JEDC project on comparing numerical solutions of models with heterogenous agents. Notes on problem A

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I tried preliminary computations for all the 1116 specifications suggested by the description of Problem A, back in the July 2004 document. I report below on my findings and make some suggestions about which cases to consider in the final versions of the papers.

I'm using the following setup: a Python scripts to generate and run in Dynare++ all different specifications of problem A. I did a third order approximation and computed the approximation error for each equation of the model for 100 points along an ellipse in the state space. Then I looked at the maximum error accross all equations described in section 1.1, to try to distinguish interesting cases. I haven't yet computed approximation errors as described in section 1.5, nor for the points described in the July 2004.

The ellipse was calibrated with one standard deviations of the endogenous variables as computed by a first order approximation. On this ellipse, stochastic shocks (innovations of autocorrelated shocks) are set to 0.

1 Specification of the models and error computation

1.1 Equilibrium conditions

$$
\begin{aligned}
\tau_n u_c(c_t^n, l_t^n) &= \lambda_t \\
\tau_n u_l(c_t^n, l_t^n) &= -\lambda_t a_t^n f_l^n(k_{t-1}^n, l_t) \\
\lambda_t \left(1 + \phi\left(\frac{i_t^n}{k_{t-1}^n} - \delta\right)\right) &= \beta E_t \Bigg\{ \lambda_{t+1} \Bigg(1 + a_{t+1}^n f_k^n\left(k_t^n, l_{t+1}^n\right) \\
&\quad + \phi\left(1 - \delta + \frac{i_{t+1}^n}{k_t^n} - \frac{1}{2}\left(\frac{i_{t+1}^n}{k_t^n} - \delta\right)\right)\left(\frac{i_{t+1}^n}{k_t^n} - \delta\right)\Bigg)\Bigg\} \\
k_t^n &= i_t^n + (1-\delta)k_{t-1}^n \\
\sum_{n=1}^{N} c_t^n + i_t^n - \delta k_{t-1}^n &= \sum_{n=1}^{N} a_t^n f^n(k_{t-1}^n, l_t^n) - \frac{\phi}{2} k_{t-1}^n \left(\frac{i_t^n}{k_{t-1}^n} - \delta\right)^2
\end{aligned}
$$

$$\ln a_t^n = \rho \ln a_{t-1}^n + \sigma\left(e_t + e_t^n\right)$$

1.2 Utility specifications

- $U(c_t^n, l^n) = \frac{{c_t^n}^{1-\frac{1}{\gamma_n}}}{1-\frac{1}{\gamma_n}}$

$$U_c^n(c_t^n, l^n) = {c_t^n}^{-\frac{1}{\gamma_n}}$$

- $U(c_t^n, l^n) = \frac{{c_t^n}^{1-\frac{1}{\gamma_n}}}{1-\frac{1}{\gamma_n}} - b_n \frac{{l_t^n}^{1-\frac{1}{\eta_n}}}{1-\frac{1}{\eta_n}}$

$$\begin{aligned} U_c^n(c_t^n, l^n) &= {c_t^n}^{-\frac{1}{\gamma_n}} \\ U_l^n(c_t^n, l^n) &= -b_n {l_t^n}^{-\frac{1}{\eta_n}} \end{aligned}$$

- $U(c_t^n, l_t^n) = \frac{\gamma_n}{\gamma_n - 1}\left({c_t^n}^{\psi}\left(L^e - l_t^n\right)^{(1-\psi)}\right)^{1-\frac{1}{\gamma_n}}$

$$\begin{aligned} U_c^n(c_t^n, l^n) &= \psi {c_t^n}^{\frac{\psi(\gamma_n - 1)}{\gamma_n} - 1}\left(L^e - l_t^n\right)^{\frac{(1-\psi)(\gamma_n - 1)}{\gamma_n}} \\ U_l^n(c_t^n, l^n) &= -(1-\psi) {c_t^n}^{\frac{\psi(\gamma_n - 1)}{\gamma_n}}\left(L^e - l_t^n\right)^{\frac{\psi(1-\gamma_n) - 1}{\gamma_n}} \end{aligned}$$

