The Conquest of U.S. Inflation: Learning and Robustness to Model Uncertainty

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- The Fed updates parameters of each model and its posterior probabilities over models
- The Fed uses dynamic programming to choose a first-period decision each period

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- Using time $t+1$ model, solve dynamic programming problem at $t+1$ and $\ldots$

Three models

- Samuelson-Solow model (a long-run tradeoff)

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- Lucas model (no tradeoff)

Variables

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- η_{it} is iid $N(0, \sigma^2_i)$.

Form of each model

$$Y_t = X_t\theta + \nu_t$$

Priors

$$p(\theta, \sigma^2) = p(\theta|\sigma^2)p(\sigma^2)$$

The marginal prior $p(\sigma^2)$ makes the error variance an inverse gamma variate.

The conditional prior $p(\theta|\sigma^2)$ makes the regression parameters a normal random vector.

Posteriors

$$p(\theta|\sigma^2, Z^{t-1}) = N(\theta_{t-1}, \sigma^2 P_{t-1}^{-1}),$$

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Posteriors

After seeing outcomes at t, the central bank's updated beliefs are

$$p(\theta|\sigma^2, Z^t) = N(\theta_t, \sigma^2 P_t^{-1})$$

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where

$$P_t = P_{t-1} + X_t X_t',$$
$$\theta_t = P_t^{-1}(P_{t-1}\theta_{t-1} + X_t Y_t).$$
$$s_t = s_{t-1} + Y_t' Y_t + \theta_{t-1}' P_{t-1}\theta_{t-1} - \theta_t' P_t \theta_t,$$
$$v_t = v_{t-1} + 1.$$

Model probabilities

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where m_{it} is the marginalized likelihood function for model i at date t.

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where m_{it} is the marginalized likelihood function for model i at date t. The conditional likelihood for model i through date t is defined via a prediction error decomposition as

$$l(Y^t, X^t, \theta, \sigma^2) = \prod_{s=1}^{t} p(Y_s|X_s, \theta, \sigma^2).$$

Model probabilities (2)

The marginalized likelihood is

$$m_{it} = \iint l(Y_i^t, X_i^t, \theta_i, \sigma_i^2) p(\theta_i, \sigma_i^2) d\theta_i d\sigma_i^2$$

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m_{it} is the normalizing constant in Bayes' theorem and can also be expressed as

$$m_{it} = \frac{l(Y_i^t, X_i^t, \theta_i, \sigma_i^2) p(\theta_i, \sigma_i^2)}{p(\theta_i, \sigma_i^2 | Z_i^t)}$$

Posterior model probabilities

$$\log w_{it+1} = \log w_{it} + \log p(Y_{it+1}|X_{it+1}, \theta_i, \sigma_i^2) - \log \frac{p(\theta_i, \sigma_i^2|Z_i^{t+1})}{p(\theta_i, \sigma_i^2|Z_i^t)}$$

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A renormalization enforces that the model weights sum to 1

$$\alpha_{it} = \frac{w_{it}}{w_{1t} + w_{2t} + w_{3t}}$$

Posterior model probabilities (2)

or

$$\alpha_{it} = [\exp R_{1i}(t) + \exp R_{2i}(t) + \exp R_{3i}(t)]^{-1}$$

where $R_{ji}(t) = (\log w_{jt} - \log w_{it})$ summarizes the weight of the evidence favoring model j relative to model i.

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State space models

$$S_{it+j} = A_i(t-1)S_{it+j-1} + B_i(t-1)x_{t+j|t-1} + C_i(t-1)\eta_{it+j} \quad (1)$$

where $(S_{it}, A_i(t-1), B_i(t-1), C_i(t-1))$ are the state vector and system arrays for model i at time t.

Grand model

$$\begin{bmatrix} S_{1t+j} \\ S_{2t+j} \\ S_{3t+j} \end{bmatrix} = \begin{bmatrix} A_1(t-1) & 0 & 0 \\ 0 & A_2(t-1) & 0 \\ 0 & 0 & A_3(t-1) \end{bmatrix} \begin{bmatrix} S_{1t+j-1} \\ S_{2t+j-1} \\ S_{3t+j-1} \end{bmatrix}$$

$$+ \begin{bmatrix} B_1(t-1) \\ B_2(t-1) \\ B_3(t-1) \end{bmatrix} x_{t+j|t-1}$$

$$+ \begin{bmatrix} C_1(t-1) & 0 & 0 \\ 0 & C_2(t-1) & 0 \\ 0 & 0 & C_3(t-1) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{bmatrix}$$

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or in a more compact notation

$$S_{Et+j} = A_E(t-1)S_{Et+j-1} + B_E(t-1)x_{t+j|t-1} + C_E(t+j)\eta_t$$

Loss function

$$\mathcal{L}(M_i)(t) = E_t \sum_{j=0}^{\infty} \beta^j \big(S'_{it+j} M'_{s_i} Q M_{s_i} S_{it+j} + x'_{t+j|t-1} R x_{t+j|t-1} \big)$$

Grand loss function

$$\mathcal{L}_E(t) = \alpha_{1t}\mathcal{L}(M_1) + \alpha_{2t}\mathcal{L}(M_2) + \alpha_{3t}\mathcal{L}(M_3)$$

$$= E_t \sum_{i=1}^{3} \alpha_{it} \sum_{j=0}^{\infty} \beta^j (S'_{it+j} M'_{s_i} Q M_{s_i} S_{it+j}$$

$$+ x'_{t+j|t-1} R x_{t+j|t-1})$$

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where

$$Q_E = \begin{bmatrix} \alpha_{1t} M'_{s_1} Q M_{s_1} & 0 & 0 \\ 0 & \alpha_{2t} M'_{s_2} Q M_{s_2} & 0 \\ 0 & 0 & \alpha_{3t} M'_{s_3} Q M_{s_3} \end{bmatrix}$$

Bellman equation

$$v_t(S_E) = \max_x \left\{ -S_E' Q_{Et} S_E - x' R x + \beta E v_t(S_E^*) \right\}$$

subject to

$$S_E^* = A_E(t-1) S_E + B_E(t-1) x + C_E(t-1) \eta$$

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Decision rule

$$x_{t|t-1} = -f_E(t-1) \cdot S_{Et-1} = -f_E(t-1)^1 S_{1t-1} - f_E^2(t-1) S_{2t-1} - f_E^3(t-1) S_{3t-1}$$

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If the composite model is ‘detectable’ and ‘stabilizable,’ then the policy rule f_E can be computed using standard algorithms

Data

- Inflation: log difference in chain weighted GDP deflator
- Unemployment: civilian unemployment rate
- quarterly, seasonally adjusted, 1948:Q1–2002:Q4
- $u_t^* = u_{t-1}^* + .075(u_t - u_{t-1}^*)$

