Finite Forecasting Function Solutions to the Stochastic Growth Model

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Heterogeneity and Macrodynamics
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Main Idea

- Transition for actual state is infinite-dimensional:

$$\Gamma_{t+1} = H(\Gamma_t, a_t, a_{t+1})$$

- Prices are only functions of m_1:

$$m_{1,t+1} = H_1(\Gamma_t, a_t, a_{t+1})$$

- Project onto lower dimension space:

$$m_{1,t+1} = \widehat{H}_1(m_{1,t}, a_t, a_{t+1})$$

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Algorithm

- Guess law of motion:

$$\log\left(m_1'\right) = A\left(a\right) + B\left(a\right)\log\left(m_1\right)$$

- Solve household problem
- Simulate
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Solving the Household Problem

$$v(k, l, m_1, a,) = \max_{k' \geq 0} \left\{ u(c) + \beta E\left[v\left(k', l', m_1', a'\right) | l, m_1, a \right] \right\}$$

- Value iteration with Howard's improvement
- Cubic and linear splines for value function
- Feasible sequential quadratic programming method for maximization

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Simulation Procedure

- Nonstochastic simulation procedure
- Store distribution as vector of point masses
- Each period redistribute mass at point (k, l) according to transition probabilities

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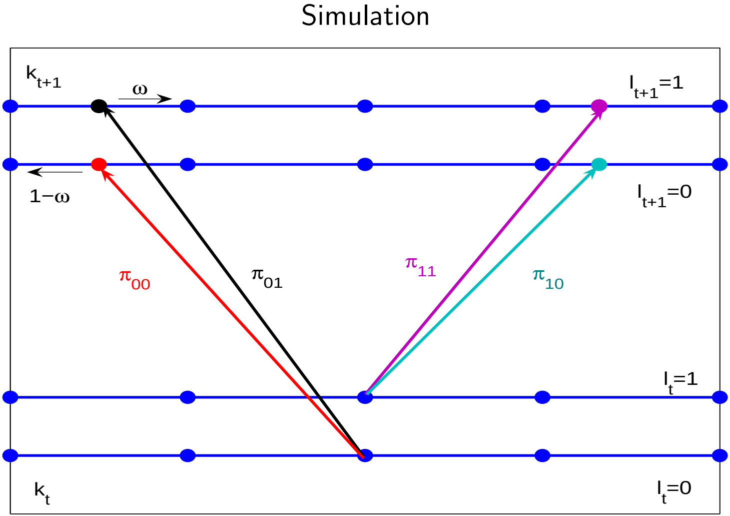


Stationary Economies

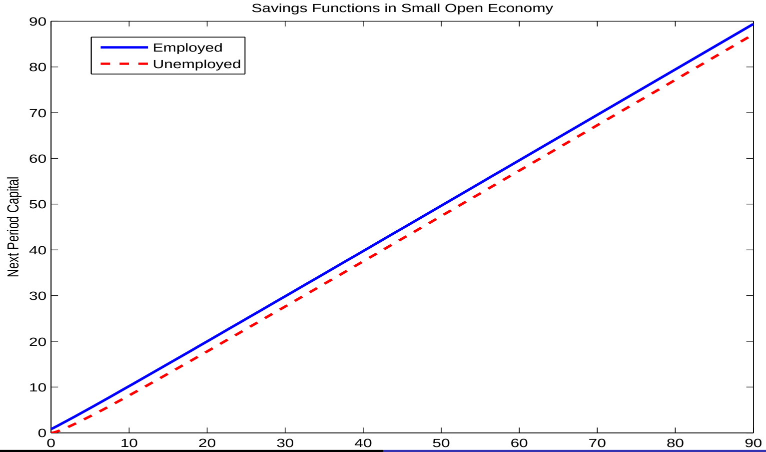
Table 1

Stationary Distributions

Economy	$m(1)$	$m(2)$	$m(3)$	$m(4)$	Bind
Small Open	10.995	0.344	-0.266	0.442	0.2
Aiyagari	37.678	0.478	0.467	0.691	0.2