- $U(c_t^n, l^n) = \frac{\gamma_n}{\gamma_n - 1}\left({c_t^n}^{1-\frac{1}{\chi_n}} + b_n\left(L^e - l_t^n\right)^{1-\frac{1}{\chi_n}}\right)^{\frac{1-\frac{1}{\gamma_n}}{1-\frac{1}{\chi_n}}}$

$$\begin{aligned} U_c^n(c_t^n, l^n) &= {c_t^n}^{-\frac{1}{\chi_n}}\left({c_t^n}^{1-\frac{1}{\chi_n}} + b_n\left(L^e - l_t^n\right)^{1-\frac{1}{\chi_n}}\right)^{\frac{1-\frac{1}{\gamma_n}}{1-\frac{1}{\chi_n}} - 1} \\ U_l^n(c_t^n, l^n) &= -b_n\left(L^e - l_t^n\right)^{-\frac{1}{\chi_n}}\left({c_t^n}^{1-\frac{1}{\chi_n}} + b_n\left(L^e - l_t^n\right)^{1-\frac{1}{\chi_n}}\right)^{\frac{1-\frac{1}{\gamma_n}}{1-\frac{1}{\chi_n}} - 1} \end{aligned}$$

1.3 Production function specifications

- $f^n(k_{t-1}^n, l_t^n) = A {k_{t-1}^n}^{\alpha}$

$$f_k^n(k_{t-1}^n, l_t^n) = \alpha A\left(\frac{k_{t-1}^n}{l_t^n}\right)^{\alpha - 1}$$

- $f^n(k_{t-1}^n, l_t^n) = A {k_{t-1}^n}^{\alpha} {l_t^n}^{\alpha - 1}$

$$\begin{aligned} f_k^n(k_{t-1}^n, l_t^n) &= \alpha A\left(\frac{k_{t-1}^n}{l_t^n}\right)^{\alpha - 1} \\ f_l^n(k_{t-1}^n, l_t^n) &= (1-\alpha) A\left(\frac{k_{t-1}^n}{l_t^n}\right)^{\alpha} \end{aligned}$$

- $f^n(k^n_{t-1}, l^n_t) = A\left(\alpha {k^n_{t-1}}^{\mu_n} + (1-\alpha){l^n_t}^{\mu_n}\right)^{\frac{1}{\mu_n}}$

$$
\begin{aligned}
f^n_k(k^n_{t-1}, l^n_t) &= \alpha A {k^n_{t-1}}^{\mu_n - 1}\left(\alpha {k^n_{t-1}}^{\mu_n} + (1-\alpha){l^n_t}^{\mu_n}\right)^{\frac{1}{\mu_n}-1} \\
f^n_l(k^n_{t-1}, l^n_t) &= (1-\alpha) A {l^n_t}^{\mu_n - 1}\left(\alpha {k^n_{t-1}}^{\mu_n} + (1-\alpha){l^n_t}^{\mu_n}\right)^{\frac{1}{\mu_n}-1}
\end{aligned}
$$

1.4 Deterministic steady state

$$
\begin{aligned}
\bar{k} &= 1 \\
\bar{l} &= 1 \\
f^n(\bar{k}, \bar{l}) &= A \\
\bar{i} &= \delta \\
\bar{c}^n &= A \\
\lambda &= 1 \\
\tau_n &= \frac{1}{u_c\left(\bar{c}, \bar{l}\right)} \\
b_n &= (1-\alpha) A^{1-\frac{1}{\gamma_n}} \qquad \text{(S2)} \\
b_n &= (1-\alpha)\frac{A^{1-\frac{1}{\chi_n}}}{\left(L^e - \bar{l}\right)^{-\frac{1}{\chi_n}}} \qquad \text{(S4)} \\
\psi &= \frac{1}{L^e - \alpha\left(L^e - \bar{l}\right) \qquad (S3)}
\end{aligned}
$$

