EABCN Training School: Monetary-Fiscal Policy Interactions

Lecture 2. Fiscal Theory of the Price Level

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THE MESSAGES

1. FTPL is a specific example of monetary-fiscal interactions
2. It challenges conventional—purely monetary—explanations of price level determination
3. Almost no doubt that there are historical episodes in which FTPL operative—the past couple of years, for example
4. Open question: how important it is in general?
5. Empirical work on FTPL is extremely hard to do well
6. Many open areas of research
7. Will give two distinct presentations on FTPL
   - fully non-linear, follows Woodford (2001)
   - linear, follows Leeper (1991)

The Messages

- FTPL requires that government debt denominated in nominal terms
- Most debt issued by advanced economies qualifies:
    - 90% of U.S. debt
    - 80% of U.K. debt
    - over 90% of Australian, Canadian, New Zealand, Swedish debt
    - most EMU-member debt in euro
    - tiny fraction of Japanese bonds
- This argues that the FTPL mechanism is potentially operative in many countries

The Model

- Endowment economy, MIUF
- Representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U \left(c_t + g_t, \frac{M_t}{P_t} \right) \right\}$$

subject to sequence of flow budget constraints

$$M_t + E_t[R_{t,t+1}(W_{t+1} - M_t)] \leq W_t + P_t y_t - T_t - P_t c_t$$

  - $W_{t+1} - M_t$: nominal value in $t+1$ of HH's bond holdings at end of t
  - $E_t[R_{t,t+1}(W_{t+1} - M_t)]$: nominal market value of state-contingent claims
  - $R_{t,t+1}$: stochastic discount factor
  - Note: $W_t^s = M_{t-1}^s + (1 + i_{t-1})B_{t-1}^s$
  - So $1 + i_t = E_t[R_{t,t+1}]^{-1}$

THE MODEL

- Can write HH's flow b.c. as

$$P_t c_t + \frac{i_t}{1+i_t} M_t + E_t[R_{t,t+1} W_{t+1}] \leq W_t + P_t y_t - T_t$$

- $i/(1+i)$ is effective cost of holding wealth as M
- HH portfolio choices also satisfy the borrowing limit

$$W_{t+1} \geq -\sum_{T=t+1}^{\infty} E_{t+1}\left[R_{t+1,T}(P_T y_T - T_T)\right]$$

  for all states in $t+1$
- Note: $R_{t+1,T} \equiv \prod_{s=t+1}^{T-1} R_{s,s+1}, R_{T,T} = 1$

The Model

- HH's flow b.c. & borrowing limit $\Rightarrow$ intertemporal b.c.

$$\sum_{T=t}^{\infty} E_t R_{t,T}\left[P_T c_T + \frac{i_T}{1+i_T} M_T\right] \leq W_t + \sum_{T=t}^{\infty} E_t R_{t,T}\left[P_T y_T - T_T\right]$$

- HH's FOCs yield

$$\frac{U_m(c_t + g_t, m_t)}{U_c(c_t + g_t, m_t)} = \frac{i_t}{1+i_t}$$

$$\frac{U_c(c_t + g_t, m_t)}{U_c(c_{t+1} + g_{t+1}, m_{t+1})} = \frac{\beta}{R_{t,t+1}} \frac{P_t}{P_{t+1}}$$

- And intertemporal b.c. at equality

The Model

- Could replace HH's intertemporal b.c. with
  - HH planned expenditure has finite present value

$$\sum_{T=t}^{\infty} E_t R_{t,T} \left[P_T c_T + \frac{i_T}{1+i_T} M_T \right] < \infty$$

  - And transversality condition on wealth

$$\lim_{T \to \infty} E_t [R_{t,T} W_T] = 0$$

- Market clearing conditions in all states for all t
  - $c_t + g_t = y_t$
  - $M_t = M_t^s$
  - $W_{t+1} = W_{t+1}^s$

Equilibrium

- Impose market clearing on liquidity preference

$$\frac{U_m\left(y_t, \frac{M_t^s}{P_t}\right)}{U_c\left(y_t, \frac{M_t^s}{P_t}\right)} = \frac{i_t}{1+i_t}$$

we can write this as

$$\frac{M_t^s}{P_t} = L(y_t, i_t), \qquad L_y > 0, L_i < 0$$

- Impose market clearing on Fisher relation (assume U separable in c and m)

$$R_{t,t+1} = \beta \frac{U_c(y_{t+1})}{U_c(y_t)} \frac{P_t}{P_{t+1}}$$

$$1 + i_t = \beta^{-1} \left\{ E_t \left[\frac{U_c(y_{t+1})}{U_c(y_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1}$$

