

Log-linearized equations are listed as below:

(Variables in upper case is its steady state value, while lower case is percentage deviation)

Two country open economy, flexible price setting (means symmetric equilibrium), pegging exchange rate regime of home currency

(1) Aggregate demand

$$y_t^w = \left[ n \frac{C}{Y^w} c_t + (1-n) \frac{C^*}{Y^w} c_t^* \right] + \left[ n \frac{I}{Y^w} i_t + (1-n) \frac{I^*}{Y^w} i_t^* \right] + \left[ n \frac{G}{Y^w} g_t + (1-n) \frac{G^*}{Y^w} g_t^* \right] + \left[ n \frac{C^e}{Y^w} c_t^e + (1-n) \frac{C^{e*}}{Y} c_t^{e*} \right] + \left[ n \frac{Mo}{Y^w} mo_t + (1-n) \frac{Mo^*}{Y^w} mo_t^* \right]$$

where

$$mo_t = r_t^k + q_{t-1} + k_t$$

$$mo_t^* = r_t^{k*} + q_{t-1}^* + k_t^*$$

are the monitoring cost in BGG.

$y_t^w$ : world demand

$c_t^e$ : entrepreneur consumption

Consumption:

$$c_t = -r_{t+1} + E_t\{c_{t+1}\}$$

$$c_t^* = -r_{t+1}^* + E_t\{c_{t+1}^*\}$$

Money demand:

$$m_t - p_t = c_t - \frac{\beta}{1-\beta} E_t r_{t+1} - \frac{\beta}{1-\beta} (E_t p_{t+1} - p_t)$$

$$m_t^* - p_t^* = c_t^* - \frac{\beta^*}{1-\beta^*} E_t r_{t+1}^* - \frac{\beta^*}{1-\beta^*} (E_t p_{t+1}^* - p_t^*)$$

Net worth and capital in BGG:

$$E_t\{r_{t+1}^k\} - r_{t+1} = -\eta [n_{t+1} - (q_t + k_{t+1})]$$

where  $\eta = \frac{R}{R^k} \frac{\Psi(R^k/R)}{\Psi'(R^k/R)}$  (according to BGG(1999), the steady state value of  $R$  and  $R^k$  are equal).

$$E_t\{r_{t+1}^{k*}\} - r_{t+1}^* = -\eta^* [n_{t+1}^* - (q_t^* + k_{t+1}^*)]$$

where  $\eta^* = \frac{R^*}{R^{k*}} \frac{\Psi^*(R^{k*}/R^*)}{\Psi'^*(R^{k*}/R^*)}$ .

Investment demand in BGG:

$$q_t = \varphi(i_t - k_t)$$

where  $\varphi = -\frac{\Phi''(\frac{1}{K})I/K}{\Phi'(\frac{1}{K})}$ , which is positive for the increasing and concavity of function  $\Phi(\cdot)$ .

$$q_t^* = \varphi^*(i_t^* - k_t^*)$$

where  $\varphi^* = -\frac{\Phi^{*''}(\frac{I^*}{K^*})I^*/K^*}{\Phi^{*'}(\frac{I^*}{K^*})}$ , which is positive for the increasing and concavity of function  $\Phi^*(\cdot)$ .

Capital demand in BGG:

$$r_{t+1}^k = (1-\zeta)(y_{t+1} - k_{t+1} - x_{t+1}) + \zeta q_{t+1} - q_t$$

where  $\zeta = \frac{Q(1-\delta)}{Q(1-\delta) + \frac{\alpha Y}{XK}}$ .

$$r_{t+1}^{k*} = (1 - \zeta^*)(y_{t+1}^* - k_{t+1}^* - x_{t+1}^*) + \zeta^* q_{t+1}^* - q_t^*$$

where  $\zeta^* = \frac{Q^*(1-\delta^*)}{Q^*(1-\delta^*) + \frac{\alpha^* Y^*}{X^* K^*}}$ .

**Entrepreneurs' consumption (high order neglected):**

$$\begin{aligned} c_t^e &= n_t \\ c_t^{e*} &= n_t^* \end{aligned}$$

**(2) Aggregate supply**

**National wholesale goods supply:**

$$\begin{aligned} y_t &= a_t + \alpha k_t + (1 - \alpha)\Omega h_t \\ y_t^* &= a_t^* + \alpha^* k_t^* + (1 - \alpha^*)\Omega^* h_t^* \end{aligned}$$

**Retailer' s supply of diversified product (flexible price setting):**

$$\begin{aligned} y_t(h) &= -\epsilon[p_t(h) - p_t] + y_t^d \\ y_t^*(f) &= -\epsilon[p_t^*(f) - p_t^*] + y_t^{d*} \end{aligned}$$

where  $y_t^d$  is  $y_t^w$  shown in the aggregate demand section.

