of the focal latent variable.
3. Model

Our model is based on the so-called EDO model developed by Chung, Kiley, and Laforte (2011). The EDO model is large scale New Keynesian DSGE model with price and wage rigidities. We modify the modeling of the labor input in EDO by incorporating not only the intensive but also the extensive margin of the labor market. Figure 1 provides a graphical overview of the model. We assume there are two final good sectors: one sector, \( kb \), produces business investment and consumer durables, while the other sector, \( cbi \), produces other goods and services. Production by firm \( j \) in each sector \( s \) is governed by a Cobb-Douglas production function with sector-specific technologies:

\[
X_{t}^{kb}(j) = \left[ Z_{m}^{m} Z_{t}^{kb} L_{t}^{kb}(j) \right]^{1-\alpha} \left( K_{t}^{u,nr,kb}(j) \right)^{\alpha}, \quad X_{t}^{cbi}(j) = \left[ Z_{m}^{m} L_{t}^{cbi}(j) \right]^{1-\alpha} \left( K_{t}^{u,nr,cbi}(j) \right)^{\alpha},
\]

where \( Z_{m}^{m} \) represents aggregate technology and \( Z_{t}^{s} \) stands for investment-specific technological change, which only affects the business investment and consumer durable sector.

We consider four categories of private demand: consumer nondurable goods and nonhousing services, consumer durable goods, residential investment, and nonresidential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer nondurable goods and services are sold directly to households; consumer durable goods, residential capital goods, and nonresidential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer nondurable goods and services and residential capital goods are purchased from the first of economy’s two final goods-producing sectors, while consumer durable goods and nonresidential capital goods are purchased from the second sector. In addition to consuming the nondurable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

In our model, household utility is given by:

\[
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta_{t} \left\{ \varsigma_{cnn}^{'} \ln[E_{t}^{cnn}(i) - h E_{t-1}^{cnn}(i)] + \varsigma_{cd}^{'} \ln[K_{t}^{cd}(i)] + \varsigma_{r}^{'} \ln[K_{t}^{r}(i)] - \Lambda_{t}^{Pref} \Theta_{t}^{H} \sum_{s=cbi,kb}^{\infty} \int_{0}^{1} \varsigma_{s}^{L} L_{t}^{s}(i) \frac{1+\gamma_{s}}{1+\gamma_{s}} \, di \right\},
\]

where \( E_{t}^{cnn} \) stands for expenditures on consumption of nondurable goods and services; \( K_{t}^{cd} \) and \( K_{t}^{r} \) represent the stocks of consumer durables and residential capital (housing), respectively; \( \Lambda_{t}^{Pref} \) is a labor supply shock; \( \Theta_{t} \) is an endogenous preference shifter whose role is...
to reconcile the existence of a long-run balance growth path with a small short-term wealth effect as in Galí, Smets, and Wouters (2011), which is key to match the joint behavior of the labor force, consumption, and wages over the business cycle\(^1\) and \(L^{cbi}\) and \(L^{kb}\) stand for the labor supplied to each productive sector.

The EDO model features several exogenous shocks to the rates of return required by the household to hold assets. In particular, the model includes an “aggregate risk-premium” and several “sector specific” risk shocks. A shocks to sectoral risk premium leads to a sizable substitution between across residential, consumer durable, and business investment; for example, an increase in the risk premiums on residential investment leads households to shift away from residential investment and toward other types of productive investment. “Aggregate risk-premium” shocks are shocks to the required rate of return on the nominal risk-free asset. Following an increase in the premium, in the absence of nominal rigidities, the households’ desire for higher real holdings of the risk-free asset would be satisfied entirely by a

\[\Theta^H_t = Z_t \Lambda_i \gamma, \text{ where } Z_t = \frac{Z_t - \nu}{\Lambda^2} \text{ and } \Lambda^2 \text{ is the shadow price of nondurable consumption.} \]

The endogenous preference shifter is defined as \(\Theta^H_t = Z_t \Lambda_i \gamma\), where \(Z_t = \frac{Z_t - \nu}{\Lambda^2}\) and \(\Lambda^2\) is the shadow price of nondurable consumption. The importance of the short-term wealth effect is determined by the parameter \(\nu \in (0, 1]\).
fall in prices, that is, the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be offset, in part, through a fall in real income, a decline which is distributed across all spending components.

