Note Title 10/18/2005

Rational expectations models

$y^t = y_t, y_{t-1}, \dots$

y^∞ ;

$f(y^\infty; \theta)$;

Whose model is it? ||| Communism

Agents in model – while agents

Planner or the govt

Nature or God , econometrician

→ Disappearance of expectations from macro theory. "impure - non Communist RE"

Wilderness of "bounded rationality"

- Sims
- Bray & David Kreps

discipline is lost. "Arrow & the Ascent of Economic Theory, G. Feiwel 1988 or 89"

data

Why go into the wilderness?

1. To understand what rational expectations is about. Least squares learners do converge to a "type" of rational expectations equilibrium. (Communism).

Macroeconomic literature - Evans & Honkapohja 2003. [Marcet & S.]

Theory of Games - Fudenberg & Levine, 1999.

Evans & H. – select among r.e.

select among ways of expressing policy rules. – refining RE equilibria.

Fudenberg & Levine – Adaptive learners ⟶

Self-confirming equilibrium

Learning $\Rightarrow$ eventual agreement on equilibrium path, but not off. RE imposes agreement everywhere.

Practical issue. Disputes in macroeconomic theory.

Want multiple models in hands of policy maker(s). —

Multiple models & self-confirming equilibrium.

LLN.

Example: Model

Multiple models.

KP time consistency example - Phillip curve.

- Rational expectation version
- Adaptive version - (1977 KP)

KP model in Stokey notation.

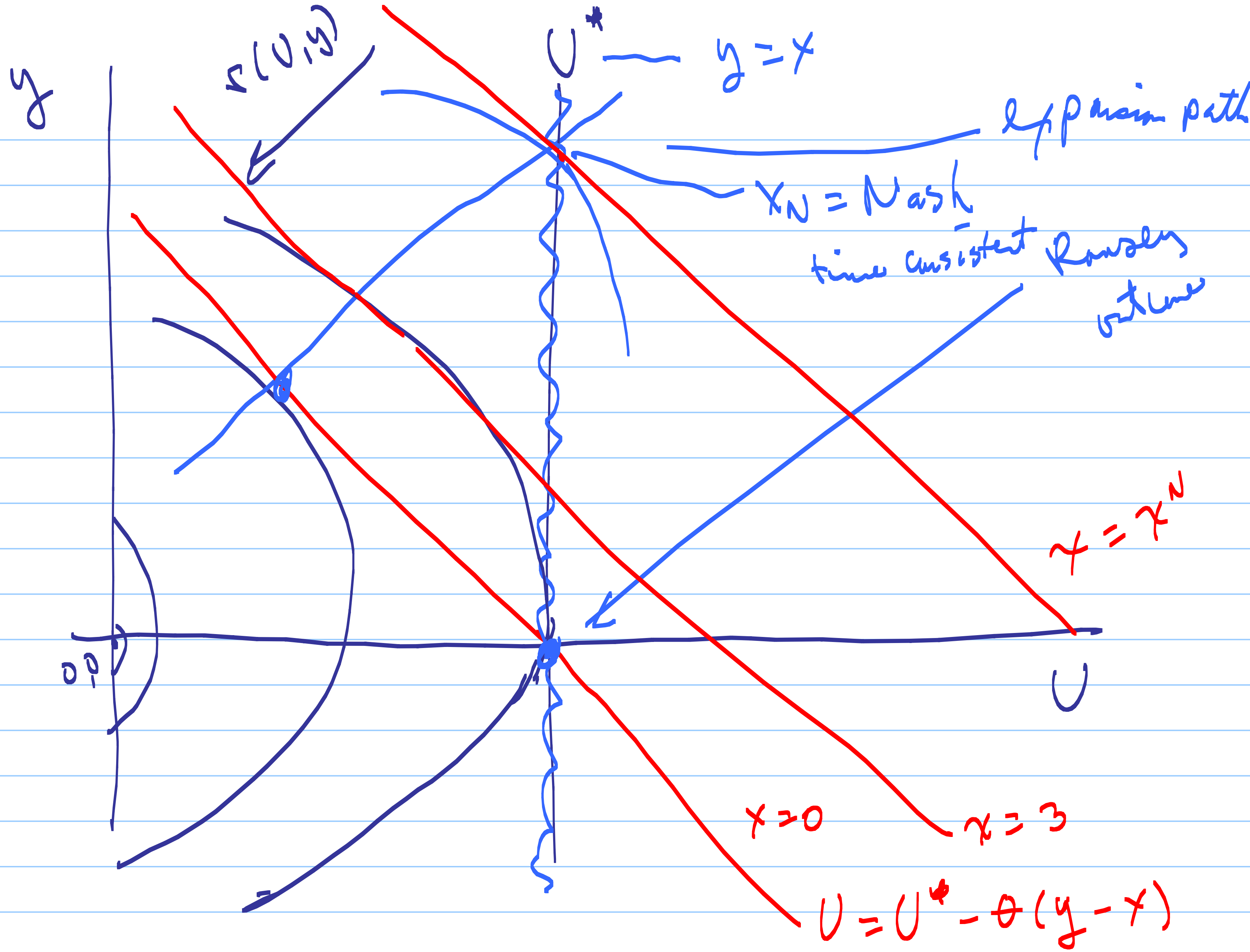
y - set by govt
inflation rate

x - set by public - private agents
expected infl'n rate.

$$U = U^* - \theta(y - x) \quad , \quad \tfrac{1}{2}(U^2 + y^2)$$

U = unempl^t , U^e = natural rate

$$r(v, y) = -\frac{1}{2}(v^2 + y^2)$$



y
r(U,y)
U*
y = x
expansion path
x_N = Nash
time consistent
Ramsey outcome
x = x^N
0
U
x = 0
x = 3
U = U* − θ(y − x)

Timing Protocols:

y - govt

x - public

$x = y$ rational expectations

if the govt goes first & sets y, public's best response is to set $x = y$.

$\min (x-y)^2$.

Conflict.

Protocol 1: govt commits (goes first)

y, $\longrightarrow$ $x = x(y) \geq y$.

Protocol 2: private agents set x "arbitrarily"

then govt chose $y = y(x)$ best

response.

