Using a higher perturbation method to solve complete market macrodynamic models

Michel Juillard

August 30, 2007

Method

- Compute Taylor expansion of decision functions from derivatives of original model
- Automatic differentiation to compute derivatives of original model
- Numerical algorithm using Faa di Bruno formula for the derivatives of composition of functions
- Same approach as Kim, Kim and Kollmann. Details may vary. Lesser use of symbolic algebra than Anderson, Levin, Swanson Mathematica implementation or Hehui Jin's program.
- Use dynare++ rewritten by Ondra Kamenik

Details of implementation

- Uses logarithm of total factor productivity (provides linear functions for autoregressive processes)
- Computes approximation in the variables themselves, not the log of the variables
- dynare(++) can't handle separately exogenous processes. Makes for unnecessarily large state space.

Model implementation

$$
\begin{aligned}
\frac{\tau^n U_c\left(c_t^n, \ell_t^n\right)}{\tau^1 U_c\left(c_t^1, \ell_t^1\right)} &= 1 \quad n = 2, \ldots, N \\
\frac{U_c\left(c_{t+1}^n, \ell_{t+1}^n\right) \beta \left(1 + exp\left(a_{t+1}^n\right) f_k\left(k_t^n, \ell_{t+1}^n\right) - \phi_2\left(k_{t+1}^n, k_t^n\right)\right)}{U_c\left(c_t^n, \ell_t^n\right) + \phi_1\left(k_t^n, k_{t-1}^n\right)} &= 1 \quad n = 1, \ldots, N \\
\frac{U_c\left(c_t^1, \ell_t^1\right) exp\left(a_t^n\right) f_\ell\left(k_{t-1}^n, \ell_t^n\right)}{U_\ell\left(c_t^n, \ell_t^n\right)} &= -1 \quad n = 1, \ldots, N \\
\frac{\sum_{n=1}^N exp\left(a_t^n\right) f\left(k_{t-1}^n, \ell_t^n\right) + k_{t-1}^n}{\sum_{n=1}^N c_t^n + k_t^n + \phi\left(k_t^n, k_{t-1}^n\right)} &= 1 \quad n = 1, \ldots, N \\
a_t^n &= \rho a_{t-1}^n + \sigma\left(e_t + e_t^n\right) \quad n = 1, \ldots, N
\end{aligned}
$$

Tests

Only three currently implemented:

- maximum error on 100 points on a sphere ± 0.3 of the steady state ($T_{0.3}$)
- maximum error on 1000 simulated points (S_{max})
- mean absolute error on 1000 simulated points (S_{mean})

Remarks

- State space points and shocks should be identical to compare accross methods
- The number of test points should increase with the number of countries

Timing

Two dual-core processors, 3Ghz

Time to compute the solution in seconds

Order 2

N	2	4	6	10
A1	0.02	0.04	0.08	0.15
A2	0.03	0.06	0.08	
A3	0.03	0.06	0.11	
A4	0.03	0.07	0.12	

Order	4	6
N	6	2
A1	13.5	37.5
A2	24.7	61.5
A3	25.4	62.6
A4	25.3	63.2

Comparisons

$\gamma \in (0.25, 1)$ $\eta \in (0.1, 0.2)$ $\sigma = 0.01$ $\phi = 0.5$ $\rho = 0.95$

		S_{max}	S_{mean}	S_{max}	S_{mean}
Model	N	JK	KKK	JK	KKK
A5	2	2.3(-4)	-5.37	6.6(-6)	-6.20
A5	10	8.9(-5)	-5.60	1.2(-5)	-6.30
A6	2	5.5(-4)	-5.33	1.6(-5)	-6.16
A6	6	8.4(-4)	-5.61	2(-5)	-6.23
A7	2	1.1(-3)	-3.03	2.3(-5)	-4.00
A7	6	1.3(-3)	-2.84	3.7(-5)	-3.55
A8	2	1.2(-3)	-3.23	2.5(-5)	-4.17
A8	6	6.1(-3)	-3.05	3.8(-4)	-3.75

- approx. in level (JK) versus log (KKK)
- simple recursion (JK) pruning (KKK)
- KKK has an equation for welfare
- Euler equation error defined differently

Comparison

$\gamma \in (0.25, 1)$ $\eta \in (0.1, 1)$ $\sigma = 0.01$ $\phi = 10$ $\rho = 0.95$

		JK			MM			KKM		
Model	N	$T_{0.3}$	S_{max}	S_{mean}	$T_{0.3}$	S_{max}	S_{mean}	$T_{0.3}$	S_{max}	S_{mean}
A5	2	2.3(-3)	3.2(-4)	1.7(-5)	5(-3)	5(-4)	2(-4)	-4.0	-5.1	-5.6
A5	10	2(-3)	3(-4)	3.3(-5)	1(-2)	2(-3)	3(-4)			
A6	2	2.1(-3)	2.6(-4)	1.7(-5)				-3.9	-4.9	-5.6
A6	6	1.5(-3)	2.4(-4)	1.9(-5)				-3.9	-4.4	-4.5
A7	2	2.1(-3)	2.6(-4)	1.7(-5)				-3.6	-4.2	-4.8
A8	2	8.4(-3)	4.2(-4)	4.5(-5)	7(-3)	7(-4)	2(-4)	-3.6	-4.1	-4.6
A8	4	1.2(-1)	2.7(-3)	4(-4)				-3.7	-4.2	-4.7
A8	6	1.4(4)	1.3(-2)	1.8(-3)	1(-2)	2(-3)	4(-4)			

Problem with the tests

- The tests are more demanding for a small number of country
- To keep the same density of test points:
  - 2 countries, 4 state variables
  - the hypersphere is dimension 3
  - $100^{1/3} = 4.64$ in each dimension
  - 10 countries, 20 state variables
  - we would need $100^{20/3} = 2.1(13)$ points