1 The linearized economy

1.1 Domestic goods-producing firms

1.1.1 The labour market

Define the deviation (not log-deviation) of unemployment as $\hat{U}_t^a = U_t^a - \bar{U}^a$. According to (**??**), and with the steady state value (**??**) substituted in, the linearized after-hiring unemployment rate is:

$$\hat{U}_t^a = -\left(1 - \bar{U}^a\right)\hat{N}_t \tag{1.1}$$

The persistent hiring cost shock (**??**) follows an AR(1) process:

$$\zeta_t^\Omega = \rho_\Omega \zeta_{t-1}^\Omega + \epsilon_t^\Omega \tag{1.2}$$

Linearizing the labour market tightness variable (**??**) and substituting (**??**) and (1.1) to give:[1]

$$\chi(1 - \bar{U}^a)\hat{\Omega}_t = -\hat{U}_t^a + (1 - \bar{\Omega})(1 - \chi)\hat{U}_{t-1}^a \tag{1.3}$$

Given the steady state condition (**??**), linearizing (**??**) gives:

$$\chi \hat{H}_t = \hat{N}_t - (1 - \chi)\hat{N}_{t-1} \tag{1.4}$$

1.1.2 Domestic intermediate-producing firms

Linearizing the production function for the intermediate producer (**??**) and substituting (**??**) gives:

$$\hat{y}_t = \bar{\lambda}^d \left(\hat{\epsilon}_t + \alpha(\hat{k}_t - \hat{\mu}_t^z) + (1 - \alpha)\hat{N}_t\right) \tag{1.5}$$

Permanent technology growth (**??**) is:

$$\hat{\mu}_t^z = \rho_{\mu^z}\hat{\mu}_{t-1}^z + \epsilon_t^z \tag{1.6}$$

The temporary technology shock (**??**) follows:

$$\hat{\epsilon}_t = \rho_\epsilon \hat{\epsilon}_{t-1} + \epsilon_t^\epsilon \tag{1.7}$$

Combining (**??**) and (**??**) gives a relationship between the real rental rate of capital and marginal cost $r_t^k = \alpha \epsilon_t k_t^{\alpha-1}(\mu_t^z)^{(1-\alpha)} N_t^{1-\alpha} mc_t^d$. Linearizing with (1.1) substituted in gives:

[1] Linearizing (**??**):

$$\begin{aligned}
H_t &= \Omega_t U_t \\
N_t - (1-\chi)N_{t-1} &= \Omega_t\left(1 - (1-\chi)N_{t-1}\right) \\
\bar{N}(1 + \hat{N}_t) - (1-\chi)\bar{N}(1 + \hat{N}_{t-1}) &= \bar{\Omega}(1 + \hat{\Omega}_t) - (1-\chi)\bar{\Omega}\bar{N}(1 + \hat{\Omega}_t + \hat{N}_{t-1}) \\
\bar{N}\hat{N}_t - (1-\chi)\bar{N}\hat{N}_{t-1} &= \bar{\Omega}\hat{\Omega}_t - (1-\chi)\bar{\Omega}\bar{N}(\hat{\Omega}_t + \hat{N}_{t-1}) \\
\hat{\Omega}_t &= \frac{\bar{N}}{(1 - (1-\chi)\bar{N})\bar{\Omega}}\hat{N}_t + \frac{\bar{N}(\bar{\Omega} - 1)(1-\chi)}{(1 - (1-\chi)\bar{N})\bar{\Omega}}\hat{N}_{t-1}
\end{aligned}$$

$$\hat{r}^k_t = \hat{\epsilon}_t - (1-\alpha)(\hat{k}_t - \hat{\mu}^z_t) - \frac{1-\alpha}{1-\bar{U}^a}\hat{U}^a_t + \widehat{mc}^d_t \tag{1.8}$$