Parameters

- $\beta = 1.04^{.25}$
- $\alpha_{10} = .98, \alpha_{20} = \alpha_{30} = .01$
- $\lambda = 16$ (equal weight on u, y)
- Training sample: first 12 years

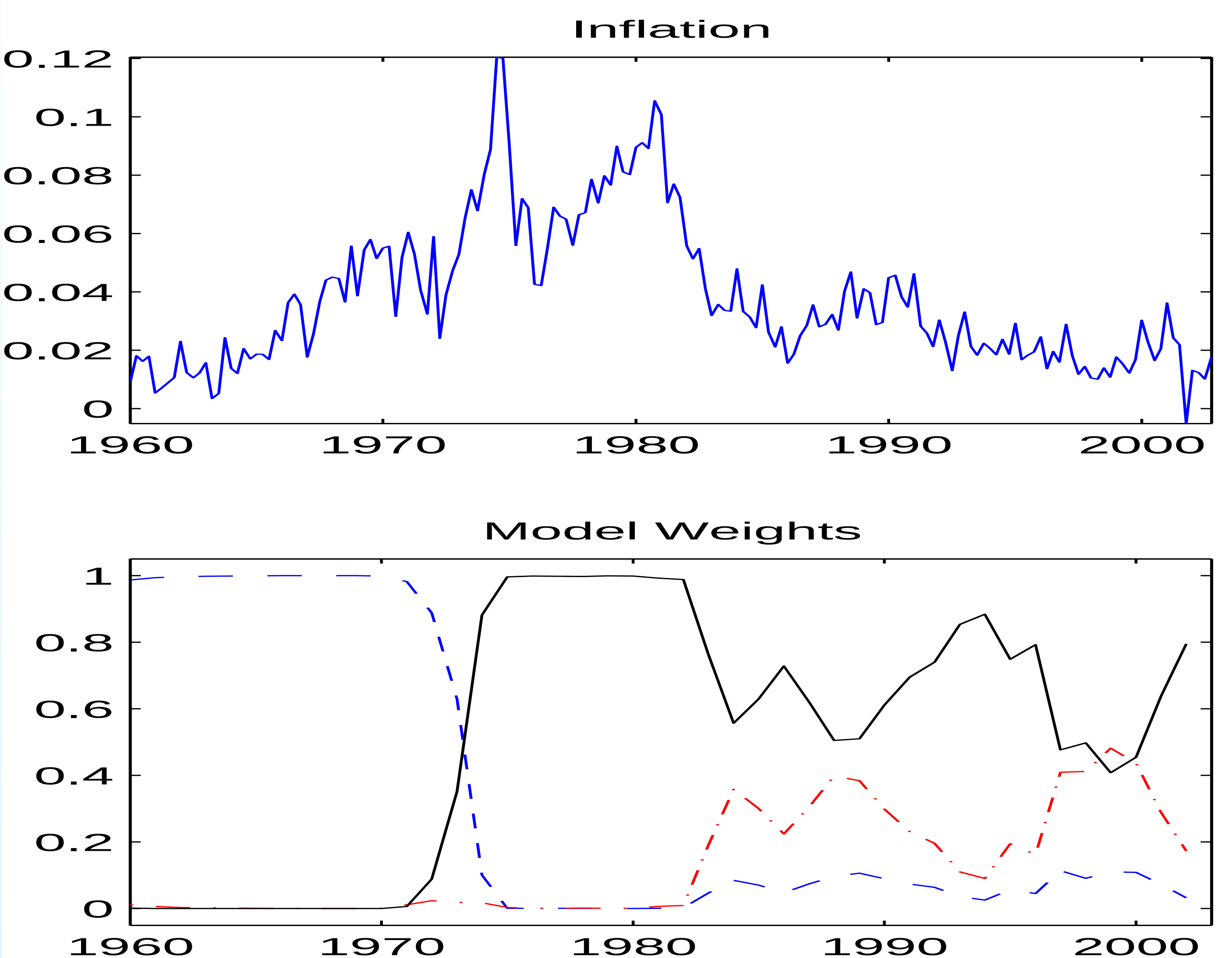
Lag orders

	Inflation	Unemployment
Samuelson-Solow	$\gamma_1 : 4$	
Solow-Tobin	$\delta_1 : 3$	
Lucas-Sargent	$\phi_1 : 0$	