Decision Rules, Small Open Economy

Savings



Decision Rules, Aiyagari Economy

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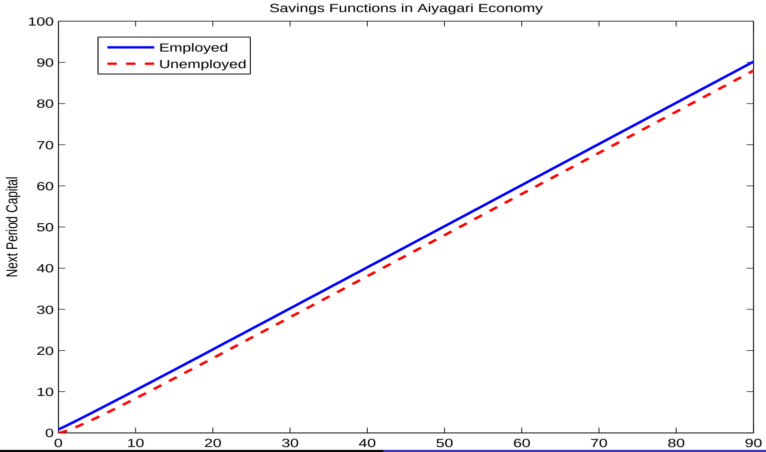
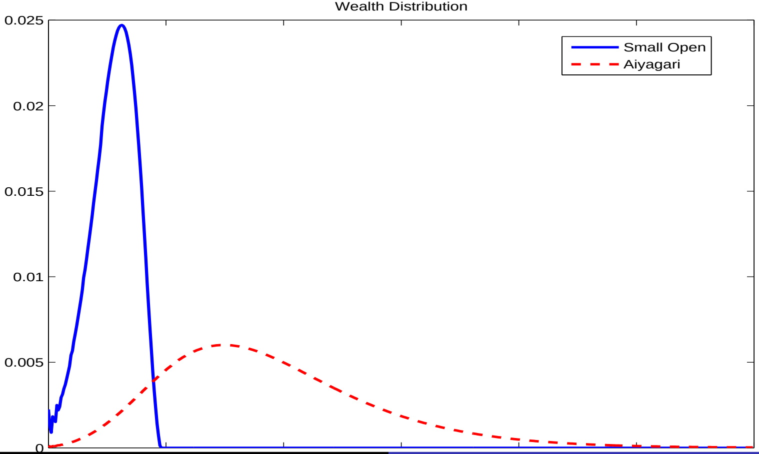


Wealth Distributions

Savings



Accuracy Tests

- Aggregate Law of Motion

$$\begin{aligned} \log\left(m_1'\right) &= 0.132 + 0.965\log\left(m_1\right) \\ R^2 &= 0.99998, \widehat{\sigma} = 0.0001, e_{\max} = 0.00002 \end{aligned}$$

$$\begin{aligned} \log\left(m_1'\right) &= 0.123 + 0.966\log\left(m_1\right) \\ R^2 &= 0.99999, \widehat{\sigma} = 0.0001, e_{\max} = 0.00003 \end{aligned}$$

- den Haan Accuracy Tests

Max Error, Simulation	1.23%
Average Error, Simulation	0.12%
Max Error, Impulse	3.16%
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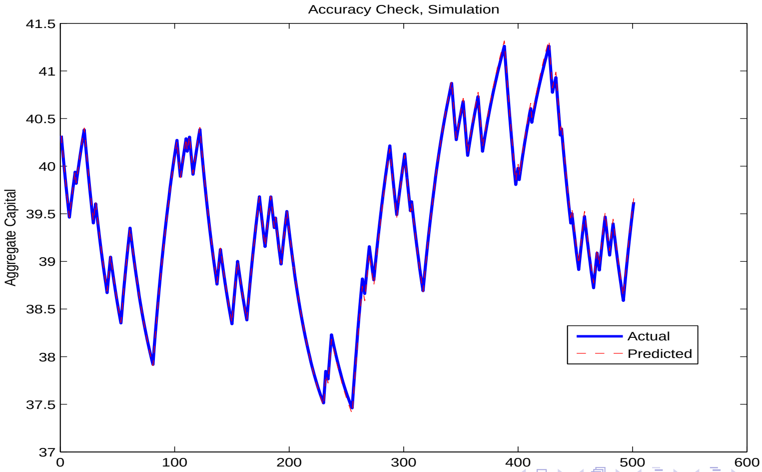

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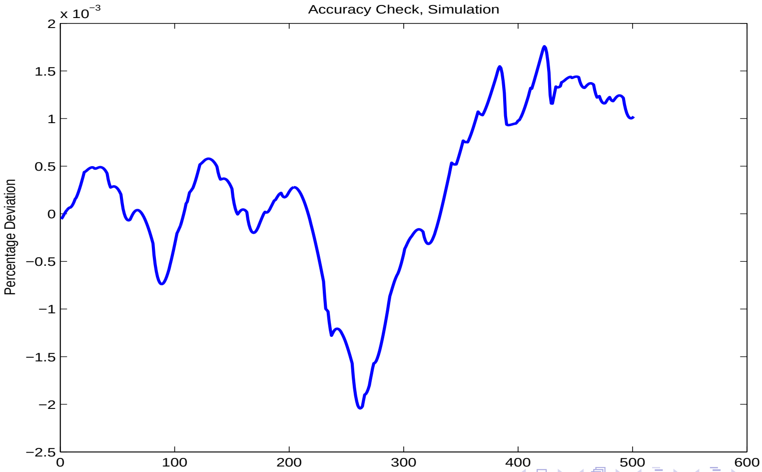

