Fiscal Activism and the Zero Nominal Interest Rate Bound∗

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Abstract

In an economy where the zero lower bound on nominal interest rates is an occasionally binding constraint and the government lacks a commitment technology, it may be desirable for society to appoint a policymaker who cares less about government spending stabilization relative to inflation and output gap stabilization than the private sector does. A policymaker of this type uses government spending more elastically to stabilize the economy. At the zero lower bound, the anticipation of aggressive fiscal expansions in future liquidity trap situations increases inflation expectations and lowers real interest rates, thereby mitigating the decline in output and inflation.

Keywords: Monetary policy, Fiscal policy, Discretion, Delegation, Liquidity trap

JEL-Codes: E52, E62, E63

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1 Introduction

In the absence of policy commitment, the zero nominal interest rate bound can be a severe drag on monetary stabilization policy. Once the policy rate hits the lower bound, standard nominal interest rate policy becomes unable to stabilize output and inflation against economic turmoil. Several studies have therefore turned the spotlight on fiscal policy in coping with liquidity traps, showing that the welfare costs associated with zero bound events can be reduced if public spending is used for macroeconomic stabilization.\(^1\) All these studies, however, maintain the assumption that the policymaker has the same preferences as society as a whole. In this paper, I ask whether the presence of an occasionally binding zero lower bound provides a rationale for appointing a policymaker whose preferences differ from those of the private sector. In doing so, I focus on the policymaker’s attitude towards the use of government purchases as a stabilization tool.

I use a standard stochastic New Keynesian model with nominal rigidities. The discretionary policymaker controls the one-period nominal interest rate and the level of government spending. The policy rate is constrained to be nonnegative. Households value private consumption as well as the provision of public goods and dislike labor. The policymaker’s preferences are similar to those of society as a whole, but the weight that he puts on the stabilization of public consumption in his objective function may differ from the one implied by households’ preferences.\(^2\)

In this model, expansionary fiscal policy is part of the optimal policy mix at the zero lower bound. Following a large adverse shock to aggregate demand the generic discretionary policymaker lowers the policy rate to zero and implements a transitory government spending stimulus. In doing so, the policymaker faces a trade-off between stabilizing inflation and the output gap, and mitigating deviations of government spending from its efficient level.\(^3\)

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\(^1\)See Eggertsson (2001), Werning (2012), Nakata (2013) and Schmidt (2013), as discussed below.

\(^2\)Given current institutional structures, in practice, this would mean that society elects a government that has certain preferences regarding the use of fiscal policy as a stabilization tool. Alternatively, this could be operationalized by the appointment of a decision-making fiscal council.

\(^3\)In the efficient equilibrium the Samuelson condition is satisfied, that is, the marginal utilities of private
Thus, in the standard discretionary equilibrium fiscal policy does not fully absorb large contractionary shocks but alleviates the impact on the output gap and inflation.

Fiscal policy is transmitted to the economy through two interrelated channels. First, a transitory government spending stimulus at the zero bound raises current aggregate demand and thereby reduces contemporaneous deflationary pressures. Second, fiscal policy works through an expectations channel. Rational forward-looking agents anticipate that government spending will be expanded in those states of the world that are associated with zero nominal interest rates. Since fiscal policy makes the downturn in these states less severe, it unfolds a stabilizing effect on agents’ expectations. Specifically, higher expected future output gaps and inflation rates mitigate the fall in the output gap and the inflation rate when the economy is in a liquidity trap.

The standard discretionary equilibrium can be improved by appointing a policymaker who is less concerned with the stabilization of public consumption than society. The basic reason for this result relies on the well-known credibility problem of discretionary policy at the zero lower bound. At the lower bound, the generic policymaker would like the private sector to expect a larger fiscal expansion in future liquidity trap situations, and, therefore, higher inflation. However, when the lower bound constraint becomes binding again the policymaker has an incentive to renege on its promise. The policy announcement is therefore not credible. Appointing a policymaker who cares less about public consumption stabilization than society does mitigates the credibility problem by making government spending more elastic. The private sector anticipates that a policymaker of this type increases government spending more aggressively whenever the zero lower bound is binding and raises its output and inflation expectations accordingly. The optimal degree of ‘fiscal activism’ balances the gains associated with improved expectations against the costs associated with larger deviations of government spending from its efficient level.

\[^4\] The credibility problem of discretionary policy at the zero lower bound and the gains from commitment to higher future inflation are typically discussed in the context of monetary policy, see Krugman (1998), and Eggertsson and Woodford (2003).
I first work with a rather stylized version of the model with sticky goods prices and flexible wages. The only source of uncertainty is a stochastic natural real rate of interest which follows a two state Markov process. The analytical results for a prominent special case where the normal non-crisis state is assumed to be an absorbing state are compared to the general case where uncertainty never dissipates.

The second part of the paper considers a more elaborate continuous state model with sticky prices and sticky nominal wages. The economy is buffeted by preference shocks to households’ discount factor as well as either technology shocks, price mark-up shocks or wage mark-up shocks. The model is calibrated to the U.S. economy and solved using global methods. Fiscal activism continuous to be welfare improving. The optimal fiscally activist policymaker is considerably less concerned with the stabilization of government spending than society. The weight that he puts on the public spending objective is about one-fourth of society’s corresponding weight. This holds true even so the higher elasticity of government spending under a fiscally activist regime also implies an excessive reliance on the fiscal tool when counteracting technology or mark-up shocks away from the zero lower bound.

The paper is most closely related to work by Eggertsson (2001), Nakata (2013) and Schmidt (2013) that studies optimal time-consistent monetary and fiscal policy in the presence of the zero lower bound using small New Keynesian models with sticky prices, flexible wages and utility-generating public goods. Eggertsson (2001) uses a two state setup with an absorbing natural real rate state to show that the optimal time-consistent policy mix consists of a transitory government spending stimulus in the liquidity trap state, and that this fiscal stimulus is welfare improving. Nakata (2013) and Schmidt (2013) find similar results in continuous state models where the zero lower bound is an occasionally binding constraint and the existence of the zero bound also affects stabilization outcomes in those states of nature where the constraint is not binding. Eggertsson (2006) and Burgert and

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5All three papers also study the optimal monetary and fiscal policy mix under commitment. Werning (2012) considers the case where monetary policy acts under discretion whereas fiscal policy is able to make credible commitments.
Schmidt (2014) extend the analysis of optimal time-consistent government spending policy in the presence of a lower bound on nominal interest rates to models where fiscal policy is non-Ricardian so that government debt becomes an endogenous state variable that can be used by discretionary policymakers to influence future policy. Open economy setups are studied by Cook and Devereux (2013) who consider a two-country model and by Bhattarai and Egorov (2014) who consider a small open economy model. In contrast to the current paper, all these studies consider a benevolent policymaker, i.e. a government that exhibits the same preferences as society as a whole. I show that the gains from fiscal stabilization policy can be enhanced by appointing a policymaker whose preferences differ from those of society in a way that renders government spending more elastic. Moreover, to the best of my knowledge, this is the first paper to study optimal monetary and fiscal policy with a zero bound on nominal interest rates in a model that features price and wage stickiness.

The paper is also related to but distinct from a broad literature on fiscal multipliers that studies the effect of an exogenous change in the fiscal instrument on GDP when monetary policy is constrained by the zero bound, see e.g. Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2011), Woodford (2011) and Coenen, Erceg, Freedman, Furceri, Kumhof, Lalone, Laxton, Linde, Mourougane, Muir, Mursula, de Resende, Roberts, Roeger, Snudden, Trabandt, and in’t Veld (2012). Woodford (2011) also investigates the welfare implications of fiscal stabilization policy at the zero bound. The fiscal policy that he considers involves a commitment to a specific level of government spending for as long as the zero bound is binding. In addition, my paper builds on earlier research that has documented the severity of the welfare costs associated with the zero lower bound under discretionary monetary policy absent any additional policy instruments, see Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2007) and Nakov (2008).

Finally, the paper is related to previous work on policy preferences and stabilization out-

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6Similar to the continuous state model analyzed in this paper, these extended models imply that, in general, government spending is used for stabilization policy at the zero lower bound as well as away from the lower bound.
comes that has emphasized the desirability of inflation-conservative central bankers along the lines of Rogoff (1985) as a remedy for the classical inflation bias of discretionary monetary policy.\(^7\) Adam and Billi (2008) analyze whether a conservative central banker remains desirable when the monetary authority has to interact with a discretionary fiscal authority. Clarida, Gali, and Gertler (1999) show that the presence of persistent cost-push shocks creates a stabilization bias in sticky-price models in which monetary policy is conducted under discretion, and that the appointment of an inflation-conservative central banker improves the trade-off between inflation and output gap stabilization. Here, instead, the focus is on the policymaker’s attitude towards the use of government spending as a stabilization tool in an environment where the presence of the zero nominal interest rate bound prevents the implementation of the efficient allocation.\(^8\)

The remainder of the paper is organized as follows. Section 2 introduces a small two state New Keynesian model and discusses the policy problem. Section 3 presents analytical and numerical results for the two state model. Section 4 presents the full continuous state model and Section 5 discusses the quantitative results. Section 6 concludes.

## 2 The two state model

The economy is represented by a small New Keynesian model. A detailed description can be found in Woodford (2003). The representative household consumes a composite private consumption good, supplies labor to the production sector in a competitive labor market and enjoys the provision of a composite public good by the government. Utility is separable in all three arguments as in Woodford (2011) and the two composite consumption goods are compiled based on the same aggregation technology.\(^9\) Monopolistically competitive firms

\(^7\)More precisely, Rogoff (1985) proposes the appointment of a weight-conservative central banker, i.e. a policymaker who puts less weight on output gap stabilization relative to inflation stabilization than society does. For a discussion of other forms of inflation conservatism see Svensson (1997).

\(^8\)In subsequent work, Nakata and Schmidt (2014) analyze the desirability of an inflation-conservative central banker as a remedy to mitigate the welfare costs associated with the presence of the zero bound.

\(^9\)Amano and Wirjanto (1998) provide empirical evidence for the U.S. in favor of additive separability in private and public consumption.
employ industry-specific labor and use a constant-return-to-scale technology to produce differentiated goods that can be used for private or public consumption. Nominal rigidities enter the model in the form of staggered price-setting as in Calvo (1983). The policymaker acts under discretion and possesses two policy instruments, the one-period, riskless nominal interest rate and government consumption, which is financed by lump-sum taxes. The steady state distortions arising from monopolistic competition are offset by an appropriate wage subsidy. Time is discrete and indexed by $t$.