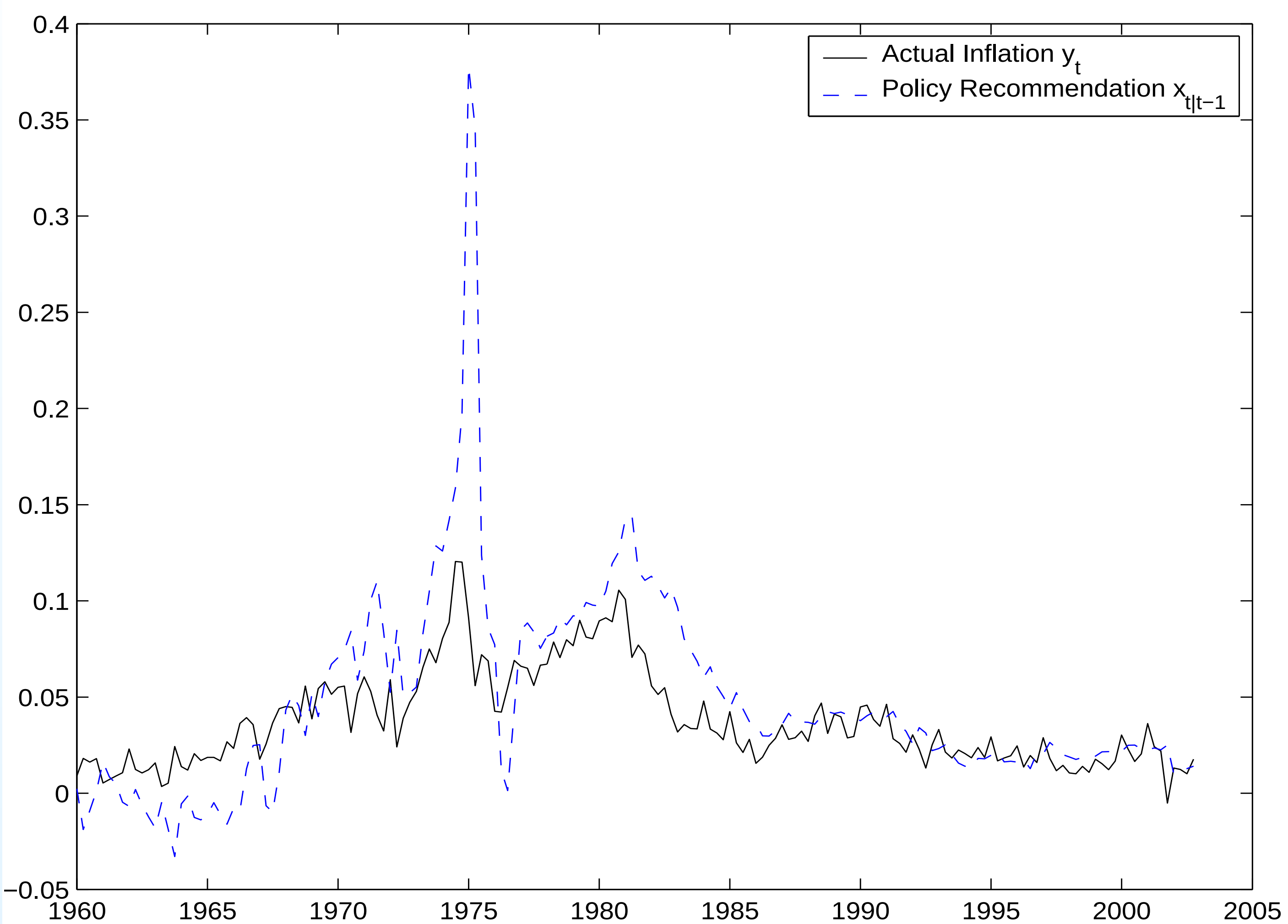
Lag orders

	Inflation	Unemployment
Samuelson-Solow	$\gamma_1 : 4$	$\gamma_2 : 2$
Solow-Tobin	$\delta_1 : 3$	$\delta_2 : 2$
Lucas-Sargent	$\phi_1 : 0$	$\phi_2 : 2$