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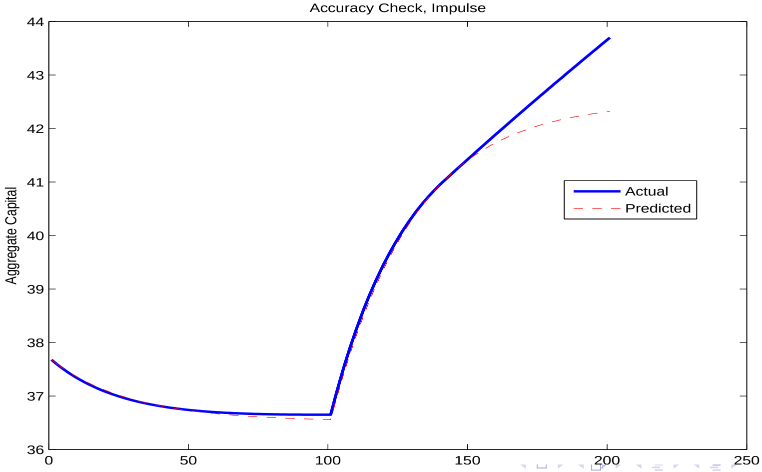

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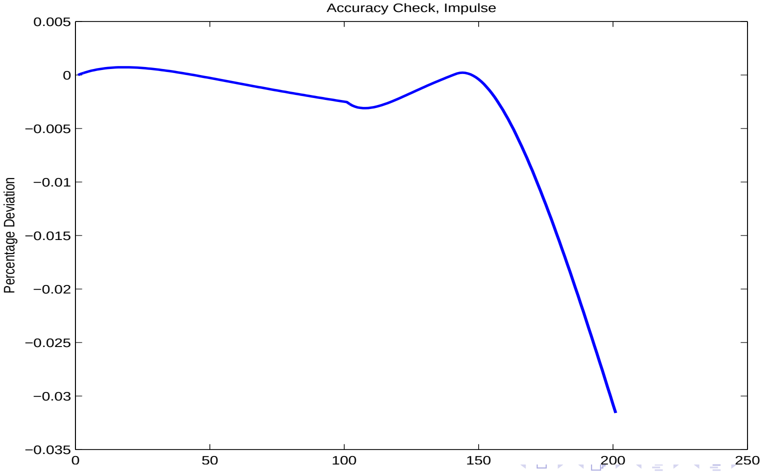

Cross-Sectional Moments

Risk Sharing Properties

x	σ_x	$\rho(x_t, C_t)$	$\rho(x_t, Y_t)$	$\rho(x_t, K_t)$	$\rho(x_t, y_t)$	$\rho(x_t, k_t)$
c_t	0.259	0.251	0.176	0.239	0.972	0.970
k_t	21.465					

x	$\rho(x_t, x_{t-1})$	$\rho(x_t, x_{t-2})$	$\rho(x_t, x_{t-3})$
c_t	0.995	0.990	0.986
k_t	1.000	0.999	0.998

Wealth Distribution

Wealth Distribution

Const	Const(g)	Const(b)	5%	10%
0.011%	0.005%	0.017%	1.3%	3.2%
$m(1)$	$m(2)$	$m(3)$	$m(4)$	$m(5)$
39.431	0.543	0.611	0.864	1.032

Algorithm

- Guess value function $v^0(b, y, a)$
- Solve household problem

$$\widehat{v}(b, y, a, q) = \max_{b' \geq \underline{b}} \left\{ u(b + ay - qb') + \beta E\left[v^n(b', y', a') \,|y, a \right] \right\}$$

- Simulate, solving equation for q_t at each t:

$$\int b'(b, a, y, q_t)\, \Gamma_t(b, y) = 0$$

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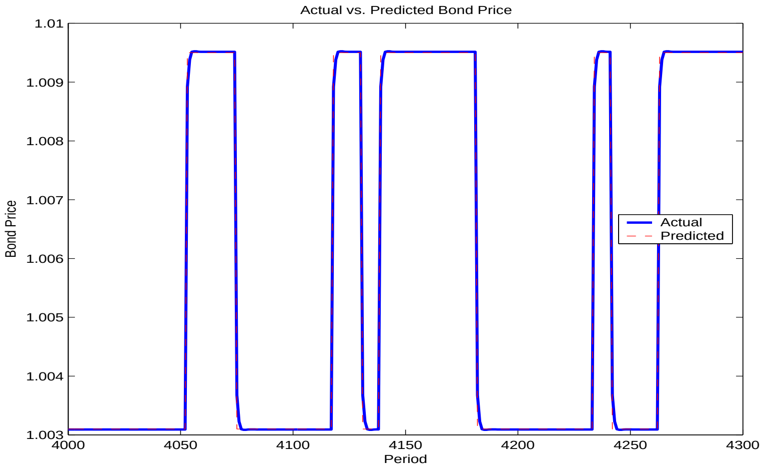

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