Learning model: 1990's adaptive expectations

"Consistency proof"

1 model , $f(y^t, \theta)$

estimator $\hat{\theta} = g(y^t)$

show $\hat{\theta} \xrightarrow[??]{} \theta$ $\boxed{\text{when } f(y^t; \theta) \text{ is true}}$

Sims - White: Kullback-Leibler information criterion -

Model(s)

truth $\qquad F(y^t, \delta)$

my model $\qquad f(y^t, \theta) \rightarrow \hat{\theta} = g(y^t)$

"consistency" proof:

$$\hat{\theta} \longrightarrow \tilde{\theta}(\delta, F) \quad \text{when } F \text{ is true.}$$

$\hat{\theta}$ minimizes $\int \ln \frac{f}{F} \, F$ KL.

limiting behavior of misspecified parameter estimates

Truth: $$U_t = U^* - \theta(y_t - x_t) + \varepsilon_{1t}$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$y_t = x_t + \varepsilon_{2t}$$

the government set x_t on the basis of its information at $t-1$.

$\rightarrow \quad U_t = U^* - \theta(y_t - x_t) + z_{1t}$

Gort's model:

smoothes x_t

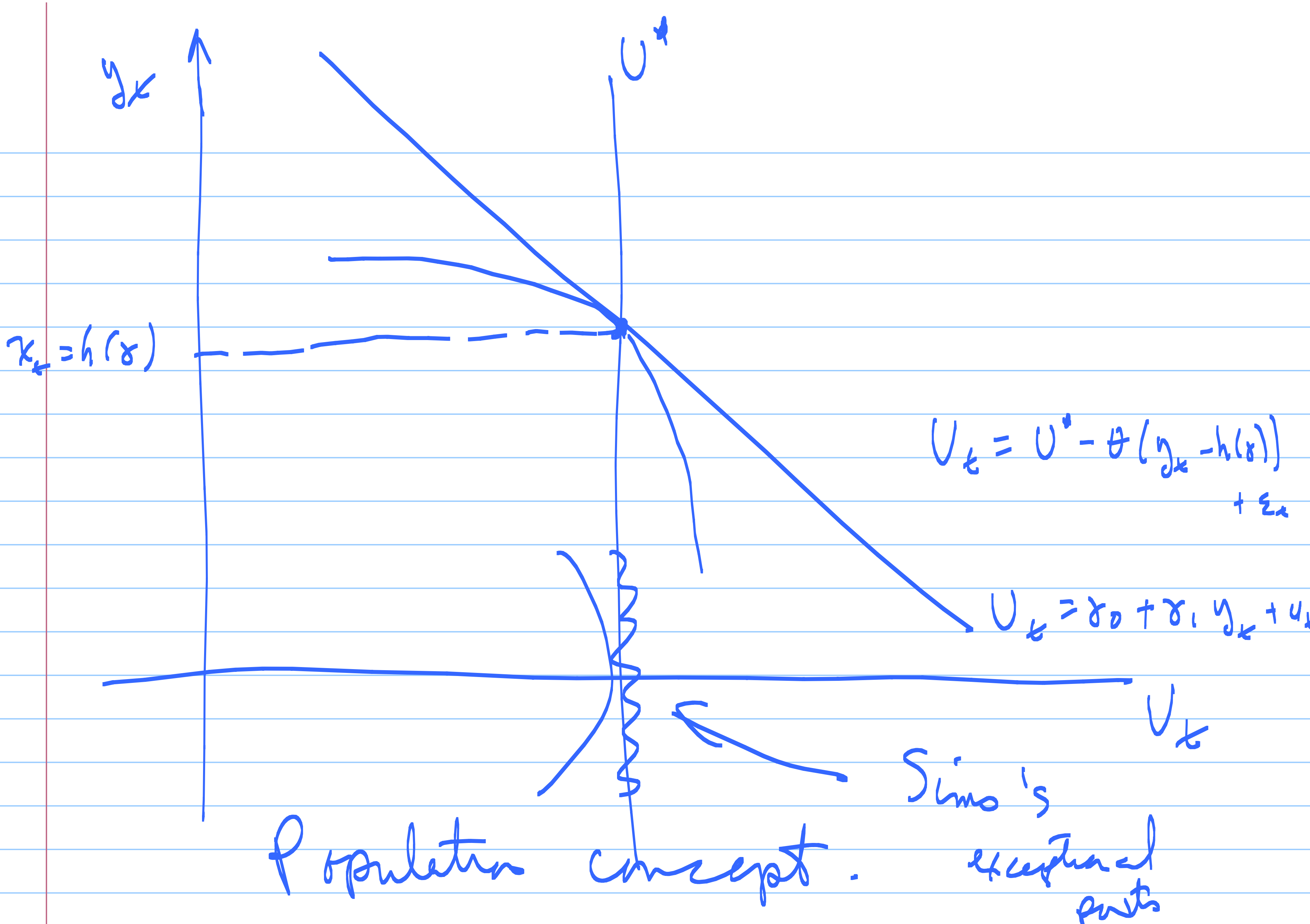
$$\rightarrow \quad U_t = \gamma_0 + \gamma_1 y_t + u_t$$

$$\leftarrow \quad u_t \perp y_t \quad , u_t \sim N(0, \sigma_u^2)$$

$$E[(U_t - \gamma_0 - \gamma_1 y_t)\begin{pmatrix}1\\ y_t\end{pmatrix}] = 0 \quad \| \quad \int \quad x_0 + \frac{1}{T}\sum_{t=1}^{T}$$

$$E \sum \beta^t r(U_t ; y_t) = E \sum_{t=0}^{\infty} \beta^t (U_t^2 + y_t^2)$$

solve this problem: $\quad x_t = h(\gamma) \quad , \gamma = [\gamma_0, \gamma_1]$