EQUILIBRIUM

- Assume government's share of output is bounded: $0 \leq g_t \leq \gamma y_t, \quad 0 < \gamma < 1$
- In equilibrium, transversality condition for wealth implies

$$\sum_{T=t}^{\infty} \beta^{T-t} E_t \frac{U_c(y_T, m_T)}{U_c(y_t, m_t)} \left[(y_T - g_T) + \frac{i_T}{1+i_T} \frac{M_T}{P_T} \right] =$$

$$\frac{W_t^s}{P_t} + \sum_{T=t}^{\infty} \beta^{T-t} E_t \frac{U_c(y_T, m_T)}{U_c(y_t, m_t)} \left[y_T - \frac{T_T}{P_T} \right]$$

Subtract $PV(y_T - g_T)$ from both sides to yield the **ubiquitous equilibrium condition**

$$\frac{W_t^s}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} E_t \frac{U_c(y_T, m_T)}{U_c(y_t, m_t)} \left[s_T + \frac{i_T}{1+i_T} \frac{M_T}{P_T} \right]$$

$s_t \equiv T_t / P_t - g_t$, net-of-interest surplus

Policy Behavior

- MP: pegs price of one-period bond $\Rightarrow \{i_t\}$ exogenous
- FP: sets $\{s_t\}$ exogenously
- All government debt riskless, one-period, nominal
- Total government liabilities at beginning of t:
  $W^s_t = M^s_{t-1} + (1 + i_{t-1})B^s_{t-1}$
- Law of motion for government liabilities

$$W^s_{t+1} = (1 + i_t)\left[W^s_t - P_t s_t - \frac{i_t}{1 + i_t} M^s_t\right]$$

- Need to ensure that hypothesized policies satisfy this law of motion

RECURSIVE SOLUTION

- Can now derive eqm recursively and obtain unique $\{W_t^s, M_t^s, P_t\}$ given exogenous processes for $\{y_t, s_t, i_t\}$

1. Given $\{y_t, i_t\}$, liquidity pref yields eqm M_t/P_t
2. Use eqm M_t/P_t in **ubiquitous eqm condition**

$$\frac{W_t^s}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} E_t \frac{\lambda(y_T, i_T)}{\lambda(y_t, i_t)} \left[s_T + \frac{i_T}{1+i_T} L(y_T, i_T) \right]$$

   where $\lambda(y, i) \equiv U_c(y, L(y, i))$
   Entire RHS exogenous & W_t predetermined $\Rightarrow P_t > 0$ (if $W_t^s > 0$)
3. Use law of motion for liabilities to get W_{t+1}^s
4. Repeat for $t + 1$

Analysis

$$\frac{W_t^s}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} E_t \frac{\lambda(y_T, i_T)}{\lambda(y_t, i_t)} \left[s_T + \frac{i_T}{1+i_T} L(y_T, i_T) \right]$$

- News that $E_t s_T \Downarrow$ implies $P_t \Uparrow$ (anticipated fiscal expansions are inflationary)
- Although $M_t^s \Uparrow$ to clear money market, this is a passive response induced by pegging i_t—M is not causing P
- Both P_t and M_t^s rise *before* fiscal changes are realized
- This is *not* the usual monetization of deficits, as in unpleasant arithmetic
- Anticipated surpluses & seigniorage symmetric: lower $E_t s_T$ or $E_t \frac{i_T}{1+i_T} L(y_T, i_T)$ both inflationary(highly irregular)

THE ECONOMICS

- How can changes in lump-sum taxes affect P?
- Answer: wealth effect
- Suppose $E_t s_T \Downarrow$
  - HH feels wealthier (lower expected future taxes)
  - able to afford more c_t, so demand for goods $\Uparrow$ (at initial prices)
  - because supply of goods fixed, $P_t \Uparrow$
  - P reaches new eqm by reducing real value of nominal assets held by HH
  - P rises to point where value of nominal assets $=$ PV expected primary surpluses b/c then HH can afford to buy exactly the quantity of goods produced
  - P adjusts until **ubiquitous eqm condition** restored
  - in eqm, no change in wealth from lower anticipated taxes

