Measuring the Effects of Fiscal Policy in a Model with Financial Frictions*

Jesús Fernández-Villaverde†

April 26, 2010

Abstract

In these notes, I explain in further detail the model in my paper "Fiscal Policy with Financial Frictions."

Keywords: DSGE models, Fiscal Policy, Financial Frictions.

*I thank the NSF for financial support.

†University of Pennsylvania, NBER, CEPR, and FEDEA, <jesusfv@econ.upenn.edu>.

1. Introduction

In these notes, I explain in further detail the model in my paper "Fiscal Policy with Financial Frictions" that appears in AER, P&P May 2010.

2. A Model of Financial Frictions with Fiscal Policy

I describe a simple model with a representative household, final and intermediate good producers, producers of capital, entrepreneurs, financial intermediaries, and a government that conducts monetary and fiscal policy. The financial frictions appear as a consequence of information asymmetries between lenders and borrowers.

2.1. Household

There is a representative household that maximizes a utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\phi_t} \left\{ \log\left(c_t - hc_{t-1}\right) - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} + \upsilon \log\left(\frac{m_{t-1}}{p_t}\right) \right\}$$

where c_t is consumption, l_t hours worked, m_{t-1}/p_t (where p_t is the price level) real money balances that the household carry into the period, β is the discount factor, h controls habit persistence, and ϕ_t is an intertemporal preference shock with law of motion:

$$\phi_t = \rho_d \phi_{t-1} + \sigma_\phi \varepsilon_{\phi,t} \text{ where } \varepsilon_{\phi,t} \sim \mathcal{N}(0,1).$$

This intertemporal shock allows me to capture changes in aggregate demand in a simple way. Empirically, it helps the Euler intertemporal equation of consumption to fit the data.

The representative household has a non-trivial portfolio decision since it can save on:

1. Money balances to carry into the next period, m_t.

2. Nominal deposits at the financial intermediary, a_t, which pay an uncontingent nominal gross interest rate R_t.

3. Nominal government bonds, d_t, which pay an uncontingent nominal gross return Rd_t.

4. Arrow securities over all possible events. Since, in equilibrium, the net supply of those securities must be zero, we do not include them in the budget constraint to save on notation. This complete markets assumption will be convenient below to price the

future flows of profits of the firms in the economy (and, for the matter, any other redundant asset, such a long-term bonds).

Given the portfolio possibilities, the household's budget constraint is given by:

$$\begin{aligned} &(1+\tau_{c,t})\, c_t + \frac{a_t}{p_t} + \frac{d_t}{p_t} + \frac{m_t}{p_t} \\ &= (1-\tau_{l,t})\, w_t l_t + (1 + (1-\tau_{R,t})(R_{t-1}-1))\frac{a_{t-1}}{p_t} + Rd_{t-1}\frac{d_{t-1}}{p_t} + \frac{m_{t-1}}{p_t} + T_t + \digamma_t + tre_t \end{aligned}$$

where real consumption is taxed at rate $\tau_{c,t}$, the real wage w_t is is taxed at a rate $\tau_{l,t}$, the net returns on deposits are taxed at rate $\tau_{R,t}$, T_t is a lump-sum transfer from the result of open market operations of the monetary authority, $\digamma_t$ are the profits of the firms in the economy (financial and non-financial) plus the intermediation costs of the financial firm, and tre_t is the net real transfer to new and from old entrepreneurs that we will describe momentarily and that takes the form:

$$tre_t = (1-\gamma_t^e)\, n_t - w^e$$

Note that the returns on public debt are not taxed. If the tax were a constant or it would be determined in period $t-1$ for returns on period t, it would be irrelevant to have the tax or not: an arbitrage condition would raise the before-tax return on public debt and leave the after-tax return unchanged, namely, the government would pay higher interest rates and recover higher taxes without any real change in allocations. If the tax for period t were announced on period t, we would be introducing a state-dependent return on public debt that it is more convenient to abstract at the moment to keep the analysis focused (and which is rarely observed in practice anyway).

The first order conditions for the problem of the household are:

$$\begin{aligned} e^{\phi_t}\frac{1}{c_t - hc_{t-1}} - \mathbb{E}_t\beta e^{\phi_{t+1}}\frac{h}{c_{t+1}-hc_t} &= (1+\tau_{c,t})\,\lambda_t \\ \lambda_t &= \beta\mathbb{E}_t\left\{\lambda_{t+1}\frac{(1+(1-\tau_{R,t+1})(R_t-1))}{\Pi_{t+1}}\right\} \\ \lambda_t &= \beta\mathbb{E}_t\left\{\lambda_{t+1}\frac{Rd_t}{\Pi_{t+1}}\right\} \\ e^{\phi_t}\psi l_t^{\vartheta} &= (1-\tau_{l,t})\, w_t\lambda_t \end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint (I omit the first order condition with respect to money holdings since it will be irrelevant for the dynamics of the model). Note that the second and third first order condition imply the arbitrage condition:

$$\mathbb{E}_t\left\{\lambda_{t+1}\frac{\left(1+\left(1-\tau_{R,t+1}\right)\left(R_t-1\right)\right)}{\Pi_{t+1}}\right\}=\mathbb{E}_t\left\{\lambda_{t+1}\frac{Rd_t}{\Pi_{t+1}}\right\}$$

This condition illustrates that, while both R_t and Rd_t are uncontingent, the after tax returns on deposits are not. Therefore, in equilibrium, there will be a premium on the after-tax returns of deposits over the return of public debt to compensate for that tax risk.

2.2. The Final Good Producer

There is one final good produced using intermediate goods according to the aggregator:

$$y_t=\left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where ε is the elasticity of substitution across goods.

The final good producer is perfectly competitive and maximize profits subject to the production function (1), taking as given all intermediate goods prices p_{ti} and the final good price p_t. Thus, the input demand functions is:

$$y_{it}=\left(\frac{p_{it}}{p_t}\right)^{-\varepsilon}y_t \qquad \forall i,$$

where y_t is the aggregate demand and price level:

$$p_t=\left(\int_0^1 p_{it}^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}.$$

2.3. Intermediate Good Producers

There is a continuum of intermediate goods producers that enjoy some market power on their own good. Each intermediate good producer i has access to a technology represented by a production function

$$y_{it}=e^{z_t}k_{it-1}^{\alpha}l_{it}^{1-\alpha}$$

where k_{it-1} is the capital rented by the firm, l_{it} is the amount of labor input rented by the firm, and where the productivity level z_t follows:

$$z_t=\rho_z z_{t-1}+\sigma_z\varepsilon_{z,t} \text{ where } \varepsilon_{z,t}\sim\mathcal{N}(0,1).$$

Cost minimizations implies:

$$k_{it-1}=\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}l_{it}$$

and that the marginal cost is:

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{w_t^{1-\alpha} r_t^{\alpha}}{e^{z_t}}$$

Since all the intermediate good producers face the same prices, market clearing imposes that:

$$\frac{k_{t-1}}{l_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}.$$

This result will be convenient below when we derive an expression for aggregate supply.

The firms are subject to a Calvo pricing mechanism. In each period, a fraction $1-\theta$ of firms can change their prices while all other firms keep the previous price. All other firms can only index their prices by past inflation. Indexation is controlled by the parameter $\chi \in [0,1]$, where $\chi = 0$ is no indexation and $\chi = 1$ is total indexation. The problem of firm i is then to solve:

$$\max_{p_{it}} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t}\left\{\left(\prod_{s=1}^{\tau}\frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}}\frac{p_{it}}{p_t} - mc_{t+\tau}\right) y_{it+\tau}\right\}$$

subject to

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau}\frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}}\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_{t+\tau},$$

where the marginal value of a dollar to the household, as determined by the ratio of Lagrangian multipliers, is treated as exogenous by the firm.

Using the fact that we deal with a symmetric equilibrium where $p_{it}^* = p_t^*$, (and after a fair amount of algebra), the relative reset price $\Pi_t^* = p_t^*/p_t$ is set such that the following conditions are satisfied:

$$\varepsilon f_t^1 = (\varepsilon - 1) f_t^2$$

$$f_t^1 = \lambda_t mc_t y_t + \beta\theta \mathbb{E}_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon} f_{t+1}^1$$

$$f_t^2 = \lambda_t \Pi_t^* y_t + \beta\theta \mathbb{E}_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon}\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) f_{t+1}^2$$

where f_t^1 and f_t^2 are two auxiliary variables. Also, given Calvo's pricing, the price index evolves as:

$$1 = \theta\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{1-\varepsilon} + (1-\theta)\,\Pi_t^{*1-\varepsilon}.$$

2.4. Capital Good Producers

Capital is created by a perfectly competitive capital good producer that buys installed capital, x_t, and adds new investment, i_t using the final good in the economy, to generate new installed capital for the next period:

$$x_{t+1} = x_t + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t$$

where $S[1] = 0$, $S'[1] = 0$, and $S''[\cdot] > 0$. The period profits of the firm are then:

$$q_t \left(x_t + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t\right) - q_t x_t - i_t = q_t \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t - i_t$$

where q_t is the relative price of capital in the period. The discounted profits for the capital good producer are then:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left(q_t \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t - i_t\right)$$

Note that this objective function does not depend on the level of x_t and hence we can make it equal to $(1-\delta) k_{t-1}$ to clear the market.

The first order condition of this problem is:

$$q_t \left(1 - S\left[\frac{i_t}{i_{t-1}}\right] - S'\left[\frac{i_t}{i_{t-1}}\right] \frac{i_t}{i_{t-1}}\right) + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left[\frac{i_{t+1}}{i_t}\right] \left(\frac{i_{t+1}}{i_t}\right)^2 = 1$$

and the law of motion for aggregate capital is:

$$k_t = (1-\delta) k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t$$

2.5. Entrepreneurs

Entrepreneurs use their (end-of-period) real wealth, n_t, and a nominal loan b_t, to purchase new installed capital k_t:

$$q_t k_t = n_t + \frac{b_t}{p_t}$$

When mapping into the data, we can think about wealth as equity and the loan as the sum of all liabilities of the firm. The presence of nominal debt opens the door for a "Fisher effect" where inflation increases (or deflation erodes) the net wealth of entrepreneurs. We will come back to this point below.