Movements in financial markets and economic activity in recent years have made clear the role that frictions in financial markets play in economic fluctuations. Imperfections in financial markets drive a wedge between the cost of riskless funds and the cost of funds facing households and firms. Because the risk-premium shocks induce a wedge between the short-term nominal risk-free rate and the rate of return on the affected risky states, these shocks may be interpreted as a reflection of financial frictions not explicitly modeled in EDO.

In the model, monetary policy is set following a Taylor-type interest rate feedback rule. Policymakers smoothly adjust the actual interest rate \( R_t \) to its target level \( \bar{R}_t \):

\[
R_t = (R_{t-1})^{\rho^r} \left( \bar{R}_t \right)^{1-\rho^r} \exp \left[ \epsilon^r_t \right],
\]

where the parameter \( \rho^r \) reflects the degree of interest rate smoothing, while \( \epsilon^r_t \) represents a monetary policy shock. The central bank’s target nominal interest rate, \( \bar{R}_t \) depends the deviation of output from its flexible price counterpart. The target equation is

\[
\bar{R}_t = \left( \tilde{X}_t^{pf} \right)^{\nu} \left( \frac{\Pi_t^c}{\Pi_t^c} \right)^{\pi} R^*_t.
\]

In equation (7), \( R^*_t \) denotes the economy’s steady-state nominal interest rate, \( \tilde{X}_t^{pf} \) is the flexible price output gap, and \( \Pi_t^c \) is the consumer price inflation, which is computed as the the weighted average of inflation in the nominal prices of the goods produced in each sector, \( \Pi_t^{p,cbi} \) and \( \Pi_t^{p,kb} \):

\[
\Pi_t^c = (\Pi_t^{p,cbi})^{1-w_{cd}}(\Pi_t^{p,kb})^{w_{cd}}.
\]

The parameter \( w_{cd} \) is the share of the durable goods in nominal consumption expenditures.

The model also includes a long-term interest rate \( (RL_t) \), which is governed by the expectations hypothesis subject to an exogenous term premiums shock:

\[
RL_t = \varphi_t \left[ \Pi_t^N R_t \right] \cdot \Upsilon_t.
\]

where \( \Upsilon \) is the exogenous term premium, governed by

\[
Ln \left( \Upsilon_t \right) = (1 - \rho^\Upsilon) Ln \left( \Upsilon_* \right) + \rho^\Upsilon Ln \left( \Upsilon_{t-1} \right) + \epsilon^\Upsilon_t.
\]

In the model, the long-term interest rate plays no allocative role; nonetheless, as stated by Edge, Kiley, and Laforte (2010), the term structure contains information on economic developments useful for forecasting and, hence, \( RL \) is included in the model.
Finally, fluctuations in economic variables are driven by 13 structural shocks belonging to the following five categories:

- **Permanent technology shocks:** This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.

- **A labor supply shock:** This shock affects the willingness to supply labor. This shock captures the dynamics of the labor force participation rate in the sample and those of employment. While EDO labels such movements labor supply shocks, an alternative interpretation would describe these as movements in the labor force and employment that reflect structural features not otherwise captured by the model.

- **Financial, or intertemporal, shocks:** This category consists of shocks to risk premiums both the premium households receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries

- **Markup shocks:** This category includes the price and wage markup shocks.

- **Other demand shocks:** This category includes the shock to autonomous demand and a monetary policy shock.