Given $\bar{\epsilon} = 1$, $\bar{\zeta}^\Omega = 1$, and define:

$$\begin{aligned}
\kappa &= \left[\bar{w}\bar{R} + B(1+\vartheta)\bar{\Omega}^\vartheta \left(1 + \beta(1-\chi)\bar{\mu}^z\bar{\pi}^d \left[\frac{\vartheta}{1-\vartheta}\bar{\Omega} - 1\right]\right)\right] \\
\kappa_1 &= -\frac{(1-\alpha)\bar{w}\bar{R}}{\kappa} \\
\kappa_2 &= -\frac{(1-\alpha)(1+\vartheta)B\bar{\Omega}^\vartheta}{\kappa} \\
\kappa_3 &= -\frac{(1-\alpha)\beta(1-\chi)\bar{\mu}^z\bar{\pi}^d(\vartheta\bar{\Omega} - 1 - \vartheta)B\bar{\Omega}^\vartheta}{\kappa} \\
\kappa_4 &= \frac{(1-\alpha)\beta(1-\chi)\bar{\mu}^z\bar{\pi}^d\vartheta(1+\vartheta)(1-\bar{\Omega})B\bar{\Omega}^\vartheta}{\kappa}
\end{aligned}$$

The linearized real marginal cost (**??**) is:[2]

$$\begin{aligned}
\widehat{mc}^d_t =& -\hat{\epsilon}_t + \alpha\hat{r}^k_t - \kappa_1\left[\hat{w}_t + \hat{R}_{t-1}\right] - \kappa_2\left[\hat{\epsilon}_t + \vartheta\hat{\Omega}_t + \hat{\zeta}^\Omega_t\right] \\
& - \kappa_3 E_t\left[\hat{\epsilon}_{t+1} + \hat{\mu}^z_{t+1} + \hat{\pi}^d_{t+1} + \hat{\zeta}^\Omega_{t+1}\right] - \kappa_4 E_t\left[\hat{\Omega}_{t+1}\right]
\end{aligned} \tag{1.9}$$

[2]

$$\begin{aligned}
\left(\overline{mc}^d\right)^{\frac{1}{1-\alpha}}\left(1 + \frac{1}{1-\alpha}\widehat{mc}^d_t\right) =& \left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(\bar{r}^k\right)^{\frac{\alpha}{1-\alpha}}\left(1 - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right) \\
& \times\left\{\bar{w}\bar{R}\left[1 + \hat{w}_t + \hat{R}_{t-1}\right] + B(1+\vartheta)\bar{\Omega}^\vartheta\left[1 + \hat{\epsilon}_t + \vartheta\hat{\Omega}_t + \hat{\zeta}^\Omega_t\right]\right. \\
& + \beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d\vartheta\bar{\Omega}^{1+\vartheta}E_t\left[1 + \hat{\mu}^z_{t+1} + \hat{\epsilon}_{t+1} + \hat{\pi}^d_{t+1} + \hat{\zeta}^\Omega_{t+1} + (1+\vartheta)\hat{\Omega}_{t+1}\right] \\
& \left. -\beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d(1+\vartheta)\bar{\Omega}^\vartheta E_t\left[1 + \hat{\mu}^z_{t+1} + \hat{\epsilon}_{t+1} + \hat{\pi}^d_{t+1} + \hat{\zeta}^\Omega_{t+1} + \vartheta\hat{\Omega}_{t+1}\right]\right\}
\end{aligned}$$

Moving $\left(1 - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right)$ into the curly bracket, ignoring the cross products and applying $\overline{mc}^d$ from (**??**) to cancel the constant terms gives:

$$\begin{aligned}
& \frac{\left(\overline{mc}^d\right)^{\frac{1}{1-\alpha}}}{1-\alpha}\widehat{mc}^d_t = \left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(\bar{r}^k\right)^{\frac{\alpha}{1-\alpha}} \\
& \times\left\{\bar{w}\bar{R}\left[\hat{w}_t + \hat{R}_{t-1} - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right] + B(1+\vartheta)\bar{\Omega}^\vartheta\left[\hat{\epsilon}_t + \vartheta\hat{\Omega}_t + \hat{\zeta}^\Omega_t - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right]\right. \\
& + \beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d\vartheta\bar{\Omega}^{1+\vartheta}E_t\left[\hat{\epsilon}_{t+1} + \hat{\mu}^z_{t+1} + \hat{\pi}^d_{t+1} + \hat{\zeta}^\Omega_{t+1} + (1+\vartheta)\hat{\Omega}_{t+1} - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right] \\
& \left. -\beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d(1+\vartheta)\bar{\Omega}^\vartheta E_t\left[\hat{\epsilon}_{t+1} + \hat{\mu}^z_{t+1} + \hat{\pi}^d_{t+1} + \hat{\zeta}^\Omega_{t+1} + \vartheta\hat{\Omega}_{t+1} - \frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right]\right\}
\end{aligned}$$

Factoring out $\left(-\frac{1}{1-\alpha}\hat{\epsilon}_t + \frac{\alpha}{1-\alpha}\hat{r}^k_t\right)$ and dividing both side by $\frac{\overline{mc}^{\frac{1}{1-\alpha}}}{1-\alpha}$ gives:

The new Keynesian Phillips curve for domestic intermediate producing firms (**??**) is:

$$\hat{\pi}_t^d = \frac{\beta}{1+\beta} E_t \left[\hat{\pi}_{t+1}^d\right] + \frac{1}{1+\beta}\hat{\pi}_{t-1}^d + \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d(1+\beta)} \left[\hat{\lambda}_t^d + \widehat{mc}_t^d\right] \tag{1.10}$$

1.1.3 Domestic-importing firms

The new Keynesian Phillips curve for the consumption-importing firms (**??**) is:

$$\hat{\pi}_t^{m,c} = \frac{\beta}{1+\beta} E_t \left[\hat{\pi}_{t+1}^{m,c}\right] + \frac{1}{1+\beta}\hat{\pi}_{t-1}^{m,c} + \frac{(1-\xi_c^m)(1-\beta\xi_c^m)}{\xi_c^m(1+\beta)} \left[\hat{\lambda}_t^{m,c} + \widehat{mc}_t^{m,c}\right] \tag{1.11}$$

where the real marginal cost $\widehat{mc}_t^{m,c}$ as defined in (**??**):

$$\widehat{mc}_t^{m,c} = \hat{P}^* + \hat{S}_t - \hat{P}_t^{m,c} = -\widehat{mc}_t^x - \hat{\gamma}_t^{x,*} - \hat{\gamma}_t^{mc,d} \tag{1.12}$$

The new Keynesian Phillips curve for the investment-importing firms **??** is:

$$\hat{\pi}_t^{m,i} = \frac{\beta}{1+\beta} E_t \left[\hat{\pi}_{t+1}^{m,i}\right] + \frac{1}{1+\beta}\hat{\pi}_{t-1}^{m,i} + \frac{(1-\xi_i^m)(1-\beta\xi_i^m)}{\xi_i^m(1+\beta)} \left[\hat{\lambda}_t^{m,i} + \widehat{mc}_t^{m,i}\right] \tag{1.13}$$

where the real marginal cost $\widehat{mc}_t^{m,i}$ as defined in (**??**):

$$\widehat{mc}_t^{m,i} = \hat{P}^* + \hat{S}_t - \hat{P}_t^{m,i} = -\widehat{mc}_t^x - \hat{\gamma}_t^{x,*} - \hat{\gamma}_t^{mi,d} \tag{1.14}$$