The purchased capital is shifted by a productivity shock ω_{t+1} that is lognormally distributed with CDF $F(\omega)$ and parameters $\mu_{\omega,t}$ and $\sigma_{\omega,t}$ such that $\mathbb{E}_t\omega_{t+1} = 1$ for all t. Therefore:

$$\mathbb{E}_t\omega_{t+1} = e^{\mu_{\omega,t+1}+\frac{1}{2}\sigma^2_{\omega,t+1}} = 1 \Rightarrow \mu_{\omega,t+1} = -\frac{1}{2}\sigma^2_{\omega,t+1}$$

The evolution of the standard deviation is such that:

$$\log\sigma_{\omega,t} = (1-\rho_\sigma)\log\sigma_\omega + \rho_\sigma\log\sigma_{\omega,t-1} + \eta_\sigma\varepsilon_{\sigma,t} \text{ where } \varepsilon_{\sigma,t} \sim \mathcal{N}(0,1)$$

The shock $t+1$ is revealed at the end of period t right before investment decisions are decided. Then:

$$\log\sigma_{\omega,t} - \log\sigma_\omega = \rho_\sigma\left(\log\sigma_{\omega,t-1} - \log\sigma_\omega\right) + \eta_\sigma\varepsilon_{\sigma,t} \Rightarrow$$
$$\widehat{\sigma}_{\omega,t} = \rho_\sigma\widehat{\sigma}_{\omega,t-1} + \eta_\sigma\varepsilon_{\sigma,t}$$

To keep track of the value of $\sigma_{\omega,t}$, we will make the dependence explicit and write $F(\omega,\sigma_{\omega,t})$.

The entrepreneur rents the capital to intermediate good producers, who pay r_{t+1}. Then, at the end of the period, the entrepreneur sells the undepreciated capital to the capital good producer at price q_{t+1}. Therefore, the average return of the entrepreneur per nominal unit invested in period t is:

$$R^k_{t+1} = \frac{p_{t+1}}{p_t}\frac{r_{t+1} + q_{t+1}(1-\delta)}{q_t}.$$

The debt contract is structured as follows. For every state with associated return on capital R^k_{t+1}, entrepreneurs have to either pay a state-contingent gross nominal interest rate R^l_{t+1} on the loan or default. If the entrepreneur defaults, it gets nothing: the financial intermediary sizes its revenue, although a proportion μ of that revenue is lost in bankruptcy procedures. Hence, the entrepreneur will always pay if it has generated enough revenue to do so. This will be the case if productivity is at least as high as a level $\overline{\omega}_{t+1}$ at which the entrepreneurs just can pay back its debt:

$$R^l_{t+1}b_t = \overline{\omega}_{t+1}R^k_{t+1}p_tq_tk_t$$

This equation tells us that $\overline{\omega}_{t+1}$ moves in the same direction than R^l_{t+1} all other variables being equal. The equation is also useful because, below, instead of characterizing the debt contract in terms of R^l_{t+1}, we will do it in terms of $\overline{\omega}_{t+1}$, which is much easier. If $\omega_{t+1} < \overline{\omega}_{t+1}$, the entrepreneur defaults, the financial intermediary monitors the entrepreneur and gets $(1-\mu)$ of the revenue of the entrepreneur. This is the mechanism proposed by Bernanke, Gertler, and Gilchrist (1999) to capture the asymmetries of information between lenders and borrowers and the need to have a costly-state verification.

The debt contract determines R^l_{t+1} to be the return such that financial intermediaries satisfy its zero profit condition in all states of the world:

$$\underbrace{\left[1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]R^l_{t+1}b_t}_{\text{Revenue if loan pays}}+\underbrace{(1-\mu)\int_0^{\overline{\omega}_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t+1}\right)R^k_{t+1}p_tq_tk_t}_{\text{Revenue if loan defaults}}=\underbrace{s_tR_tb_t}_{\text{Cost of funds}}$$

where R_t is the (non-contingent) return of households that have saved in the financial intermediary and s_t is a spread caused by the costs of intermediation (for example, the labor costs of writing the loan contract or the cost of setting up offices for the financial intermediary to receive funds from households). I assume that these costs evolve stochastically over time in such a way that the spread is:

$$s_t=1+e^{\overline{s}+\widetilde{s}_t}$$

where:

$$\widetilde{s}_t=\rho_s\widetilde{s}_{t-1}+\sigma_s\varepsilon_{s,t}\text{ where }\varepsilon_{s,t}\sim\mathcal{N}(0,1).$$

For simplicity, we will assume that the intermediation cost is rebated back to the households in a lump-sum fashion (we can imagine, for instance, that intermediation costs are wages paid back to the household on an inelastically supplied amount of intermediation know-how). Finally, note that he zero profit condition loads all the aggregate risk of delivering the right level of return to the financial intermediary through changes in $\overline{\omega}_{t+1}$ (and the associated movements in R^l_{t+1}).

To explore the debt contract further, define:

$$\underbrace{\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}_{\text{Share of entrepreneurial earnings accrued to the financial intermediary}}=\overline{\omega}_{t+1}\left(1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)+G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)$$

$$G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=\int_0^{\overline{\omega}_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t+1}\right)$$

Note that, by the properties of the lognormal distribution:

$$\begin{aligned}G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)&=\int_0^{\overline{\omega}_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t+1}\right)\\&=1-\Phi\left(\frac{\mu_{\omega,t+1}+\sigma^2_{\omega,t+1}-\log\overline{\omega}_{t+1}}{\sigma_{\omega,t+1}}\right)\\&=1-\Phi\left(\frac{\frac{1}{2}\sigma^2_{\omega,t+1}-\log\overline{\omega}_{t+1}}{\sigma_{\omega,t+1}}\right)\end{aligned}$$

where Φ is the CDF of a normal distribution. Thus, we can rewrite the zero profit condition of the financial intermediary as:

$$\left[\overline{\omega}_{t+1}\left[1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]+(1-\mu)\int_{0}^{\overline{\omega}_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t+1}\right)\right]\frac{R_{t+1}^{k}}{s_t R_t}q_t k_t=\frac{b_t}{p_t}\Rightarrow$$
$$\frac{R_{t+1}^{k}}{s_t R_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]q_t k_t=\frac{b_t}{p_t}$$

which gives a schedule relating R_{t+1}^{k} and $\overline{\omega}_{t+1}$, a key component of the model. For example, when R_{t+1}^{k} is low, $\overline{\omega}_{t+1}$ is high, which increases the payoffs to the financial intermediary to compensate the lower return on capital although it also raises default rates.[1]

Now, define the ratio of loan over wealth:

$$\varrho_t=\frac{b_t/p_t}{n_t}=\frac{q_t k_t-n_t}{n_t}=\frac{q_t k_t}{n_t}-1$$

and we get and expression for the zero profit condition of the form:

$$\frac{R_{t+1}^{k}}{s_t R_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]\frac{q_t k_t}{n_t}=\frac{b_t/p_t}{n_t}\Rightarrow$$
$$\frac{R_{t+1}^{k}}{s_t R_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right](1+\varrho_t)=\varrho_t$$

that tells us that all the entrepreneurs , regardless of their level of wealth, will have the same leverage, ϱ_t, a most convenient feature for aggregation.

The problem of the entrepreneur is then to pick ϱ_t and a schedule for $\overline{\omega}_{t+1}$ to maximize its expected net worth given the zero-profit condition of the financial intermediary:

$$\max_{\varrho_t,\overline{\omega}_{t+1}}\mathbb{E}_t\left\{\begin{array}{c}\frac{R_{t+1}^{k}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)(1+\varrho_t)\\ +\eta_t\left[\frac{R_{t+1}^{k}}{s_t R_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right](1+\varrho_t)-\varrho_t\right]\end{array}\right\}$$

with first order conditions:

$$\varrho_t\ :\ \mathbb{E}_t\frac{R_{t+1}^{k}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)+\eta_t\left[\frac{R_{t+1}^{k}}{s_t R_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]-1\right]=0$$
$$\overline{\omega}_{t+1}\ :\ -s_t\Gamma_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)+\eta_t\left[\Gamma_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]=0$$

Now, note that we can write the Lagrangian (and making the dependence on $\overline{\omega}_{t+1}$ and $\sigma_{\omega,t+1}$

[1] You can show that for interior values of $\overline{\omega}_t$, the increase in revenue is bigger than the higher losses due to default.

explicit) as:

$$\eta\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=\frac{s_t\Gamma_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}{\Gamma_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}$$

Since:

$$G_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=\overline{\omega}_{t+1}F_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)$$

$$\Gamma_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=\left(1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)-\overline{\omega}_{t+1}F_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)+G_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)$$

we get:

$$\eta\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)=s_t\frac{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu\overline{\omega}_{t+1}F_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}$$

Then, going back to the optimality condition:

$$\mathbb{E}_t\left\{\frac{R^k_{t+1}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)+\eta\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\left[\frac{R^k_{t+1}}{s_tR_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]-1\right]\right\}=0$$

and using the zero profit condition for the financial intermediary:

$$\mathbb{E}_t\frac{R^k_{t+1}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)=\mathbb{E}_t\eta\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\frac{n_t}{q_tk_t}$$

Often, this expression is also written as:

$$q_tk_t=\frac{\mathbb{E}_t\eta\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}{\mathbb{E}_t\frac{R^k_{t+1}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)}n_t=\Psi\left(\frac{R^k_{t+1}}{R_t},\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)n_t$$

which relates purchases of capital to level of wealth and the finance premium, R^k_{t+1}/R_t.