2.1 Social welfare

Society’s preferences are represented by a linear-quadratic approximation to household welfare

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \pi_t^2 + \lambda (Y_t - \Gamma G_t)^2 + \lambda_G G_t^2 \right],$$

where $E_t$ is the rational expectations operator conditional on information available in period $t$, $\pi_t$ is the inflation rate between periods $t-1$ and $t$, $Y_t$ denotes output expressed in percentage deviations from the efficient steady state level, $G_t$ represents government spending as a share of steady state output and expressed in percentage point deviations from the steady state ratio, and $\beta \in (0, 1)$ denotes the subjective discount factor. The output gap is defined as $Y_t - \Gamma G_t$, where $\Gamma = \frac{\sigma}{\sigma + \eta}$, with $\sigma > 0$ denoting the elasticity of the marginal utility of private consumption with respect to total output and $\eta > 0$ denoting the inverse of the elasticity of labor supply.

The relative weights $\lambda, \lambda_G$ that society puts on the output gap term and on the government consumption term are functions of the structural parameters

$$\lambda = \frac{\kappa}{\theta}, \quad \lambda_G = \lambda \Gamma \left( 1 - \Gamma + \frac{\nu}{\sigma} \right),$$

where $\kappa$ is the slope of the New Keynesian Phillips curve, $\theta > 1$ is the price elasticity of demand for differentiated goods, and $\nu$ denotes the elasticity of the marginal utility of public
consumption with respect to total output.\textsuperscript{10}

\section*{2.2 The policymaker}

At the beginning of time, society appoints a discretionary policymaker that from then on is in charge of monetary and fiscal policy. The policymaker’s preferences are similar to those of society as a whole, but he may attach a different weight to the stabilization of public consumption, denoted by $\tilde{\lambda}_G > 0$, than society does. Society can perfectly observe the preferences of alternative candidates and appoints the policymaker that maximizes its expected discounted lifetime utility. Appendix A shows how the parameter $\tilde{\lambda}_G$ can be linked to the underlying preference parameters in the policymaker’s utility function. In doing so, it is ensured that the deterministic steady state remains efficient.

The policy problem of a generic policymaker is as follows. Each period $t$, he chooses inflation, output, government spending and the nominal interest rate to minimize his loss function subject to the behavioral constraints of the private sector and the zero nominal interest rate bound, taking agents’ expectations as given.\textsuperscript{11} In particular, since the model features no endogenous state variable, the policymaker solves a sequence of static optimiza-

\textsuperscript{10}The slope of the New Keynesian Phillips curve is itself a function of the structural model parameters, $\kappa = \left(\frac{1-\alpha(1-\alpha\beta)}{\alpha(1+\eta\theta)}\right)^{\alpha(1+\eta\theta)} \left(\sigma + \eta\right)$, where $\alpha \in (0, 1)$ denotes the share of firms that cannot reoptimize their price in a given period.

\textsuperscript{11}Note that while the exposition of the policy problem relies on a single policymaker who controls both policy instruments, results do not change if one reformulates the problem in terms of two separate policymakers, a monetary authority and a fiscal authority.
tion problems

$$\min_{\{\pi_t, Y_t, G_t, i_t\}} \frac{1}{2} \left[ \pi_t^2 + \lambda (Y_t - \Gamma G_t)^2 + \tilde{\lambda} G_t^2 \right]$$

(2)

subject to

$$\pi_t = \kappa (Y_t - \Gamma G_t) + \beta E_t \pi_{t+1},$$

(3)

$$Y_t = G_t + E_t Y_{t+1} - E_t G_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t),$$

(4)

$$i_t \geq 0,$$

(5)

$$r^n_t \text{ given, } \{\pi_{t+j}, Y_{t+j}, G_{t+j}, i_{t+j} \geq 0\} \text{ given for } j \geq 1.$$  

(6)

Equation (4) is a New Keynesian Phillips curve summarizing the price-setting behavior of firms and equation (5) is a consumption Euler equation summarizing the representative household’s intertemporal optimization. The variable $i_t$ denotes the level of the nominal interest rate between periods $t$ and $t + 1$, and $r^n_t$ denotes the exogenous natural real rate of interest, which is observed by all agents at the beginning of period $t$.

The natural real rate $r^n_t$ follows a two state Markov process, taking either the value $r^n_H > 0$ or $r^n_L < 0$. The transition probabilities are given by

$$\text{Prob}(r^n_{t+1} = r^n_L | r^n_t = r^n_H) = p_H$$

(8)

$$\text{Prob}(r^n_{t+1} = r^n_H | r^n_t = r^n_L) = p_L.$$  

(9)

3 Results for the two state model

Using the small two state New Keynesian model, this section investigates how much weight the discretionary policymaker should put on the stabilization of government spending relative to the stabilization of the inflation rate and the output gap in his objective function, and discusses how this affects optimal policy, stabilization outcomes and welfare.

In this simple model, a generic discretionary policymaker with $\tilde{\lambda}_G > 0$ only uses govern-
ment spending as a stabilization tool when the zero lower bound is binding. Specifically, at the lower bound he implements a transitory government spending stimulus, $G_t > 0$, the size of which depends on $\lambda_G$. This prescription of the optimal time-consistent monetary and fiscal policy mix would remain unaffected by an explicit consideration of price mark-up shocks.

3.1 Analytical results for a special case

I first consider a special case where the high state of the natural real rate is an absorbing state, $p_H = 0$, and the economy is assumed to start off in the low state. This assumption allows me to solve the model in closed form.

Denote the stochastic period in which the natural real rate jumps from $r^n_L$ to $r^n_H$ by $T$. As shown in Appendix B, if $(1 - p_L)(1 - \beta p_L) > \frac{\kappa}{\sigma} p_L$, then there exists a bounded rational expectations equilibrium where $\pi_t, Y_t, G_t = 0$ and $i_t = r^m_H$ for all $t \geq T$, and $\pi_t = \pi_L, Y_t = Y_L, G_t = G_L, i_t = 0$ for all $0 \leq t < T$, with $\pi_L = \omega_\pi r^m_L < 0, Y_L = \omega_Y r^m_L, G_L = \omega_G r^m_L > 0$. The policy function parameters $(\omega_\pi, \omega_Y, \omega_G)$ are functions of the structural model parameters including $\lambda_G$ and are defined in Appendix B. While the responses of inflation and the output gap $Y_L - \Gamma G_L$ in the low state are unequivocally negative, the sign of the output response depends on the size of $\lambda_G$. For very small values of $\lambda_G$, the reduced-form parameter $\omega_Y$ becomes negative, implying that the output response is positive.

Based on the characterization of the equilibrium, we can now determine which type of policymaker—as identified by the relative weight that he puts on the stabilization of government spending—society should appoint, and how this affects the optimal fiscal policy response in a liquidity trap. Given the assumptions about the shock process, the optimal

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12See Appendix B as well as Eggertsson (2001), Nakata (2013) and Schmidt (2013).
14This assumption is made quite often in the literature on the zero lower bound, see, for instance, Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011).
value for \( \tilde{\lambda}_G \), henceforth denoted by \( \tilde{\lambda}_G^* \), minimizes

\[
\frac{1}{2} \frac{1}{1 - \beta p_L} \left[ \omega_\pi^2 + \lambda (\omega_Y - \Gamma \omega_G)^2 + \lambda_G \omega_G^2 \right] (r_L^n)^2.
\] (10)

Solving the minimization problem yields the following proposition.

**Proposition 1** Under discretionary policy, welfare can be enhanced by the appointment of a policymaker who puts less weight on the stabilization of public spending than society as a whole. The best-performing policymaker exhibits

\[
\tilde{\lambda}_G^* = \frac{((1 - p_L)(1 - \beta p_L) - \frac{\kappa}{\beta p_L}) (\kappa^2 + (1 - \beta p_L) \lambda)}{(1 - p_L)(1 - \beta p_L) \left( \frac{\kappa^2}{1 - \beta p_L} + (1 - \beta p_L) \lambda \right)} \lambda_G < \lambda_G.
\] (11)

In the low state where the economy is in a liquidity trap, the best-performing policymaker raises government spending by more than a policymaker whose preferences are identical to those of society as a whole

\[
\left| \omega_G \left( \tilde{\lambda}_G^* \right) \right| > \left| \omega_G \left( \lambda_G \right) \right|.
\] (12)

The notation \( \omega_G \left( \cdot \right) \) in Proposition 1 underlines that the value of \( \omega_G \) depends on the relative weight that the policymaker puts on the stabilization of government spending. Furthermore, the analytical expressions for \( (\omega_\pi, \omega_Y, \omega_G) \) imply that

\[
\omega_\pi \left( \lambda_G \right) > \omega_\pi \left( \tilde{\lambda}_G^* \right) > 0, \quad \omega_Y \left( \lambda_G \right) - \Gamma \omega_G \left( \lambda_G \right) > \omega_Y \left( \tilde{\lambda}_G^* \right) - \Gamma \omega_G \left( \tilde{\lambda}_G^* \right) > 0.
\]

Hence, appointing the best-performing activist policymaker instead of a policymaker who exhibits preferences identical to those of society lowers the inflation and the output gap term in the welfare-based loss function (10) and increases the public consumption term. In order to understand why the overall effect of the change in policy preferences is welfare-increasing, reconsider the optimization problem of the discretionary policymaker in Section
2.2. In the absence of a commitment device, the policymaker is unable to exploit the positive relationship between the anticipated size of the government spending stimulus in future liquidity trap situations and agents’ output and inflation expectations. Under forward-looking behavior, higher output and inflation expectations would mitigate the decline of current output and inflation in the low state. Instead, when choosing the optimal amount of government spending in the liquidity trap, the discretionary policymaker takes private agents’ expectations as given. The appointment of a policymaker who cares less about the stabilization of public consumption than society as a whole provides a device to correct for discretionary authorities’ disregard of the expectations channel.\footnote{In Appendix B, I show that the best-performing discretionary policymaker replicates the stabilization performance of a policymaker who can commit to an optimized simple feedback rule for government spending as analyzed by Woodford (2011).} Likewise, if the economy stays only for one period in the low natural rate state and if this is perfectly known by agents, then the expectations channel becomes irrelevant and there is no need to appoint an activist policymaker, $\hat{\lambda}_G^* \rightarrow \lambda_G$ as $p_L \rightarrow 0$. The next proposition states how the optimal degree of fiscal activism depends on the persistence $p_L$ of the low natural real rate shock.

