Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas

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The economy

- Central Bank chooses v_t to

$$\min \quad E_0 \sum_{t=0}^{\infty} \beta^t (U_t^2 + \lambda v_t^2), s.t.$$

- Model 1:

$$\begin{aligned} U_t &= \overline{U}_1 + A_1 U_{t-1} + B_1 \pi_t + C_{U,1} \eta_{1,t} \\ \pi_t &= v_{t-1} + C_\pi \eta_{3,t} \end{aligned}$$

- Model 2:

$$\begin{aligned} U_t &= \overline{U}_2 + A_2 U_{t-1} + B_2 (\pi_t - v_{t-1}) + C_{U,2} \eta_{2,t} \\ \pi_t &= v_{t-1} + C_\pi \eta_{3,t} \end{aligned}$$

The economy

- Central Bank chooses v_t to

$$\min \quad E_0 \sum_{t=0}^{\infty} \beta^t (U_t^2 + \lambda v_t^2), \, s.t.$$

- Model 1 (α_t):

$$\begin{aligned} U_t &= \overline{U}_1 + A_1 U_{t-1} + B_1 \pi_t + C_{U,1} \eta_{1,t} \\ \pi_t &= v_{t-1} + C_\pi \eta_{3,t} \end{aligned}$$

- Model 2 $(1 - \alpha_t)$:

$$\begin{aligned} U_t &= \overline{U}_2 + A_2 U_{t-1} + B_2 (\pi_t - v_{t-1}) + C_{U,2} \eta_{2,t} \\ \pi_t &= v_{t-1} + C_\pi \eta_{3,t} \end{aligned}$$

- Bayesian updating:

$$\alpha_t = B(\alpha_{t-1}, U_t)$$

The economy

- Central Bank chooses v_t to

$$\min \quad E_0 \sum_{t=0}^{\infty} .995^t (U_t^2 + 0.1 v_t^2), s.t.$$

- Model 1 (α_t):

$$\begin{aligned} U_t &= 0.0023 + 0.7971 U_{t-1} - 0.2761 \pi_t + 0.0054 \eta_{1,t} \\ \pi_t &= v_{t-1} + 0.0055 \eta_{3,t} \end{aligned}$$

- Model 2 $(1 - \alpha_t)$:

$$\begin{aligned} U_t &= 0.0007 + 0.8468 U_{t-1} - 0.2489 (\pi_t - v_{t-1}) + 0.0055 \eta_{2,t} \\ \pi_t &= v_{t-1} + 0.0055 \eta_{3,t} \end{aligned}$$

- Bayesian updating:

$$\alpha_t = B(\alpha_{t-1}, U_t)$$

Plan of the talk

- Use Bayes' law to get a transition equation for α_t
- Bellman equations
  1. Bayesian problem
  2. Anticipated utility
- Policy and value functions
- Experiments

Evolution of α_t

- Using Bayes' law:

$$\log \frac{\alpha_t}{1-\alpha_t} = \log \frac{\alpha_{t-1}}{1-\alpha_{t-1}} + \log \frac{p_1(U_t|U_{t-1}, v_{t-1})}{p_2(U_t|U_{t-1}, v_{t-1})}$$

- Timing protocol

Evolution of α_t

- Using Bayes' law:

$$\log \frac{\alpha_t}{1-\alpha_t} \quad = \quad \log \frac{\alpha_{t-1}}{1-\alpha_{t-1}} + \log \frac{p_1(U_t|U_{t-1}, v_{t-1})}{p_2(U_t|U_{t-1}, v_{t-1})}$$

- Timing protocol

- α_t is a martingale from the point of view of the Bayesian agent.
- If the prior attaches nonzero probability to the model that actually generates the economy, then α_t converges almost surely under that measure.

Bayesian Problem

$$V(U_t, \alpha_t) = \max_{v_t} \Big\{ -(U_t^2 + \lambda v_t^2) + \beta\alpha_t \int V(U_{1,t+1}, B(\alpha_t, U_{1,t+1}))\, dF(\varepsilon_{1,t+1}) + \beta(1-\alpha_t) \int V(U_{2,t+1}, B(\alpha_t, U_{2,t+1}))\, dF(\varepsilon_{2,t+1}) \Big\}$$

subject to:

$$U_{1,t+1} = \overline{U}_1 + A_1 U_t + B_1 v_t + C_1 \varepsilon_{1,t+1}$$
$$U_{2,t+1} = \overline{U}_2 + A_2 U_t + C_2 \varepsilon_{2,t+1}$$

Anticipated Utility

- Bellman equation:

$$\begin{aligned} W(U_t, \alpha) &= \max_{v_t} \Big\{ -(U_t^2 + \lambda v_t^2) \\ &\quad + \beta\alpha \int W(U_{1,t+1}, \alpha)\, dF(\varepsilon_{1,t+1}) \\ &\quad + \beta(1-\alpha) \int W(U_{2,t+1}, \alpha)\, dF(\varepsilon_{2,t+1}) \Big\} \end{aligned}$$

subject to:

$$\begin{aligned} U_{1,t+1} &= \overline{U}_1 + A_1 U_t + B_1 v_t + C_1 \varepsilon_{1,t+1} \\ U_{2,t+1} &= \overline{U}_2 + A_2 U_t + C_2 \varepsilon_{2,t+1} \end{aligned}$$