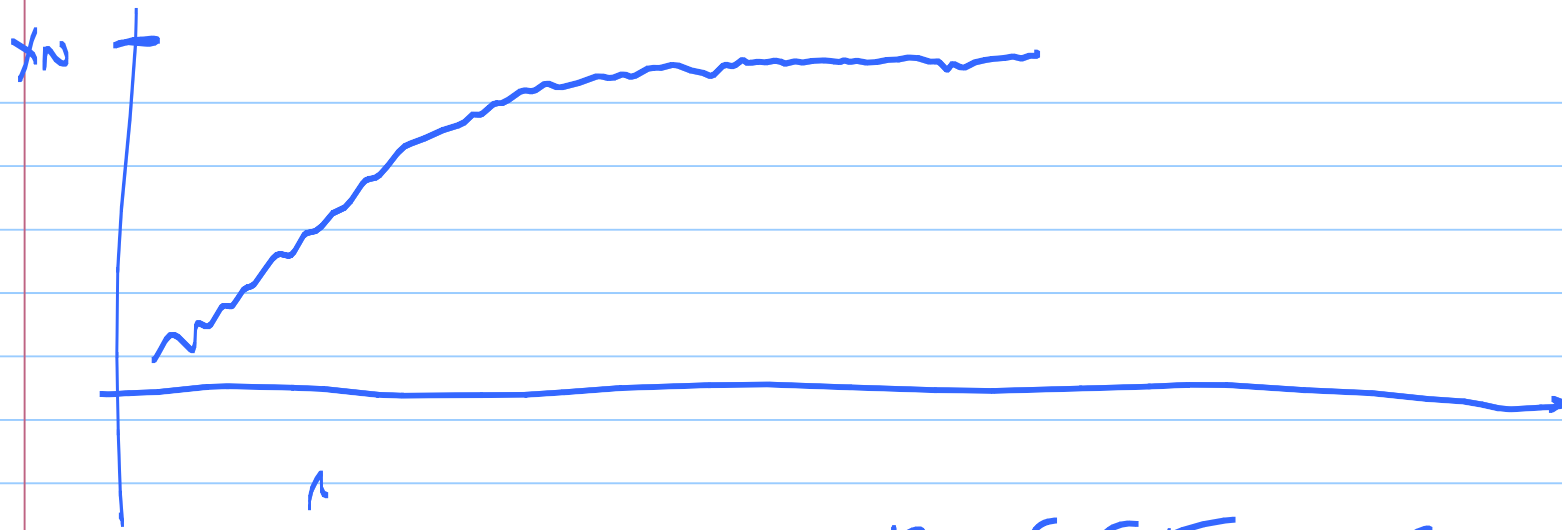
Population concept.

Learning model

- ~~Population moments~~ replace with sample moments up to t.

- $x_t = h(\gamma_t)$ $\quad$ h pretends the coefficients are known.

$\gamma_t = \gamma_{t-1}$ + revision $\leftarrow$ least squares.



$\hat{\gamma}_t \to \gamma$

The SCE γ.

E It theorem show this.

MS.

Learning convergence theorems: in a nutshell

$$(*) \qquad \gamma_t = \gamma_{t-1} + \text{junk}(\text{true model}, \text{govt models}, \text{randomness at } t)$$

↑ awful stochastic difference equation

stochastic approximation: limiting behavior of $(*)$ is governed by an ODE.

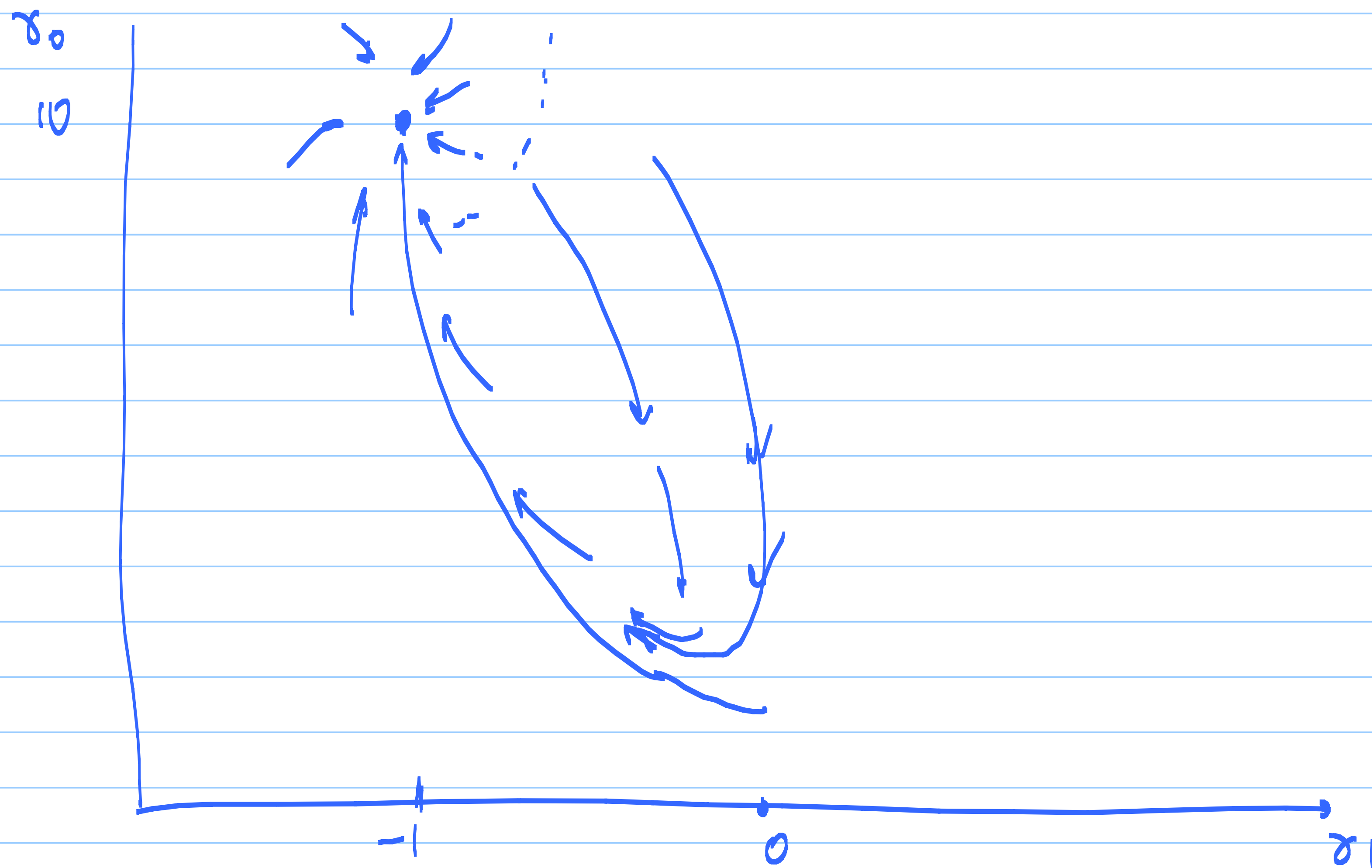
$$\begin{bmatrix} \dot{\gamma} \\ \dot{R} \end{bmatrix} = B \begin{bmatrix} \gamma \\ R \end{bmatrix}$$