Inflation and α_{it}'s

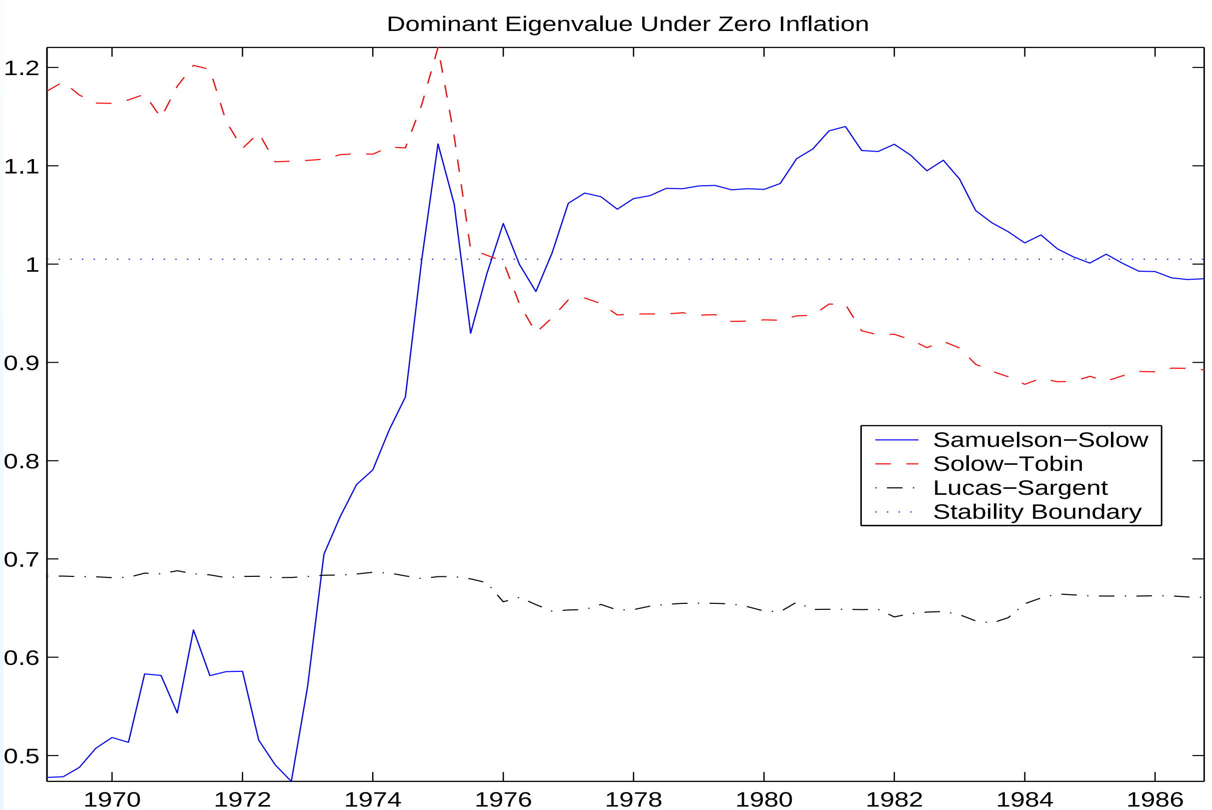
- 

Inflation and Optimal Policy

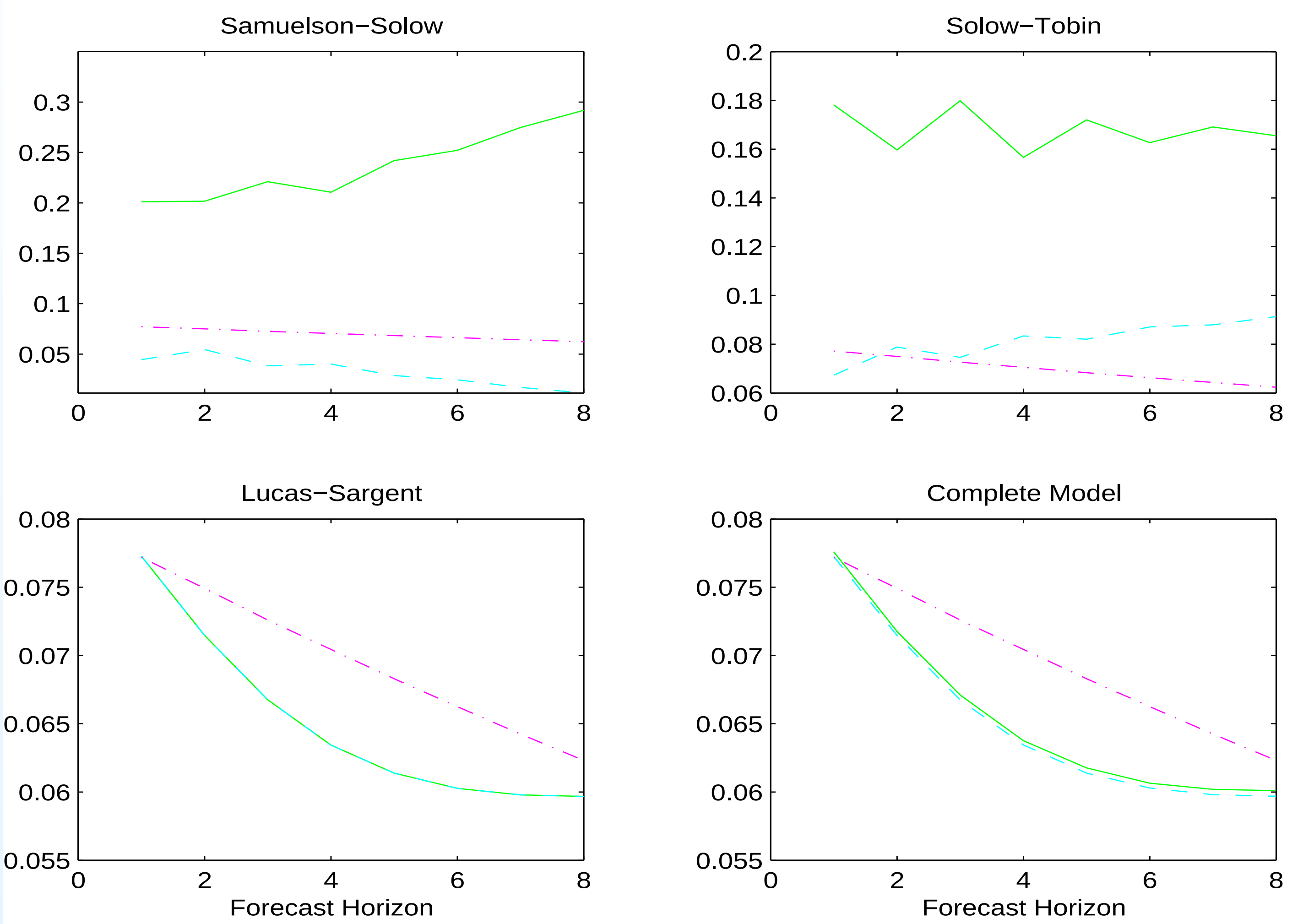


Expected Loss from a Zero Inflation Policy

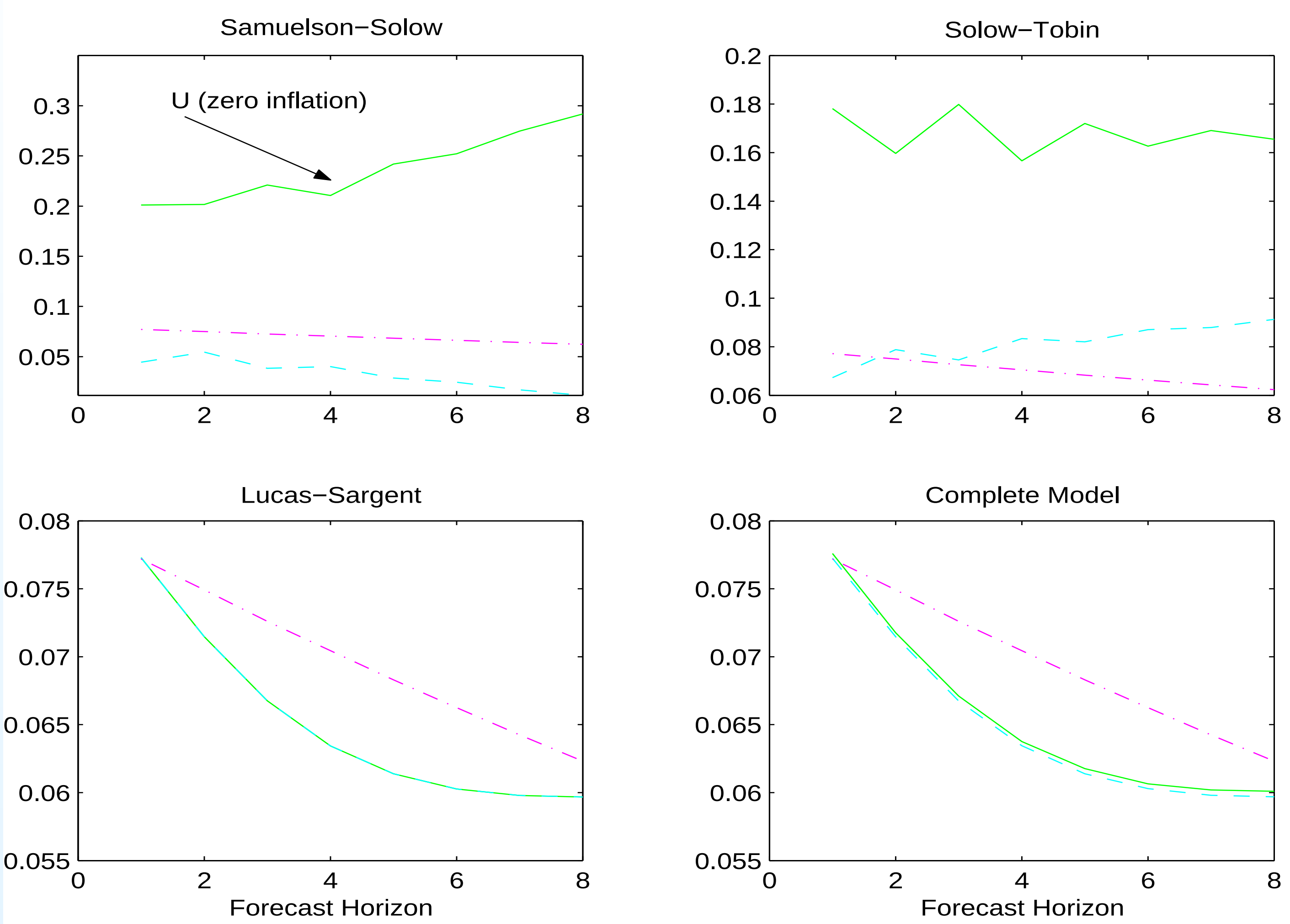
Dominant Eigenvalue Under Zero Inflation



