## EABCN Training School: Monetary-Fiscal Policy Interactions

Lecture 1. Simple Models of Policy Interactions: Some Monetary Doctrines

## Eric M. Leeper

Indiana University
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## The Messages

1. A complete specification of macro policy is necessary for determination of equilibrium
2. Complete specification includes enough information about policy behavior that agents can form expectations of the entire future paths of policy instruments
3. Monetary and fiscal policies must interact in certain ways in any equilibrium
4. Every statement about monetary policy effects is conditional on maintained assumptions about fiscal policy behavior
5. And vice versa

## The Model

- Draws on "Monetary Doctrines" in Ljungqvist-Sargent
- Shopping time monetary model
- constant endowment, $y>0$
- no uncertainty
- steady-state analysis
- lump-sum taxes/transfers
- How money gets valued unimportant to results
- Aggregate resource constraint

$$
\begin{equation*}
c_{t}+g_{t}=y \tag{1}
\end{equation*}
$$

- Preferences

$$
\begin{align*}
& \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}, l_{t}\right), 0<\beta<1  \tag{2}\\
& u_{c}, u_{l}>0 ; u_{c c}, u_{l l}<0, u_{c l} \geq 0
\end{align*}
$$

## Shopping Technology

- Households must spend time shopping, $s_{t}$, to acquire consumption goods, $c_{t}$
- Shopping/transactions technology

$$
\begin{equation*}
s_{t}=H\left(c_{t}, \frac{m_{t}}{p_{t}}\right) \tag{3}
\end{equation*}
$$

$m_{t} / p_{t}$ real money balances chosen at $t$ $H$ convex: $H, H_{c}, H_{c c}, H_{\frac{m}{p}}^{p} \geq 0, H_{\frac{m}{p}}, H_{c, \frac{m}{p}} \leq 0$

- Example: Baumol-Tobin

$$
H\left(c_{t}, \frac{m_{t}}{p_{t}}\right)=\frac{c_{t}}{m_{t} / p_{t}} \varepsilon
$$

$\varepsilon>0$ : time cost per trip to the bank

## Other Constraints

- Time constraint

$$
\begin{equation*}
l_{t}+s_{t}=1 \tag{4}
\end{equation*}
$$

- Household budget constraint

$$
\begin{equation*}
c_{t}+\frac{b_{t}}{R_{t}}+\frac{m_{t}}{p_{t}}=y-\tau_{t}+b_{t-1}+\frac{m_{t-1}}{p_{t}} \tag{5}
\end{equation*}
$$

$b$ : 1-period indexed bonds; $p$ : price level; $\tau$ : lump-sum tax

- Maximize (2) s.t. (3), (4), (5)
- Note that
- $m_{t} \geq 0$ (HH cannot issue currency)
- $b_{t} \lesseqgtr 0$ (HH can borrow or lend)
- Multipliers: $\lambda_{t}$ for (5), $\mu_{t}$ for (4)


## Optimality Conditions

- Let $R_{m t} \equiv p_{t} / p_{t+1}$, the return on fiat currency
- Arbitrage between $m$ and $b$

$$
\begin{gather*}
1-\frac{R_{m t}}{R_{t}} \geq-\frac{\mu_{t}}{\lambda_{t}} H_{\frac{m}{p}}(t) \geq 0 \\
1-\frac{R_{m t}}{R_{t}}=\frac{i_{t}}{1+i_{t}} \geq 0 \tag{6}
\end{gather*}
$$

- (6) leads to the key result that nominal interest rates are non-negative: because $R_{m t} \leq R_{t}$ (currency is dominated in rate of return)

$$
i_{t} \geq 0
$$

## Optimality Conditions

- Consumption-leisure tradeoff implies

$$
\begin{equation*}
\lambda_{t}=u_{c}(t)-u_{l}(t) H_{c}(t) \tag{7}
\end{equation*}
$$