R is the moment matrix of the data

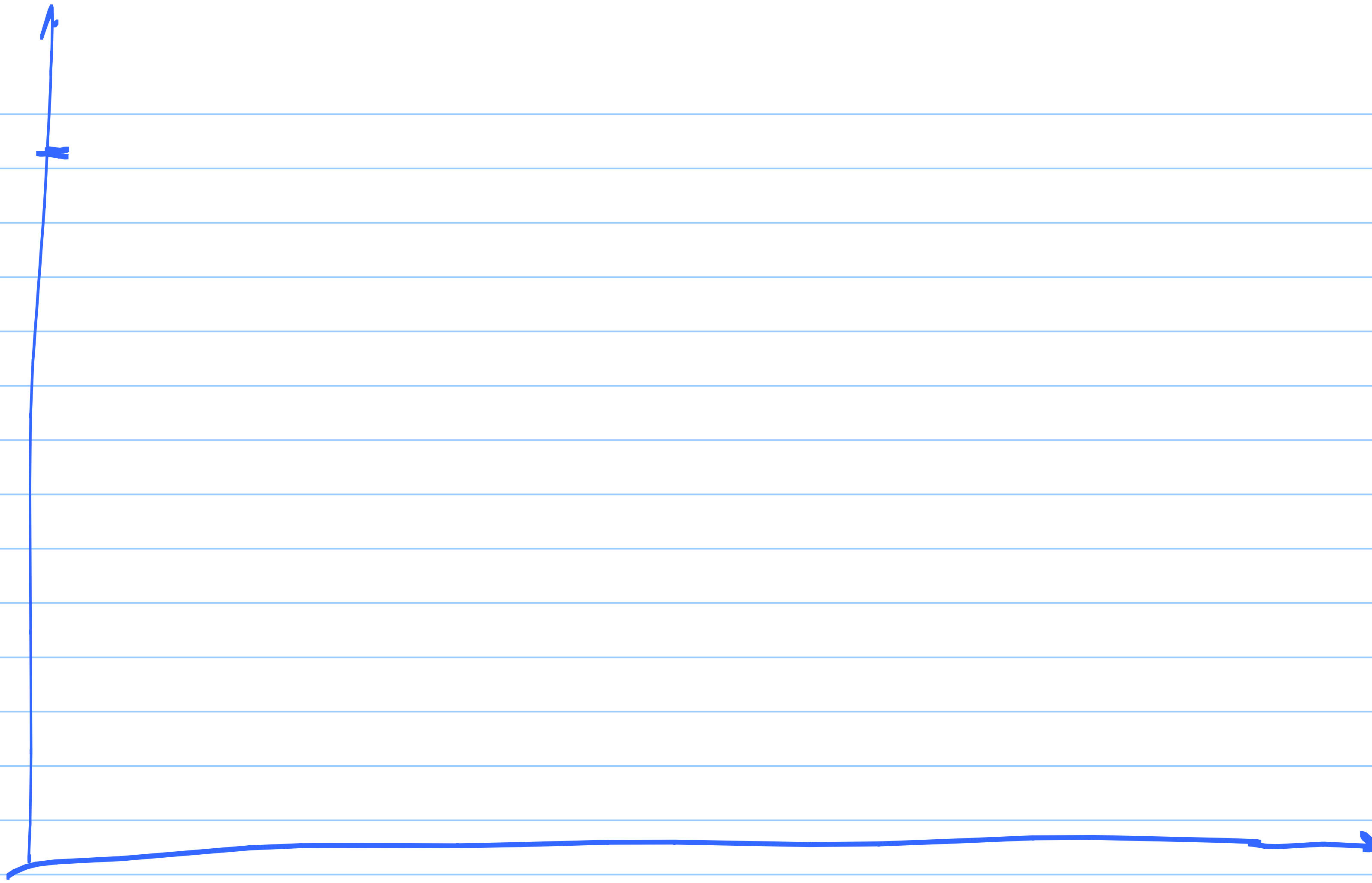
$\theta = 1$

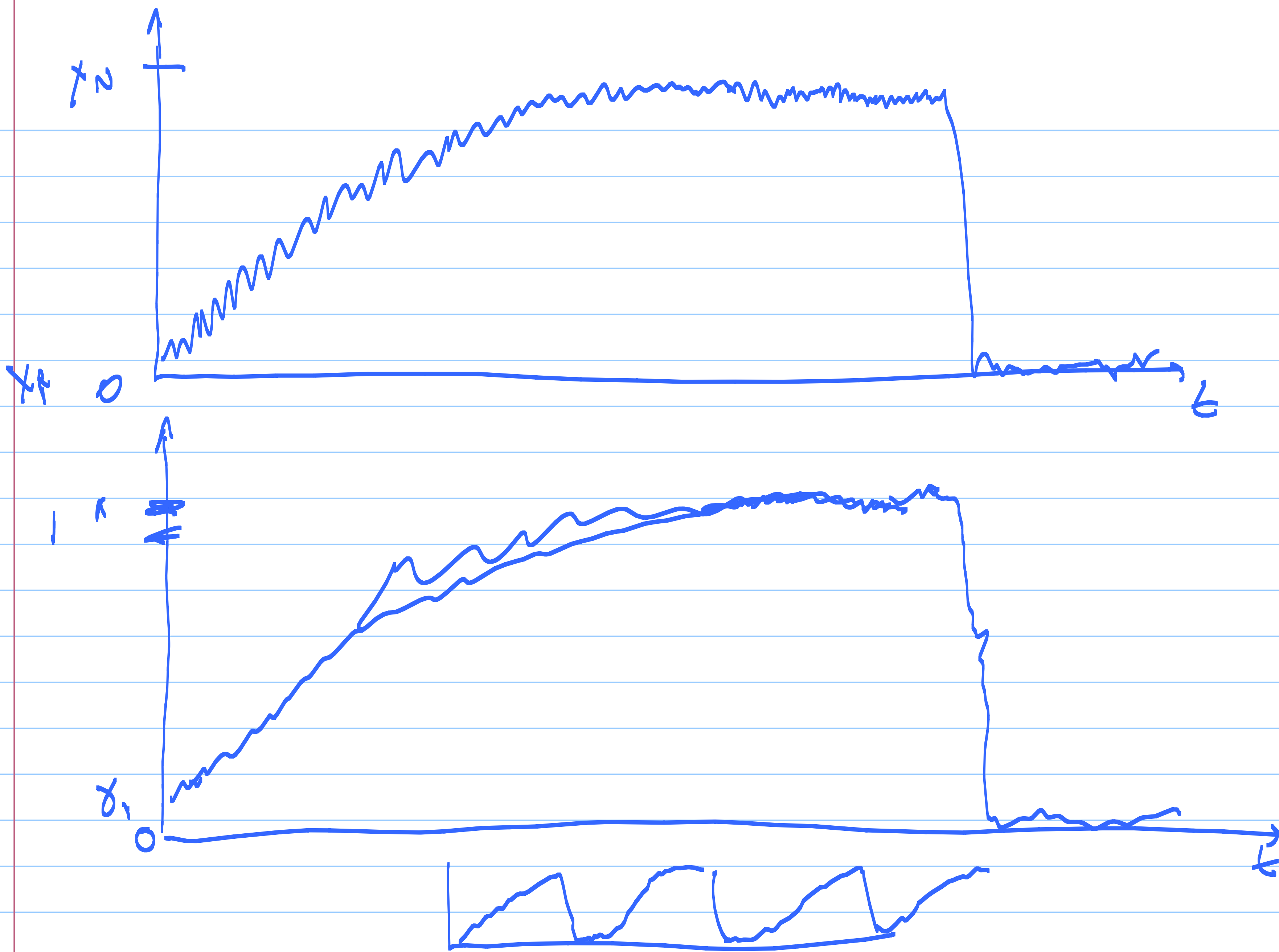
$\gamma_0 = 10, \; \gamma_1 = -1$

$\dot{\gamma} = \ell(\gamma)$