**Proposition 2** The optimal degree of fiscal activism is increasing in the persistence $p_L$ of the low natural real rate shock

$$\frac{\partial (\hat{\lambda}_G^*/\lambda_G)}{\partial p_L} < 0.$$  \hspace{1cm} (13)

**Proof** See Appendix B.

This is intuitive, the higher the conditional probability to stay in the low state in the next period the more important becomes the expectations channel.

### 3.2 Numerical results

I now consider the general case with $p_H > 0$, so that even if the natural real rate of interest is in the high state today there is a positive probability that the economy falls into a liquidity

\[12\]
I use the parameter values estimated by Denes, Eggertsson, and Gilbukh (2013) to match U.S. data during the Great Recession period as summarized in Table 1. Denes, Eggertsson, and Gilbukh (2013) assume that the high state is an absorbing state.

Table 1: Calibration - two state model

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>η</th>
<th>α</th>
<th>p_H</th>
<th>p_L</th>
<th>r_H</th>
<th>r_L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.997</td>
<td>1.69</td>
<td>0.784</td>
<td>0.0125</td>
<td>0.857</td>
<td>1/β - 1</td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>1.22</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>4.88</td>
<td>13.23</td>
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<td></td>
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</tr>
</tbody>
</table>

In this case, without fiscal stabilization policy the output gap would fall by 10% and the annualized inflation rate would fall by 2% in the low state. Instead, I set \( p_H = 0.0125 \), i.e. the probability that the natural real rate is in the low state in the next period conditional on being in the high state today is 1.25%, and the economy falls into the low state on average once every 20 years. The expected duration of such a crisis event is 7 quarters, as in Denes, Eggertsson, and Gilbukh (2013).

Figure 1 shows how the output gap, inflation, government spending and the nominal interest rate in the two states vary with the policymaker’s weight on government spending stabilization \( \lambda_G \). Solid lines represent outcomes under the baseline calibration and dashed lines represent outcomes for the special case considered before where the high state of the natural real rate is an absorbing state (\( p_H = 0 \)). The dotted vertical lines indicate outcomes when the policymaker puts the same weight on the stabilization of government spending as society as a whole (\( \lambda_G = \lambda_G \)). In the low state, the output gap and inflation are negative, the nominal interest rate is zero and government spending is positive. In the high state, the nominal interest rate is strictly positive and government spending is not used for stabilization policy. However, the output gap and the inflation rate can no longer be completely

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16 Nakata and Schmidt (2014) provide analytical characterizations of the conditions for equilibrium existence for the case where the discretionary policymaker possesses only the nominal interest rate tool.

17 In doing so I calibrate parameter \( ν \) such that the intertemporal elasticity of substitution for the public consumption good equals the intertemporal elasticity of substitution for the private consumption good as in Woodford (2011).

18 In this case, absent fiscal policy, the output gap would fall by 9.3% and the inflation rate would fall by 2.1% in the low state.
Figure 1: Policy functions - two state model

Note: The figure displays how the output gap, annualized inflation, government spending and the annualized one-period nominal interest rate vary with $\tilde{\lambda}_G$ in the two states. The solid lines represent outcomes for $p_H = 0.0125$, the dashed lines represent outcomes for $p_H = 0$. The dotted vertical lines indicate society’s weight $\lambda_G$.

stabilized if there is a positive probability to enter the low state in the future ($p_H > 0$). Instead, the output gap is strictly positive and the inflation rate is strictly negative. The latter phenomenon is sometimes referred to as deflationary bias.\textsuperscript{19} The deflationary bias in the high state arises because the presence of the zero lower bound faces policymakers with a trade-off between inflation and output gap stabilization when the constraint is not

\textsuperscript{19}See, for instance, Nakov (2008) and Nakata and Schmidt (2014). Instead, Eggertsson (2006) uses the term deflation bias to refer to the excessive amount of deflation materializing in a liquidity trap if the policymaker is unable to make credible commitments.
binding. A positive probability of ending up in the deflationary and recessionary low natural real rate state in the next period reduces firms’ expected marginal costs and households’ expected consumption. Forward-looking agents react to these dampened expectations by lowering prices and consumption already in the high state. The policymaker responds to the deflationary pressures resulting from muted inflation expectations by increasing the output gap, which requires a reduction in the nominal interest rate below the high-state level of the natural real rate.\textsuperscript{20} Instead, in the special case with $p_H = 0$ (dashed lines), the so-called divine coincidence is reestablished in the high natural real rate state, that is, the output gap and the inflation rate are completely stabilized, and the equilibrium nominal interest rate equals the natural rate. The reason is that with $p_H = 0$ the economy will forever stay in the high state once the high state is reached and uncertainty is completely resolved.

The difference between stabilization outcomes in the high state when $p_H > 0$ and when $p_H = 0$ also affects outcomes in the low state. In particular, under the calibration used here, inflation in the low state is lower and the output gap in the low state is higher when the high state is not an absorbing state. In general, the effect of an increase in $p_H$ on stabilization outcomes in the low state is ambiguous. On the one hand, an increase in $p_H$ raises the output gap in the high state which has a positive effect on the output gap in the low state via the expected output gap term in the Euler equation. On the other hand, an increase in $p_H$ lowers the inflation rate in the high state which puts upward pressure on the low-state ex-ante real interest rate and thereby unfolds a negative effect on the low-state output gap. Likewise, the inflation rate in the low state can either decline or increase when $p_H$ is increased as it depends on the expected inflation rate and the contemporaneous output gap. A necessary condition for a positive effect of $p_H$ on the low-state inflation rate is, however, that $p_H$ has a positive effect on the low-state output gap. For values of $\sigma$ sufficiently lower than in the baseline calibration, I find that an increase in $p_H$ leads to lower output gaps and lower inflation rates in the low state, whereas for values of $\sigma$ sufficiently higher than in the baseline calibration, the assumption $r_H > 0$ is no longer sufficient to ensure that the zero lower bound is only binding in the low state.
baseline calibration, increasing $p_H$ has the opposite effect on output gaps and inflation rates in the low state.

How does the policymaker’s relative weight on government spending stabilization affect policy functions? In the low natural real rate state, the government spending stimulus increases and the drops in the output gap and in the inflation rate are mitigated when the policymaker becomes more fiscally activist, i.e. when $\tilde{\lambda}_G$ is reduced. If $p_H > 0$, a higher degree of fiscal activism also improves the trade-off between output gap and inflation stabilization in the high state due to a less severe downward bias in expectations about future output and inflation. In equilibrium, both, the output gap and the inflation rate are closer to their target levels the smaller $\tilde{\lambda}_G$.

The improvement in the stabilization of the output gap and the inflation rate triggered by the appointment of a fiscally activist policymaker comes, however, at the cost of higher government spending distortions in the low state. To quantify the welfare implications of alternative policy regimes that differ in terms of parameter $\tilde{\lambda}_G$, I calculate the unconditional welfare loss in the two state model with $p_H > 0$ for a grid of $\tilde{\lambda}_G$ candidates. Figure 2 shows how the welfare loss expressed as a share of the loss under the benchmark regime $\tilde{\lambda}_G = \lambda_G$ varies with $\tilde{\lambda}_G$. The vertical dotted line indicates the benchmark regime. As in the special case with $p_H = 0$, welfare in the model with $p_H > 0$ can be improved by appointing a policymaker who is less concerned with the stabilization of public spending than society as a whole. The best-performing policymaker exhibits a considerable degree of fiscal activism, $\tilde{\lambda}_G^*/\lambda_G = 0.18$, and reduces the welfare costs associated with the existence of the zero lower bound by more than 40 percent.\footnote{A summary of the sensitivity of the results to selected parameters can be found in the Online Appendix.}

Taking stock, in the two state model with a lower bound on nominal interest rates, the appointment of a policymaker who cares less about government spending stabilization than society as a whole improves welfare under the optimal time-consistent monetary-fiscal policy. The best-performing policymaker raises government spending more aggressively in
Figure 2: Welfare loss - two state model

Note: The figure displays the welfare loss for alternative values of $\tilde{\lambda}_G$ normalized by the welfare loss under the benchmark regime ($\tilde{\lambda}_G = \lambda_G$) which is indicated by the vertical dotted line.

the crisis state than a policymaker who has the same preferences as society would do. If the normal non-crisis state is an absorbing state, as is often assumed in the literature, then the welfare gain associated with the appointment of the fiscally activist policymaker results from improved output gap and inflation stabilization outcomes in the crisis state. Instead, in the more general case where liquidity traps are recurring events the welfare gain results from improved output gap and inflation stabilization outcomes in the crisis state and in the normal state.

The next two sections extend the analysis to a more elaborate continuous state model. This model provides an empirically more plausible framework to quantify the welfare effects of fiscal activism than the two state model. It also highlights an additional dimension of the policy delegation problem that arises when public consumption is not just used for macroeconomic stabilization at the zero lower bound, as in the simple model, but also away from the lower bound.
4 The full continuous state model

The more elaborate model features price and wage rigidities. Instead of the discrete state setup used before I allow the model’s state variables to assume a continuum of values. In this setup, the states in which the zero lower bound is binding are determined endogenously and are not independent of the policy regime in place. The economy operates as follows. Intermediate goods are produced by a Cobb-Douglas function with labor and capital inputs, where capital is assumed to be constant and immobile across firms. All intermediate goods producers employ the same kinds of labor and face the same composite wage rate. Following Erceg, Henderson, and Levin (2000), the labor input used in the production of the intermediate goods is a CES aggregate of a continuum of differentiated labor services. Households set the wage rate for their specialized type of labor under monopolistic competition and nominal wage adjustments are subject to Calvo-type rigidities. The steady state distortions arising from monopolistic competition are offset by appropriate fiscal subsidies. Finally, in addition to the preference shock to households’ discount factor that implicitly drove the natural real rate of interest in the two state model, a technology shock, a price mark-up shock and a wage mark-up shock buffet the economy. Due to the curse of dimensionality when solving and simulating the model only one of the three additional shocks is incorporated at a time.