1.1.4 Domestic-exporting firms

The new Keynesian Phillips curve for the exporting firms (**??**) is:

$$\hat{\pi}_t^x = \frac{\beta}{1+\beta} E_t \left[\hat{\pi}_{t+1}^x\right] + \frac{1}{1+\beta}\hat{\pi}_{t-1}^x + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\beta)} \left[\hat{\lambda}_t^x + \widehat{mc}_t^x\right] \tag{1.15}$$

where the real marginal cost $\widehat{mc}_t^x = \hat{P}_t^d - \hat{S}_t - \hat{P}_t^x$. Lagging one period then subtracting from itself gives:

$$\widehat{mc}_t^x = \widehat{mc}_{t-1}^x + \hat{\pi}_t^d - \hat{S}_t + \hat{S}_{t-1} - \hat{\pi}_t^x \tag{1.16}$$

$$\begin{aligned}
\widehat{mc}_t = -\hat{\epsilon}_t + \alpha\hat{r}_t^k + &\frac{1-\alpha}{\left[\bar{w}\bar{R} + B(1+\vartheta)\bar{\Omega}^\vartheta \left(1+\beta(1-\delta)\bar{\mu}^z\bar{\pi}^d\left[\frac{\vartheta}{1-\vartheta}\bar{\Omega}-1\right]\right)\right]} \\
&\times\left\{\bar{w}\bar{R}\left[\hat{w}_t + \hat{R}_{t-1}\right] + B(1+\vartheta)\bar{\Omega}^\vartheta\left[\hat{\epsilon}_t + \vartheta\hat{\Omega}_t + \hat{\zeta}_t^\Omega\right]\right. \\
&+\beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d\bar{\Omega}^\vartheta(\vartheta\bar{\Omega}-1-\vartheta)E_t\left[\hat{\epsilon}_{t+1} + \hat{\mu}_{t+1}^z + \hat{\pi}_{t+1}^d + \hat{\zeta}_{t+1}^\Omega\right] \\
&\left.-(1-\bar{\Omega})\beta B(1-\chi)\bar{\mu}^z\bar{\pi}^d\vartheta(1+\vartheta)\bar{\Omega}^\vartheta E_t\left[\hat{\Omega}_{t+1}\right]\right\}
\end{aligned}$$

1.1.5 CPI inflation

According to the aggregate consumption price (??), linearizing, lagging one period and subtract from itself, then applying (??) gives:[3]

$$\hat{\pi}_t^c = \left[(1-\omega_c)\left(\bar{\gamma}^{c,d}\right)^{\eta_c-1}\right]\hat{\pi}_t^d + \left[\omega_c\left(\frac{\bar{\gamma}^{c,d}}{\bar{\lambda}^{m,c}}\right)^{\eta_c-1}\right]\hat{\pi}_t^{m,c} \tag{1.17}$$

1.2 Households

Since the steady state value of preference shocks $\bar{\zeta}^l = 1$ for $l \in \{c, N\}$, the log deviation from the steady state values are measured by $\hat{\zeta}_t^l = \frac{\zeta_t^l - 1}{1}$, which follow an AR(1) process:

$$\hat{\zeta}_t^c = \rho_{\zeta^c}\hat{\zeta}_{t-1}^c + \epsilon_t^{\zeta^c} \tag{1.18}$$

$$\hat{\zeta}_t^N = \rho_{\zeta^N}\hat{\zeta}_{t-1}^N + \epsilon_t^{\zeta^N} \tag{1.19}$$

Similar to the preference shocks, the investment-specific technology shock follows also an AR(1) process:

$$\hat{\Gamma}_t = \rho_\Gamma\hat{\Gamma}_{t-1} + \epsilon_t^\Gamma \tag{1.20}$$

Linearizing the consumption Euler equation (??) gives:[4]

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[3]Linearizing (??) gives:

$$\hat{p}_t^c = (1-\omega_c)\left(\frac{\bar{P}_t^d}{\bar{P}_t^c}\right)^{1-\eta_c}\hat{p}_t^d + \omega_c\left(\frac{\bar{P}_t^{m,c}}{\bar{P}_t^d}\frac{\bar{P}_t^d}{\bar{P}_t^c}\right)^{1-\eta_c}\hat{p}_t^{m,c} = (1-\omega_c)(\bar{\gamma}^{c,d})^{\eta_c-1}\hat{p}_t^d + \omega_c\left(\frac{\bar{\gamma}^{c,d}}{\bar{\gamma}^{mc,d}}\right)^{\eta_c-1}\hat{p}_t^{m,c}$$