1.5 Approximation errors

$$
\begin{aligned}
R_j &= \frac{\tau_n u_c(c^n_t, l^n_t) - \lambda_t}{\tau_n u_c(c^n_t, l^n_t)} \qquad j = 1, \ldots, n \\
R_j &= \frac{\tau_n u_l(c^n_t, l^n_t) + \lambda_t a^n_t f^n_l(k^n_{t-1}, l_t)}{\tau_n u_l(c^n_t, l^n_t)} \qquad j = n+1, \ldots, 2n \\
R_j &= \Bigg[\lambda_t\left(1 + \phi\left(\frac{i^n_t}{k^n_{t-1}} - \delta\right)\right) - \beta E_t\Bigg\{\lambda_{t+1}\Bigg(1 + a^n_{t+1} f^n_k\left(k^n_t, l^n_{t+1}\right) \\
&\quad + \phi\left(1 - \delta + \frac{i^n_{t+1}}{k^n_t} - \frac{1}{2}\left(\frac{i^n_{t+1}}{k^n_t} - \delta\right)\right)\left(\frac{i^n_{t+1}}{k^n_t} - \delta\right)\Bigg)\Bigg\}\Bigg] / \left(\lambda_t\left(1 + \phi\left(\frac{i^n_t}{k^n_{t-1}} - \delta\right)\right)\right)^{-1} \\
&\qquad j = 2n+1, \ldots, 3n \\
R_j &= \frac{k^n_t - i^n_t - (1-\delta)k^n_{t-1}}{k^n_t} \qquad j = 3n+1, \ldots, 4n \\
R_{4n+1} &= \frac{\sum_{n=1}^{N} c^n_t + i^n_t - \delta k^n_{t-1} - a^n_t f^n(k^n_{t-1}, l^n_t) + \frac{\phi}{2} k^n_{t-1}\left(\frac{i^n_t}{k^n_{t-1}} - \delta\right)^2}{\sum_{n=1}^{N} c^n_t + i^n_t - \delta k^n_{t-1}}
\end{aligned}
$$

$$R_j = \frac{a_t^n - e^{\rho \ln a_{t-1}^n + \sigma(e_t + e_t^n)}}{a_t^n} \qquad j = 4n+2, \ldots, 5n+1$$

1.6 Remark

For specification A3, A4, A7 and A8, it isn't possible to express the Euler equation in consumption units. For this reason, I suggest that we report the error of approximation that corresponds to the above equations.

2 Suggestions for diminishing the number of cases to study

1. Specification A2 and A6 generate unit roots for almost all combinations of parameters. This is the only one. It makes it almost impossible to do long simulations. I suggest to drop specifications A2 and A6.
2. As expected the size of approximation errors increases with the variance of the exogenous variables. I suggest to limit ourselves to two polar cases: $(\sigma = 0.001, \rho = 0.8)$ and $(\sigma = 0.01, \rho = 0.95)$.
3. I couldn't detect any clear pattern concerning the effect of the number of countries on the approximation error. I suggest to have only $N = 2, 6, 10$.
4. The shape and adjustment cost parameters $gamma, \phi$ have an ambiguous effect for this experiment: increasing the curvature of the problem increases the difficulty of approximation but at the same time diminishes the variance of most endogenous variables and diminishes the size of the ellipse on which the approximation errors are computed. Maybe we should think of some other way of specifying those ellipses: compute them once for all specification. In any case, I suggest to study only extreme cases: $\gamma = 0.25, 1$, $\phi = 0.5, 10$.
5. Problem A1 is too easy and will serve only as a benchmark, I would keep only the logarithmic utility case.
6. In summary, number of cases:

$Model$		$Cases$
$A1$	$2 \times 2 \times 3$	12
$A3$	$2 \times 2 \times 2 \times 3$	24
$A4$	$2 \times 2 \times 3$	12
$A5$	$2 \times 2 \times 3$	12
$A7$	$2 \times 2 \times 3$	12
$A8$	$2 \times 2 \times 3$	12
$Total$		84

There may still be too many combinations of variance, persistence and adjustment cost.

3 Remark on the accuracy tests

1. For accuracy test 1, we should concentrate on $r \in \{0.01, 0.1, 0.3\}$.

2. It may also be interesting to compute the error of approximation on ellipses computed from the (first order approximated) variance of the variables. Then the scale a could $s \in \{1, 2, 3\}$ the standard deviation in the direction of each variable.

3. For accuracy test 2, we should run sequences with T=1000 if feasible, otherwise the maximum feasible.

4. For the DenHaan–Marcet statistic, we should compute it according to the specification in the July 2004 document and be prepared to compute a "surprise" specification just after the August conference.