Finally, at the end of each period, a fraction γ^e_t of entrepreneurs survives to next period while the rest die and their capital is taxed at a 100 percent rate by the government. The dead entrepreneurs are substituted by a new cohort of entrepreneurs that enter with initial real net wealth w^e (a transfer that, for simplicity in our derivations, the surviving entrepreneurs also get even if they went bankrupt in the period). Therefore, the average net wealth n_t (here we are equating average wealth with the wealth of the entrepreneur since all the entrepreneurs get the same ϱ_t) evolves as:

$$p_t n_t = \gamma_t^e \left[R_t^k p_{t-1} q_{t-1} k_{t-1} - s_{t-1} R_{t-1} b_{t-1} - \mu \int_0^{\overline{\omega}_t} \omega dF\left(\omega, \sigma_{\omega,t}\right) R_t^k p_{t-1} q_{t-1} k_{t-1} \right] + p_t w^e \Rightarrow$$

$$n_t = \gamma_t^e \frac{1}{\Pi_t} \left[R_t^k q_{t-1} k_{t-1} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}} - \mu G\left(\overline{\omega}_t, \sigma_{\omega,t}\right) R_t^k q_{t-1} k_{t-1} \right] + w^e \Rightarrow$$

$$n_t = \gamma_t^e \frac{1}{\Pi_t} \left[\left(1 - \mu G\left(\overline{\omega}_t, \sigma_{\omega,t}\right)\right) R_t^k q_{t-1} k_{t-1} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right] + w^e$$

The share γ_t^e is equal to:

$$\gamma_t^e = \frac{1}{1 + e^{-\overline{\gamma}^e - \widetilde{\gamma}_t^e}}$$

where $\widetilde{\gamma}_t^e$ follows:

$$\widetilde{\gamma}_t^e = \rho_\gamma \widetilde{\gamma}_{t-1}^e + \sigma_\gamma \varepsilon_{\gamma,t} \text{ where } \varepsilon_{\gamma,t} \sim \mathcal{N}(0,1).$$

This transformation ensures that γ_t^e is bounded in the unit interval while $\overline{\gamma}^e$ controls the mean of deaths.

We also summarize, for future convenience, the properties of the functions that depend on $\overline{\omega}_{t+1}$:

$$\Gamma\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right) = \overline{\omega}_{t+1}\left(1 - F\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right)\right) + G\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right)$$

$$\Gamma_\omega\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right) = 1 - F\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right)$$

$$G\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right) = 1 - \Phi\left(\frac{\frac{1}{2}\sigma_{\omega,t+1}^2 - \log \overline{\omega}_{t+1}}{\sigma_{\omega,t+1}}\right)$$

$$G_\omega\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right) = \overline{\omega}_{t+1} F_\omega\left(\overline{\omega}_{t+1}, \sigma_{\omega,t+1}\right)$$

2.6. Financial Intermediary

There is a representative, competitive financial firm that intermediates between households and entrepreneurs. We can think about that firm as including banks but also other financial institutions as venture capital firms or investment funds commonly engaged in the matching of savers and investors. The financial intermediary loans to entrepreneurs a nominal amount b_t at rate R_{t+1}^l, but recovers only an (uncontingent) rate R_t because of default and intermediation costs. Therefore, the financial intermediary pays interest R_t to the households. Also, we have, by market clearing, that loans must be equal to deposits (since all our debts are short-term we can abstract from reserve requirements for the financial intermediary):

$$a_t = b_t$$

2.7. The Government

The government determines monetary and fiscal policy. To keep the investigation focused, in a first pass, I abstract from the interactions between monetary and fiscal policy (for instance, I will assume that the results of open market operations are distributed in a lump-sum fashion to households and not transferred to the general revenue of the government). The current balance sheet of the Federal Reserve Bank and the dangers it entails to the U.S. Treasury suggests, though, that such an abstraction is only a provisional simplification that should be removed in the close future.

2.7.1. Monetary Policy

The government sets the nominal interest rates according to the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{y_t}{y}\right)^{\gamma_y}\right)^{(1-\gamma_R)} \exp\left(\sigma_m m_t\right)$$

through open market operations that are financed through lump-sum transfers T_t. The variable Π represents the target level of inflation (equal to inflation in the steady state), y is the steady state level of output, and $R = \frac{\Pi}{\beta}$ the steady state nominal gross return of capital. The term ε_{mt} is a random shock to monetary policy distributed according to $\mathcal{N}(0,1)$.

2.7.2. Fiscal Policy

The government intertemporal budget constraint is given by:

$$\frac{d_t}{p_t} = g_t + Rd_{t-1}\frac{d_{t-1}}{p_t} - tax_t$$

where:

$$tax_t = \tau_{c,t}c_t + \tau_{l,t}w_t l_t + \tau_{R,t}\left(R_{t-1} - 1\right)\frac{a_{t-1}}{p_t}$$

are tax revenues. Note that we can rewrite the budget constraint as:

$$\begin{aligned} \frac{d_t}{p_t} &= g_t + Rd_{t-1}\frac{d_{t-1}}{p_{t-1}}\frac{p_{t-1}}{p_t} - tax_t \\ &= g_t + \frac{Rd_{t-1}}{\Pi_{t-1}}\frac{d_{t-1}}{p_{t-1}} - tax_t \end{aligned}$$

that makes explicit the reduction in real public debt caused by inflation.

Government expenditure follows an autoregressive process:

$$\widehat{g}_t = \gamma_g \widehat{g}_{t-1} + d_g \frac{d_{t-1}}{p_t y_t} + \sigma_g \varepsilon_{g,t}$$

where $\widehat{g}_t$ are the log deviations with respect to the mean of the process:

$$\widehat{g}_t = \log \frac{g_t}{\overline{g}}$$

and d_g determines the sensitivity of expenditures to the ratio of public debt brought into the period over nominal output. A negative value of d_g ensures that the model have a determined equilibrium.

Taxes follow:

$$\begin{aligned}
\widehat{\tau}_{c,t} &= \gamma_c \widehat{\tau}_{c,t-1} - \sigma_{\tau c} \varepsilon_{\tau c,t} \\
\widehat{\tau}_{l,t} &= \gamma_l \widehat{\tau}_{l,t-1} + \sigma_{\tau l} \varepsilon_{\tau l,t} \\
\widehat{\tau}_{R,t} &= \gamma_R \widehat{\tau}_{R,t-1} + \sigma_{\tau k} \varepsilon_{\tau k,t}
\end{aligned}$$

where

$$\begin{aligned}
\widehat{\tau}_{c,t} &= \log \frac{1+\tau_{c,t}}{1+\overline{\tau}_c} \\
\widehat{\tau}_{l,t} &= \log \frac{1-\tau_{l,t}}{1-\overline{\tau}_l} \\
\widehat{\tau}_{R,t} &= \log \frac{1-\tau_{R,t}}{1-\overline{\tau}_R}
\end{aligned}$$

We sign with a minus the innovations to consumption taxes to think about them as an expansionary fiscal policy shock, as it is the case with the other two taxes.

2.8. Aggregation

Using the equality of capital-labor ratio across firms, some algebra steps give us an expression for aggregate demand:

$$y_t = c_t + i_t + g_t + \mu G\left(\overline{\omega}_t, \sigma_{\omega,t}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1}$$

and another for aggregate supply:

$$y_t = \frac{1}{v_t} e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha}$$

where $v_t = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$ is the inefficiency created by price dispersion. By the properties of the index under Calvo's pricing, this inefficiency evolves as:

$$v_t = \theta \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{-\varepsilon} v_{t-1} + (1-\theta)\, \Pi_t^{*-\varepsilon}.$$

3. Equilibrium

A definition of equilibrium in this economy is standard and the following equations can be solved for the 32 variables: c_t, λ_t, l_t, r_t, w_t, f_t^1, f_t^2, mc_t, Π_t, Π_t^*, $\overline{\omega}_t$, b_t/p_t, n_t, q_t, k_t, d_t, R_t, Rd_t, R_t^k, y_t, v_t, i_t, ϕ_t, z_t, g_t,tax_t, $\widehat{\tau}_{c,t}$, $\widehat{\tau}_{l,t}$, $\widehat{\tau}_{R,t}$, $\widetilde{s}_t$, $\widetilde{\gamma}_t^e$, and $\sigma_{\omega,t}$ (plus the accounting definitions of $\widehat{\tau}_{c,t}$, $\widehat{\tau}_{l,t}$, $\widehat{\tau}_{R,t}$, $\widetilde{s}_t$, and $\widetilde{\gamma}_t^e$, the money holding condition and the value of R_t^l, whose dynamics are irrelevant for the rest of the variables).

- The first order conditions of the household:

$$e^{\phi_t} \frac{1}{c_t - hc_{t-1}} - \mathbb{E}_t \beta e^{\phi_{t+1}} \frac{h}{c_{t+1} - hc_t} = (1 + \tau_{c,t})\, \lambda_t$$

$$\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{(1 + (1 - \tau_{R,t+1})(R_t - 1))}{\Pi_{t+1}} \right\}$$

$$\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{Rd_t}{\Pi_{t+1}} \right\}$$

$$e^{\phi_t} \psi l_t^{\vartheta} = (1 - \tau_{l,t})\, w_t \lambda_t$$

- The first order conditions of the intermediate firms:

$$\varepsilon f_t^1 = (\varepsilon - 1) f_t^2$$

$$f_t^1 = \lambda_t mc_t y_t + \beta\theta \mathbb{E}_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{-\varepsilon} f_{t+1}^1$$

$$f_t^2 = \lambda_t \Pi_t^* y_t + \beta\theta \mathbb{E}_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2$$

$$k_{t-1} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} l_t$$

$$mc_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{e^{z_t}}$$

- Price index evolves:

$$1 = \theta \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon} + (1-\theta)\, \Pi_t^{*1-\varepsilon}$$

- Capital good producers:

$$q_t\left(1-S\left[\frac{i_t}{i_{t-1}}\right]-S'\left[\frac{i_t}{i_{t-1}}\right]\frac{i_t}{i_{t-1}}\right)+\beta\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}S'\left[\frac{i_{t+1}}{i_t}\right]\left(\frac{i_{t+1}}{i_t}\right)^2=1$$

$$k_t=(1-\delta)\,k_{t-1}+\left(1-S\left[\frac{i_t}{i_{t-1}}\right]\right)i_t$$

- Entrepreneur problem:

$$R_{t+1}^k=\Pi_{t+1}\frac{r_{t+1}+q_{t+1}\,(1-\delta)}{q_t}$$

$$\mathbb{E}_t\frac{R_{t+1}^k}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right)=\left(\mathbb{E}_t s_t\frac{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu\overline{\omega}_{t+1}F_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}\right)\frac{n_t}{q_tk_t}$$

$$\frac{R_{t+1}^k}{s_tR_t}\left[\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu G\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right]=\frac{q_tk_t-n_t}{q_tk_t}$$

$$q_tk_t=n_t+\frac{b_t}{p_t}$$

$$n_t=\gamma_t^e\frac{1}{\Pi_t}\left[\left(1-\mu G\left(\overline{\omega}_t,\sigma_{\omega,t}\right)\right)R_t^kq_{t-1}k_{t-1}-s_{t-1}R_{t-1}\frac{b_{t-1}}{p_{t-1}}\right]+w^e$$

- The government follows is Taylor rule:

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\gamma_R}\left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi}\left(\frac{y_t}{y}\right)^{\gamma_y}\right)^{(1-\gamma_R)}\exp\left(\sigma_m m_t\right)$$

and its budget constraint:

$$\frac{d_t}{p_t}=g_t+\frac{Rd_{t-1}}{\Pi_{t-1}}\frac{d_{t-1}}{p_{t-1}}-tax_t$$

with expenditure and taxes:

$$\widehat{g}_t=\gamma_g\widehat{g}_{t-1}+d_g\frac{d_{t-1}}{p_ty_t}+\sigma_g\varepsilon_{g,t}$$

$$tax_t=\tau_{c,t}c_t+\tau_{l,t}w_tl_t+\tau_{R,t}\left(R_{t-1}-1\right)\frac{a_{t-1}}{p_t}$$

and taxes:

$$\begin{aligned}\widehat{\tau}_{c,t}&=\gamma_c\widehat{\tau}_{c,t-1}-\sigma_{\tau c}\varepsilon_{\tau c,t}\\ \widehat{\tau}_{l,t}&=\gamma_l\widehat{\tau}_{l,t-1}+\sigma_{\tau l}\varepsilon_{\tau l,t}\\ \widehat{\tau}_{R,t}&=\gamma_R\widehat{\tau}_{R,t-1}+\sigma_{\tau k}\varepsilon_{\tau k,t}\end{aligned}$$

- Market clearing:

$$y_t = c_t + i_t + g_t + \mu G\left(\overline{\omega}_t, \sigma_{\omega,t}\right)\left(r_t + q_t\left(1-\delta\right)\right)k_{t-1}$$
$$y_t = \frac{1}{v_t} e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha}$$
$$v_t = \theta\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} v_{t-1} + \left(1-\theta\right)\Pi_t^{*-\varepsilon}$$

- Stochastic processes:

$$\phi_t = \rho_d \phi_{t-1} + \sigma_\phi \varepsilon_{\phi,t}$$
$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$$
$$\widetilde{s}_t = \rho_s \widetilde{s}_{t-1} + \sigma_s \varepsilon_{s,t}$$
$$\widetilde{\gamma}_t^e = \rho_\gamma \widetilde{\gamma}_{t-1}^e + \sigma_\gamma \varepsilon_{\gamma,t}$$
$$\log \sigma_{\omega,t} = \left(1-\rho_\sigma\right)\log \sigma_\omega + \rho_\sigma \log \sigma_{\omega,t-1} + \eta_\sigma \varepsilon_{\sigma,t}$$

4. Steady State

We define $\overline{b} = b/p$ as the steady state level of real private debt. Before finding the steady state, note that Π is a parameter and that we can set up all the stochastic processes and taxes to their mean. Also, we will pick:

$$g = \overline{g} = tax = \overline{tax} = \overline{\tau}_c c + \overline{\tau}_l wl + \overline{\tau}_R \left(R - 1\right) \overline{b}$$

which implies that $\overline{d} = d/p = 0$. Then, the steady state equilibrium conditions for the household are:

$$\frac{1 - \beta h}{1 - h} \frac{1}{c} = \left(1 + \overline{\tau}_c\right) \lambda$$
$$R = 1 + \frac{1}{1 - \tau_R} \left(\frac{\Pi}{\beta} - 1\right)$$
$$Rd = \frac{\Pi}{\beta}$$
$$\psi l^{\vartheta} = \left(1 - \overline{\tau}_l\right) w\lambda$$

for the firm, the law of motion for prices, and capital producers:

$$\varepsilon f^1 = (\varepsilon - 1) f^2$$
$$f^1 = \lambda mcy + \beta\theta\Pi^{\varepsilon(1-\chi)} f^1$$
$$f^2 = \lambda\Pi^* y + \beta\theta\Pi^{(\varepsilon-1)(1-\chi)} f^2$$
$$\frac{k}{l} = \frac{\alpha}{1 - \alpha} \frac{w}{r}$$
$$mc = \left(\frac{1}{1 - \alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w^{1-\alpha} r^{\alpha}$$
$$1 = \theta\Pi^{(\varepsilon-1)(1-\chi)} + \left(1 - \theta\right) \Pi^{*1-\varepsilon}$$
$$q = 1$$
$$i = \delta k$$

Entrepreneur problem (where we already use $q = 1$):

$$R^k = \Pi \left(1 + r - \delta\right)$$

$$\frac{R^k}{R}\left(1 - \Gamma\left(\overline{\omega}, \sigma_\omega\right)\right) = s\frac{1 - F\left(\overline{\omega}, \sigma_\omega\right)}{1 - F\left(\overline{\omega}, \sigma_\omega\right) - \mu G_\omega\left(\overline{\omega}, \sigma_\omega\right)}\frac{n}{k}$$

$$\frac{R^k}{sR}\left[\Gamma\left(\overline{\omega}, \sigma_\omega\right) - \mu G\left(\overline{\omega}, \sigma_\omega\right)\right] = \frac{\overline{b}}{k}$$

$$\overline{b} + n = k$$

$$n = \gamma\frac{1}{\Pi}\left[\left(1 - \mu G\left(\overline{\omega}, \sigma_\omega\right)\right)R^k k - sR\overline{b}\right] + w^e$$

the market clearing conditions:

$$y = c + i + g + \mu G\left(\overline{\omega}, \sigma_\omega\right)\left(1 + r - \delta\right)k$$

$$y = \frac{1}{v}k^\alpha l^{1-\alpha}$$

$$v = \theta \Pi^{\varepsilon(1-\chi)}v + (1 - \theta)\Pi^{*-\varepsilon}$$

and the government budget balance:

$$g = \overline{\tau}_c c + \overline{\tau}_l wl + \overline{\tau}_R R\overline{b}$$

We start working on these equations. First, from the firms's conditions, we have that:

$$mc = \frac{\varepsilon - 1}{\varepsilon}\frac{1 - \beta\theta\Pi^{\varepsilon(1-\chi)}}{1 - \beta\theta\Pi^{(\varepsilon-1)(1-\chi)}}\Pi^*$$

Second, the relationship between inflation and optimal relative prices is:

$$\Pi^* = \left(\frac{1 - \theta\Pi^{(\varepsilon-1)(1-\chi)}}{1 - \theta}\right)^{\frac{1}{1-\varepsilon}}$$

and the value of distortions:

$$v = \frac{1 - \theta}{1 - \theta\Pi^{\varepsilon(1-\chi)}}\Pi^{*-\varepsilon}$$

To solve for the rest of the steady state, I calibrate $\frac{\overline{b}}{k} = b\_k$ and $l = 1/3$. To calibrate $\frac{\overline{b}}{k}$, note that, in the U.S. economy:

$$\frac{n}{k - n} \approx 2$$

Thus:

$$\frac{n}{k-n} = \frac{k-\overline{b}}{\overline{b}} = \frac{k}{\overline{b}} - 1 = 2 \Rightarrow \frac{\overline{b}}{k} = \frac{1}{3}$$

Now, we can use:

$$\frac{R^k}{sR}\left[\Gamma\left(\overline{\omega},\sigma_\omega\right) - \mu G\left(\overline{\omega},\sigma_\omega\right)\right] = b\_k$$

$$\frac{R^k}{R} = s\frac{1}{1-\Gamma\left(\overline{\omega},\sigma_\omega\right)}\frac{1-F\left(\overline{\omega},\sigma_\omega\right)}{1-F\left(\overline{\omega},\sigma_\omega\right)-\mu G_\omega\left(\overline{\omega},\sigma_\omega\right)}\left(1-b\_k\right)$$

to solve for R^k and $\overline{\omega}$.[2] A simpler system is:

$$b\_k = \underbrace{\frac{\Gamma\left(\overline{\omega},\sigma_\omega\right) - \mu G\left(\overline{\omega},\sigma_\omega\right)}{1-\Gamma\left(\overline{\omega},\sigma_\omega\right)}\frac{1-F\left(\overline{\omega},\sigma_\omega\right)}{1-F\left(\overline{\omega},\sigma_\omega\right)-\mu G_\omega\left(\overline{\omega},\sigma_\omega\right)}}_{\Omega(\overline{\omega})}\left(1-b\_k\right)$$

and then:

$$b\_k = \frac{\Omega\left(\overline{\omega},\sigma_\omega\right)}{1+\Omega\left(\overline{\omega},\sigma_\omega\right)} \Rightarrow \overline{\omega} = f\left(b\_k,\sigma_\omega\right)$$

$$R^k = \frac{b\_k * sR}{\Gamma\left(\overline{\omega},\sigma_\omega\right) - \mu G\left(\overline{\omega},\sigma_\omega\right)}$$

With this, we can get:

$$r = \frac{R^k}{\Pi} - 1 + \delta$$

With r,

$$w = (1-\alpha)\left(\left(\frac{1}{\alpha}\right)^\alpha \frac{1}{mc} r^\alpha\right)^{\frac{1}{\alpha-1}}$$

and with r and $l = 1/3$

$$k = \frac{\alpha}{1-\alpha}\frac{w}{r}l$$

$$\overline{b} = b\_k * k$$

$$n = k - \overline{b}$$

$$i = \delta k$$

$$y = \frac{1}{v}k^\alpha l^{1-\alpha}$$

[2] Given $\overline{\omega}$, we pick the right σ_ω^2 (and then $\mu_\omega = -\frac{1}{2}\sigma_\omega^2$) given our observation of $F\left(\overline{\omega}\right)$ from the data.