γ

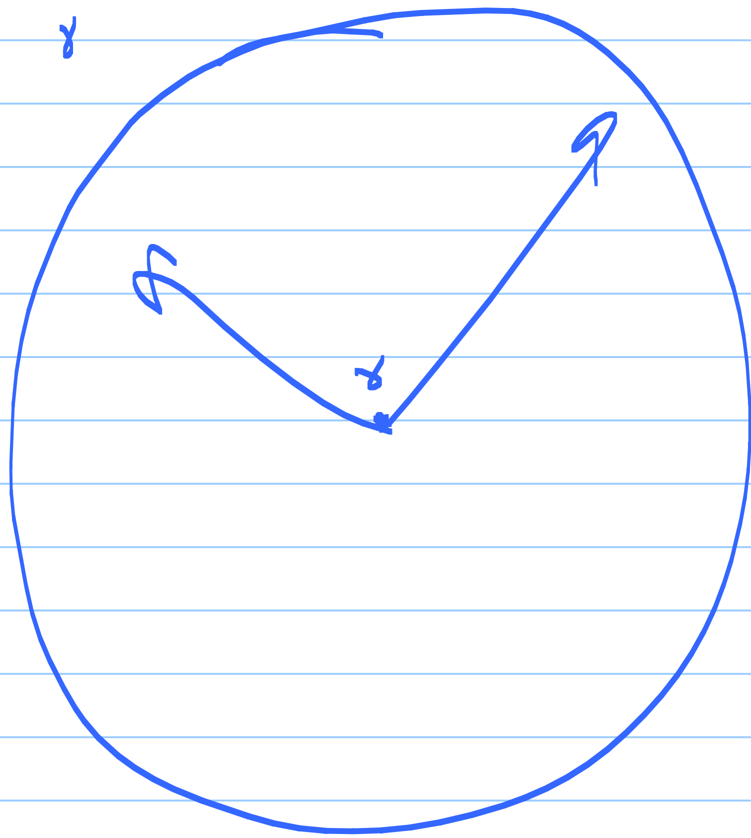




x_N
x_f
0
t
1
y_1
0
t

Escape dynamics: Large deviation theory

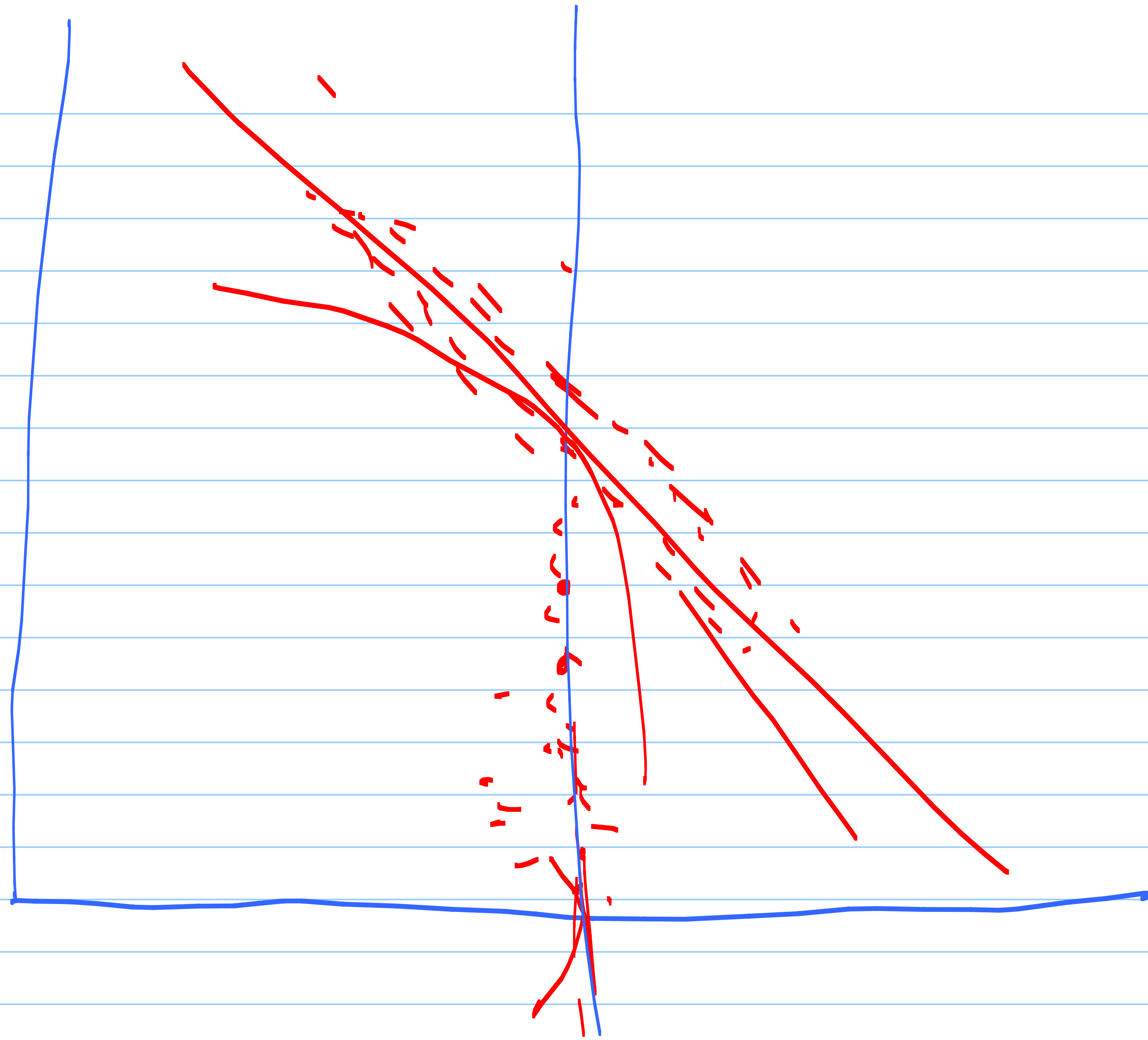
γ



SCE

$$\dot{\gamma} = \ell(\gamma) + v$$