Anticipated Utility

- Bellman equation:

$$
\begin{aligned}
W\left(U_t, \alpha\right) &= \max_{v_t} \Big\{ -(U_t^2 + \lambda v_t^2) \\
&\quad + \beta\alpha \int W\left(U_{1,t+1}, \alpha\right) dF(\varepsilon_{1,t+1}) \\
&\quad + \beta(1-\alpha) \int W\left(U_{2,t+1}, \alpha\right) dF(\varepsilon_{2,t+1}) \Big\}
\end{aligned}
$$

- Policy function

$$
v_t = w(U_t, \alpha)
$$

Anticipated Utility

- Bellman equation:

$$
\begin{aligned}
W(U_t, \alpha) \quad &= \quad \max_{v_t} \Big\{ -(U_t^2 + \lambda v_t^2) \\
&\quad + \beta\alpha \int W(U_{1,t+1}, \alpha)\, dF(\varepsilon_{1,t+1}) \\
&\quad + \beta(1-\alpha) \int W(U_{2,t+1}, \alpha)\, dF(\varepsilon_{2,t+1}) \Big\}
\end{aligned}
$$

- Policy function

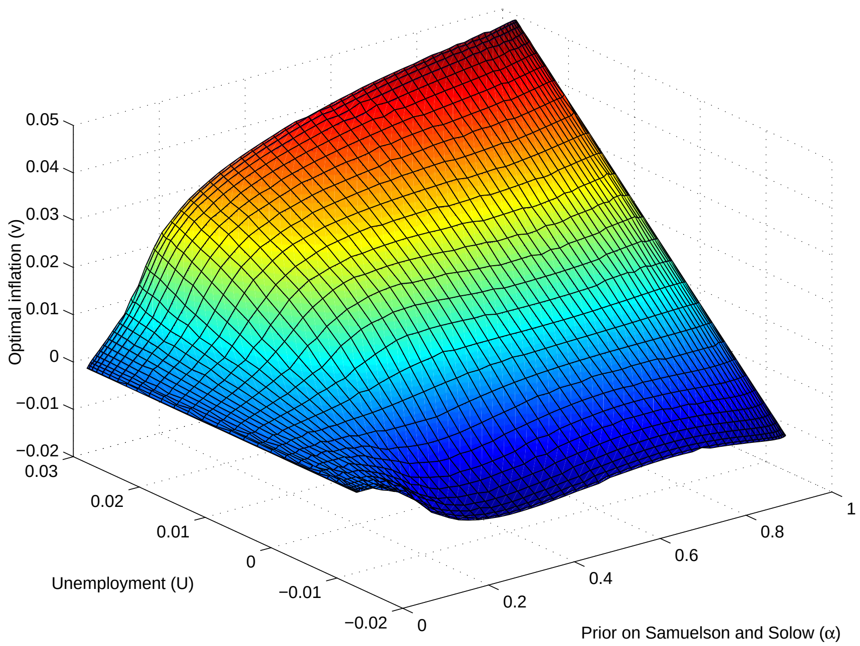
$$
\begin{aligned}
v_t &= w(U_t, \alpha_t) \\
\alpha_t &= B(\alpha_{t-1}, U_t)
\end{aligned}
$$

Value function for anticipated utility

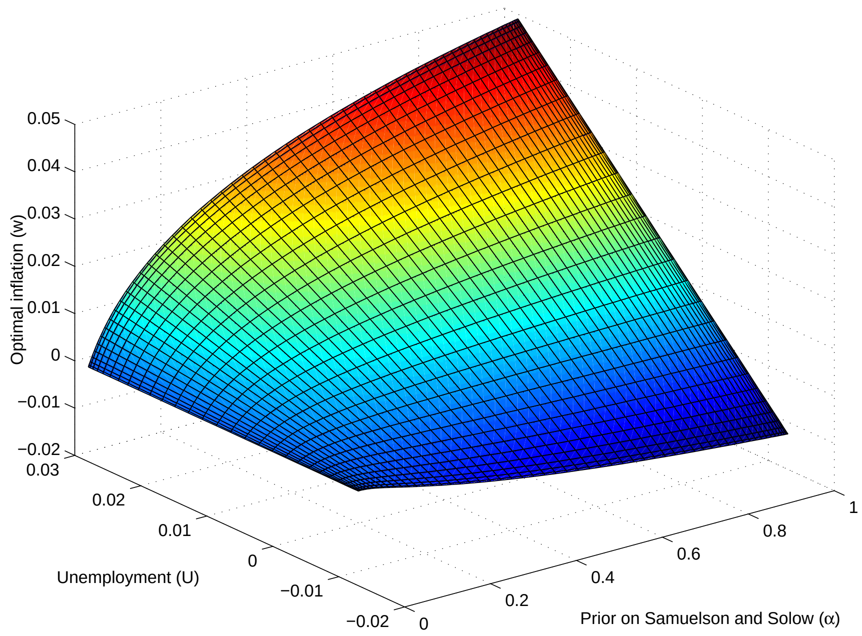
$$
\begin{aligned}
W(s_t;\alpha_t) &= r(s_t, w(s_t,\alpha_t)) \\
&+ \beta\alpha_t \int W(U_{1,t+1}, B(\alpha_t, U_{1,t+1}))dF(\epsilon_{1,t+1}) \\
&+ \beta(1-\alpha_t) \int W(U_{2,t+1}, B(\alpha_t, U_{2,t+1}))dF(\epsilon_{2,t+1}).
\end{aligned}
$$

$$
\begin{aligned}
U_{1,t+1} &= A_1 s_t + B_1 w(s_t,\alpha_t) + C_1\epsilon_{1,t+1} \\
U_{2,t+1} &= A_1 s_t + B_1 w(s_t,\alpha_t) + C_1\epsilon_{1,t+1}
\end{aligned}
$$