[4]Equation (??) can be simplified to:

$$\frac{\zeta_t^c\mu_t^z}{c_t\mu_t^z - bc_{t-1}} - E_t\left[\frac{\beta b\zeta_{t+1}^c}{c_{t+1}\mu_{t+1}^z - bc_t}\right] = \psi_t^z\gamma_t^{c,d}$$

Multiplying both sides by $(c_t\mu_t^z - bc_{t-1})(c_{t+1}\mu_{t+1}^z - bc_t)$, applying steady state value $\bar{\zeta}^c = 1$ and linearizing gives:

$$\begin{aligned}
&E_t\left[\bar{c}\,(\bar{\mu}^z)^2\,[\hat{\zeta}_t^c + \hat{\mu}_t^z + \hat{c}_{t+1} + \hat{\mu}_{t+1}^z] - \bar{c}b\bar{\mu}^z[\hat{\zeta}_t^c + \hat{\mu}_t^z + \hat{c}_t] - \bar{c}\beta b\bar{\mu}^z[\hat{\zeta}_{t+1}^c + \hat{\mu}_t^z + \hat{c}_t] + \bar{c}\beta b^2[\hat{\zeta}_{t+1}^c + \hat{c}_{t-1}]\right]\\
&= E_t\Big[\bar{c}^2\bar{\psi}^z\bar{\gamma}^{c,d}\{(\bar{\mu}^z)^2\,[\hat{\psi}_t^z + \hat{\gamma}_t^{c,d} + \hat{c}_t + \hat{c}_{t+1} + \hat{\mu}_t^z + \hat{\mu}_{t+1}^z] - b\bar{\mu}^z[\hat{\psi}_t^z + \hat{\gamma}_t^{c,d} + \hat{c}_{t-1} + \hat{c}_{t+1} + \hat{\mu}_{t+1}^z]\\
&\qquad -b\bar{\mu}^z[\hat{\psi}_t^z + \hat{\gamma}_t^{c,d} + 2\hat{c}_t + \hat{\mu}_t^z] + b^2[\hat{\phi}_t^z + \hat{\gamma}_t^{c,d} + \hat{c}_t + \hat{c}_{t-1}]\}\Big]
\end{aligned}$$

Dividing both side by $\bar{c}$, using equation (??) for the steady state and collecting common terms gives:

$$\begin{aligned}
&E_t\left[\bar{\mu}^z(\bar{\mu}^z - b)\hat{\zeta}_t^c + (\bar{\mu}^z)^2\,[\hat{\mu}_t^z + \hat{c}_{t+1} + \hat{\mu}_{t+1}^z] - (1+\beta)b\bar{\mu}^z[\hat{\mu}_t^z + \hat{c}_t] - \beta b(\bar{\mu}^z - b)\hat{\zeta}_{t+1}^c + \beta b^2\hat{c}_{t-1}\right]\\
&= E_t\left[\frac{\bar{\mu}^z - \beta b}{\bar{\mu}^z - b}\left\{(\bar{\mu}^z - b)^2\,[\hat{\psi}_t^z + \hat{\gamma}_t^{c,d}] + (\bar{\mu}^z - b)^2\,\hat{c}_t + \bar{\mu}^z(\bar{\mu}^z - b)(\hat{c}_{t+1} + \hat{\mu}_t^z + \hat{\mu}_{t+1}^z) - b(\bar{\mu}^z - b)\hat{c}_{t-1}\right\}\right]
\end{aligned}$$

Expanding RHS and collecting common terms gives (1.21).