Now:

$$
\begin{aligned}
c &= y - \delta k - \mu G(\overline{\omega}, \sigma_\omega)(1 + r - \delta) k - g \\
&= \frac{1}{1 + \overline{\tau}_c} \left(y - \delta k - \mu G(\overline{\omega}, \sigma_\omega)(1 + r - \delta) k - \overline{\tau}_l w l - \overline{\tau}_R (R - 1) \overline{b} \right)
\end{aligned}
$$

and the four auxiliary conditions:

$$
\begin{aligned}
\psi l^\vartheta &= (1 - \overline{\tau}_l) w \lambda \\
\lambda &= \frac{1 - \beta h}{1 - h} \frac{1}{(1 + \overline{\tau}_c) c} \\
f^1 &= \frac{mc \lambda y}{1 - \beta \theta \Pi^{\varepsilon(1-\chi)}} \\
f^2 &= \frac{\Pi^* \lambda y}{1 - \beta \theta \Pi^{(\varepsilon - 1)(1-\chi)}} \\
tax &= \overline{\tau}_c c + \overline{\tau}_l w l + \overline{\tau}_R (R - 1) \overline{b}
\end{aligned}
$$

Now, we have two equations left:

$$
\begin{aligned}
n &= \gamma_e \frac{1}{\Pi} \left[(1 - \mu G(\overline{\omega}, \sigma_\omega)) R^k k - s R \overline{b} \right] + w^e \\
\psi l^\vartheta &= (1 - \overline{\tau}_l) w \lambda
\end{aligned}
$$

and we use them to back-up the values of w^e and ψ that justify our calibration

$$
\begin{aligned}
\psi &= \frac{(1 - \overline{\tau}_l) w \lambda}{l^\vartheta} \\
w^e &= n - \gamma_e \frac{1}{\Pi} \left[(1 - \mu G(\overline{\omega}, \sigma_\omega)) R^k k - s R \overline{b} \right]
\end{aligned}
$$

Finally, we calibrate $\overline{s}$ and $\overline{\gamma}^e$. Note that $e^{\overline{s}} = s - 1$ and

$$
\gamma^e = \frac{1}{1 + e^{-\overline{\gamma}^e}}
$$

using the fact that γ^e is observable as follows:

$$
e^{-\overline{\gamma}^e} = \frac{1 - \gamma^e}{\gamma^e}
$$

5. Loglinearized Equilibrium Conditions

The loglinearized equilibrium conditions are:

1. Marginal utility of consumption:

$$\phi_t - \frac{1+\beta h^2}{1-h}\widehat{c}_t - \frac{\beta h}{1-h}\widehat{c}_{t+1} + \frac{h}{1-h}\widehat{c}_{t-1} = (1-\beta h)\left(\widehat{\tau}_{c,t} + \widehat{\lambda}_t\right)$$

2. Intertemporal condition, deposits:

$$\widehat{\lambda}_t = \mathbb{E}_t\left\{\widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} + \left(1 - \frac{\beta}{\Pi}\right)\left(\widehat{\tau}_{R,t+1} + \frac{R}{R-1}\widehat{R}_t\right)\right\}$$

3. Intertemporal condition, public debt:

$$\widehat{\lambda}_t = \mathbb{E}_t\left\{\widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} + \widehat{Rd}_t\right\}$$

4. Marginal utility of labor:

$$\widehat{\phi}_t + \vartheta\widehat{l}_t = \widehat{\tau}_{l,t} + \widehat{w} + \widehat{\lambda}_t$$

5. Auxiliary functions:

$$\widehat{f}_t^1 = \widehat{f}_t^2$$

6. Recursive equation for prices 1:

$$\widehat{f}_t^1 = \left(1 - \beta\theta\Pi^{\varepsilon(1-\chi)}\right)\left(\widehat{\lambda}_t + \widehat{mc}_t + \widehat{y}_t\right) + \beta\theta\Pi^{\varepsilon(1-\chi)}\mathbb{E}_t\left(\varepsilon\left(\widehat{\Pi}_{t+1} - \chi\widehat{\Pi}_t\right) + \widehat{f}_{t+1}^1\right)$$

7. Recursive equation for prices 2:

$$\begin{aligned}\widehat{f}_t^2 \;=\;& \left(1 - \beta\theta\Pi^{(\varepsilon-1)(1-\chi)}\right)\left(\widehat{\lambda}_t + \widehat{\Pi}_t^* + \widehat{y}_t\right) + \\ & \beta\theta\Pi^{(\varepsilon-1)(1-\chi)}\mathbb{E}_t\left((\varepsilon-1)\left(\widehat{\Pi}_{t+1} - \chi\widehat{\Pi}_t\right) + \widehat{\Pi}_t^* - \widehat{\Pi}_{t+1}^* + \widehat{f}_{t+1}^2\right)\end{aligned}$$

8. FOC of firms with respect to capital and labor:

$$\widehat{k}_{t-1} = \widehat{w}_t + \widehat{l}_t - \widehat{r}_t$$

9. Marginal cost:

$$\widehat{mc}_t = \alpha\widehat{r} + (1-\alpha)\,\widehat{w}_t - z_t$$

10. Evolution of prices:

$$\widehat{\Pi}_t - \chi\widehat{\Pi}_{t-1} = \frac{1-\theta}{\theta}\left(\Pi^*\Pi^{(1-\chi)}\right)^{(1-\varepsilon)}\widehat{\Pi}_t^*$$

11. Adjustment cost:

$$\widehat{q}_t = S''[1]\left(\widehat{i}_t - \widehat{i}_{t-1}\right) - \beta S''[1]\,\mathbb{E}_t\left(\widehat{i}_{t+1} - \widehat{i}_t\right)$$

12. Law of motion for private capital:

$$\widehat{k}_t = (1-\delta)\,\widehat{k}_{t-1} + \delta\widehat{i}_t$$

13. Return on capital:

$$\widehat{R}^k_{t+1} = \widehat{\Pi}_{t+1} + \frac{\Pi r}{R^k}\widehat{r}_{t+1} + \frac{\Pi(1-\delta)}{R^k}\widehat{q}_{t+1} - \widehat{q}_t$$

14. Entrepreneur 1:

$$\mathbb{E}_t\widehat{R}^k_{t+1} - \widehat{R}_t + \omega_a\mathbb{E}_t\widehat{\overline{\omega}}_{t+1} + \sigma_a\widehat{\sigma}_{\omega,t+1} = \frac{s-1}{s}\widetilde{s}_t + \widehat{n}_t - \widehat{q}_t - \widehat{k}_t$$

15. Entrepreneur 2:

$$\widehat{R}^k_t - \widehat{R}_{t-1} - \frac{s-1}{s}\widehat{s}_{t-1} + \omega_b\widehat{\overline{\omega}}_t + \sigma_b\widehat{\sigma}_{\omega,t} = \widehat{\overline{b}}_{t-1} - \widehat{q}_{t-1} - \widehat{k}_{t-1}$$

16. Entrepreneur 3:

$$\widehat{q}_t + \widehat{k}_t = \frac{n}{k}\widehat{n}_t + \frac{\overline{b}}{k}\widehat{\overline{b}}_t$$

17. Wealth evolution:

$$\begin{aligned}\widehat{n}_t \;=\;& a_1\left((1-\gamma^e)\,\widetilde{\gamma}^e_t - \widehat{\Pi}_t\right) + a_2\left(\omega_c\widehat{\overline{\omega}}_t + \sigma_c\widehat{\sigma}_{\omega,t}\right) + \\ & a_3\left(\widehat{R}^k_t + \widehat{q}_{t-1} + \widehat{k}_{t-1}\right) + a_4\left(\widehat{R}_{t-1} + \frac{s-1}{s}\widehat{s}_{t-1} + \widehat{\overline{b}}_{t-1}\right)\end{aligned}$$

18. Taylor rule:

$$\widehat{R}_t = \gamma_R\widehat{R}_{t-1} + (1-\gamma_R)\left(\gamma_\Pi\widehat{\Pi}_t + \gamma_y\widehat{y}_t\right) + \sigma_m\varepsilon_{mt}$$

19. Government budget constraint:

$$\overline{d}_t = g\widehat{g}_t + \beta\overline{d}_{t-1} - tax\widehat{tax}_t$$

20. Taxes:

$$\begin{aligned} \widehat{taxtax}_t &= \overline{\tau}_c c\left(\widehat{c}_t+\widehat{\tau}_{c,t}\right)+c\widehat{\tau}_{c,t}+\overline{\tau}_l wl\left(\widehat{w}_t+\widehat{l}_t+\widehat{\tau}_{l,t}\right)-wl\widehat{\tau}_{l,t} \\ &+\overline{\tau}_R\left(R-1\right)\overline{b}\left(\widehat{\overline{b}}_{t-1}+\widehat{\tau}_{R,t}\right)+\overline{\tau}_R\overline{b}R\widehat{R}_{t-1}-\left(R-1\right)\overline{b}\widehat{\tau}_{R,t} \end{aligned}$$

21. Resource constraint:

$$\begin{aligned} \widehat{y}_t &= \frac{c}{y}\widehat{c}_t+\frac{i}{y}\widehat{i}_t+\frac{g}{y}\widehat{g}_t+ \\ &\frac{\mu}{y}G\left(\overline{\omega}\right)\left[\left(rk\left(\widehat{r}_t+\widehat{k}_{t-1}\right)+\left(1-\delta\right)k\left(\widehat{q}_t+\widehat{k}_{t-1}\right)\right)+\left(r+1-\delta\right)k\left(\omega_c\widehat{\overline{\omega}}_t+\sigma_c\widehat{\sigma}_{\omega,t}\right)\right] \end{aligned}$$

22. Production function:

$$\widehat{y}_t = z_t+\alpha\widehat{k}_{t-1}+\left(1-\alpha\right)\widehat{l}_t-\widehat{v}_t$$

23. Evolution of price dispersion inefficiency:

$$\widehat{v}_t = \theta\Pi^{\varepsilon(1-\chi)}\left(\varepsilon\left(\widehat{\Pi}_t-\chi\widehat{\Pi}_{t-1}\right)+\widehat{v}_{t-1}\right)-\varepsilon\left(1-\theta\Pi^{\varepsilon(1-\chi)}\right)\widehat{\Pi}_t^*$$

24. Government expenditure:

$$\widehat{g}_t = \gamma_g\widehat{g}_{t-1}+d_g\overline{d}_{t-1}+\sigma_g\varepsilon_{g,t}$$

25. Tax on consumption:

$$\widehat{\tau}_{c,t} = \gamma_c\widehat{\tau}_{c,t-1}-\sigma_{\tau c}\varepsilon_{\tau c,t}$$

26. Tax on labor income:

$$\widehat{\tau}_{l,t} = \gamma_l\widehat{\tau}_{l,t-1}+\sigma_{\tau l}\varepsilon_{\tau l,t}$$

27. Tax on deposit returns:

$$\widehat{\tau}_{R,t} = \gamma_R\widehat{\tau}_{R,t-1}+\sigma_{\tau k}\varepsilon_{\tau k,t}$$

28. Intertemporal shock:

$$\phi_t = \rho_d\phi_{t-1}+\sigma_\phi\varepsilon_{\phi,t}$$

29. Productivity process:

$$z_t = \rho_z z_{t-1}+\sigma_z\varepsilon_{z,t}$$

30. Dispersion process:

$$\widehat{\sigma}_{\omega,t} = \rho_\sigma \widehat{\sigma}_{\omega,t-1} + \eta_\sigma \varepsilon_{\sigma,t}$$

31. Spread:

$$\widetilde{s}_t = \rho_s \widetilde{s}_{t-1} + \sigma_s \varepsilon_{s,t}$$

32. Entrepreneurs entry:

$$\widetilde{\gamma}_t^e = \rho_\gamma \widetilde{\gamma}_{t-1}^e + \sigma_\gamma \varepsilon_{\gamma,t}$$

6. Experiments

I will simulate now the effect of different fiscal policy shocks. The problem is how to set up experiments that are meaningfully comparable. Saying, for instance, that a 10 percent increase in government expenditure has a bigger effect on output than a 1 percent reduction in taxes is not particularly informative. A natural choice would be to look at shocks that have an equivalent effect on the budget, that is, a reduction on taxes that lowers revenue in the same amount than the increase in expenditure we are comparing it to. But behind this simple logic we face a number of difficultie: when do we measure this effect on the budget? At impact? Over time?