Private sector behavior is summarized by the following system of equations

\[ Y_t = G_t + E_t Y_{t+1} - E_t G_{t+1} - \frac{1}{\sigma} (\pi_t - E_t \pi_{t+1} - \Delta r^n_t) \]  \tag{14}
\[ \pi_t = \kappa_p \left( \frac{\gamma}{1-\gamma} Y_t + w_t \right) + \beta E_t \pi_{t+1} + u_t \]  \tag{15}
\[ \pi^W_t = \kappa_w \left( \sigma + \frac{\eta}{1-\gamma} \right) (Y_t - \Gamma G_t) - w_t \right) + \beta E_t \pi^W_{t+1} + \epsilon_t \]  \tag{16}
\[ \pi^W_t = w_t - w_{t-1} + \pi_t - \Delta w^n_t. \]  \tag{17}

---

22 There is perfect risk-sharing among households specialized in different labor services.
23 Whereas in the simple model with flexible wages the introduction of technology or mark-up shocks would not have changed the general result that a generic discretionary policymaker uses government spending only as a stabilization tool when the zero lower bound is binding, these shocks do in general trigger a fiscal policy response even if the zero bound is not binding in the more complex model considered here.
Equation (14) is the familiar consumption Euler equation. Equation (15) summarizes the price setting behavior of firms, where $u_t$ is a price mark-up shock. Equation (16) summarizes the nominal wage setting behavior of households, where $\pi_t^W$ is nominal wage inflation, $w_t$ is the composite real wage rate, and $e_t$ is a wage mark-up shock. Equation (17) relates nominal wage inflation to the change in the real wage rate, the price inflation rate and the change in the natural level of the wage rate $\Delta w^n_t$. Here, output, government spending (as a share of steady state output) and the real wage rate are expressed in percentage deviations from their efficient equilibrium counterparts.

In what follows, when considering the model variant with the technology shock, non-stationary variables in the original non-linear model have been normalized by the aggregate productivity level before taking a log-linear approximation. The other three exogenous state variables follow stationary autoregressive processes

$$r^n_t = \rho_r r^n_{t-1} + (1 - \rho_r) r^n + \epsilon^n_t$$
$$u_t = \rho_u u_{t-1} + \epsilon^u_t$$
$$e_t = \rho_e e_{t-1} + \epsilon^e_t,$$

In the presence of the technology shock, output, government spending and the real wage rate are not constant in the efficient equilibrium.

In order to be able to stationarize the model it is assumed that the technology shock enters the utility function as a scaling factor.
where $\epsilon^x_t, x \in \{r, u, e\}$, are i.i.d. $\mathcal{N}(0, \sigma^2_x)$ innovations, and where I have written the process for the preference shock directly in terms of the natural real rate of interest since the specification of the technology process above implies that only fluctuations in the preference shock have an effect on the natural real rate.

In the presence of wage stickiness, the linear quadratic approximation to household welfare features an additional term related to wage inflation

$$
E_t \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \pi^2_{t+j} + \lambda (Y_t + \Gamma G_t + \Gamma G_t)^2 + \lambda_G G_t^2 + \lambda_W (\pi^W_{t+j})^2 \right],
$$

where the relative weights are again functions of the structural parameters

$$
\lambda = \kappa_p \left( \sigma + \eta + \gamma - \frac{1}{\theta} \right), \quad \lambda_G = \lambda \Gamma \left( 1 - \Gamma + \frac{\nu}{\sigma} \right), \quad \lambda_W = \frac{(1 - \gamma) \theta_W}{\kappa_w (\sigma + (\eta + \gamma)/(1 - \gamma))}.
$$

As before, society may appoint a policymaker who puts a different relative weight on government spending stabilization, denoted by $\bar{\lambda}_G$, than itself does. Appendix C describes the optimization problem of such a generic discretionary policymaker and lists the first order conditions. The presence of an endogenous state variable—the real wage rate—implies that the policymaker takes agents’ expectations no longer as given. While he cannot make any direct promises regarding future policy, he takes into account how the current real wage rate affects expectations through agents’ decision rules. I use a projection method with finite elements to solve the model numerically. This method allows for an accurate treatment of expectation terms, which is crucial for the analysis. The computational algorithm is described in Appendix D.

The model parameterization is summarized in Table 2. Preference parameters take similar values as in the two state model. The share of labor in total income $1 - \gamma$ is set to 0.7 in line with U.S. data. The Calvo parameter representing the degree of nominal wage stickiness is set to 0.72, based on the estimates discussed in Christiano, Eichenbaum, and Rebelo (2011). Given this additional source of nominal rigidity, I set the Calvo parameter representing the
degree of price stickiness to a slightly lower value than in the two state model. The implied value of the reduced form parameter \( \kappa_p \) lies within the point estimates in Altig, Christiano, Eichenbaum, and Linde (2011). The persistence parameter of the natural real rate is set to 0.8 and the standard deviation of the innovation is chosen such that the economy is at the zero lower bound in about one-fourth of the simulated periods under the benchmark regime.\(^{26}\)

The technology shock is calibrated according to the estimate by Amano and Shukayev (2012) who fit a Cobb-Douglas production function to U.S. aggregate capital, labor hours, and real GDP. The law of motion of the price mark-up shock is calibrated according to the estimate by Ireland (2011) for the U.S. economy. For the wage mark-up shock, I choose a somewhat smaller standard deviation to avoid the zero interest rate bound to be binding in the risky steady state. The model matches the observed volatility of inflation and short-term interest rates in the U.S. over the previous two decades pretty well, though since it abstracts from capital investment it underestimates the volatility of real GDP growth.\(^{27}\)

5 Results for the continuous state model

This section determines the optimal degree of fiscal activism for the full continuous state model, and analyzes the implications for stabilization outcomes and equilibrium dynamics.

\(^{26}\)A similar parameterization of the preference shock is used by Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2015).

\(^{27}\)If the economy is buffeted by preference and technology shocks (price mark-up shocks), the unconditional standard deviations of annualized inflation, the annualized short-term nominal interest rate, and annualized quarterly real GDP growth are 0.58 (0.61), 2.30 (2.31) and 0.98 (1.27), respectively. In the data, for the period from 1995-Q1 until 2015-Q2 the standard deviation of annualized quarterly U.S. inflation as measured by the CPI less food and energy is 0.62, the standard deviation of the quarterly short-term nominal interest rate as measured by the effective federal funds rate is 2.37 (annualized), and the standard deviation of the annualized quarterly real GDP growth rate is 2.55.
In the Online Appendix I consider an extended continuous state model with inertia in the price inflation rate and present results from a sensitivity analysis with respect to selected parameters.

5.1 Optimal degree of fiscal activism

To determine the optimal degree of fiscal activism I consider a grid of policymakers that differ in terms of parameter $\tilde{\lambda}_G$ and calculate for each candidate the average of the discounted welfare loss across 2000 simulations with a length of 1050 periods each, where the first 50 periods are discarded as burn-in periods.\(^{28}\)

Figure 3 plots the welfare losses associated with the alternative policymaker candidates for three variants of the model: the model with preference shocks and technology shocks (solid line), the model with preference shocks and price mark-up shocks (dashed line), and the model with preference shocks and wage mark-up shocks (dash-dotted line). Losses are normalized by the loss in the benchmark regime $\tilde{\lambda}_G = \lambda_G$, which is indicated by a vertical dotted line. In all three model variants welfare is increased when a fiscally activist policymaker takes office. The largest welfare gains occur in the model with preference and wage mark-up shocks and the smallest in the model with preference and price mark-up shocks. Table 3 reports the optimal degree of fiscal activism and the welfare gains from appointing the best-performing fiscally activist policymaker instead of the benchmark policymaker for the alternative model versions. The optimal relative weight on government spending stabilization in the policymaker’s loss function is markedly lower than the weight that society puts on the stabilization of government spending, the ratio $\tilde{\lambda}_G^*/\lambda_G$ is between 0.22 and 0.25. The last two rows show that the zero lower bound binds less frequently under the best-performing fiscally activist regime than under the benchmark regime. As discussed below, due to the downward bias in agents’ inflation and output expectations, the zero lower bound is in gen-

\(^{28}\)Average discounted losses are reported to keep the welfare analysis parsimonious. It is important to keep in mind that society’s loss function is conditional. The precise welfare ranking of alternative policy regimes thus depends on the initial values of the state variables and the transition dynamics.
Figure 3: Welfare loss - continuous state model

Note: The figure displays the welfare loss for the model with preference shocks and technology shocks (solid line), the model with preference shocks and price mark-up shocks (dashed line), and the model with preference shocks and wage mark-up shocks (dash-dotted line). The vertical dotted line indicates the benchmark regime ($\tilde{\lambda}_G = \lambda_G$). All losses are normalized by the welfare loss under the benchmark regime, respectively.

eral not only binding when the natural real rate is negative but also when it is positive and sufficiently close to zero. Fiscal activism mitigates the deflationary bias and thereby reduces the conditional threshold values for the natural real rate of interest below which the zero lower bound becomes binding. At the same time, the lower bound is always binding when the natural real rate is negative (abstracting from other shocks), which is why there is an inherent limit as to how much the appointment of a fiscally activist policymaker can reduce the frequency of lower bound events.

Figure 4 tries to disentangle how on the one hand the presence of the zero lower bound and on the other hand the presence of disturbances that trigger fiscal policy responses away from the lower bound impact on the welfare analysis. The solid line shows how society’s welfare loss varies with $\tilde{\lambda}_G$ when the economy is subject to preference and price mark-up shocks, the dashed line shows the loss when the economy is only subject to preference shocks, and
the dash-dotted line shows the loss when the economy is subject to both shocks but the zero lower bound on nominal interest rates is ignored. All losses are normalized by the welfare loss under the benchmark regime, respectively. Three observations are in order. First, without the zero lower bound, the appointment of a fiscally activist policymaker does not increase welfare, that is, fiscal activism does not improve the stabilization trade-offs associated with price mark-up shocks. Second, the presence of price mark-up shocks makes the most extreme fiscally activist candidates unappealing. Third, for values not close to zero, varying $\tilde{\lambda}_G$ has only very small effects on the welfare costs associated with price mark-up shocks. Indeed, the optimal degree of fiscal activism turns out to be the same in the models with and without price mark-up shocks (see also Table 3).

Next, I discuss how fiscal activism affects stabilization outcomes and equilibrium dynamics. In so doing, I focus on the model with preference and price mark-up shocks.

### 5.2 Risky steady state

I first consider the implications of fiscal activism for the risky steady state. The risky steady state describes the equilibrium in which state variables stay constant and the realization of shocks is zero, but agents account for the uncertainty resulting from future shocks.²⁰²⁰

²⁰This explains why the relative welfare gains from fiscal activism are larger in the model with preference shocks only than in the model with preference and price mark-up shocks, see Table 3.
³⁰See Coeurdacier, Rey, and Winant (2011).
Figure 4: Welfare loss - the role of asymmetric shocks and the lower bound

Note: The figure displays the welfare loss for the model with preference and price mark-up shocks (solid line), the model with preference shocks only (dashed line) and the model with both shocks but without the zero lower bound (dash-dotted line) as a function of $\tilde{\lambda}_G$. The vertical dotted line indicates the benchmark regime ($\tilde{\lambda}_G = \lambda_G$). All losses are normalized by the welfare loss under the benchmark regime, respectively.