$$\hat{c}_t = \frac{b\bar{\mu}^z}{(\bar{\mu}^z)^2 - \beta b^2} E_t\left[\hat{c}_{t-1} + \beta \hat{c}_{t+1}\right] - \frac{b\bar{\mu}^z}{(\bar{\mu}^z)^2 - \beta b^2} E_t\left[\hat{\mu}^z_t - \beta \hat{\mu}^z_{t+1}\right]$$
$$+ \frac{\bar{\mu}^z - b}{(\bar{\mu}^z)^2 - \beta b^2} E_t\left[\bar{\mu}^z \hat{\zeta}^c_t - \beta b \hat{\zeta}^c_{t+1}\right] - \frac{(\bar{\mu}^z - \beta b)(\bar{\mu}^z - b)}{(\bar{\mu}^z)^2 - \beta b^2}\left[\hat{\psi}^z_t + \hat{\gamma}^{c,d}_t\right] \tag{1.21}$$

The optimal asset holding **??** can be linearized to:

$$\hat{\psi}^z_t = \hat{R}_t + E_t[\hat{\psi}^z_{t+1} - \hat{\mu}^z_{t+1} - \hat{\pi}^d_{t+1}] \tag{1.22}$$

Given (**??**), $a(\bar{u}) = 0$ and $a'(\bar{u}) = \bar{r}^k$ at the steady state, the ready to install asset price (**??**) can be linearized to:

$$\hat{P}^{k'}_t = E_t\left[\frac{(1-\delta)\beta}{\bar{\mu}^z}\hat{P}^{k'}_{t+1} + \frac{\bar{\mu}^z - \beta(1-\delta)}{\bar{\mu}^z}\hat{r}^k_{t+1} + \hat{\psi}^z_{t+1} - \hat{\psi}^z_t - \hat{\mu}^z_{t+1}\right] \tag{1.23}$$

With (**??**) substituted, the optimality condition for investment (**??**) can be linearized as:[5]

$$\hat{P}^{k'}_t = \hat{\gamma}^{i,d}_t - \hat{\Gamma}_t - \tilde{f}_1(i_t, i_{t-1}, \mu^z_t) - \beta E_t\left[\tilde{f}_2(i_{t+1}, i_t, \mu^z_{t+1})\right]$$

where $\tilde{f}_1(i_t, i_{t-1}, \mu^z_t)$ and $\tilde{f}_2(i_{t+1}, i_t, \mu^z_{t+1})$ are the differences between the first order Taylor expansion of $f_1(.)$ and $f_2(.)$ with the respective steady state $\bar{f}_1(.)$ and $\bar{f}_2(.)$, which can be derived from (**??**) and (**??**).[6] Substituting into the above equation gives:

[5]Take first order Taylor expansion of (**??**), ignoring cross products gives:

$$\bar{\gamma}^{i,d}(1 + \hat{\gamma}^{i,d}_t) - \bar{\Gamma}\bar{P}^{k'}\bar{f}_1(\bar{i}, \bar{i}, \bar{\mu}^z)\left(1 + \hat{P}^{k'}_t + \hat{\Gamma}_t + \tilde{f}_1(i_t, i_{t-1}, \mu^z_t)\right) = \frac{\beta\bar{\Gamma}\bar{\psi}^z\bar{P}^{k'}}{\bar{\psi}^z} E_t\left[\tilde{f}_2(i_{t+1}, i_t, \mu^z_{t+1})\right] +$$
$$\beta\bar{f}_2(\bar{i}, \bar{i}, \bar{\mu}^z) E_t\left[\frac{\bar{\Gamma}\bar{\psi}^z\bar{P}^{k'}}{\bar{\psi}^z} + \frac{\bar{\Gamma}\bar{\psi}^z}{\bar{\psi}^z}\hat{P}^{k'}_{t+1} + \frac{\bar{\Gamma}\bar{P}^{k'}}{\bar{\psi}^z}\hat{\psi}^z_{t+1} - \frac{\bar{\Gamma}\bar{\psi}^z\bar{P}^{k'}}{(\bar{\psi}^z)^2}\hat{\psi}^z_t + \frac{\bar{\psi}^z\bar{P}^{k'}}{\bar{\psi}^z}\hat{\Gamma}_{t+1}\right]$$

Then applying (**??**), $\bar{\Gamma} = 1$, $\bar{f}_1(\bar{i}, \bar{i}, \bar{\mu}^z) = 1$, and $\bar{f}_2(\bar{i}, \bar{i}, \bar{\mu}^z) = 0$.