deterministic

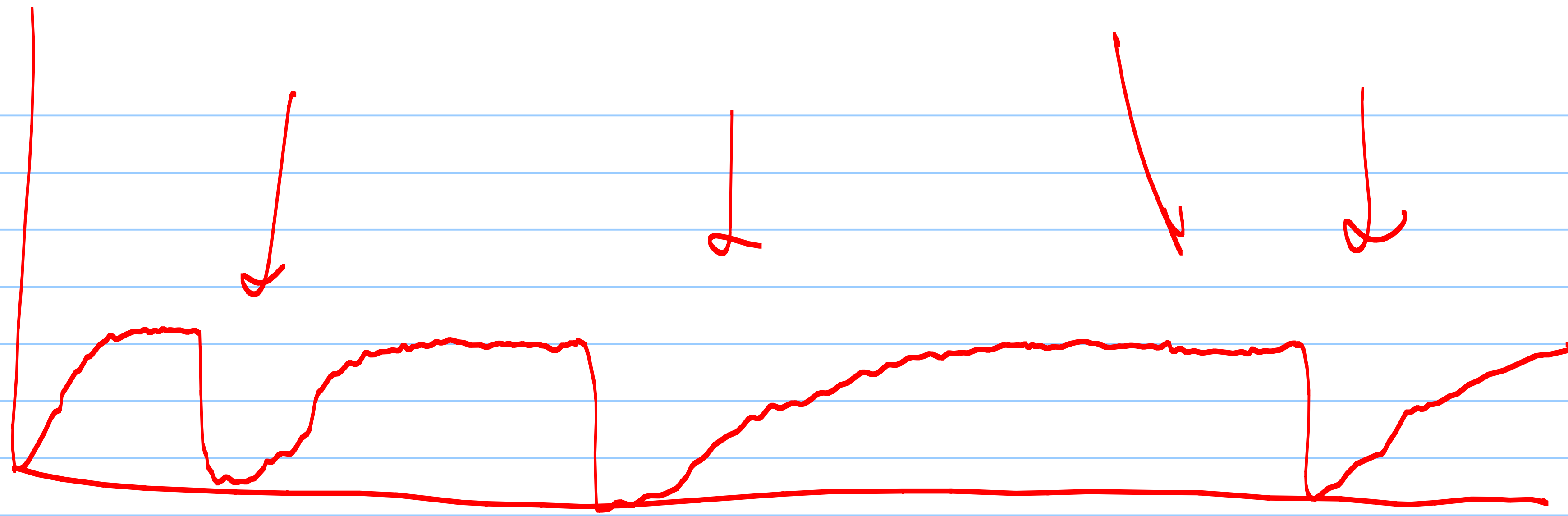


$$\gamma_t = \gamma_{t-1} + \frac{1}{t}(\text{stuff at } t)$$ constant weight

$$\gamma_t = \gamma_{t-1} + c\,(\text{stuff at } t)$$

"as if"