Table 4 compares the risky steady state of the benchmark regime and the best-performing fiscally activist regime. Notice, first, that the nominal and the real interest rates in the risky steady state of the two regimes are lower than in the deterministic steady state (2.50%). Likewise, under both regimes the risky steady state inflation rate is negative and the output gap is positive, reflecting the stabilization trade-off between these two target variables in the presence of uncertainty about future zero lower bound events, as discussed in detail in the context of the two state model. Comparing the risky steady states of the two regimes shows that the stabilization trade-off between inflation and the output gap improves under the fiscally activist regime, that is, the deflationary bias and the output gap overshooting are smaller than under the benchmark regime. The output gap boost that is needed under the benchmark regime to dampen the spillovers from deflationary expectations to the inflation rate requires a nominal interest rate level that is 0.5 percentage points below the deterministic


Table 4: Risky steady state - model with preference and price mark-up shocks

<table>
<thead>
<tr>
<th>Regime</th>
<th>Benchmark ((\tilde{\lambda}_G = \lambda_G))</th>
<th>Fiscally activist ((\tilde{\lambda}_G = \tilde{\lambda}_G^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal interest rate (annualized)</td>
<td>1.97</td>
<td>2.17</td>
</tr>
<tr>
<td>Real interest rate (annualized)</td>
<td>2.08</td>
<td>2.22</td>
</tr>
<tr>
<td>Price inflation (annualized)</td>
<td>-0.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>Wage inflation (annualized)</td>
<td>-0.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.18</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note: The risky steady state of the output gap is expressed in deviations from the respective deterministic steady state.

steady state, implying a smaller buffer to the zero lower bound than under the fiscally activist regime.

5.3 Policy functions

To provide a more general characterization of the equilibrium under the two regimes, we next consider the equilibrium responses to exogenous shocks, again using the model with preference and price mark-up shocks. Figure 5 displays equilibrium responses to the natural real rate of interest ranging from minus to plus four unconditional standard deviations. The lagged real wage rate is set to its unconditional average value, respectively, and the price mark-up shock is set to zero. Solid lines represent equilibrium responses under the best-performing fiscally activist regime and dashed lines represent responses under the benchmark regime. For low realizations of the natural real rate the zero lower bound constraint is binding. Under the benchmark regime, a low natural real rate can lead to considerable drops in the output gap, price inflation and wage inflation. The decline in the target variables is smaller under the fiscally activist regime, and the larger the adverse shock, the more distinct is the difference in the equilibrium responses. For the calibration used here, the government spending stimulus under the fiscally activist regime is larger than under the benchmark regime in most states in which the zero lower bound is binding. However, due to general equilibrium effects and unlike in the two state model a higher elasticity of government
Figure 5: Equilibrium responses to the natural real rate

Note: The figure displays equilibrium responses to the natural real rate ranging from minus to plus four unconditional standard deviations. The solid line represents the fiscally activist regime ($\tilde{\lambda}_G = \tilde{\lambda}_G^* \lambda^G$). The dashed line represents the benchmark regime ($\tilde{\lambda}_G = \lambda^G$). The lagged real wage rate is kept fixed at its average value, respectively, and the price mark-up shock is set to zero. Interest rates and inflation rates are expressed in annualized terms.

spending does not necessarily translate into a higher fiscal stimulus in all zero lower bound states.

The figure also shows that the natural real rate does not need to be negative to push the economy into a liquidity trap. In states where the natural real rate is strictly positive but the zero lower bound is binding the policymaker is able to adjust the nominal interest rate to mimic the natural rate but hits the lower bound due to the additional adverse effects coming from the downward bias in agents’ expectations. The last panel shows that the natural real rate threshold below which the zero bound is binding is lower under the fiscally activist regime than under the benchmark regime. This is because fiscal activism reduces the deflationary bias of discretionary policy as discussed next.

In those states where the zero lower bound is not binding, the output gap is positive
whereas price and wage inflation continue to fall below target. These target deviations are more pronounced under the benchmark regime than under the fiscally activist regime. Since inflation declines less in a liquidity trap when an activist policymaker took office, inflation expectations under such a regime are closer to target in all states of the world, which in turn unfolds a stabilizing effect on current private sector decisions about allocations and prices in all states.

Figure 6 displays equilibrium responses to the price mark-up shock ranging from minus to plus four unconditional standard deviations. The lagged real wage rate is set to its unconditional average value, respectively, and the natural real rate of interest is set to its steady state value. As before, solid lines represent equilibrium responses under the best-performing fiscally activist regime and dashed lines represent responses under the benchmark regime. The last panel shows the Lagrange multiplier associated with equation (17), where a positive value implies that the policymaker in period $t$ would have liked to be faced with a higher lagged real wage rate $w_{t-1}$. Price mark-up shocks have strong effects on price inflation whereas the pass-through to wage inflation is relatively muted. The type of the policy regime has important implications for the equilibrium policy mix. Whereas the benchmark policymaker varies the nominal interest rate aggressively with the price mark-up shock and indeed encounters the zero lower bound constraint for large deflationary shocks, the equilibrium policy rate response is much less elastic under the optimal fiscally activist regime. At the same time, the latter regime uses the fiscal instrument more actively even so deviations of government spending from its efficient level are associated with welfare costs for society.

5.4 A liquidity trap scenario

Finally, we want to study how the economy behaves in a liquidity trap scenario, using again the model with natural real rate (i.e. preference) shocks and price mark-up shocks. Figure

31 Notice that the benchmark monetary and fiscal policy regime already reduces the deflationary bias compared to the case where the discretionary policymaker possesses only the nominal interest rate tool, see Schmidt (2013).
Figure 6: Equilibrium responses to the price mark-up shock

Note: The figure displays equilibrium responses to the price mark-up shock ranging from minus to plus four unconditional standard deviations. The solid line represents the fiscally activist regime ($\tilde{\lambda}_G = \tilde{\lambda}^*_G$). The dashed line represents the benchmark regime ($\tilde{\lambda}_G = \lambda_G$). The lagged real wage rate is kept fixed at its average value, respectively, and the natural real rate is set to its steady state value. Interest rates and inflation rates are expressed in annualized terms.

7 shows impulse responses to a natural rate shock of minus three unconditional standard deviations for the benchmark regime ($\tilde{\lambda}_G = \lambda_G$). In period 0 the economy is in the risky steady state and in period 1 the shock materializes. The actual paths in the absence of any further shocks are represented by solid lines, the expected paths conditional on information available in period 1 are represented by dashed lines, and the corresponding confidence intervals are represented by the blue-shaded areas. The shock drives the natural real rate of interest into negative territory and triggers a reduction of the policy rate to zero where it remains for eight quarters. The economy starts to contract, and, despite the government spending stimulus, the output gap, price inflation and wage inflation fall below their target values. Due to the zero lower bound on the nominal interest rate the expected path of the real interest rate is more contractionary than the actual real interest rate path. It is the
Figure 7: Impulse responses - benchmark regime

Note: The figure displays impulse responses to a negative natural real rate shock of minus three unconditional standard deviations. Solid lines represent actual paths in the case of no further shocks, dashed lines represent expected paths conditional on information in period 1 and the shaded areas indicate 50%, 75% and 90% confidence intervals. Interest rates and inflation rates are expressed in annualized terms.

Figure 7 also shows how the truncation of the probability distribution of the nominal interest rate at zero leads to positive skewness of the confidence intervals for the nominal and the real interest rate. Likewise, the distributions of the output gap and the wage inflation rate are negatively skewed. The distribution of the price inflation rate is much less skewed since it directly depends on the distribution of the price mark-up shock. In particular, agents attach positive probabilities to positive price inflation rates in the future. If, instead, there were no price mark-up shocks agents would anticipate that the policymaker never allows inflation to rise above target.

Figure 8 shows impulse responses to the same natural real rate shock for the optimal
fiscally activist regime. The policymaker raises government spending more aggressively and

Figure 8: Impulse responses - fiscally activist regime

![Impulse responses graphs]

Note: The figure displays impulse responses to a negative natural real rate shock of minus three unconditional standard deviations. Solid lines represent actual paths in the case of no further shocks, dashed lines represent expected paths conditional on information in period 1 and the shaded areas indicate 50%, 75% and 90% confidence intervals. Interest rates and inflation rates are expressed in annualized terms.

is able to lower the real interest rate to a level closer to zero than under the benchmark regime. Consequently, the initial drop in the output gap, price inflation and wage inflation is less severe. At the same time, due to the superior stabilization performance, the fiscally activist policymaker is able to raise the policy rate from zero one quarter earlier than the benchmark policymaker.

6 Conclusion

The credibility problem of discretionary monetary policy at the zero lower bound—the inability to commit to higher future inflation—also persists under the jointly optimal discretionary
monetary-fiscal policy. I show that the credibility problem can be mitigated and welfare be improved by the appointment of a monetary-fiscal policymaker who is less concerned with the stabilization of government spending than society.

The implementation of the proposed delegation scheme would most likely require some institutional changes. For instance, whereas in principle the fiscally activist policymaker could be operationalized by the appointment of a decision-making fiscal council, in practice most fiscal councils are non-decision-making institutions that have only an advisory role.

Finally, in terms of the modeling framework, an interesting avenue for future work would be the incorporation of capital to allow for a decomposition of total government spending into public consumption and public investment.

References


Appendix

A Mapping the policymaker’s relative weight on government spending stabilization into his preferences

Suppose, society has the following standard period utility function

$$\frac{C_t^{1-\hat{\sigma}} - 1}{1 - \hat{\sigma}} + \xi_G \frac{G_t^{1-\hat{\nu}} - 1}{1 - \hat{\nu}} - \xi_N \int_0^1 N_t(i)^{1+\eta} \frac{d\bar{i}}{1 + \eta}.$$

In the efficient steady state equilibrium, the marginal utility of private consumption has to equal the marginal utility of public consumption

$$C^{-\hat{\sigma}} = \xi_G G^{-\hat{\nu}},$$

where $\hat{\sigma} = \sigma^C_Y$ and $\hat{\nu} = \nu^G_Y$.