New Model

- Seek to characterize FTPL more generally by relaxing extreme policy ass'ns
- Also give policy behavior conventional parametric representation
- Allow us to characterize the MP/FP behavior that is consistent with existence & uniqueness of eqm
- Cost of generality: focus on local dynamics within a neighborhood of steady state, rather than previous global results
- Essentially the same model as above

THE MODEL

- Representative HH, endowment, MIUF; both the endowment, y, and government consumption, g, are constant; $g = 0$
- Agent chooses sequences $\{c_t, M_t, B_t\}$ to solve

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \delta \log\left(M_t/p_t\right)\right], \quad 0 < \beta < 1, \delta > 0$$

subject to

$$c_t + \frac{M_t + B_t}{p_t} + \tau_t = y + \frac{M_{t-1} + R_{t-1}B_{t-1}}{p_t}$$

M is nominal money balances, B is nominal one-period government debt, which pays gross nominal interest at rate R, c is consumption, and τ is lump-sum taxes (if positive) and transfers (if negative)

THE MODEL

- Aggregate resource constraint for this economy is

$$c_t + g_t = y$$

- FOCs imply the Fisher and money-demand equilibrium relations

$$\frac{1}{R_t} = \beta E_t \left[\frac{1}{\pi_{t+1}} \right]$$

$$m_t = \delta c \left[\frac{R_t - 1}{R_t} \right]^{-1}$$

where $\pi_t = p_t / p_{t-1}$ and $m_t = M_t / p_t$

Policy Behavior

- Government policy is sequences $\{M_t, B_t, \tau_t\}$ that satisfy the government's budget identity

$$b_t + m_t + \tau_t = g + \frac{R_{t-1}b_{t-1} + m_{t-1}}{\pi_t}$$

where $b_t = B_t/p_t$

- Fiscal policy obeys

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t$$

where $\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_{\psi t}$ is an exogenous shock

- Monetary policy obeys

$$R_t = \alpha_0 + \alpha \pi_t + \theta_t$$

where $\theta_t = \rho_\theta \theta_{t-1} + \varepsilon_{\theta t}$ is an exogenous shock

Solving the Model

- Reduce the model to a dynamical system in (π_t, b_t)
- Define the forecast error $\eta_{t+1} = \pi_{t+1} - E_t \pi_{t+1}$
- Write the linearized system to be solved as

$$\begin{bmatrix} 1 & 0 \\ \varphi_1 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t+1} \\ b_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha\beta & 0 \\ \varphi_2 & \beta^{-1} - \gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 \\ \varphi_3 & -1 \end{bmatrix} \begin{bmatrix} \theta_{t+1} \\ \psi_{t+1} \end{bmatrix} + \begin{bmatrix} \beta & 0 \\ \varphi_4 & 0 \end{bmatrix} \begin{bmatrix} \theta_t \\ \psi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta_{t+1}$$

which holds for $t \geq 0$

$$\varphi_1 = \frac{\delta y}{\bar{R} - 1} \left[\frac{1}{\beta\bar{\pi}} - \frac{\alpha}{\bar{R} - 1} \right] + \frac{\bar{b}}{\beta\bar{\pi}} \qquad \varphi_3 = \frac{\delta y}{(\bar{R} - 1)^2}$$
$$\varphi_2 = -\frac{\alpha}{\bar{\pi}} \left[\frac{\delta y}{(\bar{R} - 1)^2} - \bar{b} \right] \qquad \varphi_4 = \frac{\varphi_2}{\alpha} = -\frac{1}{\bar{\pi}} \left[\frac{\delta y}{(\bar{R} - 1)^2} - \bar{b} \right]$$

SOLVING THE MODEL

- Using Sims's (2001) notation, let $x_t = (\pi_t, b_t)'$ and $z_t = (\theta_t, b_t)'$ and write system as

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + \Phi_0 z_{t+1} + \Phi_1 z_t + \Pi \eta_{t+1}$$