- Return on bonds can be expressed as

$$
\begin{equation*}
R_{t}=\frac{1}{\beta}\left[\frac{u_{c}(t)-u_{l}(t) H_{c}(t)}{u_{c}(t+1)-u_{l}(t+1) H_{c}(t+1)}\right] \tag{8}
\end{equation*}
$$

- (6) yields

$$
\begin{equation*}
\left(\frac{R_{t}-R_{m t}}{R_{t}}\right) \lambda_{t}=-\mu_{t} H_{\frac{m}{p}}(t) \tag{9}
\end{equation*}
$$

## Money Demand

- Combining FOCs [(7),(8),(9)]

$$
\left(1-\frac{R_{m t}}{R_{t}}\right)\left[\frac{u_{c}(t)}{u_{l}(t)}-H_{c}(t)\right]+H_{\frac{m}{p}}(t)=0
$$

- Evaluate $u_{c}(t), u_{l}(t)$ at $l_{t}=1-H\left(c_{t}, m_{t} / p_{t}\right)$ to get the implicitly defined money demand function

$$
\begin{equation*}
\frac{m_{t}}{p_{t}}=F\left(c_{t}, \frac{R_{m t}}{R_{t}}\right)=F\left(c_{t}, i_{t}\right) \tag{10}
\end{equation*}
$$

- Straightforward to show in (10) that $F_{c}>0, F_{i}<0$


## Government \& EQuilibrium

- Government finances $\left\{g_{t}\right\}$ s.t.

$$
\begin{equation*}
g_{t}=\tau_{t}+\frac{B_{t}}{R_{t}}-B_{t-1}+\frac{M_{t}-M_{t-1}}{p_{t}} \tag{11}
\end{equation*}
$$

- A price system is a pair of positive sequences $\left\{R_{t}, p_{t}\right\}_{t=0}^{\infty}$
- Take as exogenous $\left\{g_{t}, \tau_{t}\right\}_{t=0}^{\infty}$ and $B_{-1}=b_{-1}$, $M_{-1}=m_{-1}>0$.
An equilibrium is a price system, and sequences $\left\{c_{t}, B_{t}, M_{t}\right\}_{t=0}^{\infty}$ such that
- the household's optimum problem is solved with

$$
b_{t}=B_{t}, m_{t}=M_{t}
$$

- the government's budget constraint is satisfied
- $c_{t}+g_{t}=y$


## Policy Experiments

- Need a complete specification of policy
- Will give definite meaning to concepts of
- "short run": initial date
- "long run": stationary equilibrium
- Assume

$$
\begin{array}{rlrl}
g_{t} & =g & t \geq 0 \\
\tau_{t} & =\tau & t \geq 1 \\
B_{t} & =B & t \geq 0
\end{array}
$$

We permit $\tau_{0} \neq \tau, B_{-1} \neq B$

- Economy in stationary eqm for $t \geq 1$ but starts from a different position at $t=0$
- Reduces dynamics to 2 periods: now $(t=0)$ \& future $(t \geq 1)$


## Stationary Equilibrium

- Seek an equilibrium with

$$
\begin{aligned}
p_{t} / p_{t+1} & =R_{m} \quad t \geq 0 \\
R_{t} & =R \quad t \geq 0 \\
c_{t} & =c \quad t \geq 0 \\
s_{t} & =s \quad t \geq 0
\end{aligned}
$$

which imply that

$$
\begin{aligned}
R & =\beta^{-1} \\
\frac{m_{t}}{p_{t}} & =F\left(c, R_{m} / R\right)=f\left(R_{m}\right), \quad f^{\prime}>0
\end{aligned}
$$

## Two EQUILIBRIUM Conditions

1. Impose eqm on government budget constraint at $t \geq 1$

$$
g-\tau+\frac{B(R-1)}{R}=f\left(R_{m}\right)\left(1-R_{m}\right)
$$

(Future)
2. Impose eqm on government budget constraint at $t=0$

$$
\frac{M_{-1}}{p_{0}}=f\left(R_{m}\right)-\left(g+B_{-1}-\tau_{0}\right)+\frac{B}{R}
$$