The policymaker’s period utility function is similar to the one of society but the preference parameters related to the utility provided by public goods, $\tilde{\xi}_G$ and $\tilde{\nu}$, may differ from those of society

$$\frac{C_t^{1-\hat{\sigma}} - 1}{1 - \hat{\sigma}} + \tilde{\xi}_G \frac{G_t^{1-\tilde{\nu}} - 1}{1 - \tilde{\nu}} - \xi_N \int_0^1 N_t(i)^{1+\eta} \frac{d\bar{i}}{1 + \eta}.$$

In the flexible-price steady state equilibrium, it holds

$$C^{-\hat{\sigma}} = \tilde{\xi}_G G^{-\tilde{\nu}}.$$

Suppose that $\tilde{\nu} \neq \hat{\nu}$. In order to ensure that the policymaker replicates the allocation of the efficient steady state equilibrium, choose $\tilde{\xi}_G$ such that

$$\tilde{\xi}_G = \xi_G (G^{ess})^{\hat{\nu} - \tilde{\nu}},$$
where \(G^{ess}\) is the level of government spending in the efficient steady state.

The policymaker’s relative weight on government spending stabilization \(\tilde{\lambda}_G\) in his quadratic loss function is then linked to the preference parameter \(\tilde{\nu}\) as follows

\[
\tilde{\lambda}_G = \lambda \Gamma \left(1 - \Gamma + \frac{\tilde{\nu}}{\sigma} \left(\frac{G}{Y}\right)^{-1}\right).
\]

\section*{B Proofs and analytical results for the two state model}

\subsection*{Optimal time-consistent fiscal policy}

The consolidated first order conditions of the discretionary policymaker’s optimization problem in the simple model read

\[(1 - \Gamma) \left[\kappa \pi_t + \lambda (Y_t - \Gamma G_t)\right] + \tilde{\lambda}_G G_t = 0 \quad (B.1)\]

\[i_t \left(\kappa \pi_t + \lambda (Y_t - \Gamma G_t)\right) = 0 \quad (B.2)\]

\[i_t \geq 0 \quad (B.3)\]

\[\kappa \pi_t + \lambda (Y_t - \Gamma G_t) \leq 0, \quad (B.4)\]

as well as the New Keynesian Phillips curve and the consumption Euler equation. If the zero lower bound on the nominal interest rate is binding, condition (B.4) becomes a strict inequality \(\kappa \pi_t + \lambda (Y_t - \Gamma G_t) < 0\). Rewriting condition (B.1)

\[G_t = -\frac{1 - \Gamma}{\tilde{\lambda}_G} \left[\kappa \pi_t + \lambda (Y_t - \Gamma G_t)\right], \quad \text{if } 1 - \Gamma, \tilde{\lambda}_G > 0,
\]

where \(1 - \Gamma, \tilde{\lambda}_G > 0\), and hence \(G_t > 0\).

If the zero lower bound on the nominal interest rate is not binding, condition (B.4) holds with equality. In order for conditions (B.1) and (B.4) to hold simultaneously, the government spending gap has to be zero, \(G_t = 0\).
Equilibrium

We first consider the long-run equilibrium. Suppose, \( i_t > 0 \) for all \( t \geq T \). We then know that \( G_t = 0 \). Hence, the remaining equilibrium conditions for periods \( t \geq T \) can be simplified to

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa Y_t \tag{B.5}
\]

\[
Y_t = E_t Y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^p_H) \tag{B.6}
\]

\[
Y_t = -\frac{\kappa}{\lambda} \pi_t. \tag{B.7}
\]

Substitute (B.7) into (B.5)

\[
\pi_t = \frac{\lambda \beta}{\kappa^2 + \lambda} E_t \pi_{t+1}. \tag{B.8}
\]

Since inflation is a free endogenous variable, it follows from \( 0 < \frac{\lambda \beta}{\kappa^2 + \lambda} < 1 \) that there exists a unique rational expectations equilibrium. It is then straightforward to verify that the policy functions for the long-run equilibrium are given by the expressions stated in Proposition ??.

We now turn to the short-run equilibrium. Using the previous results, it holds

\[
E_t (\pi_{t+1} | t < T) = p_L E_t^L \pi_{t+1}^L, \tag{B.9}
\]

where \( E_t^L \) denotes the expectations operator conditional on the natural rate shock in period \( t \) being in state \( L \), i.e. \( t < T \), and \( \pi_{t+1}^L \) is the inflation rate in period \( t + 1 \) conditional on \( t+1 < T \). Using similar notation for \( Y_t, G_t \) and \( i_t \), given \( t < T \), we can rewrite the optimality
conditions as

\[ \pi_t^L = \kappa (Y_t^L - \Gamma G_t^L) + p_L E_t^L \pi_{t+1}^L \]  
(B.10)

\[ Y_t^L = p_L E_t^L Y_{t+1}^L + G_t^L - p_L E_t^L G_{t+1}^L - \frac{1}{\sigma} (i_t^L - p_L E_t^L \pi_{t+1}^L - r_t^n) \]  
(B.11)

\[ i_t^L = 0 \]  
(B.12)

\[ G_t^L = -\frac{1 - \Gamma}{\lambda G} \left[ \kappa \pi_t^L + \lambda (Y_t^L - \Gamma G_t^L) \right] \]  
(B.13)

\[ 0 > \kappa \pi_t^L + \lambda (Y_t^L - \Gamma G_t^L) \]  
(B.14)

where I have made use of the fact that \( i_t > 0 \) is not a solution for \( r_t^n = r_L^n < 0 \), which is straightforward to verify. I first show that a unique bounded rational expectations equilibrium exists in the short run and then derive the closed-form expressions for the policy functions.

**Existence of a unique equilibrium**

Case I: \( \lambda G \neq \lambda \Gamma (1 - \Gamma) \).

Substitute (B.12) and (B.13) into (B.10) and (B.11). We then obtain a system of two equations with two unknowns

\[ A z_t^L = B E_t^L z_{t+1}^L + C r_t^n, \]  
(B.15)

where \( z_t^L = [\pi_t^L, Y_t^L]' \) and

\[ A = \begin{pmatrix} 1 - \frac{\kappa^2 \Gamma (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)} & -\kappa \left( 1 + \frac{\lambda \Gamma (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)} \right) \\ \kappa \left( 1 - \frac{\Gamma (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)} \right) & 1 + \frac{\lambda (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)} \end{pmatrix}, \]

\[ B = \begin{pmatrix} \frac{\beta p_L}{\sigma + \frac{\kappa (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)}} p_L & 0 \\ 0 & \left( 1 + \frac{\lambda (1 - \Gamma)}{\lambda G - \lambda \Gamma (1 - \Gamma)} \right) p_L \end{pmatrix}. \]

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The form of matrix $C$ is omitted since it is not required for what follows. Define $\Omega \equiv B^{-1}A$. Since $z_t^L$ consists of two free endogenous variables, the system (B.15) has a unique bounded solution if and only if $\Omega$ exhibits two eigenvalues outside the unit circle. The characteristic polynomial of $\Omega$ is given by $P(\delta) = \delta^2 - \phi_1 \delta + \phi_0$, where

$$\phi_0 = \frac{\tilde{\lambda}_G + (1 - \Gamma)^2 (\lambda + \kappa^2)}{(\tilde{\lambda}_G + (1 - \Gamma)^2 \lambda) \beta p_L^2}$$

$$\phi_1 = \frac{(1 + \beta) (\tilde{\lambda}_G + (1 - \Gamma)^2 \lambda) + \frac{\kappa}{\sigma} \tilde{\lambda}_G + (1 - \Gamma)^2 \kappa^2}{(\tilde{\lambda}_G + (1 - \Gamma)^2 \lambda) \beta p_L}.$$

Then,

$$P(1) = \frac{\tilde{\lambda}_G ((1 - p_L) (1 - \beta p_L) - \frac{\kappa}{\sigma} p_L) + (1 - \Gamma)^2 (1 - p_L) ((1 - \beta p_L) \lambda + \kappa^2)}{(\tilde{\lambda}_G + (1 - \Gamma)^2 \lambda) \beta p_L^2}$$

$$P(-1) = \frac{\tilde{\lambda}_G ((1 + p_L) (1 + \beta p_L) + \frac{\kappa}{\sigma} p_L) + (1 - \Gamma)^2 (1 + p_L) ((1 + \beta p_L) \lambda + \kappa^2)}{(\tilde{\lambda}_G + (1 - \Gamma)^2 \lambda) \beta p_L^2}.$$

Note that under Assumption 1 $P(1) > 0$ and $P(-1) > 0$. Continuity implies that the characteristic equation has an even number of roots inside the unit circle. Suppose, both eigenvalues would lie inside the unit circle. Then, $|\text{det}(\Omega)| < 1$. However, $\text{det}(\Omega) = \phi_0 > 1$. Hence, $\Omega$ has two eigenvalues outside the unit circle and the system (B.15) has a unique bounded solution.

Case II: $\tilde{\lambda}_G = \lambda \Gamma (1 - \Gamma)$.

Then, equation (B.13) reduces to $\kappa \pi_t^L + \lambda Y_t^L = 0$. Substituting this expression and (B.12) into (B.10) and (B.11), we again obtain a system of two equations with two unknowns, where
now $z^L_t = [\pi^L_t, G^L_t]'$ and

$$A = \begin{pmatrix} 1 + \frac{\kappa^2}{\chi} & \kappa \Gamma \\ \frac{\kappa}{\chi} & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \beta p_L & 0 \\ (\frac{\kappa}{\chi} - \frac{1}{\sigma}) p_L & p_L \end{pmatrix}.$$ 

Define $\Omega \equiv B^{-1}A$. Since $z^L_t$ consists of two free endogenous variables, the system has a unique bounded solution if and only if $\Omega$ exhibits two eigenvalues outside the unit circle.