Optimal Policy v. Zero Inflation, 1975.Q4



Optimal Policy v. Zero Inflation, 1975.Q4



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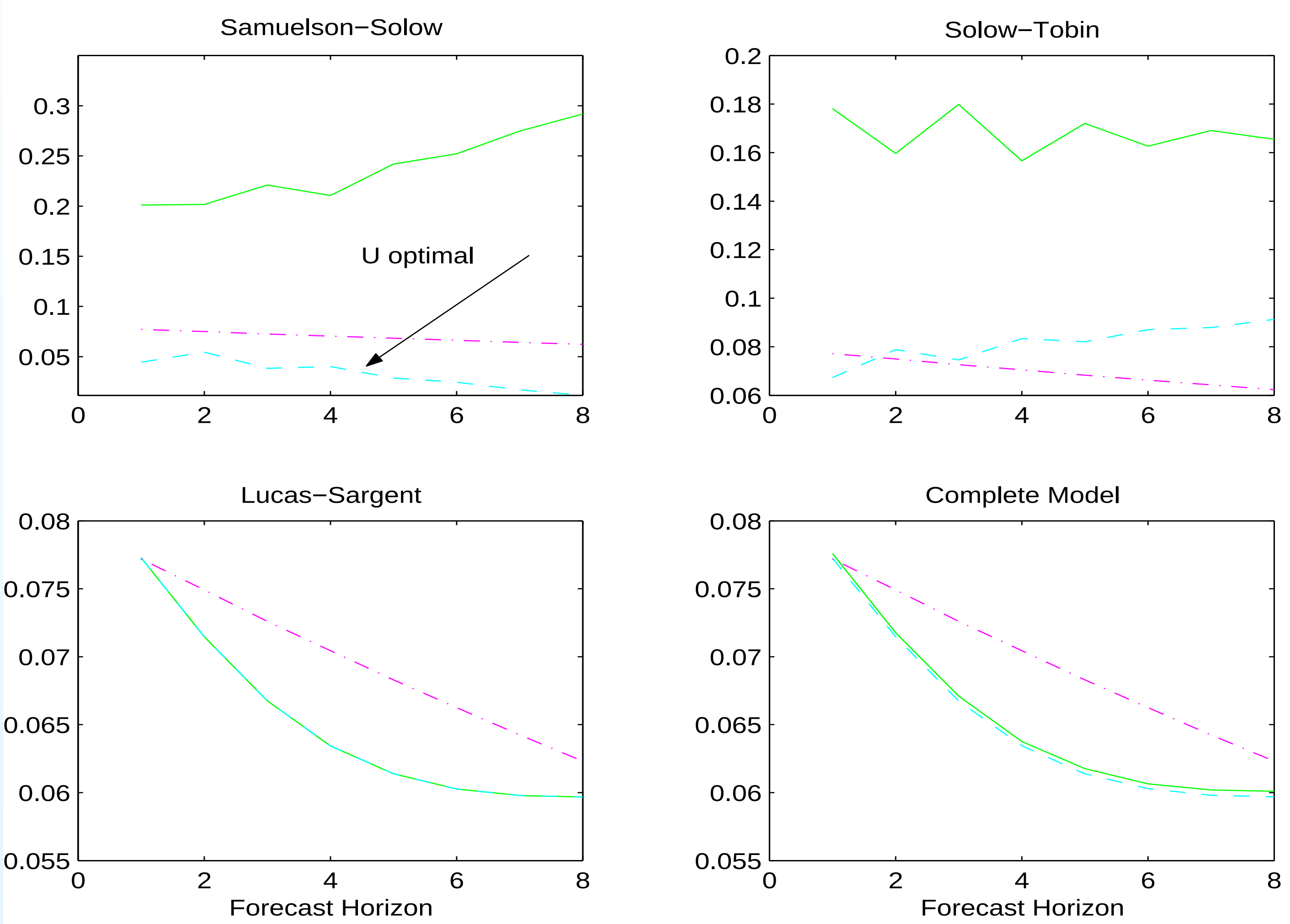
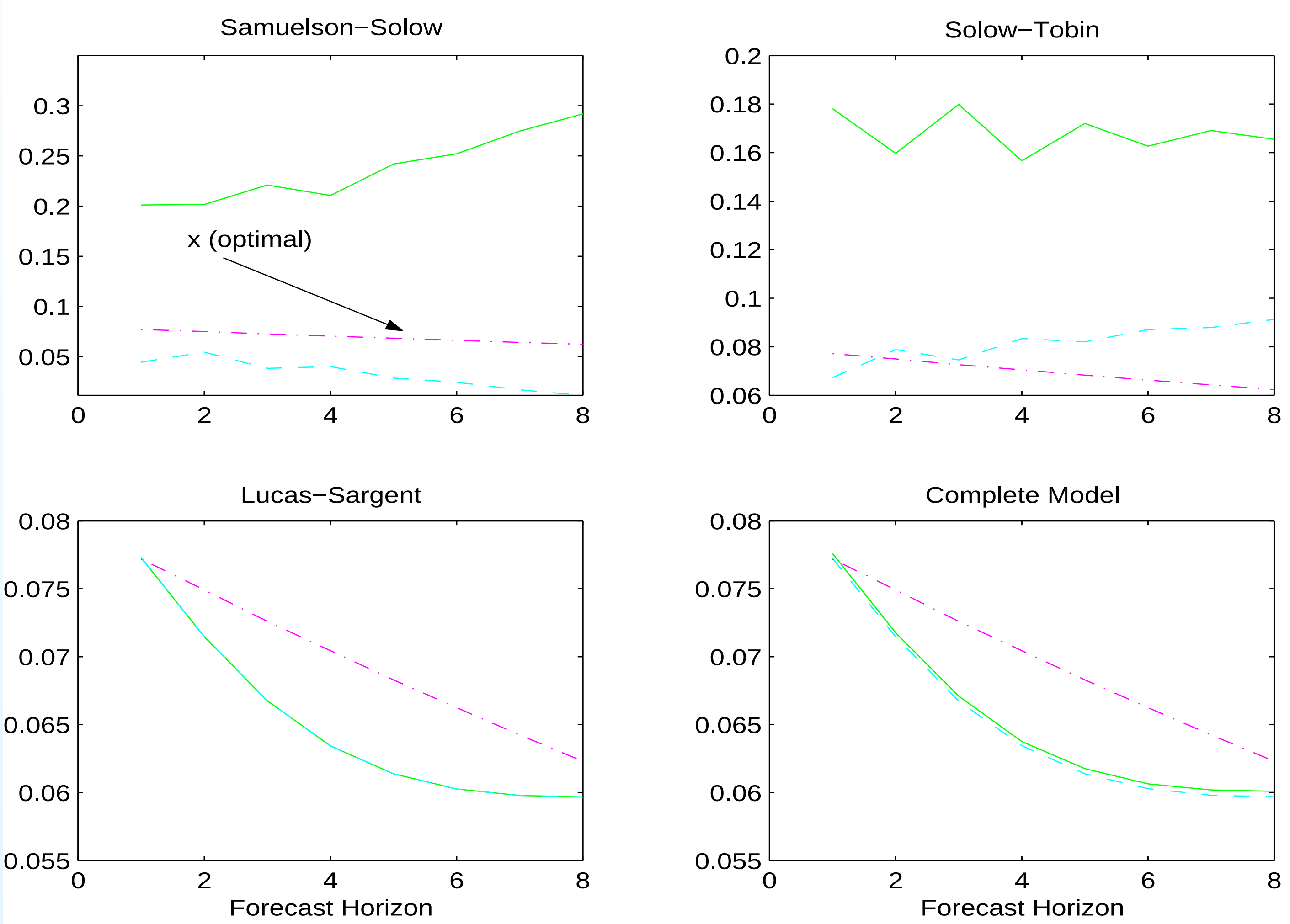
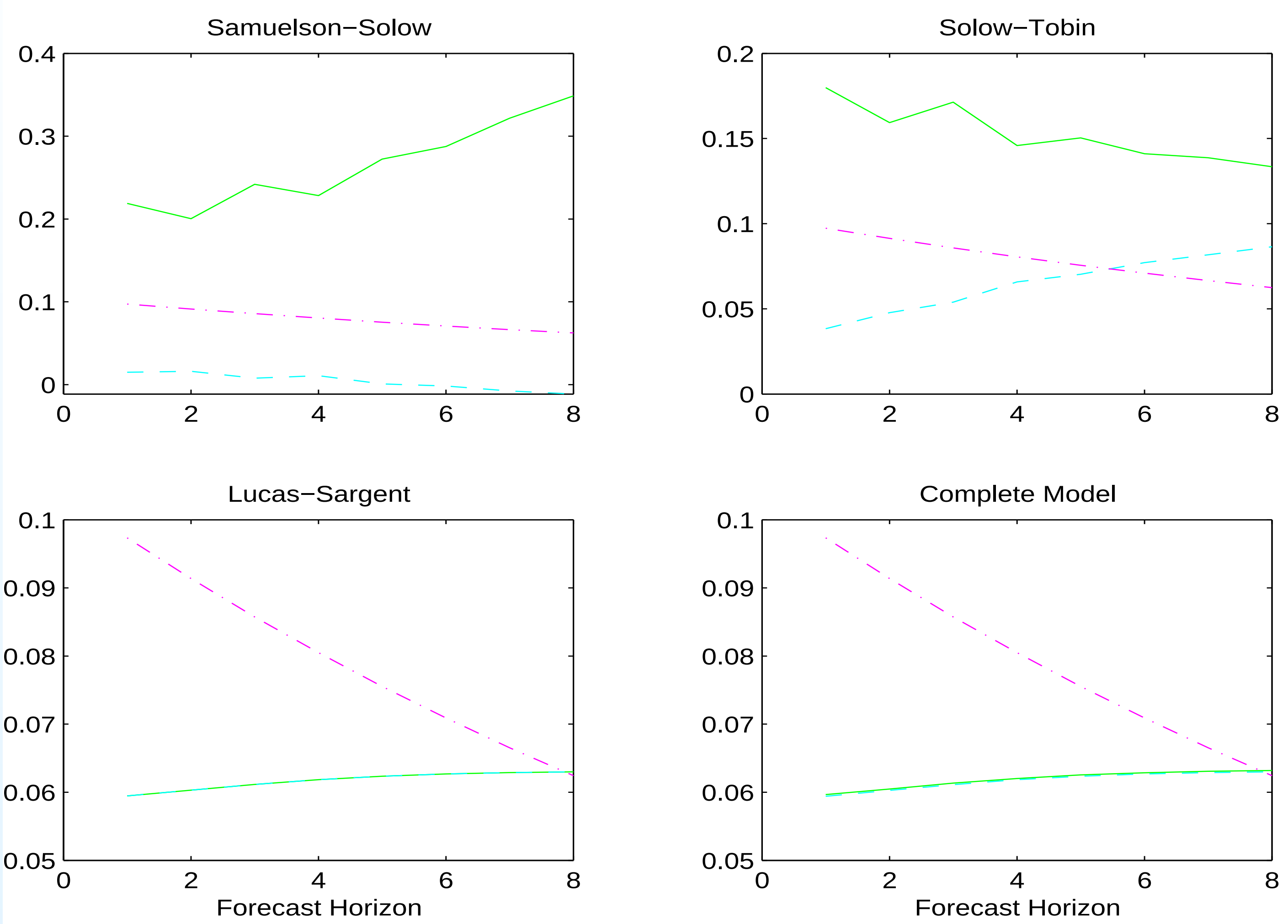