- Eigensystem analysis focuses on the transition matrix

$$\Gamma_0^{-1}\Gamma_1 = \left[\begin{array}{cc} \alpha\beta & 0 \\ -\alpha\beta\varphi_1 + \varphi_2 & \beta^{-1} - \gamma \end{array} \right]$$

- Eigenvalues are $\alpha\beta$ & $\beta^{-1} - \gamma$
- With a single forecast error, η_{t+1}, need one unstable root for a unique eqm to exist

Characterizing Equilibria

- Four regions of policy parameter space are of interest

$$\text{I}: \quad |\alpha\beta| \geq 1 \text{ and } \left|\beta^{-1} - \gamma\right| < 1 \Longrightarrow \text{ Unique Eqm}$$

$$\text{II}: \quad |\alpha\beta| < 1 \text{ and } \left|\beta^{-1} - \gamma\right| \geq 1 \Longrightarrow \text{ Unique Eqm}$$

$$\text{III}: \quad |\alpha\beta| < 1 \text{ and } \left|\beta^{-1} - \gamma\right| < 1 \Longrightarrow \text{ Multiple Eq/Sunspots}$$

$$\text{IV}: \quad |\alpha\beta| \geq 1 \text{ and } \left|\beta^{-1} - \gamma\right| \geq 1 \Longrightarrow \text{ No Bounded Eqm}$$

- Nature of eqm very different across regions
- Region I: monetarist & Ricardian
- Region II: non-monetarist & FTPL
- Region III: non-monetarist & FTPL in all eq
- Region IV: no eqm unless θ_t & ψ_t correlated in right way

Characterizing Equilibria

- Stack x and z; let $Y_t = (\pi_t, b_t, \theta_t, \psi_t)', \xi_t = (\eta_t, 0, \varepsilon_{\theta_t}, \varepsilon_{\psi t})'$

$$\begin{bmatrix} \pi_{t+1} \\ b_{t+1} \\ \theta_{t+1} \\ \psi_{t+1} \end{bmatrix} =$$

$$\begin{bmatrix} \alpha\beta & 0 & \beta & 0 \\ -\alpha\beta\varphi_1 + \varphi_2 & \beta^{-1} - \gamma & \rho_\theta\varphi_3 - \beta\varphi_1 + \varphi_4 & -\rho_\psi \\ 0 & 0 & \rho_\theta & 0 \\ 0 & 0 & 0 & \rho_\psi \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t \\ \theta_t \\ \psi_t \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\varphi_1 & 0 & \varphi_3 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_{t+1} \\ 0 \\ \varepsilon_{\theta t+1} \\ \varepsilon_{\psi t+1} \end{bmatrix}$$

which holds for $t = 0, 1, 2, ...$

Characterizing Equilibria

- More compactly,

$$Y_{t+1} = AY_t + C\xi_{t+1}$$

  which implies

$$Y_t = A^t Y_0 + \sum_{s=0}^{t-1} A^s C\xi_{t-s}$$

- Jordan decomposition of A implies $A^s = P\Lambda^s P^{-1}$, where the eigenvalues are along the diagonal of Λ
- Let $P^{j\cdot}$ be the jth row of P^{-1} and let $P_{\cdot j}$ be the jth column of P
- Then system is

$$Y_t = \sum_{j=1}^{n} P_{\cdot j}\lambda_j^t P^{j\cdot} Y_0 + \sum_{j=1}^{n} P_{\cdot j} \sum_{s=0}^{t-1} \lambda_j^s P^{j\cdot} C\xi_{t-s}$$

Characterizing Equilibria

$$Y_t = \sum_{j=1}^{n} P_{\cdot j} \lambda_j^t P^{j\cdot} Y_0 + \sum_{j=1}^{n} P_{\cdot j} \sum_{s=0}^{t-1} \lambda_j^s P^{j\cdot} C \xi_{t-s}$$

- To eliminate explosive eigenvalues (ones where $|\lambda_j| > 1$) we need to impose for each explosive j :