Bishop $\quad \gamma_t = \gamma_{t-1} + c\, a_t \quad , a_t \sim N(0; V)$



Constant gain

$$U_t = \gamma_0 + \gamma_1 y_t + u_t$$

two dimension γ model

let gut is model be

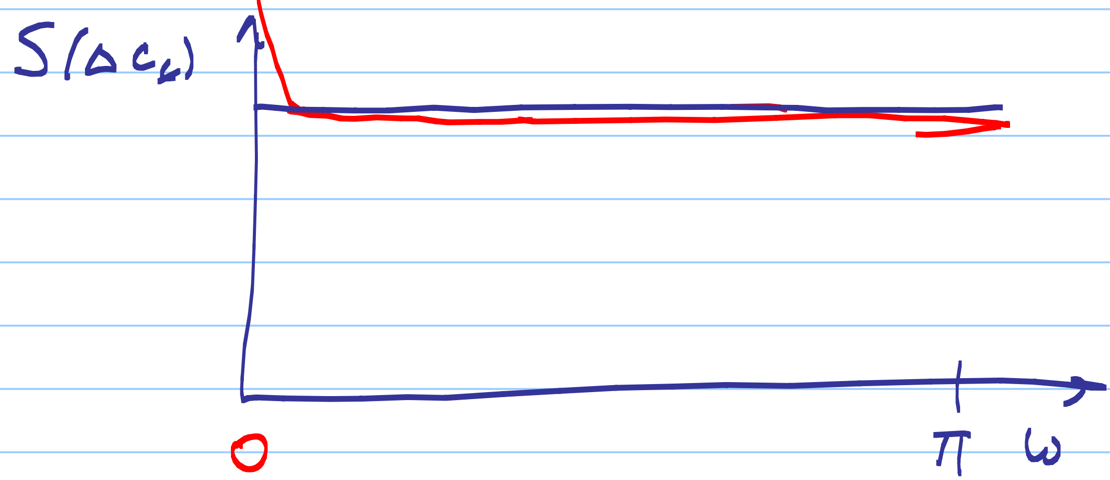
$$U_t = \gamma_0 + \gamma_1 y_t + \gamma_2 y_{t-1} + \gamma_3 y_{t-3}$$

$$+ \dots \beta_1 U_{t-1} + \beta_2 U_{t-2} + u_t$$

$$x_t = h(\gamma; y_{t-1}, \dots, U_{t-1}, \dots)$$

Long run risks & Asset pricing:

$$\Delta c_t = \log C_t - \log C_{t-1}$$



$$(c_t, d_t)$$

$$\Delta c_t = \mu_c + x_{t-1} + \varepsilon_{ct}$$

$$\Delta d_t = \mu_d + \lambda_{dx} x_{t-1} + \varepsilon_{dt}$$

$$x_t = \rho x_{t-1} + \varepsilon_{xt}$$

$$|\rho| < 1, \quad \rho \approx 1 \qquad \varepsilon_{jt} \sim \mathcal{N}(0, \sigma_j^2)$$

$\sigma_{\varepsilon x}^2 \sim$ small but positive.

$$\Rightarrow \Delta c_t = \alpha + \frac{(1 - \beta L)}{(1 - \alpha L)} \varepsilon_{ct} \qquad \beta \approx \alpha$$

Preferences: Epstein-Zin

$$U_t = \left[(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}$$

$$\theta = \frac{1-\gamma}{1-1/\psi}$$

γ = coefficient of relative risk aversion

ψ = IES

$$1/\psi = \gamma \implies U_t = \sum_{t=0}^{\infty} \delta^t C_t^{1-\gamma}$$

To do: Combine E-Z preferences with long run risk model

$$r_{c,t+1} = \log R_{c,t+1}$$

$$R_{c,t+1} = \frac{(1+v_{c,t+1})\, e^{\Delta c_{t+1}}}{v_{c,t}}$$

$$v_{ct} = \frac{P_{ct}}{C_t}$$

After algebra

$$v_{c,t} = E_t\left[\delta^\theta \exp\left[-\frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c\,t+1}\right] \cdot (1+v_{c\,t+1})\exp[\Delta c_{t+1}]\right]$$

$$v_{d,t} = E_t\left[\delta^\theta \exp\left[-\frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c\,t+1}\right] (1+v_{d\,t+1})\exp[\Delta d_{t+1}]\right]$$

$$v_{dt} = \frac{P_{dt}}{D_t}$$

$$\Delta c_t = \mu_c + x_{t-1} + \varepsilon_{ct}$$ *

$$\Delta d_t = \mu_d + \lambda_{dx} x_{t-1} + \varepsilon_{dt}$$ *

$$x_t = \rho_x x_{t-1} + \varepsilon_{xt}$$ *

see $\uparrow$ x_{t-1}

don't see

$$\Delta c_t = \mu_c + \hat{x}_{t-1} + a_{ct}$$

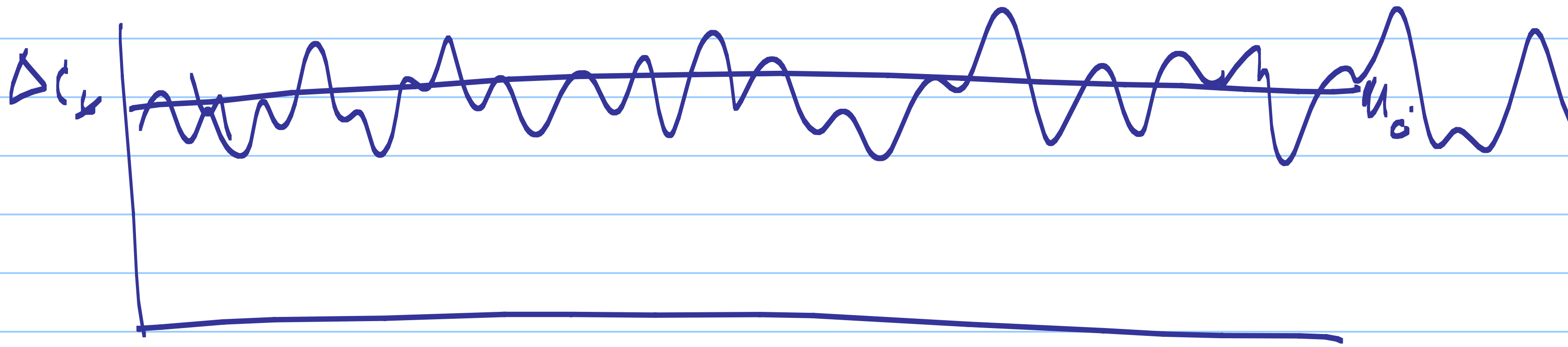
$$\Delta d_t = \mu_d + \lambda_{dx} \hat{x}_{t-1} + a_{dt}$$