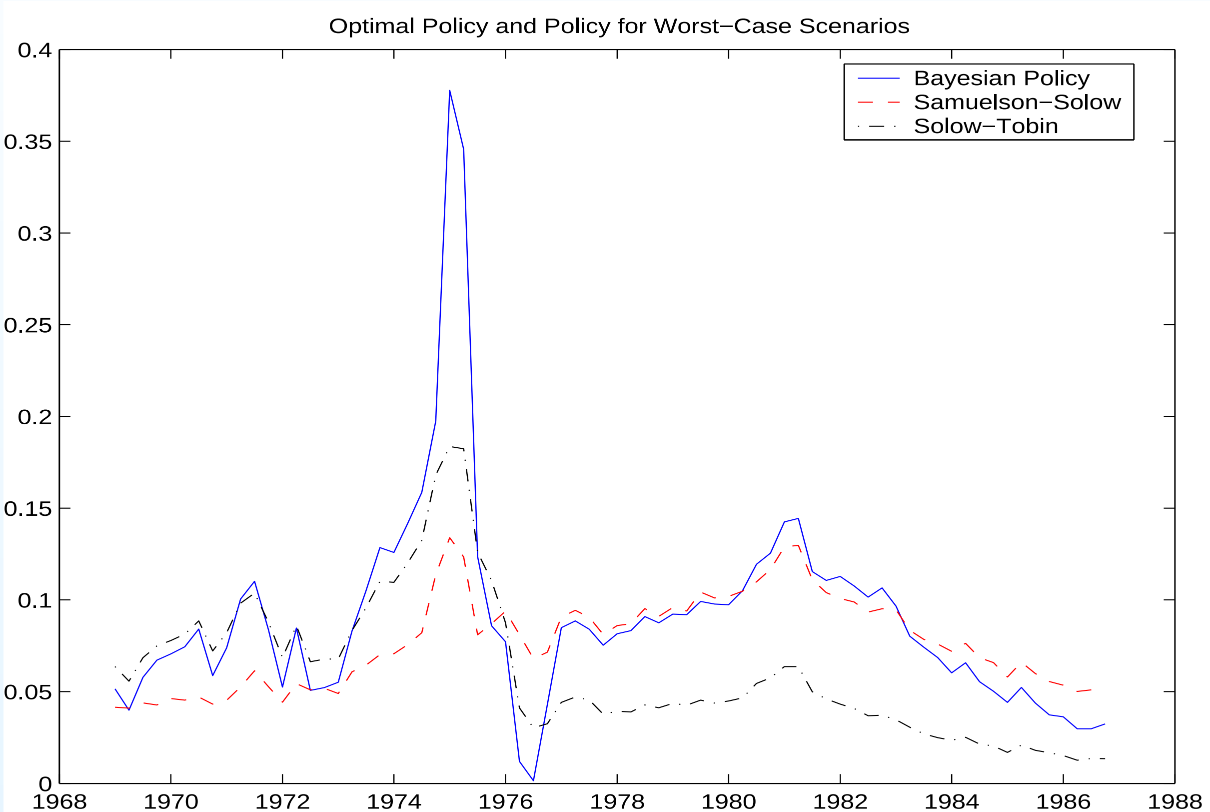
Optimal Policy v. Zero Inflation, 1975.Q4