Stability Conditions

$$P^{j\cdot} Y_t = 0, \qquad t = 0, 1, 2, ...$$

or, equivalently,

$$P^{j\cdot} Y_0 = 0,$$

$$P^{j\cdot} C \xi_t = 0, \qquad t = 1, 2, ...$$

Existence & Uniqueness of Equilibrium

$$P^{j\cdot}Y_0 = 0,$$

$$P^{j\cdot}C\xi_t = 0, \qquad t = 1, 2, ...$$

- Recall we introduced η_{t+1}, endogenous forecast error
- A unique solution requires a unique linear mapping from the ε's to η (the exogenous structural errors to the endogenous reduced-form error)
- If the model supports more than one such mapping, the solution is not unique
- If the model fails to generate a mapping (perhaps because it produces two or more mutually exclusive mappings), then no equilibrium exists

Existence & Uniqueness of Equilibrium

$$P^{j\cdot}Y_0 = 0,$$

$$P^{j\cdot}C\xi_t = 0, \qquad t = 1, 2, ...$$

- Rules of thumb for existence and uniqueness are:
  1. If there are q distinct (linearly independent) expectational errors—the η's—then we need q unstable eigenvalues, which provide q additional restrictions
  2. If there are fewer than q unstable roots, the model is underdetermined and the solution is not unique—there are too few additional restrictions to determine the q η's
  3. If there are more than q unstable roots, the model is overdetermined and no solution exists. This is because too many additional restrictions are produced.

Existence & Uniqueness of Equilibrium

- In this model, $q = 1,$ so we need only one unstable root to uniquely determine the equilibrium
- Note: these are *local* results; global conditions harder to confirm (see Benhabib, Schmitt-Grohe, Uribe)
- Can show that

$$P^{-1} = \begin{bmatrix} 1 & 0 & \frac{\beta}{\alpha\beta-\rho_\theta} & 0 \\ \frac{\alpha\beta\varphi_1-\varphi_2}{\alpha\beta-(\beta^{-1}-\gamma)} & 1 & \frac{1}{\beta^{-1}-\gamma-\rho_\theta}\left[\frac{\alpha\beta^2\varphi_1-\beta\varphi_2}{\alpha\beta-(\beta^{-1}-\gamma)}+\rho_\theta\varphi_3-\beta\varphi_1+\varphi_4\right] & \frac{\rho_\psi}{\rho_\psi-(\beta^{-1}-\gamma)} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The first two rows of this matrix give us the stability conditions associated with Regions I and II, where unique equilibria exist

Active & Passive Policy Behavior

- An *active* policy authority is free to pursue its objectives, unconstrained by the state of government debt
  - decision rule may depend on past, current, or expected future variables
- A *passive* policy authority is constrained by the behavior of the active authority and the private sector and must be consistent with equilibrium
  - decision rule necessarily depends on state of government indebtedness, as summarized by current and past variables
- *Active* forward-looking & *passive* backward-looking consistent with Simon's "rule vs. discretion" perspective as put forth by Friedman and with Sargent-Wallace's terminology

Region I: Active MP & Passive FP

- When $|\alpha\beta| \geq 1$ and $|\beta^{-1} - \gamma| < 1$, the first row of P^{-1} is the eigenvector associated with the unstable eigenvalue
- Stability condition is

$$P^{1\cdot}Y_t = \begin{pmatrix} 1 & 0 & \frac{\beta}{\alpha\beta-\rho_\theta} & 0 \end{pmatrix} Y_t = 0, \qquad t = 0, 1, 2, ...$$

implying that in equilibrium

$$\begin{aligned} \pi_t &= -\frac{\beta}{\alpha\beta - \rho_\theta}\theta_t \\ E_t\pi_{t+1} &= -\frac{\beta\rho_\theta}{\alpha\beta - \rho_\theta}\theta_t \\ R_t &= -\frac{\rho_\theta}{\alpha\beta - \rho_\theta}\theta_t \end{aligned}$$

Region I: Active MP & Passive FP

- Surprise inflation determined by the mapping from ε to η :

$$P^{1\cdot}C\xi_t = \begin{pmatrix} 1 & 0 & \frac{\beta}{\alpha\beta-\rho_\theta} & 0 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\varphi_1 & 0 & \varphi_3 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_t \\ 0 \\ \varepsilon_{\theta t} \\ \varepsilon_{\psi t} \end{bmatrix} = 0$$

which implies

$$\eta_t = -\frac{\beta}{\alpha\beta - \rho_\theta}\varepsilon_{\theta t}, \qquad t = 1, 2, ...$$

- Inflation entirely a monetary phenomenon
- Tax disturbances do not affect inflation or interest rates
- What fiscal behavior enables MP to control price level?