Policy functions

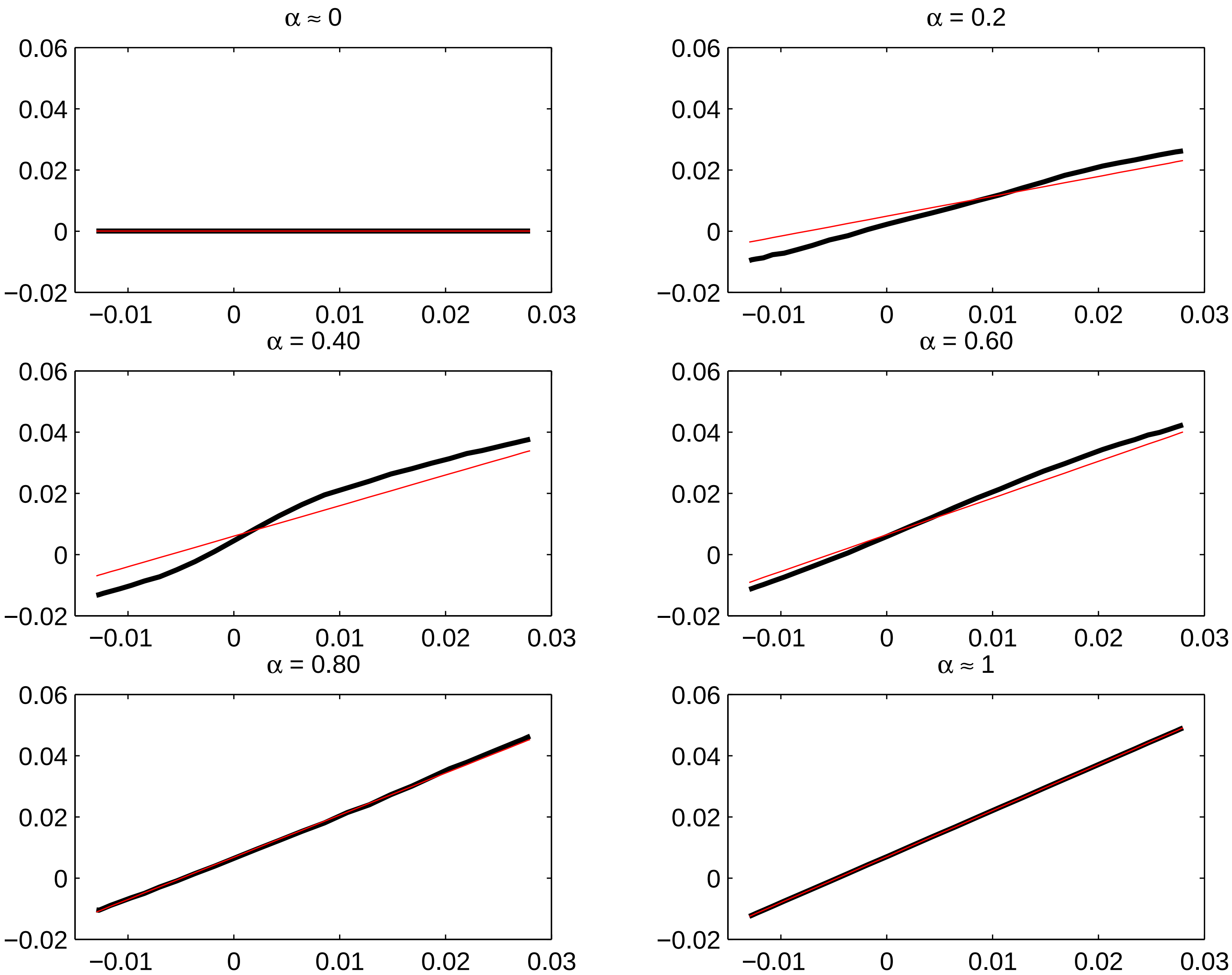


(a) Bayesian Problem

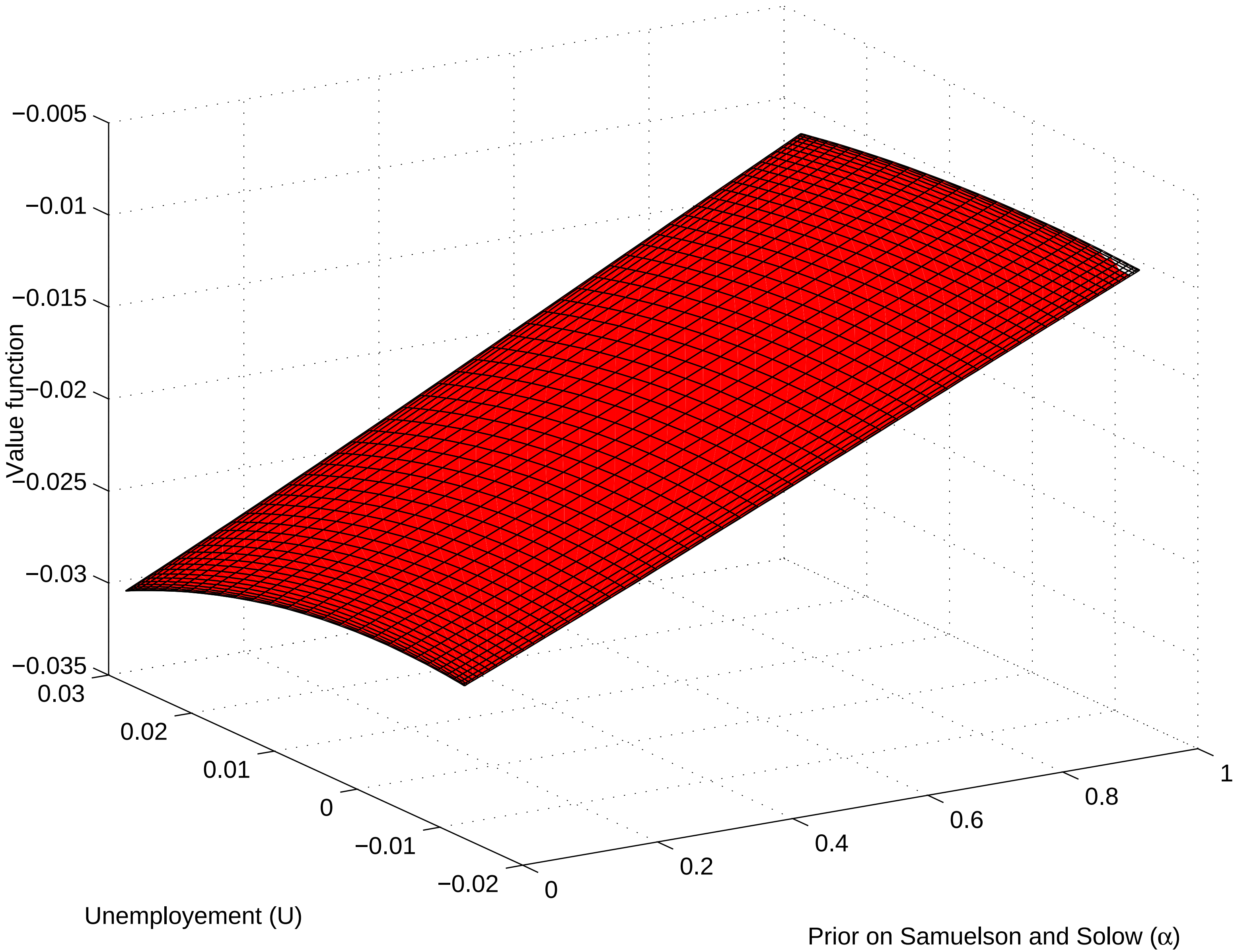