[6]Applying the first order Taylor expansion to (**??**) then subtracting $\bar{f}_1(\bar{i}, \bar{i}, \bar{\mu}^z) = 1$ to give:

$$\begin{aligned}\hat{f}_1(i_t, i_{t-1}, \mu^z_t) = &\left[-\tilde{S}''(.)\left(\frac{i_t(\mu^z_t)^2}{i^2_{t-1}}\right) - \tilde{S}'(.)\left(\frac{\mu^z_t}{i_{t-1}}\right) - \tilde{S}'(.)\left(\frac{\mu^z_t}{i_{t-1}}\right)\right](i_t - \bar{i}) \\ &+ \left[\tilde{S}''(.)\left(\frac{i^2_t(\mu^z_t)^2}{i^3_{t-1}}\right) + \tilde{S}'(.)\left(\frac{i_t\mu^z_t}{i^2_{t-1}}\right) + \tilde{S}'(.)\left(\frac{i_t\mu^z_t}{i^2_{t-1}}\right)\right](i_{t-1} - \bar{i}) \\ &+ \left[-\tilde{S}''(.)\left(\frac{i^2_t\mu^z_t}{i^2_{t-1}}\right) - \tilde{S}'(.)\left(\frac{i_t}{i_{t-1}}\right) - \tilde{S}'(.)\left(\frac{i_t}{i_{t-1}}\right)\right](\mu^z_t - \bar{\mu}^z)\end{aligned}$$

At steady state, $\bar{i}_t = \bar{i}_{t-1} = \bar{i}$, $\tilde{S}'(.) = 0$, evaluating the above equation at steady state implies:

$$\hat{f}_1(i_t, i_{t-1}, \mu^z_t) = \left[-\tilde{S}''(.)(\bar{\mu}^z)^2\right]\hat{i}_t + \left[\tilde{S}''(.)(\bar{\mu}^z)^2\right]\hat{i}_{t-1} + \left[-\tilde{S}''(.)(\bar{\mu}^z)^2\right]\hat{\mu}^z_t$$

Similarly, taking the first order Taylor expansion of (**??**), then subtracting $\bar{f}_2(\bar{i}, \bar{i}, \bar{\mu}^z) = 0$ to give:

$$\hat{P}_t^{k'} = \hat{\gamma}_t^{i,d} - \hat{\Gamma}_t + (\bar{\mu}^z)^2 \tilde{S}''(.) \left[\left(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^z\right) - \beta E_t \left[\left(\hat{i}_{t+1} - \hat{i}_t + \hat{\mu}_{t+1}^z\right)\right]\right] \tag{1.24}$$

The linearized capital accumulation (**??**) is:[7]

$$\hat{\bar{k}}_{t+1} = \frac{(1-\delta)}{\bar{\mu}^z}(\hat{\bar{k}}_t - \hat{\mu}_t^z) + \left(1 - \frac{(1-\delta)}{\bar{\mu}^z}\right)\left(\hat{\Gamma}_t + \hat{i}_t\right) \tag{1.25}$$

With (**??**), the linearized optimal condition for capital (**??**) is:

$$\hat{u}_t = \hat{k}_t - \hat{\bar{k}}_t \tag{1.26}$$

From (**??**), we can obtain:

$$\hat{u}_t = \frac{1}{\sigma_a}\hat{r}_t^k \tag{1.27}$$

where $\sigma_a \equiv \frac{a''(1)}{a'(1)}$. Given the steady state relationship (**??**), the linearized optimal real balance (**??**) is:

$$\hat{q}_t = -\frac{1}{\sigma_q}\hat{\psi}_t^z - \frac{\bar{R}}{\sigma_q(\bar{R}-1)}\hat{R}_{t-1} \tag{1.28}$$