(Current)

- Given $(g, \tau, B)$, (Future) $\Rightarrow R_{m}$-inflation rate
- Given $\left(g, \tau_{0}, B\right)$ \& initial conditions $\left(M_{-1}, B_{-1}\right)$, (Current) $\Rightarrow$ $p_{0}$-initial price level
- Have completely determined eqm $\left\{p_{t}\right\}_{t=0}^{\infty}$
- Now consider alternative policies and how they affect price-level determination


## Deriving EQuilibria Graphically



## 1. Sustained Deficits Cause Inflation



Let $D=g-\tau+\frac{B(R-1)}{R}$ and consider $D^{\prime}>D^{*}$

## 1. Sustained Deficits Cause Inflation

Seigniorage Revenue vs. $\mathrm{R}_{\mathrm{m}}$

"normal" side: $D^{\prime}>D^{*} \Rightarrow R_{m}^{\prime}<R_{m}^{*}$ (classical doctrine)

## 2. Zero Inflation Policy

- $\pi=0 \Rightarrow R_{m}=1 \Rightarrow$ seigniorage $=0$
- (Future) $\Rightarrow$

$$
g-\tau+\frac{B(R-1)}{R}=0
$$

or

$$
\frac{B}{R}=\frac{\tau-g}{R-1}=\sum_{t=1}^{\infty} R^{-t}(\tau-g)
$$

- Real value of interest bearing government debt = present value of net-of-interest primary surpluses
- Of course, this generalizes to any fixed inflation rate policy (e.g., inflation targeting)
- It is strange-and troubling-that no country that adopted inflation targeting simultaneously adopted fiscal policies that are consistent with it


## 3. Unpleasant Monetarist Arithmetic

- A little history—US FP in early 1980s
- Consider an open-market sale of bonds at $t=0$, $-d\left(M_{0} / p_{0}\right)=d B_{0}>0$
- Hold fiscal policy— $\left(g, \tau_{0}, \tau\right)$ —fixed
- OM sale raises $B$ in eqm conditions (Current) \& (Future)
- Higher debt service in the future, but FP fixed
- Future seigniorage must rise: $f\left(R_{m}\right)\left(1-R_{m}\right)$ rises by $\frac{R-1}{R} d B$
- Stationary $\pi$ rises ( $R_{m}$ falls) unambiguously


## 3. Unpleasant Monetarist Arithmetic

$$
\begin{equation*}
\frac{M_{-1}}{p_{0}}=f\left(R_{m}\right)-\left(g+B_{-1}-\tau_{0}\right)+\frac{B}{R} \tag{Current}
\end{equation*}
$$

- By (Current), effect on $p_{0}$ can be anything
- if $f^{\prime}\left(R_{m}\right)$ small, $p_{0}$ falls (usual result)
- if $f^{\prime}\left(R_{m}\right)$ large, $p_{0}$ rises (extreme unpleasantness)
- Tighter money via OMO—at best-temporarily lowers $p$ but at the cost of permanently raising $\pi$


## 4. Quantity Theory of Money

- Classic quantity theory of money experiment is a helicopter drop of money
- change $M_{-1}$ to $\lambda M_{-1}, \lambda>0$
- holding fiscal policy- $\left(g, \tau_{0}, \tau, B\right)$-fixed
- By (Current), if $p_{0} \rightarrow \lambda p_{0}$, then $M_{-1} / p_{0}$ unchanged

$$
\frac{\lambda M_{-1}}{\lambda p_{0}}=f\left(R_{m}\right)-\left(g+B_{-1}-\tau_{0}\right)+\frac{B}{R}
$$

(Current)