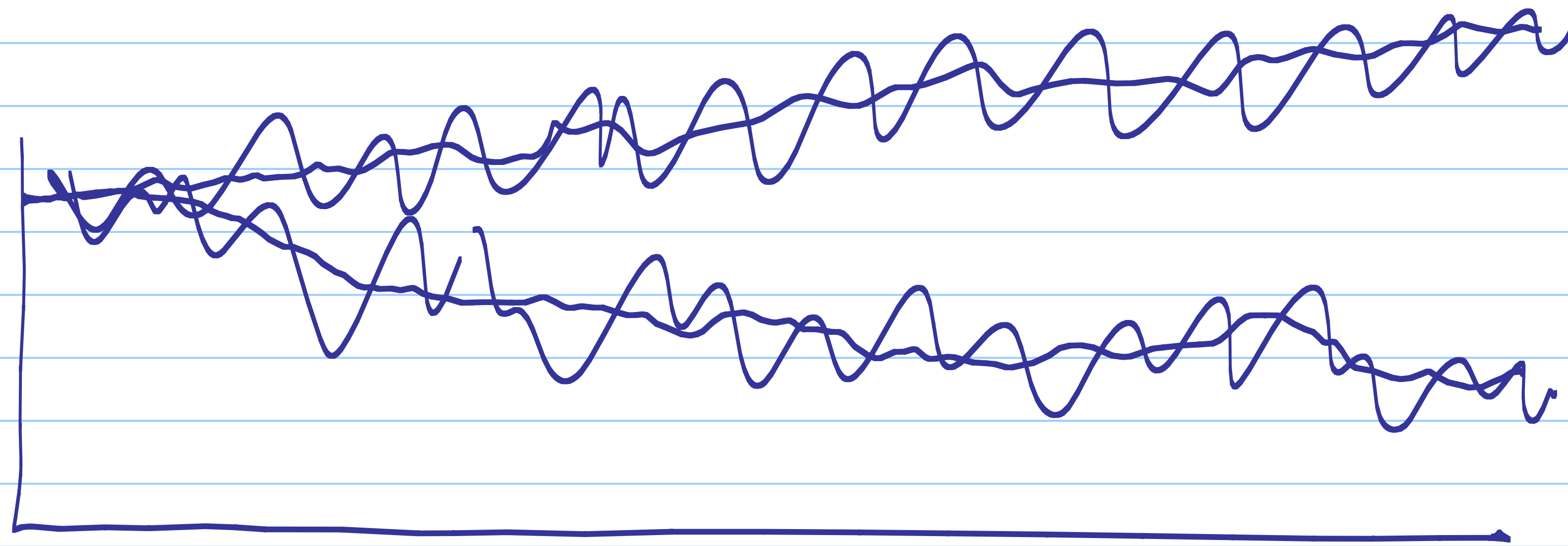
$$K = \begin{pmatrix} K_c \\ K_d \end{pmatrix}'$$

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + K_c a_{ct} + K_d a_{dt}$$

$$G = \begin{bmatrix} 1 \\ \lambda_{dx} \end{bmatrix}$$

$$E\begin{pmatrix} a_c \\ a_d \end{pmatrix}\begin{pmatrix} a_c \\ a_d \end{pmatrix}' = G \Sigma G + \begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_d^2 \end{pmatrix}$$





$$E_t(\Delta c_{t+j} - \quad)^2$$

$$x_t = A \overset{n\times 1}{x_{t-1}} + \overbrace{B}^{n\times l} \varepsilon_t$$

$$\overset{p\times 1}{y_t} = C x_{t-1} + \overbrace{D}^{p\times l} \varepsilon_t \quad , \quad \varepsilon_t \sim \mathcal{N}(0, I)$$

$$, \quad x_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$$

$$\hat{x}_t = A \hat{x}_{t-1} + K a_t \qquad E\, a_t a_t' = C \Sigma C' + DD'$$

$$y_t = C \hat{x}_{t-1} + a_t \, ,$$

$$a_t = y_t - E[y_t \mid y^{t-1}] \, , \quad \hat{x}_t = E\, x_t \mid y^{t-1}$$

$$\Sigma = E(x_t - \hat{x}_t)(x_t - \hat{x}_t)' \, ,$$

Most of the code & a write up

are available from

Riccardo Colacito's web page

"google"

A class of general equilibrium models:

LQ . general equilibrium

↕

partial equilibrium

Sherwin Rosen

cattle cycles — JPE '74

housing " 88

engineers JPE 2003

Lagrangian

o/r

Planner:

$$-\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left[(s_t - b_t)\cdot(s_t - b_t) + l_t^2 \right]$$

$$\begin{cases} s_t = \Lambda h_{t-1} + \Pi c_t \\ h_t = \Delta_h h_{t-1} + \Theta_h d_t \\ b_t = U_b z_t \\ z_t = A_{22} z_{t-1} + C_2 \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I) \end{cases}$$

Technology:

$$\begin{cases} \Phi_c c_t + \Phi_i i_t + \Phi_g g_t = \Gamma k_{t-1} + d_t \\ k_t = \Delta_k k_{t-1} + \Theta_i i_t \\ g_t \cdot g_t = \ell_t^2 \end{cases}$$

intermediate goods

generalized adjustment costs

$$d_t = U_d z_t$$

get FONC:

$$-\Phi_0' \mu_t^d + \Theta_h' \mu_t^h + \Pi' \mu_t^s = 0$$

$$-\mu_t^h + \beta E_t (\Delta_h' \mu_{t+1}^h + \Lambda' \mu_{t+1}^s) = 0$$

$$-\Phi_i' \mu_t^d + \Theta_k' \mu_t^k = 0$$

$$-\mu_t^k + \beta (E_t \Delta_k' \mu_{t+1}^k + \Gamma' \mu_{t+1}^d) = 0$$

$$-s_t + b_t - \mu_t^s = 0$$

$$-g_t - \Phi_g' \mu_t^d = 0$$

$$\sum_{t=1}^{\infty} \beta^t \left\{ -\frac{1}{2} (c_t - b_t)^2 \right\}$$

$$c_t + k_t = \gamma k_{t-1} + d_t$$

$$k_t = \delta_k k_{t-1} + i_t$$

$$+ \quad \Phi_1 i_t = g_t$$ adjustment costs —

$\beta, \gamma, \delta_k, \Phi,$ —

Lucas-Prescott 1971.