(b) Anticipated Utility

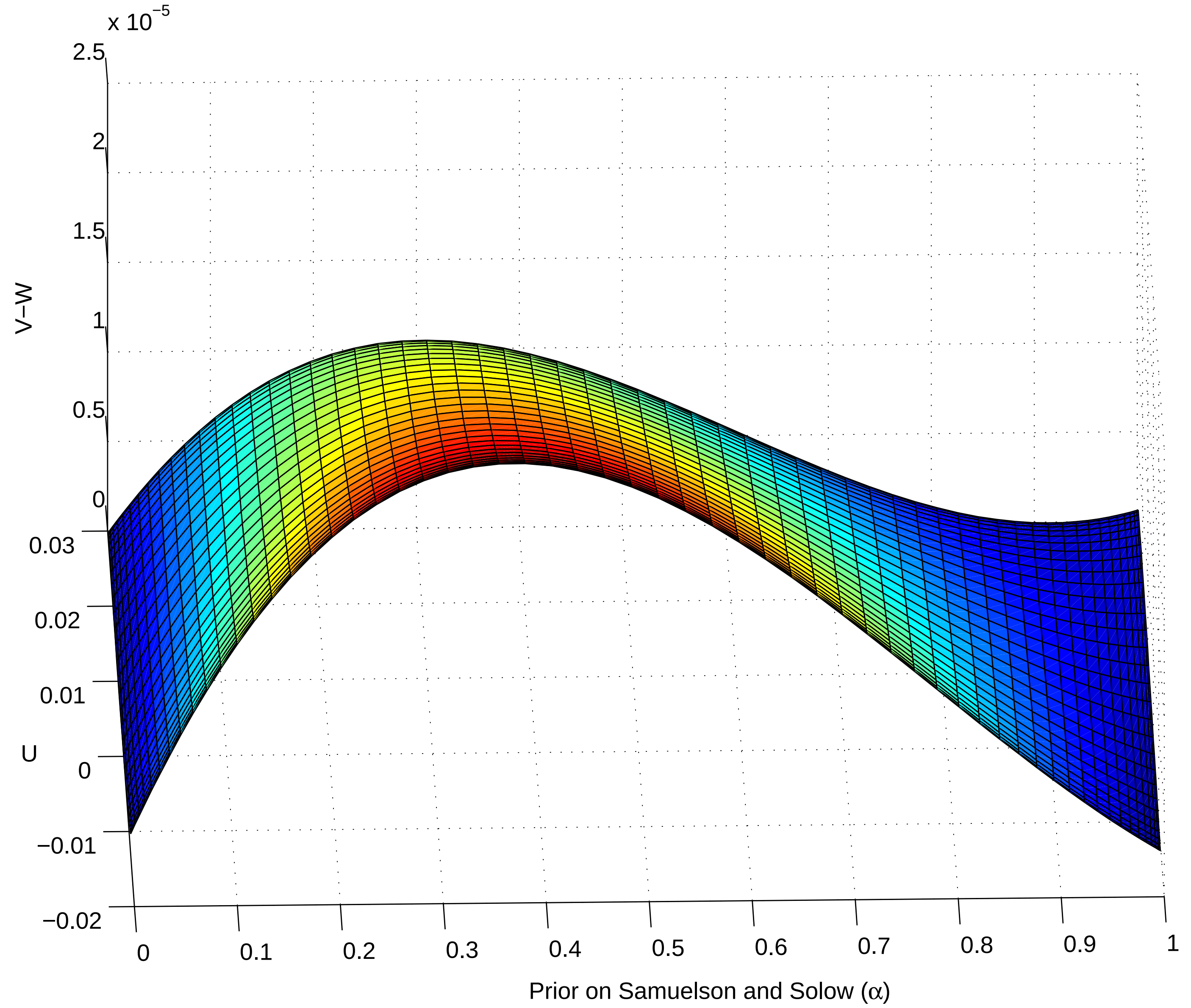
Policy functions (Slices)