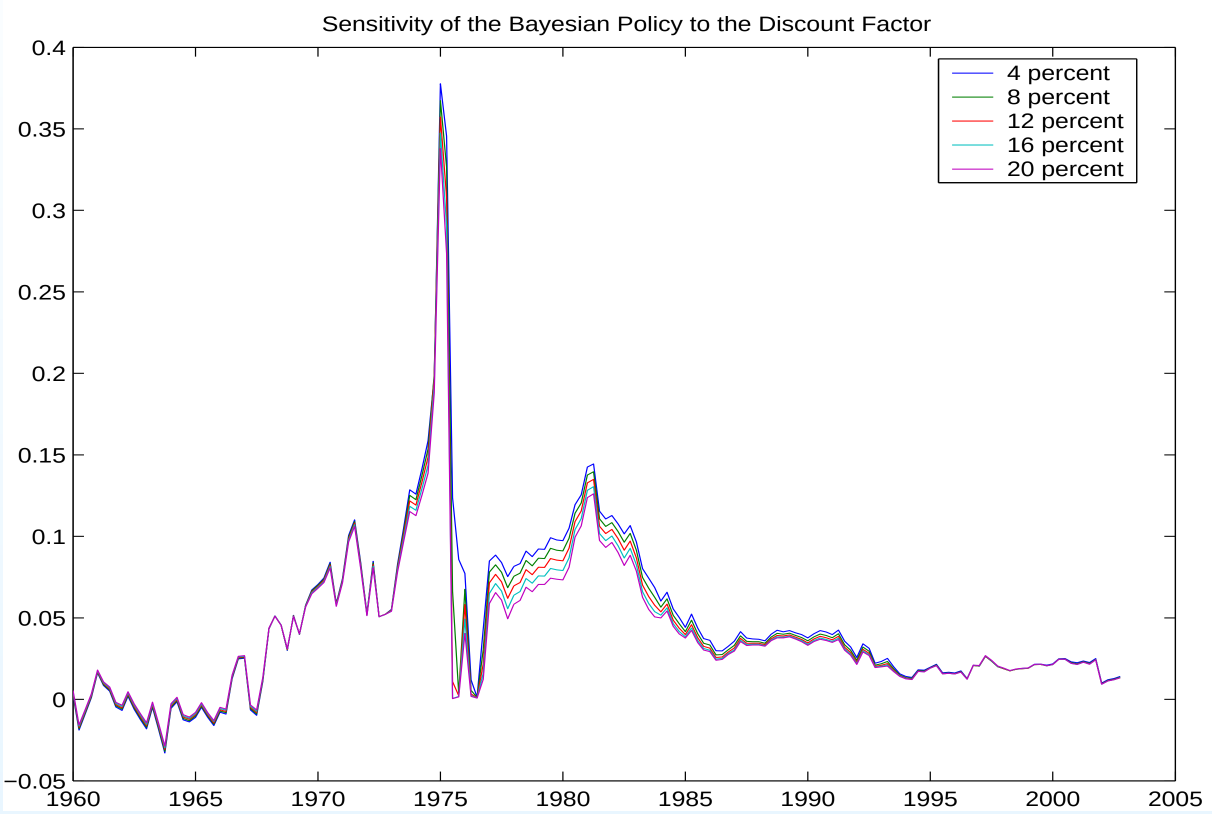
Optimal Policy v. Zero Inflation, 1979.Q4



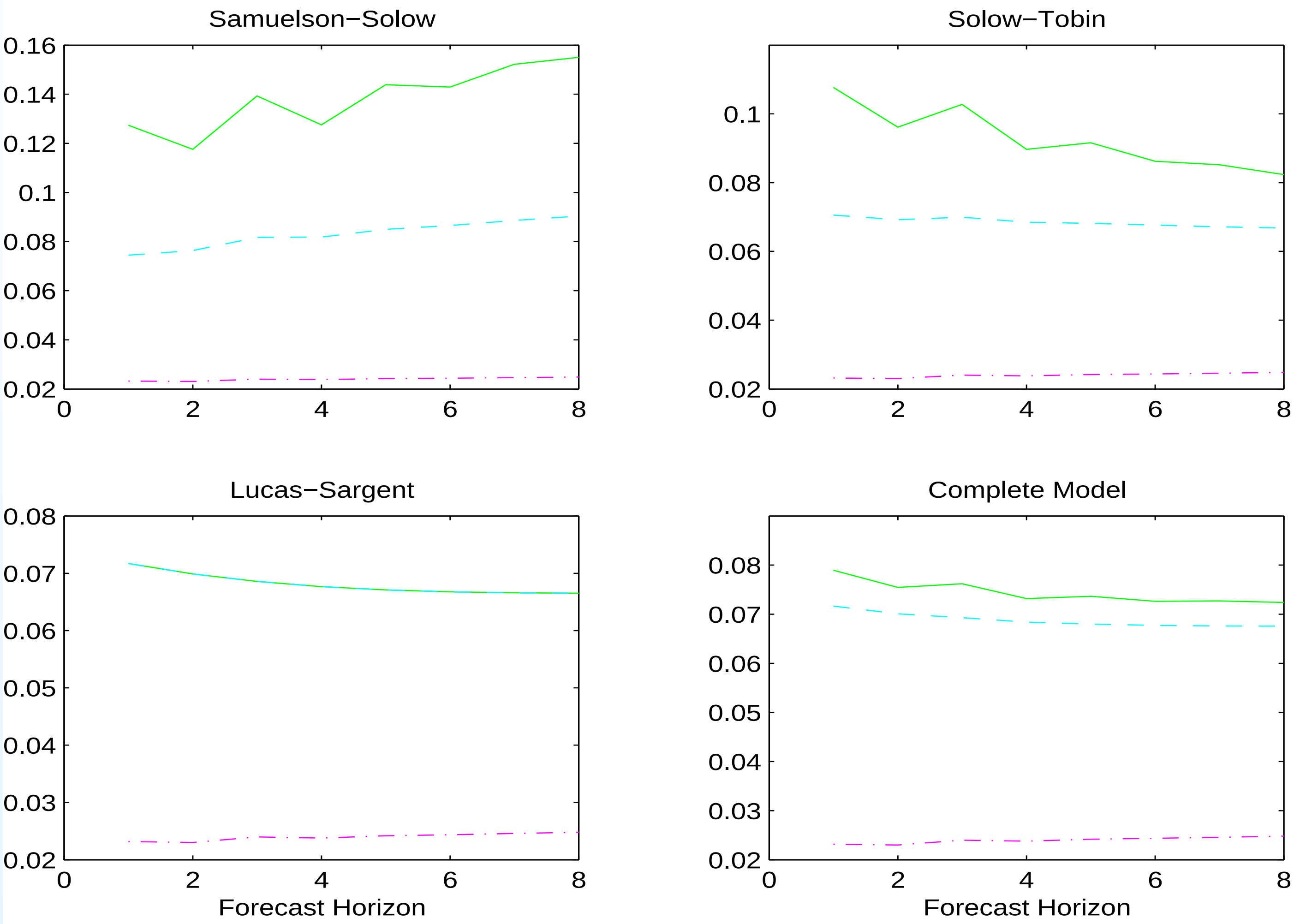
Optimal Policy and Policy for Worst-Case Scenarios