A possibility that is close to much of the policitical discussion (but certainly not the only reasonable one) is to use a static scoring rule. For example, we can look at the reduction in the tax on labor income that will generate a reduction of revenue at impact, starting in the steady state, equal to (minus) the increase in government expenditure that we are considering. Formally:

$$0.01 * \overline{g} = - \bigtriangledown \tau_l * wl \Rightarrow$$
$$\bigtriangledown \tau_l = -0.01 * \frac{\overline{g}}{wl}$$

Now, note that

$$\begin{aligned}
\widehat{\tau}_{l,t} &= \log \frac{1-\tau_{l,t}}{1-\overline{\tau}_l} \\
&= \log \frac{1-\overline{\tau}_l - \bigtriangledown \tau_l}{1-\overline{\tau}_l} \\
&= \log \left(1 - \frac{\bigtriangledown \tau_l}{1-\overline{\tau}_l}\right) \\
&= \log \left(1 + 0.01 * \frac{1}{1-\overline{\tau}_l} \frac{\overline{g}}{wl}\right)
\end{aligned}$$

Similarly, for the tax on return on deposits:

$$0.01 * \overline{g} = - \bigtriangledown \tau_R * (R-1)\,\overline{b}$$

delivers:

$$\widehat{\tau}_{R,t} = \log\left(1 + 0.01 * \frac{1}{1-\overline{\tau}_l}\frac{\overline{g}}{(R-1)\,\overline{b}}\right)$$

and for the tax on consumption:

$$0.01 * \overline{g} = - \bigtriangledown \tau_c * c$$

gives:

$$\widehat{\tau}_{c,t} = \log\left(1 + 0.01 * \frac{1}{1-\overline{\tau}_c}\frac{\overline{g}}{c}\right)$$

7. Appendix I: Useful Facts about the Lognormal Distribution

If a random variables ω_{t+1} is lognormally distributed with CDF $F(\omega)$ and parameters μ_ω and σ_ω, we have:

$$\mathbb{E}\omega = e^{\mu_\omega + \frac{1}{2}\sigma_\omega^2}$$

Also, the partial expectation:

$$\int_{\overline{\omega}}^{\infty} \omega dF(\omega) = e^{\mu_\omega + \frac{1}{2}\sigma_\omega^2}\Phi\left(\frac{\mu_\omega + \sigma_\omega^2 - \ln\overline{\omega}}{\sigma_\omega}\right)$$

Then:

$$\mathbb{E}\omega = \int_0^{\infty} \omega dF(\omega) = \int_0^{\overline{\omega}} \omega dF(\omega) + \int_{\overline{\omega}}^{\infty} \omega dF(\omega) = \int_0^{\overline{\omega}} \omega dF(\omega) + e^{\mu_\omega + \frac{1}{2}\sigma_\omega^2}\Phi\left(\frac{\mu_\omega + \sigma_\omega^2 - \ln\overline{\omega}}{\sigma_\omega}\right)$$

Therefore

$$\begin{aligned}\int_0^{\overline{\omega}} \omega dF(\omega) &= \mathbb{E}\omega - e^{\mu_\omega + \frac{1}{2}\sigma_\omega^2}\Phi\left(\frac{\mu_\omega + \sigma_\omega^2 - \ln\overline{\omega}}{\sigma_\omega}\right) \\ &= e^{\mu_\omega + \frac{1}{2}\sigma_\omega^2}\left(1 - \Phi\left(\frac{\mu_\omega + \sigma_\omega^2 - \ln\overline{\omega}}{\sigma_\omega}\right)\right)\end{aligned}$$

Also,

$$F(\overline{\omega}) = \int_0^{\overline{\omega}} \frac{1}{\omega\sigma_\omega\sqrt{2\pi}} e^{-\frac{\left(\ln\omega+\frac{1}{2}\sigma_\omega^2\right)^2}{2\sigma_\omega^2}} d\omega$$

$$F_\omega(\omega) = \frac{1}{\omega\sigma_\omega\sqrt{2\pi}} e^{-\frac{\left(\ln\omega+\frac{1}{2}\sigma_\omega^2\right)^2}{2\sigma_\omega^2}}$$

and then:

$$\begin{aligned} F_{\omega\omega}(\omega) &= -\frac{1}{\sigma_\omega\sqrt{2\pi}} e^{-\frac{(\ln\omega-\mu_\omega)^2}{2\sigma_\omega^2}} \frac{1}{\omega^2} - \frac{1}{\omega\sigma_\omega\sqrt{2\pi}} e^{-\frac{(\ln\omega-\mu_\omega)^2}{2\sigma_\omega^2}} \frac{\ln\omega-\mu_\omega}{\sigma_\omega^2}\frac{1}{\omega} \\ &= -\frac{1}{\omega}F_\omega(\omega) - \frac{1}{\omega}F_\omega(\omega)\frac{\ln\omega-\mu_\omega}{\sigma_\omega^2} \\ &= -\frac{1}{\omega}F_\omega(\omega)\left(1+\frac{\ln\omega-\mu_\omega}{\sigma_\omega^2}\right) \end{aligned}$$

8. Appendix II: Useful Facts about Loglinearization

Imagine we want to loglinearize:

$$a_t f(b_t) = c_t g(d_t)$$

Then:

$$a e^{\widehat{a}_t} f\left(b e^{\widehat{b}_t}\right) = c e^{\widehat{c}_t} g\left(d e^{\widehat{d}_t}\right)$$

and:

$$a f(b)\widehat{a}_t + a f'(b) b \widehat{b}_t = c g(d)\widehat{c}_t + c g'(d) d \widehat{d}_t \Rightarrow$$

$$\widehat{a}_t + \frac{f'(b) b}{f(b)}\widehat{b}_t = \widehat{c}_t + \frac{g'(d) d}{g(d)}\widehat{d}_t$$

A particular case of interest is when we loglinearize:

$$y_t = \frac{1}{1+e^{-a-\widetilde{y}_t}}$$

that results in:

$$y\widehat{y}_t = \frac{e^{-a}}{(1+e^{-a})^2}\widetilde{y}_t$$

Since

$$y = \frac{1}{1+e^{-a}}$$

we have:

$$y\widehat{y}_t = e^{-a} y^2 \widetilde{y}_t \Rightarrow$$
$$\widehat{y}_t = e^{-a} y \widetilde{y}_t = (1-y)\,\widetilde{y}_t$$

9. Appendix III: Loglinearization

The loglinearization of all the equilibrium conditions in the model is rather straightforward except four of them, which require somewhat further work.

9.1. Equation 1: Entrepreneur FOC

We start with:

$$\mathbb{E}_t \frac{R^k_{t+1}}{R_t}\left(1-\Gamma\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)\right) = \mathbb{E}_t s_t \frac{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}{1-F\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)-\mu\overline{\omega}_{t+1}F_\omega\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)}\frac{n_t}{q_t k_t} \Rightarrow$$
$$\mathbb{E}_t \frac{R^k_{t+1}}{R_t}\Psi^1\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right) = \frac{n_t}{q_t k_t}\mathbb{E}_t \Psi^2\left(\overline{\omega}_{t+1},\sigma_{\omega,t+1}\right)$$

which loglinearizes to:

$$\mathbb{E}_t \widehat{R}^k_{t+1} - \widehat{R}_t + \mathbb{E}_t\left(\frac{\Psi^1_\omega\left(\overline{\omega},\sigma_\omega\right)\overline{\omega}}{\Psi^1\left(\overline{\omega},\sigma_\omega\right)} - \frac{\Psi^2_\omega\left(\overline{\omega},\sigma_\omega\right)\overline{\omega}}{\Psi^2\left(\overline{\omega},\sigma_\omega\right)}\right)\widehat{\overline{\omega}}_{t+1} + \left(\frac{\Psi^1_{\sigma_\omega}\left(\overline{\omega},\sigma_\omega\right)\sigma_\omega}{\Psi^1\left(\overline{\omega},\sigma_\omega\right)} - \frac{\Psi^2_{\sigma_\omega}\left(\overline{\omega},\sigma_\omega\right)\sigma_\omega}{\Psi^2\left(\overline{\omega},\sigma_\omega\right)}\right)\widehat{\sigma}_{\omega,t+1}$$
$$= \frac{s-1}{s}\widetilde{s}_t + \widehat{n}_t - \widehat{q}_t - \widehat{k}_t$$

or:

$$\mathbb{E}_t \widehat{R}^k_{t+1} - \widehat{R}_t + \omega_a \mathbb{E}_t \widehat{\overline{\omega}}_{t+1} + \sigma_a \widehat{\sigma}_{\omega,t+1} = \frac{s-1}{s}\widetilde{s}_t + \widehat{n}_t - \widehat{q}_t - \widehat{k}_t$$

where:

$$s_t = 1 + e^{\overline{s}+\widetilde{s}_t}$$

implies that:

$$\widehat{s}_t = \frac{s-1}{s}\widetilde{s}_t$$

and

$$\frac{\Psi^1_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^1(\overline{\omega},\sigma_\omega)} = -\frac{\Gamma_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{1-\Gamma(\overline{\omega},\sigma_\omega)} = \frac{1-F(\overline{\omega},\sigma_\omega)}{\Gamma(\overline{\omega},\sigma_\omega)-1}\overline{\omega}$$

$$\frac{\Psi^2_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^2(\overline{\omega},\sigma_\omega)} = \left(-\frac{F_\omega(\overline{\omega},\sigma_\omega)}{1-F(\overline{\omega},\sigma_\omega)} + \frac{F_\omega(\overline{\omega},\sigma_\omega)+\mu\overline{\omega}F_{\omega\omega}(\overline{\omega},\sigma_\omega)+\mu F_\omega(\overline{\omega},\sigma_\omega)}{1-F(\overline{\omega},\sigma_\omega)-\mu\overline{\omega}F_\omega(\overline{\omega},\sigma_\omega)}\right)\overline{\omega}$$

$$\omega_a = \frac{\Psi^1_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^1(\overline{\omega},\sigma_\omega)} - \frac{\Psi^2_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^2(\overline{\omega},\sigma_\omega)}$$

and

$$\sigma_a = \frac{\Psi^1_{\sigma_\omega}(\overline{\omega},\sigma_\omega)\sigma_\omega}{\Psi^1(\overline{\omega},\sigma_\omega)} - \frac{\Psi^2_{\sigma_\omega}(\overline{\omega},\sigma_\omega)\sigma_\omega}{\Psi^2(\overline{\omega},\sigma_\omega)}$$

a coefficient we will compute numerically.