**Labor demand and labor supply:**

$$\begin{aligned} y_t - h_t - x_t - c_t &= h_t \frac{H}{1 - H} \\ y_t^* - h_t^* - x_t^* - c_t^* &= h_t^* \frac{H^*}{1 - H^*} \end{aligned}$$

Price evolution (flexible price setting):

$$\begin{aligned} p_t &= n p_t(h) + (1 - n)[e_t + p_t^*(f)] \\ p_t^* &= n[p_t(h) - e_t] + (1 - n)p_t^*(f) \end{aligned}$$

which imply that PPP holds for global consumption goods:

$$p_t = p_t^* + e_t$$

Optimal retail price setting:

$$\begin{aligned} p_t(h) &= p_t - x_t \\ p_t^*(f) &= p_t^* - x_t^* \end{aligned}$$

**(3) Evolution of state variables**

**Capital formation:**

$$k_{t+1} = \delta i_t + (1 - \delta)k_t$$

(actually it is

$k_{t+1} = \delta(i_t \frac{1}{KQ\Phi(\frac{1}{K})} - k_t \frac{1}{KQ\Phi(\frac{1}{K})} + k_t) + (1 - \delta)k_t$ . Therefore, it is implied by equation that at

steady state,  $\frac{1}{KQ\Phi(\frac{1}{K})} = 1$ ; if  $Q=1$ , then it means that  $\frac{1}{K} = \Phi\left(\frac{1}{K}\right) = \delta$ )

$$k_{t+1}^* = \delta^* i_t^* + (1 - \delta^*)k_t^*$$

**Net worth evolution :**

$$\begin{aligned}
n_t &= \frac{\rho}{\beta} n_{t-1} + \left( \frac{\rho}{\beta} - \frac{\rho K}{\beta N} \right) r_t + \left( \frac{\rho K}{\beta N} + \frac{\rho K}{N} \left( R^k - \frac{1}{\beta} \right) \right) r_t^k + \frac{\rho K}{N} \left( R^k - \frac{1}{\beta} \right) q_{t-1} \\
&\quad + \frac{\rho K}{N} \left( R^k - \frac{1}{\beta} \right) k_t + (1 - \alpha)(1 - \Omega) \frac{Y}{XN} (y_{t+1} - x_{t+1}) + v E n_{t+1}^* \\
n_t^* &= \rho^* R^* (1 - v) n_{t-1}^* + \left( (1 - v) \rho^* R^* - \frac{\rho^* R^* K^*}{N^*} \right) r_t^* + \left( \frac{\rho^* R^* K^*}{N^*} + \frac{\rho^* K^*}{N^*} (R^{k^*} - R^*) \right) r_t^{k^*} \\
&\quad + \frac{\rho^* K^*}{N^*} (R^{k^*} - R^*) q_{t-1}^* + \frac{\rho^* K^*}{N^*} (R^{k^*} - R^*) k_t^* \\
&\quad + (1 - \alpha^*)(1 - \Omega^*) \frac{Y^*}{X^* N^*} (y_{t+1}^* - x_{t+1}^*)
\end{aligned}$$

**Foreign reserve of home country:**

$$f_t = (1 + R^g) f_{t-1} + \frac{TB}{F} tb_t + \frac{v N^* P^* E}{F} (n_t^* + p_t^* + e_t)$$

where  $F$ ,  $R^g$ ,  $TB$  are all in foreign nominal term, while  $N^*$  is in real term deflated by contemporary foreign consumer price index  $P^*$ .  $tb_t$  is calculated as follows (**flexible price**).

**Trade balance of home country:**

$$\begin{aligned}
tb_t &= \frac{nPY^w}{TB} \left( \frac{P(h)}{P} \right)^{1-\epsilon} [\epsilon p_t + y_t^w + (1 - \epsilon) p_t(h)] - \frac{PC}{TB} (p_t + c_t) - \frac{PI}{TB} (p_t + i_t) - \frac{PG}{TB} (p_t + g_t) \\
&\quad - \frac{PC^e}{TB} (p_t + c_t^e) - \frac{PMo}{TB} (p_t + r_t^k + q_{t-1} + k_t)
\end{aligned}$$

where  $Mo_t = \mu \int_0^{\bar{\omega}} \omega dF(\omega) R_t^k Q_{t-1} K_t$ , and  $nPY^w \left( \frac{P(h)}{P} \right)^{1-\epsilon}$  is home country' s nominal GDP.

#### (4) Monetary policy rule, government budget and shock processes

**Aggregate TFP:**

$$\begin{aligned}
a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\
a_t^* &= \rho_a^* a_{t-1}^* + \varepsilon_t^{a^*}
\end{aligned}$$

**Fiscal policy:**

$$\begin{aligned}
g_t &= \rho_g g_{t-1} + \varepsilon_t^g \\
g_t^* &= \rho_g^* g_{t-1}^* + \varepsilon_t^{g^*}
\end{aligned}$$

**Money supply:**

$$\begin{aligned}
m_t - m_{t-1} &= e_t + (f_t - f_{t-1}) + \xi_t \\
m_t^* - m_{t-1}^* &= \xi_t^*
\end{aligned}$$

(which means that when the home country is increasing its position of foreign treasury bond, it will cause the money base in foreign country shrinks to a same magnitude by open market

operation; in addition, these two equations reveal the relative position of the two countries: foreign country' s monetary policy is free for its own currency is being used as reserves in home country, while home country has the responsibility to purchase or sell its reserves in foreign currency to stabilize the predetermined exchange rate)

$$e_t = 0$$