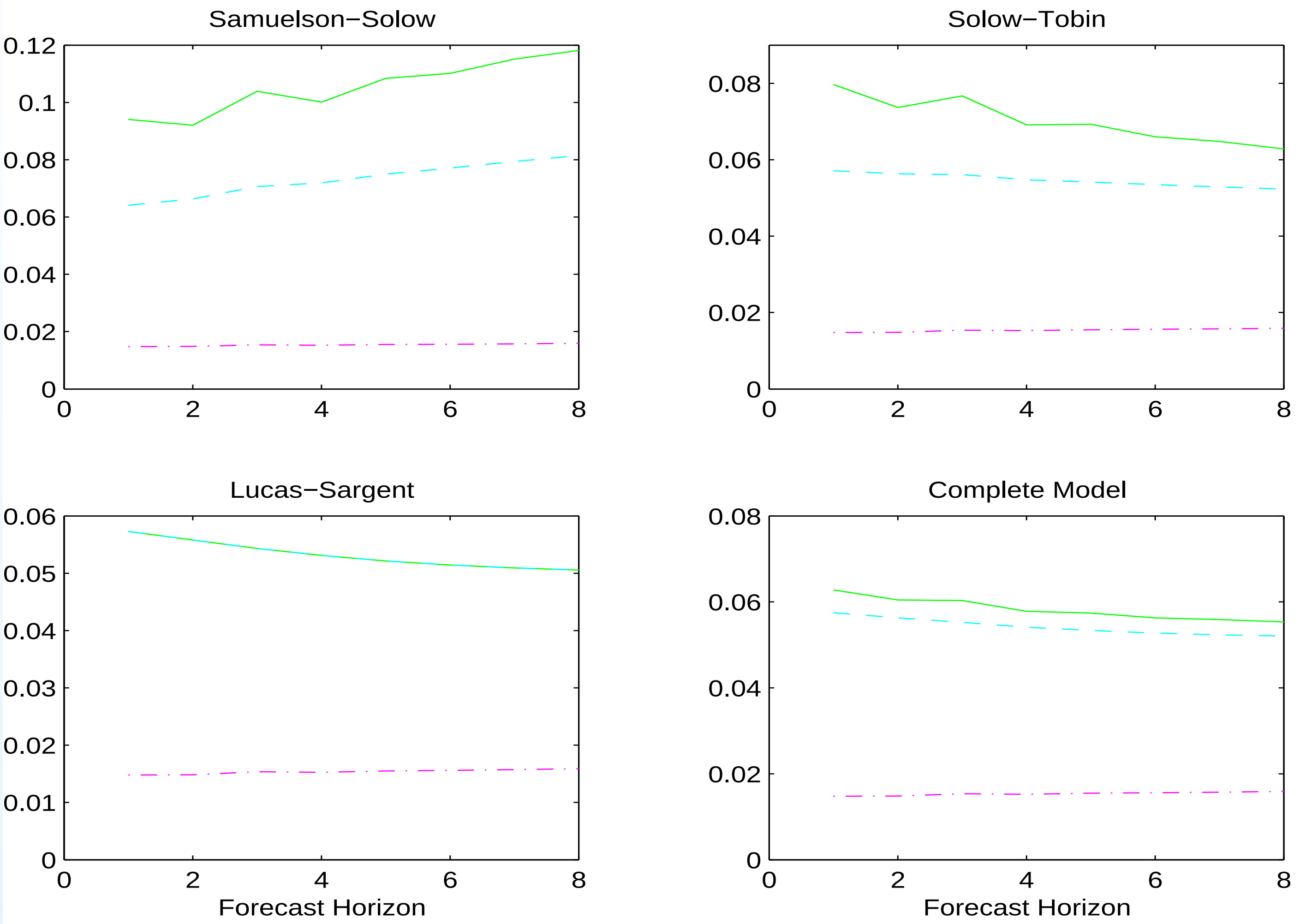
Sensitivity of Bayesian Policy to the Discount Rate



Optimal Policy v. Zero Inflation, 1992.Q4



Optimal Policy v. Zero Inflation, 2002.Q4



1978

Okun and Perry (1978) summarize things as follows:

> "Thus, the mainline model and its empirical findings reaffirm that there is a slow-growth, high unemployment cure for inflation, but that it is an extremely expensive one. ...Using one of Perry's successful equations as an example, an extra percentage point of unemployment would lower the inflation rate by only about 0.3 percentage point after one year and by 0.7 percentage point if maintained for three years. That extra point of unemployment would cost over a million jobs and some $60 billion of real production each year."

1978 again

Okun and Perry also summarize Perry's reasons for rejecting Feller's guesses of much lower costs in terms of unemployment that could be attained through a credible disinflationary policy:

> Perry "believes that much of the [inflation] inertia is backward-looking rather than forward looking, and so is not susceptible to even convincing demonstrations that demand will be restrained in the future. [Perry's] own empirical evidence shows that wage developments are better explained in terms of the recent past history of wages and prices than on any assumption that people are predicting the future course of wages and prices in a way that differs from the past." (page 6)

More 1978 beliefs

Perry (1978, pp. 50-51) elaborates forcefully on his argument against an expectational interpretation of Phillips curve dynamics. Okun (1978a, p.284) says that “recession will slow inflation, but only at the absurd cost in production of roughly $200 billion per point.” Okun (1978b) lists the models on which this estimate is based. Many distinguished economists are represented on the list. At that time, $200 billion amounted to roughly 10 percent of GDP. Inflation averaged 7.4 percent from 1974 to 1979, and extrapolating to zero inflation implies a total cost of almost three-quarters of a year’s GDP.