9.2. Equation 2: Zero Profits for the Financial Intermediary

The second equation is:

$$\frac{R^k_{t+1}}{s_t R_t}\left[\Gamma(\overline{\omega}_{t+1},\sigma_{\omega,t+1}) - \mu G(\overline{\omega}_{t+1},\sigma_{\omega,t+1})\right] = \frac{b_t/p_t}{q_t k_t} \Rightarrow$$

$$\frac{R^k_{t+1}}{s_t R_t}\Psi^3(\overline{\omega}_{t+1},\sigma_{\omega,t+1}) = \frac{\overline{b}_t}{q_t k_t}$$

which loglinearizes to:

$$\widehat{R}^k_{t+1} - \widehat{R}_t - \widehat{s}_t + \frac{\Psi^3_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^3(\overline{\omega},\sigma_\omega)}\widehat{\overline{\omega}}_{t+1} + \frac{\Psi^3_{\sigma_\omega}(\overline{\omega},\sigma_\omega)\sigma_\omega}{\Psi^3(\overline{\omega},\sigma_\omega)}\widehat{\sigma}_{\omega,t+1} = \widehat{\overline{b}}_t - \widehat{q}_t - \widehat{k}_t$$

Then

$$\widehat{R}^k_{t+1} - \widehat{R}_t - \frac{s-1}{s}\widetilde{s}_t + \omega_b\widehat{\overline{\omega}}_{t+1} + \sigma_b\widehat{\sigma}_{\omega,t+1} = \widehat{\overline{b}}_t - \widehat{q}_t - \widehat{k}_t$$

where

$$\omega_b = \frac{\Psi^3_\omega(\overline{\omega},\sigma_\omega)\overline{\omega}}{\Psi^3(\overline{\omega},\sigma_\omega)} = \frac{1-F(\overline{\omega},\sigma_\omega)-\mu\overline{\omega}F_\omega(\overline{\omega},\sigma_\omega)}{\Gamma(\overline{\omega},\sigma_\omega)-\mu G(\overline{\omega},\sigma_\omega)}\overline{\omega}$$

$$\sigma_b = \frac{\Psi^3_{\sigma_\omega}(\overline{\omega},\sigma_\omega)\sigma_\omega}{\Psi^3(\overline{\omega},\sigma_\omega)}$$

where σ_b will be computed numerically.

Also, note that since this equation holds state by state, it is better to write it as:

$$\widehat{R}^k_t - \widehat{R}_{t-1} - \frac{s-1}{s}\widetilde{s}_{t-1} + \omega_b\widehat{\overline{\omega}}_t + \sigma_b\widehat{\sigma}_{\omega,t} = \widehat{\overline{b}}_{t-1} - \widehat{q}_{t-1} - \widehat{k}_{t-1}$$

9.3. Equation 3: Law of Motion for Wealth

Since

$$\gamma_t^e = \frac{1}{1+e^{-\overline{\gamma}^e - \widetilde{\gamma}_t^e}}$$

we have that:

$$\widehat{\gamma}_t^e = e^{-\overline{\gamma}^e}\gamma^e \widetilde{\gamma}_t^e = \frac{1-\gamma^e}{\gamma^e}\gamma^e \widetilde{y}_t = (1-\gamma^e)\,\widetilde{\gamma}_t^e$$

Then:

$$n_t = \gamma_t^e \frac{1}{\Pi_t}\left[\left(1-\mu G\left(\overline{\omega}_t, \sigma_{\omega,t+1}\right)\right) R_t^k q_{t-1} k_{t-1} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}}\right] + w^e$$

that loglinearizes to:

$$\widehat{n}_t = a_1\left(\left(1-\gamma^e\right)\widetilde{\gamma}_t^e - \widehat{\Pi}_t\right) + a_2\left(\omega_c \widehat{\overline{\omega}}_t + \sigma_c \widehat{\sigma}_{\omega,t}\right) + a_3\left(\widehat{R}_t^k + \widehat{q}_{t-1} + \widehat{k}_{t-1}\right) + a_4\left(\widehat{R}_{t-1} + \frac{s-1}{s}\widetilde{s}_{t-1} + \widehat{\overline{b}}_{t-1}\right)$$

where

$$a_1 = \frac{\gamma_e}{\Pi n}\left(1-\mu G\left(\overline{\omega}, \sigma_\omega\right)\right) R^k k - sR\overline{b}$$
$$a_2 = -\frac{\gamma_e}{\Pi n}\mu R^k k$$
$$a_3 = \frac{\gamma_e}{\Pi n}\left(1-\mu G\left(\overline{\omega}, \sigma_\omega\right)\right) R^k k$$
$$a_4 = -\frac{\gamma_e}{\Pi n} sR\overline{b}$$
$$\omega_c = \frac{G_\omega\left(\overline{\omega}, \sigma_\omega\right)\overline{\omega}}{G\left(\overline{\omega}, \sigma_\omega\right)} = \frac{\overline{\omega}^2 F_\omega\left(\overline{\omega}, \sigma_\omega\right)}{G\left(\overline{\omega}, \sigma_\omega\right)}$$
$$\sigma_c = \frac{G_{\sigma_\omega}\left(\overline{\omega}, \sigma_\omega\right)\sigma_\omega}{G\left(\overline{\omega}, \sigma_\omega\right)}$$

where $s = 1 + e^{\overline{s}}$.

9.4. Equation 4: Aggregate Demand

Finally, we have the aggregate demand:

$$y_t = c_t + i_t + g_t + \mu G\left(\overline{\omega}_t, \sigma_{\omega,t+1}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1}$$

loglinearizes to:

$$\begin{aligned}
\widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{g}{y}\widehat{g}_t \\
&\quad + \frac{\mu}{y} G\left(\overline{\omega}, \sigma_\omega\right) \left(rk\left(\widehat{r}_t + \widehat{k}_{t-1}\right) + (1-\delta) k \left(\widehat{q}_t + \widehat{k}_{t-1}\right)\right) \\
&\quad + (r+1-\delta) k \frac{\mu}{y} \left(G_\omega\left(\overline{\omega}, \sigma_\omega\right) \overline{\omega}\widehat{\overline{\omega}}_t + G_{\sigma_\omega}\left(\overline{\omega}, \sigma_\omega\right) \sigma_\omega\right) \widehat{\sigma}_{\omega,t} \\
&= \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{g}{y}\widehat{g}_t \\
&\quad + \frac{\mu}{y} G\left(\overline{\omega}, \sigma_\omega\right) \left[\left(rk\left(\widehat{r}_t + \widehat{k}_{t-1}\right) + (1-\delta) k \left(\widehat{q}_t + \widehat{k}_{t-1}\right)\right) + (r+1-\delta) k \left(\omega_c \widehat{\overline{\omega}}_t + \sigma_c \widehat{\sigma}_{\omega,t}\right)\right]
\end{aligned}$$

10. Appendix IV: Some Additional Algebra

Loglinearization of the first order condition for consumption:

$$e^{\phi_t} \frac{1}{c_t - hc_{t-1}} - \mathbb{E}_t \beta e^{\phi_{t+1}} \frac{h}{c_{t+1} - hc_t} = \left(1 + \tau_{c,t}\right) \lambda_t$$

$$e^{\phi_t} \frac{1}{ce^{\widehat{c}_t} - hce^{\widehat{c}_{t-1}}} - \mathbb{E}_t \beta e^{\phi_{t+1}} \frac{h}{ce^{\widehat{c}_{t+1}} - hce^{\widehat{c}_t}} = \left(1 + \overline{\tau}_c\right) \lambda e^{\widehat{\tau}_{c,t} + \widehat{\lambda}_t}$$

$$e^{\phi_t} \frac{1}{e^{\widehat{c}_t} - he^{\widehat{c}_{t-1}}} \frac{1}{c} - \mathbb{E}_t \beta e^{\phi_{t+1}} \frac{h}{e^{\widehat{c}_{t+1}} - he^{\widehat{c}_t}} \frac{1}{c} = \frac{1-\beta h}{1-h} \frac{1}{c} e^{\widehat{\tau}_{c,t} + \widehat{\lambda}_t}$$

$$e^{\phi_t} \frac{1}{e^{\widehat{c}_t} - he^{\widehat{c}_{t-1}}} - \mathbb{E}_t \beta h e^{\phi_{t+1}} \frac{1}{e^{\widehat{c}_{t+1}} - he^{\widehat{c}_t}} = \frac{1-\beta h}{1-h} e^{\widehat{\tau}_{c,t} + \widehat{\lambda}_t}$$

$$\frac{1}{1-h}\phi_t - \left(\frac{1}{1-h}\right)^2 \left(1 + \beta h^2\right) \widehat{c}_t - \beta h \left(\frac{1}{1-h}\right)^2 \widehat{c}_{t+1} + h \left(\frac{1}{1-h}\right)^2 \widehat{c}_{t-1} = \frac{1-\beta h}{1-h} \left(\widehat{\tau}_{c,t} + \widehat{\lambda}_t\right)$$

$$\phi_t - \frac{1+\beta h^2}{1-h}\widehat{c}_t - \frac{\beta h}{1-h}\widehat{c}_{t+1} + \frac{h}{1-h}\widehat{c}_{t-1} = (1-\beta h) \left(\widehat{\tau}_{c,t} + \widehat{\lambda}_t\right)$$