The characteristic polynomial of $\Omega$ is given by $P(\delta) = \delta^2 - \phi_1 \delta + \phi_0$, where

$$\phi_0 = \frac{1}{\beta p_L^2} \left( 1 + \frac{(1 - \Gamma) \kappa^2}{\lambda} \right),$$

$$\phi_1 = \frac{(1 + \beta + \frac{\kappa}{\sigma} \Gamma) \lambda + (1 - \Gamma) \kappa^2}{\lambda \beta p_L}.$$ 

Then,

$$P(1) = \frac{(1 - p_L)(1 - \beta p_L) - \Gamma \frac{\kappa}{\sigma} p_L + (1 - p_L)(1 - \Gamma) \frac{\kappa^2}{\chi}}{\beta p_L^2},$$

$$P(-1) = \frac{(1 + p_L)(1 + \beta p_L) + \Gamma \frac{\kappa}{\sigma} p_L + (1 + p_L)(1 - \Gamma) \frac{\kappa^2}{\chi}}{\beta p_L^2}.$$ 

Note that under Assumption 1 $P(1) > 0$ and $P(-1) > 0$. Continuity implies that the characteristic equation has an even number of roots inside the unit circle. Suppose, both eigenvalues would lie inside the unit circle. Then, $|det(\Omega)| < 1$. However, $det(\Omega) = \phi_0 > 1$. Hence, $\Omega$ has two eigenvalues outside the unit circle and there exists a unique bounded solution.
Closed-form solution

The closed-form solution can be derived using the method of undetermined coefficients. The minimum-state-variable solution has the form

\[
\begin{align*}
\pi_t &= \omega_\pi r_L^n \\
Y_t &= \omega_Y r_L^n \\
G_t &= \omega_G r_L^n,
\end{align*}
\]

where \((\omega_\pi, \omega_Y, \omega_G)\) are the coefficients to be determined. Substituting this guess into equations (B.10) to (B.13), one obtains

\[
\begin{align*}
\omega_\pi &= \frac{\kappa \tilde{\lambda}_G}{\lambda_G ((1-p_L)(1-\beta p_L) - \frac{\kappa}{\sigma} p_L) + (1-p_L)(1-\Gamma)^2 ((1-\beta p_L) \lambda + \kappa^2) \sigma}\quad 1 \\
\omega_Y &= \frac{1}{\lambda_G ((1-p_L)(1-\beta p_L) - \frac{\kappa}{\sigma} p_L) + (1-p_L)(1-\Gamma)^2 ((1-\beta p_L) \lambda + \kappa^2) \sigma} \\
\omega_G &= -\frac{1}{\lambda_G ((1-p_L)(1-\beta p_L) - \frac{\kappa}{\sigma} p_L) + (1-p_L)(1-\Gamma)^2 ((1-\beta p_L) \lambda + \kappa^2) \sigma}.
\end{align*}
\]

Proof of Proposition 2

Let us first rewrite the expression for the optimal relative weight on government spending stabilization as follows

\[
\tilde{\lambda}_G^* = \left(1 - \frac{\kappa p_L ((1-p_L)\beta \kappa + \kappa^2 \sigma^{-1} + \lambda \sigma^{-1} (1-\beta p_L))}{(1-p_L) (\kappa^2 + (1-\beta p_L)^2 \lambda)} \right) \lambda_G.
\]
Taking the partial derivative of $\tilde{\lambda}_G^*/\lambda_G$ with respect to $p_L$ and collecting terms leads to

$$
\frac{\partial \left( \tilde{\lambda}_G^*/\lambda_G \right)}{\partial p_L} = - \left[ (1 - p_L)^2 (\beta \kappa^2 + \beta \kappa \lambda \sigma^{-1}) (\kappa^2 + (1 - \beta p_L)^2 \lambda + 2 \beta \lambda p_L (1 - \beta p_L)) 
+ (\kappa \lambda \sigma^{-1} (1 - \beta) + \sigma^{-1} \kappa^3) (\kappa^2 + (1 - \beta p_L)^2 \lambda + 2 \beta \lambda p_L (1 - \beta p_L) (1 - p_L)) \right] 
\left[ (1 - p_L) (\kappa^2 + (1 - \beta p_L)^2 \lambda) \right]^{-2}.
$$

Note that this expression is strictly negative for all $0 < p_L < 1$. This completes the proof.

**Comparison to an optimized feedback rule for government spending**

Suppose, the policymaker could commit to a simple forward-looking rule for government spending of the form $G_t = \tau r^n_t$ for $t < T$ and $G_t = 0$ for all $t \geq T$, whereas monetary policy is conducted under discretion. Then it is optimal to set $\tau = \omega_G \left( \tilde{\lambda}_G^* \right)$.

Proof: The short-run equilibrium conditions read

$$
G_t^L = \tau r^n_t \tag{B.16}
$$
$$
\pi_t^L = \kappa \left( Y_t^L - \Gamma G_t^L \right) + \beta p_L E_t^L \pi_{t+1}^L \tag{B.17}
$$
$$
Y_t^L = p_L \pi_t^L Y_{t+1}^L = G_t^L + G_{t+1}^L - p_L E_t^L \pi_{t+1}^L - \frac{1}{\sigma} \left( i_t^L - p_L E_t^L \pi_{t+1}^L - r^n_t \right) \tag{B.18}
$$
$$
0 = i_t^L \left( \kappa \pi_t^L + \lambda \left( Y_t^L - \Gamma G_t^L \right) \right) \tag{B.19}
$$
$$
0 \leq i_t^L \tag{B.20}
$$
$$
0 \geq \kappa \pi_t^L + \lambda \left( Y_t^L - \Gamma G_t^L \right). \tag{B.21}
$$

First, suppose that $i_t^L = 0$. Substituting government spending rule (B.16) into (B.17) and
(B.18), and focusing on the minimum-state-variable solution, we obtain

\[ \pi_t^L = \frac{\kappa}{\sigma} + (1 - p_L) (1 - \Gamma) \kappa \tau r^*_L \]

\[ Y_t^L = \frac{1}{\sigma} (1 - \beta p_L) + \left( (1 - p_L) (1 - \beta p_L) - \frac{\kappa}{\sigma} p_L \right) \tau r^*_L. \]  

(B.22)  

(B.23)

Substituting the solution functions into the welfare-based loss function (10) and using standard optimization theory, the optimal value for \( \tau \) satisfies

\[ \tau^* = -\frac{(1 - p_L) (1 - \Gamma) (\kappa^2 + \lambda (1 - \beta p_L)^2)}{((1 - p_L) (1 - \Gamma))^2 (\kappa^2 + \lambda (1 - \beta p_L)^2) + ((1 - p_L) (1 - \beta p_L) - \frac{\kappa}{\sigma} p_L)^2 \lambda G \sigma^2} \]

It is then easy to verify that \( \tau^* = \omega_G \left( \tilde{A}_G^* \right) \).

To complete the proof, it is shown that any \( \tau \) that implies \( i_t^L > 0 \) is not optimal. Suppose, to the contrary, that \( i_t^L > 0 \). Substituting the government spending rule into (B.17), (B.18) and (B.21), which has to hold with equality, one obtains

\[ \pi_t^L = 0 \]

\[ Y_t^L = \Gamma \tau r^*_L. \]

From (B.18) then follows that \( i_t^L > 0 \) if and only if \( \tau < -\frac{1}{(1 - p_L) (1 - \Gamma) \sigma} \frac{1}{\sigma} < \tau^* \).

Finally, using the welfare-based loss function (10) it is straightforward to verify that the welfare loss for \( \tau < -\frac{1}{(1 - p_L) (1 - \Gamma) \sigma} \frac{1}{\sigma} \) exceeds the welfare loss for \( \tau = \tau^* \).

C  Policy problem in the full continuous state model

Each period \( t \), the generic discretionary policymaker minimizes his loss function from period \( t \) onwards, taking the decision rules of the private sector and of future policymakers as given. I focus on stationary Markov-perfect equilibria, where the vector of state variables \( s_t \) in the full model consists of the composite real wage rate of the previous period, the natural real
rate of interest (discount factor shock), the technology shock, the price mark-up shock, and the wage mark-up shock. The policy problem reads

\[
V(s_t) = \min \frac{1}{2} \left[ \pi_t^2 + \lambda (Y_t - \Gamma G_t)^2 + \lambda G_t^2 + \lambda W (\pi_t^W)^2 \right] + \beta E_t V(s_{t+1})
- \phi_t^{IS} \left[ Y_t - G_t - E_t Y(s_{t+1}) + E_t G(s_{t+1}) + \frac{1}{\sigma} (i_t - E_t \pi(s_{t+1}) - r_t^i) \right]
- \phi_t^{PCP} \left[ \pi_t - \kappa_p \left( \frac{\gamma}{1-\gamma} Y_t + w_t \right) - \beta E_t \pi(s_{t+1}) - u_t \right]
- \phi_t^{PCW} \left[ \pi_t^W - \kappa_w \left( \sigma + \frac{\eta}{1-\gamma} (Y_t - \Gamma G_t) - w_t \right) - \beta E_t \pi^W(s_{t+1}) - e_t \right]
- \phi_t^w \left[ \pi_t^W - w_t + w_{t-1} - \pi_t + \Delta w_t^p \right]
- \phi_t^{ZLB} i_t,
\]

taking into account the laws of motion of the exogenous shocks. The functions \(V(s_{t+1})\), \(Y(s_{t+1})\), \(G(s_{t+1})\), \(\pi(s_{t+1})\), and \(\pi^W(s_{t+1})\) are the policymaker’s continuation value, output, government spending, the price inflation rate and the wage inflation rate that the policymaker expects to be realized in period \(t + 1\) in equilibrium, contingent on the realizations of the exogenous shocks in period \(t + 1\).
The consolidated first order conditions are

\[(1 - \Gamma) \left[ \lambda(Y_t - \Gamma G_t) + \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \left( \lambda W \pi_t^W - \phi_t^w \right) \right] + \frac{\kappa_p \gamma}{1 - \gamma} (\pi_t + \phi_t^w) + \lambda G G_t = 0 \]  

(C.1)

\[
\left( \beta \frac{\partial E_t \pi(s_{t+1})}{\partial w_t} + \kappa_p \right) (\pi_t + \phi_t^w) + \left( \beta \frac{\partial E_t \pi^W(s_{t+1})}{\partial w_t} - \kappa_w \right) \left( \lambda W \pi_t^W - \phi_t^w \right) + \phi_t^w \\
- \beta E_t \phi^w(s_{t+1}) + \left( \frac{\partial E_t Y(s_{t+1})}{\partial w_t} - \frac{\partial E_t G(s_{t+1})}{\partial w_t} \right) \frac{1}{\sigma} \left( \frac{\partial E_t \pi(s_{t+1})}{\partial w_t} \right) \left( \lambda(Y_t - \Gamma G_t) \right) + \frac{\kappa_p \gamma}{1 - \gamma} (\pi_t + \phi_t^w) + \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \left( \lambda W \pi_t^W - \phi_t^w \right) = 0 \\
\lambda(Y_t - \Gamma G_t) + \frac{\kappa_p \gamma}{1 - \gamma} (\pi_t + \phi_t^w) + \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \left( \lambda W \pi_t^W - \phi_t^w \right) \leq 0 \]  

(C.2)

\[
i_t \geq 0 \]  

(C.3)

\[
\left[ \lambda(Y_t - \Gamma G_t) + \frac{\kappa_p \gamma}{1 - \gamma} (\pi_t + \phi_t^w) + \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \left( \lambda W \pi_t^W - \phi_t^w \right) \right] i_t = 0, \]  

(C.5)

as well as the behavioral constraints (14) - (17).