- Nothing happens to growth rate of money, $R_{m}$, or $\pi$
- Produces "neutrality of money" (not "superneutrality")
- Tobin's gremlins: required to leave portfolios unperturbed by $M$ drop


## 5. A Neutral Open-Market Operation

- Redefine OMO from that used in unpleasant arithmetic to give MA fiscal powers so OMO have QT effects
- Denote initial eqm by $\bar{x}$; new eqm by $\hat{x}$
- Consider OMO that decreases $M_{0}$ and increases $B$ and $\tau$ (with $\bar{\tau}_{0}=\hat{\tau}_{0}$ ) such that

$$
\left(1-\frac{1}{R}\right)(\hat{B}-\bar{B})=\hat{\tau}-\bar{\tau}
$$

- If future taxes obey this for $t \geq 1$, then (Future) satisfied at initial $R_{m}$ (that is, $\left.-d \tau+d B(1-1 / R)=0\right)$
- Highlights a key aspect of conventional MP analysis (e.g., in new Keynesian models)
- lump-sum taxes in future adjust by just enough to service any additional interest payments arising from the OMO's effects on $B$
- FP "held constant" via unchanged gross-of-interest deficit


## 6. The Optimum Quantity of Money

- Given stationary $(g, B)$, Friedman argued that agents are better off with higher stationary real money balances (ones associated with higher rates of return on money)
- By running sufficiently large gross surpluses ( $g-\tau+B(R-1) / R<0$ ), government can attain any $R_{m} \in(1,1 / \beta)$
- So given $(g, B)$, choose $\tau$ to get required surplus to hit the target $R_{m}$
- Use proceeds of tax to retire currency (achieve negative growth of $M$ )
- Pursues Friedman's optimal policy of saturating economy with real balances


## 6. The Optimum Quantity of Money

- Social value of real balances in model comes from reducing shopping time
- Optimum quantity of $M$ minimizes time spent shopping
- Suppose there is a satiation point in real balances $\psi(c)$ for any $c$

$$
H_{m / p}\left(c, m_{t} / p_{t}\right)=0 \text { for } m_{t} / p_{t} \geq \psi(c)
$$

- Can achieve this only by setting $R=R_{m}$ (since $\left.\mu_{t}, \lambda_{t}>0\right)$
- If $H(c, m / p)=\frac{c}{m / p} \varepsilon$, can only approximate Friedman's rule since money demand insatiable


## 7. One Big Open-Market Operation

- Consider a large OM purchase of private indebtedness at $t=0$
- gives government a portfolio of interest-earning claims on private sector
- permits the government to run a gross-of-interest surplus
- government uses surplus to reduce money supply and create deflation
- this raises return on money $>1$
- idea underlies some optimal fiscal policy results
- Impose $g-\tau \geq 0$ so cannot achieve deflation through direct taxation
- Proposal: $M_{0} \uparrow, B \downarrow$ with $B<0$


## 7. One Big Open-Market Operation

- Given $(g, \tau)$, use (Future) to pick $B$ consistent with desired $R_{m}\left(1 \leq R_{m} \leq 1 / \beta\right)$

$$
\frac{M_{-1}}{p_{0}}=\underbrace{\left(\frac{R-R_{m}}{1-R_{m}}\right) \frac{B}{R}}_{>0}+\underbrace{\left(\frac{1}{1-R_{m}}\right)(g-\tau)-\left(g+B_{-1}-\tau_{0}\right)}_{\leqq 0}
$$

- The candidate policy is an equilibrium policy if $\left(g, \tau, \tau_{0}, B_{-1}\right)$ are such that $R H S>0$ so that there exists a $p_{0}>0$ that solves this
- Example: $g-\tau=0 \& g+B_{-1}-\tau_{0}=0$ (balance budget $t \geq 1$
- then $R H S>0$ and it's feasible to get $1<R_{m}<1 / \beta$
- Note: Cannot get $R_{m}=1 / \beta$ since then $R=R_{m}$ and government earns no arbitrage income and cannot finance deflation