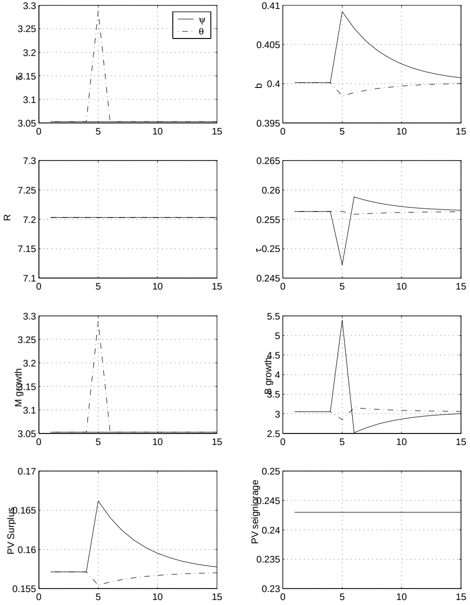
Region I: Active MP & Passive FP

- Equilibrium sequences of $\{\tau_t, b_t\}$ are determined by the (stable) difference equation in debt and the tax rule
- Because these sequences are irrelevant for inflation, the equilibrium exhibits Ricardian equivalence
- A cut in taxes due to a negative realization of ψ_t raises b_t, which raises future lump-sum taxes
- When the policy shocks are $i.i.d.$, equilibrium debt evolves according to

$$b_t = \left[\frac{1}{\alpha\beta\bar{\pi}}\left(\frac{\delta y}{\bar{R}-1} + \bar{b}\right)\right]\varepsilon_{\theta t} - \varepsilon_{\psi t} + \text{variables dated } t-1$$

- Higher ε_θ raises R_t, lowering m_t and raising b_t—this is an open-market sale—and through the tax rule, raises expected future taxes
- Although taxes appear to be irrelevant, tax policy is far from irrelevant, as it supports monetary policy

REGION I: ACTIVE MP & PASSIVE FP



Region II: Active FP & Passive MP

- When $|\alpha\beta| < 1$ and $|\beta^{-1} - \gamma| \geq 1,$ the second row of P^{-1} is the eigenvector associated with the unstable eigenvalue
- We focus on the special case in the FT literature
  - Assume $\alpha = \gamma = \rho_\theta = \rho_\psi = 0$
  - $\alpha = 0 \Rightarrow$ the nominal interest rate is exogenous
  - $\gamma = 0 \Rightarrow$ the net-of-interest fiscal surplus is exogenous
  - shocks don't change expected taxes—essential to FT
- The stability condition is

$$P^{2\cdot} Y_t = \begin{pmatrix} 0 & 1 & -\frac{\delta y}{(\bar{R}-1)^2} & 0 \end{pmatrix} Y_t = 0, \qquad t = 0, 1, 2, ...$$

  implying that

$$b_t = \frac{\delta y}{(\bar{R}-1)^2} \theta_t, \qquad t = 0, 1, 2, ...$$

- Shocks to taxes have no impact on the real value of government debt

REGION II: ACTIVE FP & PASSIVE MP

- How can it happen that tax shocks do not change the value of debt?
- Consider the mapping from ε to η :

$$P^{2\cdot}C\xi_t = \begin{pmatrix} 0 & 1 & -\frac{\delta y}{(\bar{R}-1)^2} & 0 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\varphi_1 & 0 & \varphi_3 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_t \\ 0 \\ \varepsilon_{\theta t} \\ \varepsilon_{\psi t} \end{bmatrix} = 0$$

$t = 1, 2, ...$ so

$$\eta_t = -\frac{1}{\varphi_1}\varepsilon_{\psi t}, \qquad t = 1, 2, ...$$

where $\varphi_1 > 0$

- A cut in taxes ($\varepsilon_{\psi t} < 0$) raises the forecast error and current inflation