### 3.1. Natural versus Neutral Rate of Interest

### 4. Estimation

[to be added: prior, posterior, shock decomposition of observables]

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy’s equilibrium, casts this resulting system in its state-space representation for the set of observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

The model is estimated using 13 data series over the sample period from 1984:Q4 to 2015:Q3. The series are the following:

1. The growth rate of real gross domestic product ($\Delta GDP$);
2. The growth rate of real consumption expenditure on nondurables and services ($\Delta C$);
3. The growth rate of real consumption expenditure on durables ($\Delta CD$);
4. The growth rate of real residential investment expenditure ($\Delta Res$);
5. The growth rate of real business investment expenditure ($\Delta I$);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,\text{total}}$);
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,\text{core}}$);
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods ($\Delta P_{cd}$);
9. Hours, which equals hours of all persons in the nonfarm business sector from the Bureau of Labor Statistics ($H$);
10. Civilian employment-population ratio, defined as civilian employment from the Current Population Survey (household survey) divided by the noninstitutional population, age 16 and over ($N$);
11. Labor force participation rate;
12. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index ($\Delta RW$); and
13. The federal funds rate ($R$).

Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.
5. Results: Drivers of Natural and Neutral Rate of Interest

Figure 2 presents the time series estimates for the natural, blue line, and neutral, brown line, rate of interest. The movements in the natural and neutral rates are highly correlated. These two interest rates present a cyclical pattern: they rise during booms and decline abruptly in downturns. This decline is especially pronounced in the Great Recession, when both rates fall into negative territory. The recovery following the recession is gradual. Our estimate for the the natural and the neutral rates display pronounced high frequency variation, which reflects the fact that both rates fluctuate quarterly in reaction to transitory shocks that hit the economy. The model implied measure of the neutral rate exhibits less variation than the natural rate of interest. For comparison, we include the actual series for the real rate in Figure 2. We conclude that the natural and neutral rate of interest, which are latent variables, trace quite well the low frequency movements in the realized real rate. The model implied series for the natural and neutral rates, however, present larger cyclical variation.

![Real Interest Rates](image-url)

Figure 2. Natural and Neutral Rate of Interest
Figures 3 displays the historical shock decomposition of the natural rate of interest. The business cycle variation in the natural rate is largely driven by shocks to the aggregate risk premium, while shocks to the capital-specific risk premium and aggregate technology drive most of the lower-frequency variation. As explained above, in the EDO model, the aggregate risk premium shock appears as a wedge only in the Fisher equation, so it passes through one-for-one into the natural rate. The historical shock decomposition of the neutral rate of interest is reported in Figure 4. The most remarkable differences between the drivers of the neutral versus the natural rate of interest are the smaller role of productivity shocks in driving the neutral rate and the larger countercyclical role played by preference shocks. The combination of these two forces account for the lower cyclical variation of the neutral rate when compared to the natural rate of interest.
Figure 3. Shock Decomposition of the Natural Rate of Interest
Figure 4. Shock Decomposition of the Neutral Rate of Interest
Figure 5 displays the shock contributions of the natural rates of interest in terms of movements in input data. The left upper panel indicates that the natural rate is mostly informed by labor market variables: the employment-to-population ratio and the labor force participation rate. The remaining panels display the model-implied natural rate of interest when fed by one shock at a time. We conclude that the low frequency variation in the natural rate of interest is driven by the aggregate risk premium shock, while higher frequency variation is largely driven by productivity shocks. Let us take a closer look to the model-implied natural rate of interest when feeding the model only with aggregate risk shocks. This panel allow us to conclude that the model extracts most of the information to generate the natural rate of interest from the employment-to-population ratio and from the labor force participation rate. While the former informs procyclically the aggregate-risk-only natural rate, labor force participation informs this series countercyclically, that is, it reduces the overall movements in the series. Finally, give our definition for the natural rate, there is no role for monetary policy shocks and markup shocks.
Figure 5. Natural Rate of Interest
6. Policy Analysis
[to be added]

7. Conclusion
[to be added]

References