\section*{D Computational algorithm for the continuous state model}

This section explains the solution algorithm exemplarily for the model with preference and price mark-up shocks. Let \( Z = [\pi, Y, G, \pi^W, w, i, \phi^w]' \) and \( S = [r^n, u, w_{-1}] \). I approximate \( Z \) by a linear combination of \( N \) basis functions \( \psi_n, n = 1, ..., N \). In matrix notation,

\[
Z(S) \approx C \Psi(S), \]  

(D.1)
where

\[
C = \begin{pmatrix}
    c_1^\pi & \cdots & c_N^\pi \\
    c_1^Y & \cdots & c_N^Y \\
    c_1^G & \cdots & c_N^G \\
    c_1^{\pi W} & \cdots & c_N^{\pi W} \\
    c_1^w & \cdots & c_N^w \\
    c_1^i & \cdots & c_N^i \\
    c_1^{\phi w} & \cdots & c_N^{\phi w}
\end{pmatrix}, \quad \Psi (S) = \begin{pmatrix}
    \psi_1 (S) \\
    \vdots \\
    \psi_N (S)
\end{pmatrix}.
\]

The coefficients \(c_n^j, n = 1, 2, \ldots, N; j \in \{\pi, Y, G, \pi W, w, i, \phi^w\}\), are set such that (D.1) holds exactly at \(N\) selected collocation nodes collected in the \(N \times 3\) matrix \(\mathbf{S}\)

\[
Z (\mathbf{S}) = C \Psi (\mathbf{S}),
\]

where \(Z (\mathbf{S})\) is a \(7 \times N\) matrix and \(\Psi (\mathbf{S})\) is a \(N \times N\) matrix. I use cubic splines as basis functions.

The iterative solution algorithm to obtain the policy function approximations is based on two loops. In the outer loop with counter \(l_1\) I iterate on the partial derivatives of the expectations functions with respect to the endogenous state variable(s). In the inner loop with counter \(l_2\) I iterate on the policy functions. When convergence in the inner loop is achieved, the new guess for the policy functions is used to update the guess for the partial derivatives of the expectations functions. The algorithm works as follows:

1. Start with an initial guess on the coefficient matrix \(C^{(0,0)}\) and the partial derivatives of the expectation functions for price inflation, wage inflation, output and government spending with respect to the real wage rate.

2. For fixed \(C^{(l_1,l_2)}\) in iteration \(l_2\) of the inner loop, use the collocation coefficients to determine the composite real wage rate \(w^{(l_1,l_2)} (\mathbf{S}) = C (5,:)^{(l_1,l_2)} \Psi (\mathbf{S})\), where \(C (5,:)^{(l_1,l_2)}\) refers to the 5th row of matrix \(C^{(l_1,l_2)}\). Then, update the expectations functions
using a Gaussian quadrature scheme to discretize the two normally distributed random variables

\[
\begin{align*}
E_{\pi}^{(l_1,l_2)} (\mathbf{S}) &= \sum_{k=1}^{m} \varpi_k C (1,:)^{(l_1,l_2)} \Psi \left( \hat{S}_k^{(l_1,l_2)} \right) \\
E_Y^{(l_1,l_2)} (\mathbf{S}) &= \sum_{k=1}^{m} \varpi_k C (2,:)^{(l_1,l_2)} \Psi \left( \hat{S}_k^{(l_1,l_2)} \right) \\
E_G^{(l_1,l_2)} (\mathbf{S}) &= \sum_{k=1}^{m} \varpi_k C (3,:)^{(l_1,l_2)} \Psi \left( \hat{S}_k^{(l_1,l_2)} \right) \\
(E_{\pi W})^{(l_1,l_2)} (\mathbf{S}) &= \sum_{k=1}^{m} \varpi_k C (4,:)^{(l_1,l_2)} \Psi \left( \hat{S}_k^{(l_1,l_2)} \right) \\
(E_{\phi W})^{(l_1,l_2)} (\mathbf{S}) &= \sum_{k=1}^{m} \varpi_k C (7,:)^{(l_1,l_2)} \Psi \left( \hat{S}_k^{(l_1,l_2)} \right),
\end{align*}
\]

where

\[
\hat{S}_k^{(l_1,l_2)} = \left[ \rho_r \bar{S}(:, 1) + \iota_N (1 - \rho_r) r^n + \iota_N \epsilon^r (k), \rho_u \bar{S}(:, 2) + \iota_N \epsilon^u (k), w^{(l_1,l_2)} (S) \right],
\]

with \( \iota_N \) being a vector of ones of length \( N \), \([\epsilon^r, \epsilon^u] \) is a \( m \times 2 \) matrix of quadrature nodes and \( \varpi \) is a vector of length \( m \) containing the quadrature weights.

3. Assume first that the zero lower bound constraint is not violated at any collocation node. The optimality conditions for the discretionary policy regime then imply

\[
Z^{(l_1,l_2)} \left( \mathbf{S} (q,:) \right) = A_{(q)}^{-1} \cdot B \cdot \mathbf{S} (q,:) + A_{(q)}^{-1} \cdot D \cdot E Z^{(l_1,l_2)} \left( \mathbf{S} (q,:) \right),
\]
for \( q = 1, \ldots, N \), where

\[
A(q) = \begin{pmatrix}
1 & -\frac{\kappa p \gamma}{1 - \gamma} & 0 & 0 & -\kappa p & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & \sigma^{-1} & 0 \\
0 & -\kappa w \left( \sigma + \frac{\eta}{1 - \gamma} \right) & \Gamma_{\kappa w} \left( \sigma + \frac{\eta}{1 - \gamma} \right) & 1 & \kappa w & 0 & 0 \\
-1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
\frac{\kappa p \gamma}{1 - \gamma} & (1 - \Gamma) \lambda & -(1 - \Gamma) \lambda \Gamma + \lambda_G & (1 - \Gamma) \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \lambda w & 0 & 0 & A(5, 7) \\
A(6, 1) & \Upsilon^{(l_1)}(q) \lambda & -\Upsilon^{(l_1)}(q) \lambda \Gamma & A(6, 4) & 0 & 0 & A(6, 7) \\
\frac{\kappa p \gamma}{1 - \gamma} & \lambda & -\lambda \Gamma & \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \lambda w & 0 & 0 & A(7, 7)
\end{pmatrix},
\]

with

\[
A(5, 7) = \frac{\kappa p \gamma}{1 - \gamma} - (1 - \Gamma) \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right), \quad A(6, 1) = \beta \frac{\partial \pi^{(l)}_w}{\partial w}(q) + \kappa_p + \Upsilon^{(l_1)}(q) \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \lambda w, \quad A(6, 4) = \beta \lambda w \frac{\partial \pi^{(l_1)}_w}{\partial w}(q) - \kappa_w \lambda w + \Upsilon^{(l_1)}(q) \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \lambda w, \quad A(6, 7) = \beta \left( \frac{\partial \pi^{(l_1)}_w}{\partial w}(q) - \frac{\partial \pi^{(l_1)}_w}{\partial w}(q) \right) + \kappa_p + \kappa_w + 1 + \Upsilon^{(l_1)}(q) \left( \frac{\kappa p \gamma}{1 - \gamma} - \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right) \right), \quad A(7, 7) = \frac{\kappa p \gamma}{1 - \gamma} - \kappa_w \left( \sigma + \frac{\eta}{1 - \gamma} \right), \quad \text{and} \quad \Upsilon^{(l_1)}(q) = \frac{\partial \pi^{(l_1)}_w}{\partial w}(q) - \frac{\partial \pi^{(l_1)}_w}{\partial w}(q) + \frac{1}{\sigma} \frac{\partial \pi^{(l_1)}_w}{\partial w}(q),
\]

and

\[
B = \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad D = \begin{pmatrix}
\beta & 0 & 0 & 0 & 0 \\
0 & \sigma^{-1} & 1 & -1 & 0 \\
0 & 0 & 0 & \beta & 0 \\
0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & \beta & 0 \\
0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & \beta
\end{pmatrix},
\]

as well as

\[
E\tilde{Z}^{(l_1,l_2)} = [(E\pi^{(l_1,l_2)})', (EY^{(l_1,l_2)})', (EG^{(l_1,l_2)})', ((E\pi^{W})^{(l_1,l_2)})', ((E\phi^{W})^{(l_1,l_2)})'].
\]

4. For those \( q \) for which the zero lower bound constraint is violated, that is, \( \tilde{i}^{(l_1,l_2)}(\tilde{S}(q,:)) < 0 \), the last row in matrix \( A(q) \) is replaced with \([0, 0, 0, 0, 0, 1, 0] \).

5. Update \( C^{(l_1,l_2+1)} = Z(\tilde{S})^{(l_1,l_2)} \Psi(\tilde{S})^{-1} \). Then, go back to Step 2. unless

\[
\|\text{vec}(C^{(l_1,l_2+1)} - C^{(l_1,l_2)})\|_{\infty} < \delta
\]

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for some small \( \delta > 0 \). It is useful to use a dampening parameter when updating the guess.

6. When convergence in the inner loop is achieved, set \( l_2 = 0 \), and update the partial derivatives of the expectations functions for price inflation, wage inflation, output and government spending with respect to the real wage rate

\[
\frac{\partial E_\pi^{(l+1)}}{\partial w} (S) = \sum_{k=1}^{m} \varpi_k C(1, \cdot)^{(l,0)} \Psi_w \left( \hat{S}_k^{(l,0)} \right)
\]

\[
\frac{\partial EY^{(l+1)}}{\partial w} (S) = \sum_{k=1}^{m} \varpi_k C(2, \cdot)^{(l,0)} \Psi_w \left( \hat{S}_k^{(l,0)} \right)
\]

\[
\frac{\partial EG^{(l+1)}}{\partial w} (S) = \sum_{k=1}^{m} \varpi_k C(3, \cdot)^{(l,0)} \Psi_w \left( \hat{S}_k^{(l,0)} \right)
\]

\[
\frac{\partial (E_\pi^W)^{(l+1)}}{\partial w} (S) = \sum_{k=1}^{m} \varpi_k C(4, \cdot)^{(l,0)} \Psi_w \left( \hat{S}_k^{(l,0)} \right)
\]

where \( \Psi_w(\cdot) \) represents the first derivative of the basis functions with respect to the third argument. Then go back to Step 2 unless the guesses for the partial derivatives of the expectations functions have converged, using a criterion similar to the one used in Step 5.

The collocation nodes are equally distributed with a support covering \( \pm 4 \) unconditional standard deviations of the exogenous state variables. I use MATLAB routines from the CompEcon toolbox of Miranda and Fackler (2002) to obtain the Gaussian quadrature approximation of the innovations to the exogenous shocks, and to evaluate the spline functions.