REGION II: ACTIVE FP & PASSIVE MP

- Taking expectations conditional on information at time t, the intertemporal government budget constraint is

$$\frac{B_t}{p_t} = E_t \left\{ \frac{\pi_{t+1}}{R_t}[\tau_{t+1} + s_{t+1} - g] + \sum_{s=1}^{\infty} \left(\prod_{k=1}^{s} \pi_{t+k+1} R_{t+k}^{-1} \right) [\tau_{t+s+1} - s_{t+s+1} - g] \right\}$$

where s is seigniorage revenues

- First term on the right side involves θ_t and future (unrealized) shocks, while the second term involves only future shocks
- With $\rho_\theta = \rho_\psi = 0$, all future shocks are unanticipated, so conditional on information at t, only θ_t can affect the real value of government debt at t
- θ_t induces an open-market operation

Region II: Active FP & Passive MP

- How does a tax shock leave the real value of debt unchanged?
  - A surprise tax cut at t is financed by issuing new *nominal* government debt
  - Monetary policy pegs R_t, so it can't change $\Rightarrow$ no change in expected inflation (and seigniorage)
  - FP does not allow future taxes to change ($\gamma = 0$)
  - At the initial (pre-shock) prices and interest rates, the cut in taxes, with no prospect of higher future taxes, leaves households feeling wealthier
  - Higher perceived wealth leads households to try to raise their consumption paths
  - The increase in demand for consumption goods can only result in higher goods prices
  - The price level rises until the change in wealth disappears
  - In equilibrium, there is no change in real wealth and the complete impact of the tax cut is a rise in current inflation

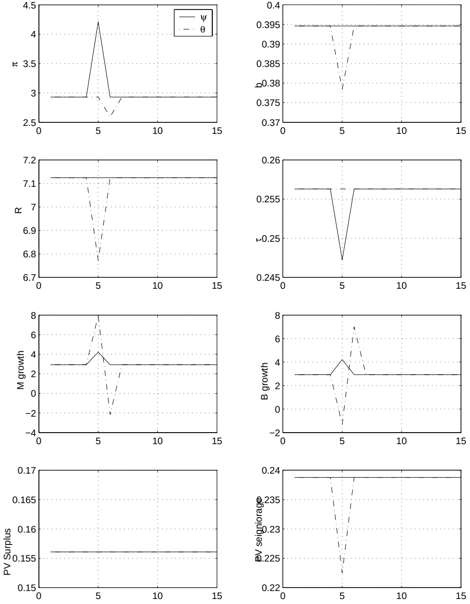
Region II: Active FP & Passive MP

- In this region the inflation process is stable and

$$\pi_t = \beta\varepsilon_{\theta t-1} - \frac{1}{\varphi_1}\varepsilon_{\psi t} \qquad t = 0, 1, 2, ...$$

- A monetary policy shock that raises R_t ($\varepsilon_{\theta t} > 0$) and has only a delayed effect on inflation
  - The delayed effect is "perverse" by conventional monetary standards, as a higher interest rate at t *raises* inflation at $t+1$
  - Another way to see this is consider the price-level effects of higher expected seigniorage (s_{t+s+1})
  - If no policies adjust at date t, under the current assumptions on policy behavior, p_t must *fall*
  - Of course, this is unpleasant monetarist arithmetic

REGION II: ACTIVE FP & PASSIVE MP



Region III: Passive FP & Passive MP

- When $|\alpha\beta| < 1$ and $|\beta^1 - \gamma| < 1$, there are no unstable eigenvalues
- Eqm is indeterminate and there are no restrictions imposed on η_{t+1}—any mapping from $(\varepsilon_{\theta,t+1}, \varepsilon_{\psi,t+1})$ to η_{t+1} is an eqm
- Eqm also admits bounded sunspot solutions
- Intuition: both monetary & fiscal policy are stabilizing debt
- Neither policy is attending to price level determination
- This—implicitly—is the policy region underlying the famous Sargent-Wallace (1975) result about indeterminacy under an interest rate peg (repeated in Sargent's textbook (1979,1987)
- As we have seen, if FP is active, the eqm is determinate

Wrap Up

- Price level determination is intrinsically about both monetary policy *and fiscal policy*
- P determination cannot be understood without bringing both macro policies into the picture
- Why care about P determination?
  - first step in understanding macro policy effects
- FTPL is one manifestation of a policy mix in which lump-sum tax shocks can affect P
- Under the FTPL
  - MP determines *expected* π
  - FP determines *realized* or *actual* π
- Price level indeterminacy can be an outgrowth of doubly passive macro policies