## 8. A Ricardian Experiment

- Consider a debt-financed tax cut at $t=0$, with future taxes adjusting
- MP held fixed: no change in $\left\{M_{t}\right\}_{t=0}^{\infty}$
- $-d \tau_{0}=\frac{1}{R} d B \& d \tau=\frac{R-1}{R} d B$
- Both (Current) \& (Future) satisfied at initial $R_{m}, p_{0}$
- Lump-sum taxes in future adjust by just enough to service any additional interest payments arising from the tax cut's effects on $B$
- Of course, lump-sum essential
- A central neutrality result in fiscal policy


## 9. LJUNGQVIST-SARGENT's "Fiscal Theory of Price Level"

- FTPL is intrinsically about nominal government debt
- LS couch FTPL in terms of indexed (real) debt
- FTPL changes assumptions about which variables the government sets
- MP commits to set PV seigniorage, $f\left(R_{m}\right)\left(1-R_{m}\right) /(R-1)$, so $B$ endogenous
- Equivalent to pegging nominal interest rate (or $\pi$ or $R_{m}^{-1}$ )
- A little history
- CBs actually have pegged $i$
- early rational expectations literature: pegged $i \Rightarrow$ price level indeterminacy


## 9. LJUNGQVIST-SARGENT's "FTPL"

- Rewrite (Future) as

$$
\begin{aligned}
\frac{B}{R} & =\frac{1}{R-1}\left[(\tau-g)+f\left(R_{m}\right)\left(1-R_{m}\right)\right] \\
& =\sum_{t=1}^{\infty} R^{-t}(\tau-g)+f\left(R_{m}\right) \frac{1-R_{m}}{R-1}
\end{aligned}
$$

- Subst. into (Current): imposes that future policy restricts current policy through the value of debt

$$
\begin{aligned}
\frac{M_{-1}}{p_{0}}+B_{-1} & =\sum_{t=0}^{\infty} R^{-t}\left(\tau_{t}-g_{t}\right)+f\left(R_{m}\right)\left(1+\frac{1-R_{m}}{R-1}\right) \\
& =\sum_{t=0}^{\infty} R^{-t}\left(\tau_{t}-g_{t}\right)+\sum_{t=1}^{\infty} R^{-t} f\left(R_{m}\right)\left(R-R_{m}\right)
\end{aligned}
$$

## 9. LJUNGQVIST-SARGENT's "FTPL"

- Repeat equilibrium condition

$$
\frac{M_{-1}}{p_{0}}+B_{-1}=\sum_{t=0}^{\infty} R^{-t}\left(\tau_{t}-g_{t}\right)+\sum_{t=1}^{\infty} R^{-t} f\left(R_{m}\right)\left(R-R_{m}\right)
$$

- Government chooses $\left(g, \tau, \tau_{0}, R_{m}\right)$ (recall: $i=\left(\beta R_{m}\right)^{-1}$ )
- $B$ determined by expected surpluses plus seigniorage
- This condition yields eqm $p_{0}$ for given $M_{-1}$
- Use money demand in eqm to solve for

$$
\frac{M_{0}}{p_{0}}=F\left(y-g, R_{m} / R\right)
$$

- A quantity theory demand for money $\nRightarrow$ can control $\left\{p_{t}\right\}$ by controlling $\left\{M_{t}\right\}$


## Wrap Up

- These doctrines, though simple, highlight the centrality of monetary-fiscal policy interactions for the nature of eqm
- Although this general point has been known, we often ignore it
- introduces inconvenient considerations
- makes policy analysis much harder
- prescribing both MP \& FP is many times harder than prescribing MP, assuming FP-i.e., lump-sum taxes-will adjust to ensure fiscal sustainability
- The doctrines should have made clear that once you deviate from this kind of FP, lots of interesting things can happen