Loglinearization of the Euler condition for deposits:

$$\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{(1 + (1 - \tau_{R,t+1})(R_t - 1))}{\Pi_{t+1}} \right\}$$

$$\lambda e^{\widehat{\lambda}_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda}{\Pi} e^{\widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1}} \left(1 + (1 - \overline{\tau}_R) e^{\widehat{\tau}_{R,t+1}} \left(R e^{\widehat{R}_t} - 1 \right) \right) \right\}$$

$$1 = \beta \frac{(1 + (1 - \tau_R)(R - 1))}{\Pi}$$

$$\frac{\Pi}{\beta} - 1 = \frac{\Pi - \beta}{\beta} = (1 - \tau_R)(R - 1)$$

$$\widehat{\lambda}_t = \mathbb{E}_t \left\{ \frac{\beta}{\Pi} (1 + (1 - \overline{\tau}_R)(R - 1)) \left(\widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} \right) + \frac{\beta}{\Pi} (1 - \overline{\tau}_R)(R - 1) \widehat{\tau}_{R,t+1} + \frac{\beta}{\Pi} (1 - \overline{\tau}_R) R \widehat{R}_t \right\}$$

$$\widehat{\lambda}_t = \mathbb{E}_t \left\{ \widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} + \left(1 - \frac{\beta}{\Pi} \right) \left(\widehat{\tau}_{R,t+1} + \frac{R}{R - 1} \widehat{R}_t \right) \right\}$$

Loglinearization of the Euler condition for public debt:

$$\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{Rd_t}{\Pi_{t+1}} \right\}$$

$$\lambda e^{\widehat{\lambda}_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda}{\Pi} Rd e^{\widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} + \widehat{Rd}_t} \right\}$$

$$\widehat{\lambda}_t = \widehat{\lambda}_{t+1} - \widehat{\Pi}_{t+1} + \widehat{Rd}_t$$

Loglinearization for adjustment costs:

$$q_t \left(1 - S \left[\frac{i_t}{i_{t-1}} \right] - S' \left[\frac{i_t}{i_{t-1}} \right] \frac{i_t}{i_{t-1}} \right) + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S' \left[\frac{i_{t+1}}{i_t} \right] \left(\frac{i_{t+1}}{i_t} \right)^2 = 1$$

$$e^{\widehat{q}_t} \left(1 - S \left[e^{\widehat{i}_t - \widehat{i}_{t-1}} \right] - S' \left[e^{\widehat{i}_t - \widehat{i}_{t-1}} \right] e^{\widehat{i}_t - \widehat{i}_{t-1}} \right) + \beta \mathbb{E}_t e^{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{q}_{t+1}} S' \left[e^{\widehat{i}_{t+1} - \widehat{i}_{t-1}} \right] \left(e^{\widehat{i}_{t+1} - \widehat{i}_t} \right)^2 = 1$$

$$\widehat{q}_t = S''[1] \left(\widehat{i}_t - \widehat{i}_{t-1} \right) - \beta S''[1] \mathbb{E}_t \left(\widehat{i}_{t+1} - \widehat{i}_t \right)$$

Loglinearization for intertemporal budget constraint of government:

$$\frac{d_t}{p_t} = g_t + \frac{Rd_{t-1}}{\Pi_{t-1}} \frac{d_{t-1}}{p_{t-1}} - tax_t$$

$$\overline{d}_t = g e^{\widehat{g}_t} + \frac{Rd}{\Pi} e^{\widehat{Rd}_{t-1} - \widehat{\Pi}_{t-1}} \overline{d}_{t-1} - tax e^{\widehat{tax}_t}$$

$$\overline{d}_t = g \widehat{g}_t + \frac{Rd}{\Pi} \overline{d}_{t-1} - tax \widehat{tax}_t$$

$$\overline{d}_t = g \widehat{g}_t + \beta \overline{d}_{t-1} - tax \widehat{tax}_t$$

Loglinearization for taxes. First, note that:

$$\begin{aligned}\widehat{\tau}_{c,t} &= \log\frac{1+\tau_{c,t}}{1+\overline{\tau}_c} \Rightarrow \tau_{c,t} = (1+\overline{\tau}_c)\,e^{\widehat{\tau}_{c,t}} - 1 \\ \widehat{\tau}_{l,t} &= \log\frac{1-\tau_{l,t}}{1-\overline{\tau}_l} \Rightarrow \tau_{l,t} = 1-(1-\overline{\tau}_l)\,e^{\widehat{\tau}_{l,t}} \\ \widehat{\tau}_{R,t} &= \log\frac{1-\tau_{R,t}}{1-\overline{\tau}_R} \Rightarrow \tau_{R,t} = 1-(1-\overline{\tau}_R)\,e^{\widehat{\tau}_{R,t}}\end{aligned}$$

Then:

$$tax_t = \tau_{c,t}c_t + \tau_{l,t}w_t l_t + \tau_{R,t}\left(R_{t-1}-1\right)\overline{b}_{t-1}$$

$$tax e^{\widehat{tax}_t} = \left((1+\overline{\tau}_c)\,e^{\widehat{\tau}_{c,t}} - 1\right) c e^{\widehat{c}_t} + \left(1-(1-\overline{\tau}_l)\,e^{\widehat{\tau}_{l,t}}\right) w l e^{\widehat{w}_t+\widehat{l}_t}$$

$$+\left(1-(1-\overline{\tau}_R)\,e^{\widehat{\tau}_{R,t}}\right)\left(R e^{\widehat{R}_{t-1}} - 1\right)\overline{b} e^{\widehat{\overline{b}}_{t-1}}$$

or

$$\begin{aligned}tax\widehat{tax}_t &= \overline{\tau}_c c\widehat{c}_t + (1+\overline{\tau}_c)\,c\widehat{\tau}_{c,t} + \overline{\tau}_l wl\left(\widehat{w}_t + \widehat{l}_t\right) - (1-\overline{\tau}_l)\,wl\widehat{\tau}_{l,t} \\ &\quad +\overline{\tau}_R(R-1)\,\overline{b}\widehat{\overline{b}}_{t-1} + \overline{\tau}_R R\overline{b}\widehat{R}_{t-1} - (1-\overline{\tau}_R)(R-1)\,\overline{b}\widehat{\tau}_{R,t} \\ &= \overline{\tau}_c c\left(\widehat{c}_t + \widehat{\tau}_{c,t}\right) + c\widehat{\tau}_{c,t} + \overline{\tau}_l wl\left(\widehat{w}_t + \widehat{l}_t + \widehat{\tau}_{l,t}\right) - wl\widehat{\tau}_{l,t} \\ &\quad +\overline{\tau}_R(R-1)\,\overline{b}\left(\widehat{\overline{b}}_{t-1} + \widehat{\tau}_{R,t}\right) + \overline{\tau}_R R\overline{b}\widehat{R}_{t-1} - (R-1)\,\overline{b}\widehat{\tau}_{R,t}\end{aligned}$$

Loglinearization for price dispersion:

$$v_t = \theta\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} v_{t-1} + (1-\theta)\,\Pi_t^{*-\varepsilon}$$

$$v e^{\widehat{v}_t} = \theta\Pi^{\varepsilon(1-\chi)} e^{\varepsilon\left(\widehat{\Pi}_t - \chi\widehat{\Pi}_{t-1}\right)+\widehat{v}_{t-1}} v_t + (1-\theta)\,\Pi^{*-\varepsilon} e^{-\varepsilon\widehat{\Pi}_t^*}$$

$$\widehat{v}_t = \theta\Pi^{\varepsilon(1-\chi)}\left(\varepsilon\left(\widehat{\Pi}_t - \chi\widehat{\Pi}_{t-1}\right) + \widehat{v}_{t-1}\right) - \frac{(1-\theta)\,\Pi^{*-\varepsilon}}{\frac{1-\theta}{1-\theta\Pi^{\varepsilon(1-\chi)}}\Pi^{*-\varepsilon}}\varepsilon\widehat{\Pi}_t^*$$

$$\widehat{v}_t = \theta\Pi^{\varepsilon(1-\chi)}\left(\varepsilon\left(\widehat{\Pi}_t - \chi\widehat{\Pi}_{t-1}\right) + \widehat{v}_{t-1}\right) - \left(1-\theta\Pi^{\varepsilon(1-\chi)}\right)\varepsilon\widehat{\Pi}_t^*$$

References

[1] Bernanke, Ben, Mark Gertler, and Simon Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework." *Handbook of Macroeconomics*, volume 3.

[2] Carlstrom, Charles and Timothy Fuerst (1997). "Agency Costs, Net Worth and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review.*

[3] Chari, V.V. Lawrence Christiano, and Martin Eichenbaum (1995). "Inside Money, Outside Money and Short-Term Interest Rates" *Journal of Money, Credit, and Banking.*

[4] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2009). "Financial Factors in Economic Fluctuations." Mimeo.

[5] Curdia, Vasco, and Michael Woodford (2008). "Credit Frictions and Optimal Monetary Policy." Mimeo.

[6] Diamond, D. and P. Dybvig (1983). "Bank Runs, Deposit Insurance and Liquidity." *Journal of Political Economy* 91, 401-419.

[7] Diamond, D. and R. Rajan (2001). "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking." *Journal of Political Economy* 109, 287-327.

[8] Diaz-Gimenez, Javier, Edward Prescott, Terry Fitzgerald, and Fernando Álvarez (1992). "Banking in Computable General Equilibrium Economies." *Journal of Economic Dynamics and Control* 16(3-4), pages 533-559 (see also the staff report version at the Minneapolis Fed).

[9] Gerali, Andrea, Stefano Neri, Luca Sessa, and Federico Signoretti (2009). "Credit and Banking in a DSGE Model of the Euro Area." Mimeo.

[10] Jermann, Urban and Vincenzo Quadrini (2009). "Financial Innovation and Macroeconomic Volatility." Mimeo.

[11] Kiyotaki, Nobuhiro and John Moore (1997). "Credit Cycles," *Journal of Political Economy*, 105, 211-248.

[12] Kiyotaki, Nobuhiro and John Moore (2008). "Liquidity Business Cycles and Monetary Policy." Mimeo.