Value functions



Value of experimentation

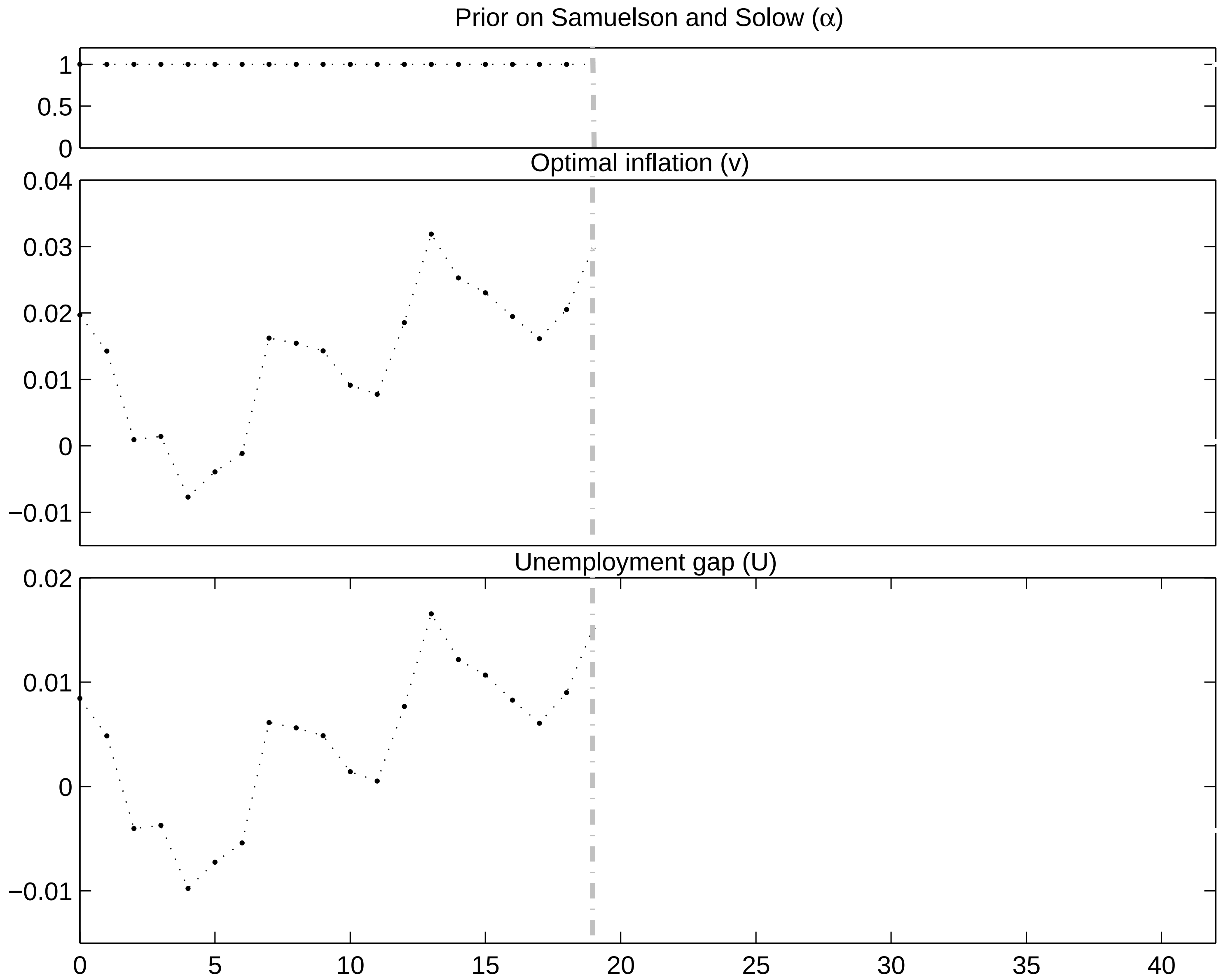


Learning Experiments

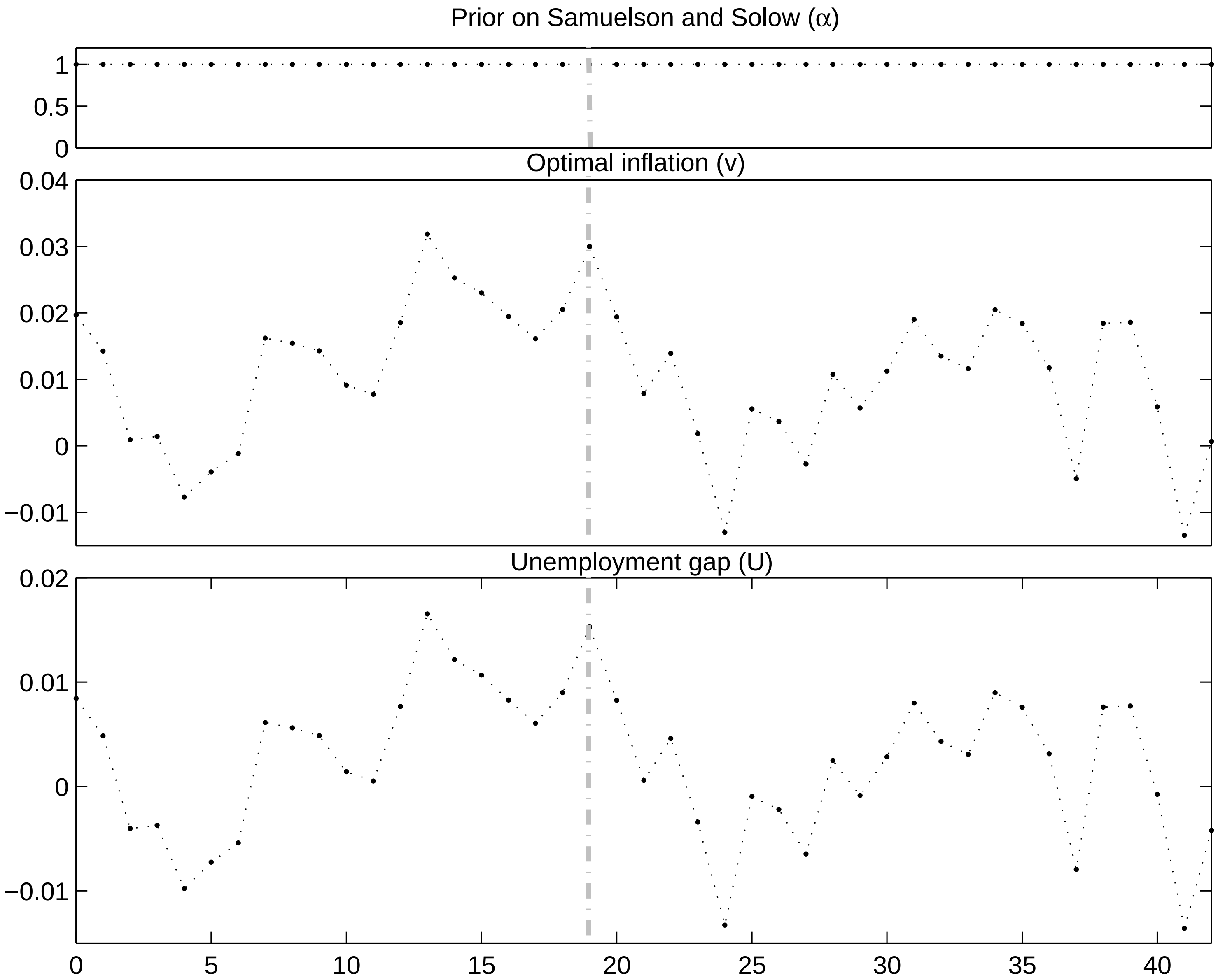
Suppose that one model generates the data:

- How long does it take to learn it?
- How much faster can we learn it with experimentation?
- How different are inflation and unemployment in the learning process?

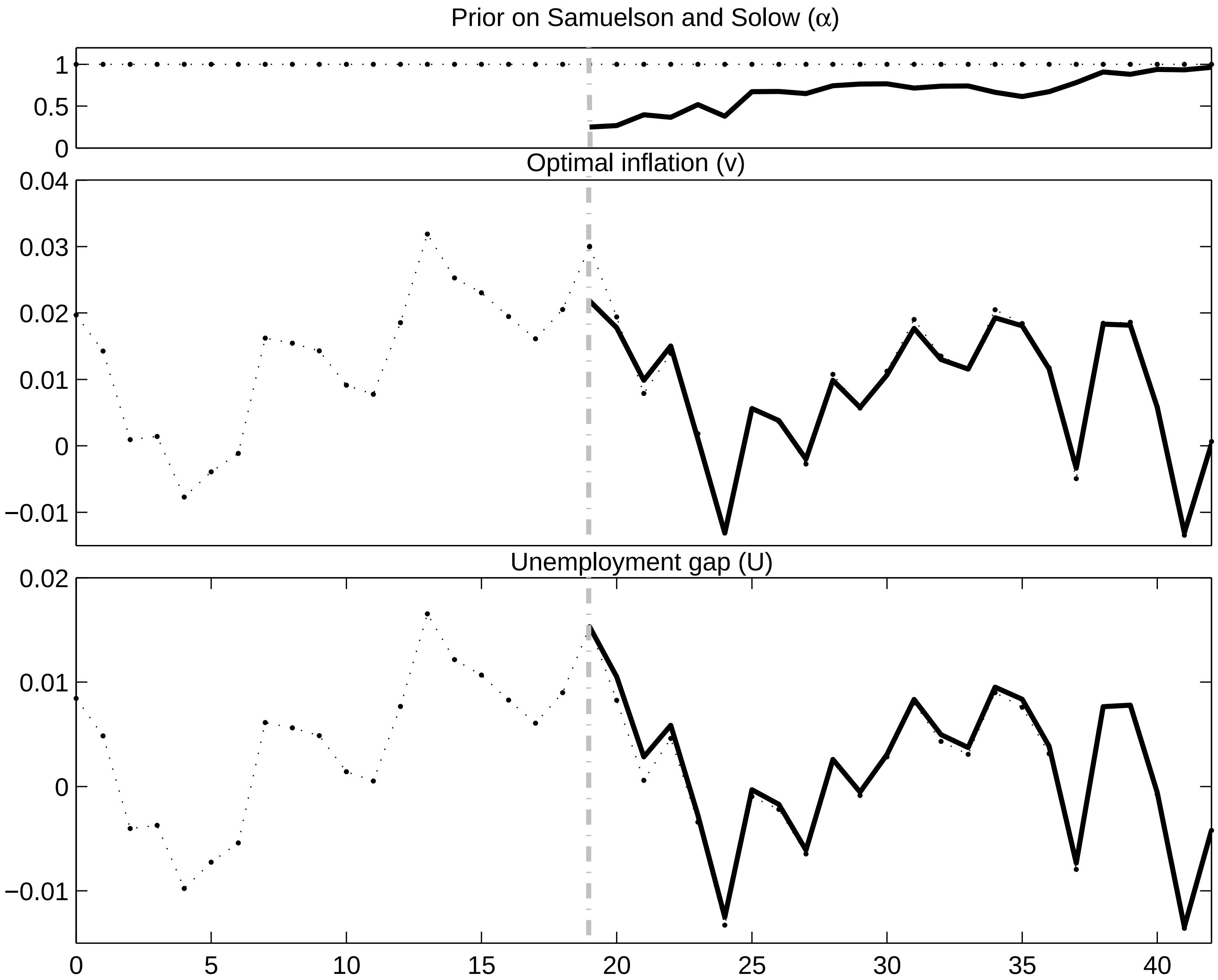
Forgetting Lucas



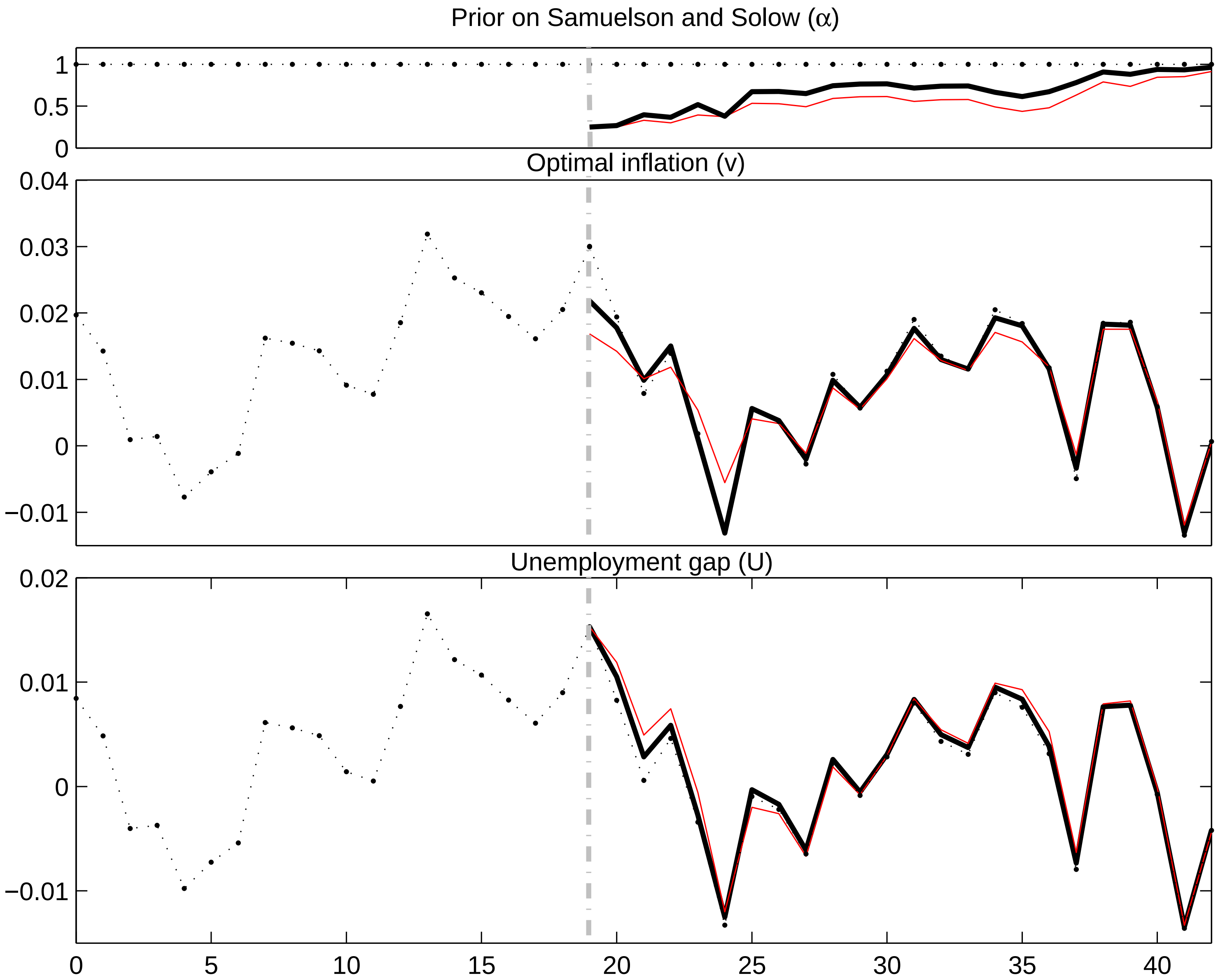
Forgetting Lucas



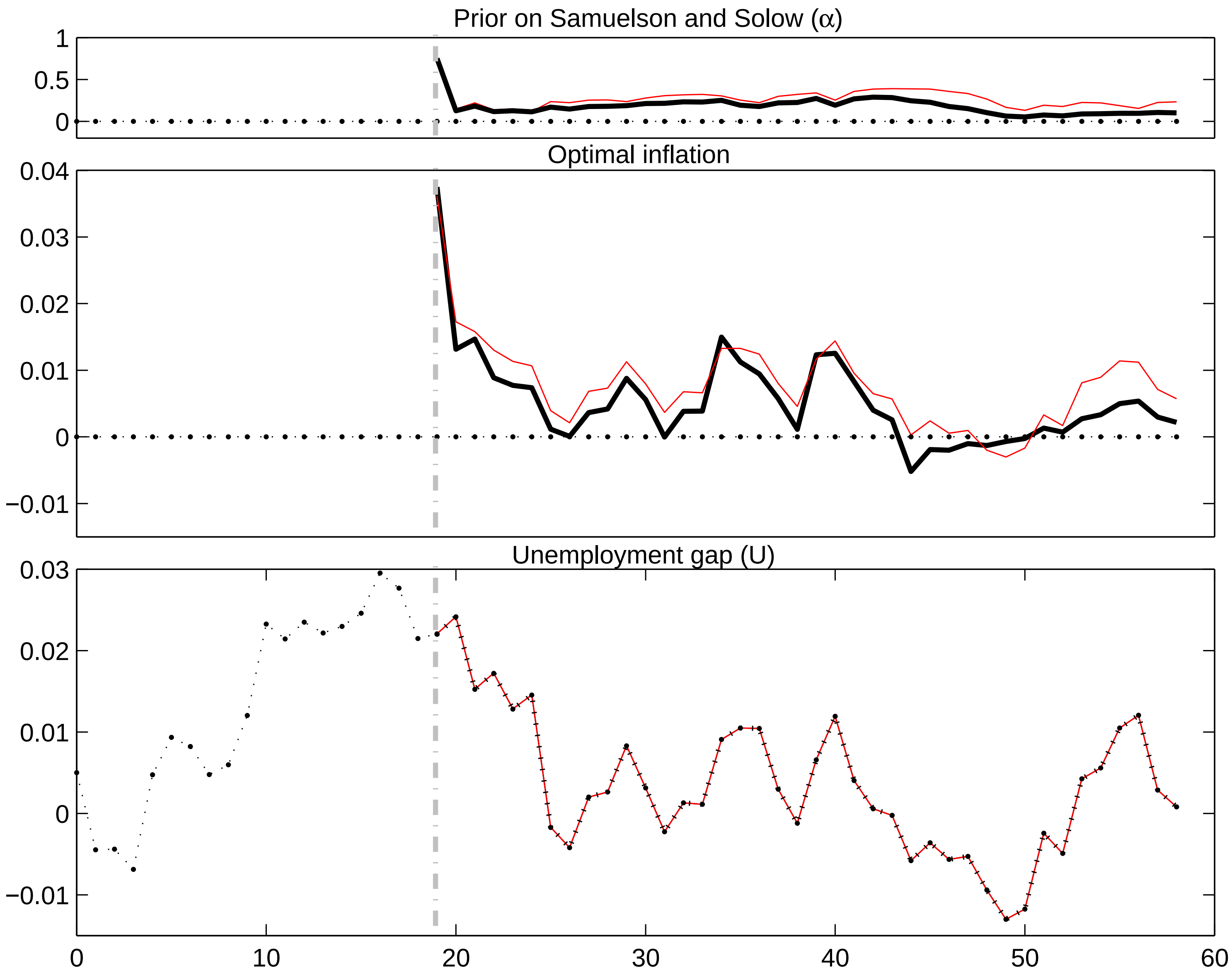
Forgetting Lucas



Forgetting Lucas



Forgetting Samuelson and Solow



Conclusions

- Benefits of experimentation, but not too big.
- Samuelson-Solow is less sensitive to small doubts.
- Limitations:
  1. Only two models are on the table.
  2. Models' parameters are assumed to be known.
- How do results vary with λ?