$$\begin{aligned}
\hat{f}_2(i_{t+1}, i_t, \mu_{t+1}^z) = &\left[\tilde{S}''(.)\frac{\left(i_{t+1}\mu_{t+1}^z\right)^2}{i_t^3} + 2\tilde{S}'(.)\frac{i_{t+1}\mu_{t+1}^z}{(i_t)^2}\right](i_{t+1} - \bar{i}) \\
&- \left[\tilde{S}''(.)\left(\frac{i_{t+1}\mu_{t+1}^z}{i_t^2}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{t+1}^z + 2\tilde{S}'(.)\frac{1}{i_t}\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{t+1}^z\right](i_t - \bar{i}) \\
&+ \left[\tilde{S}''(.)\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{t+1}^z + \tilde{S}'(.)\left(\frac{i_{t+1}}{i_t}\right)^2\right](\mu_t^z - \bar{\mu}^z)
\end{aligned}$$

Evaluating at steady state gives:

$$\hat{f}_2(i_{t+1}, i_t, \mu_{t+1}^z) = \left[\tilde{S}''(.)\,(\bar{\mu}^z)^2\right]\hat{i}_{t+1} - \left[\tilde{S}''(.)\,(\bar{\mu}^z)^2\right]\hat{i}_t + \left[\tilde{S}''(.)\,(\bar{\mu}^z)^2\right]\hat{\mu}_{t+1}^z$$

[7]The capital accumulation (**??**) can be linearized as (with Δ_t equals 0 for all t at equilibrium):

$$\hat{\bar{k}}_{t+1} = (1-\delta)\frac{1}{\bar{\mu}^z}(\hat{\bar{k}}_t - \hat{\mu}_t^z) + \frac{\bar{f}(\bar{i},\bar{i},\bar{\mu}^z)}{\bar{\bar{k}}}(\Gamma_t - \bar{\Gamma}) + \frac{\bar{\Gamma}}{\bar{\bar{k}}}\tilde{f}(i_t, i_{t-1}, \mu_t^z)$$

where $\tilde{f}(.) = f(.) - \bar{f}(.)$ Evaluating $\tilde{f}(i_t, i_{t-1}, \mu_t^z)$ at the steady state gives:

$$\begin{aligned}
\tilde{f}(i_t, i_{t-1}, \mu_t^z) &= \left[-\frac{\mu_t^z i_t}{i_{t-1}}\tilde{S}'(.) + \left(1 - \tilde{S}(.)\right)\right](i_t - \bar{i}) + \left[\tilde{S}'(.)\frac{i_t^2\mu_t^z}{i_{t-1}^2}\right](i_{t-1} - \bar{i}) - \left[\frac{i_t^2}{i_{t-1}}\tilde{S}'(.)\right](\mu_t^z - \bar{\mu}^z) \\
&= i_t - \bar{i}
\end{aligned}$$

Noting $\bar{f}(.) = \bar{i}$, combining with (**??**) gives (1.25).

Combining the optimality condition of domestic bond holding (**??**) and foreign bond holding (**??**) gives: $\frac{S_{t+1}}{S_t}\Phi(a_t, \tilde{\phi}_t) = \frac{R_t}{R_t^*}$. Given the functional form of the risk premium (**??**), log-linearizing gives the uncovered interest rate parity (with risk premium) condition:

$$\hat{R}_t - \hat{R}_t^* = E_t\left[\hat{S}_{t+1} - \hat{S}_t\right] - \tilde{\phi}_a \hat{a}_t + \hat{\tilde{\phi}}_t \tag{1.29}$$

where $\hat{\tilde{\phi}}_t$ is the time-varying shock to the risk premium, which is assumed to follow a stationary AR(1) process:

$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}} \hat{\tilde{\phi}}_{t-1} + \epsilon_t^{\tilde{\phi}} \tag{1.30}$$

1.3 Wage rigidities

1.3.1 Nominal wage rigidity

The wage determination process under nominal wage rigidity (**??**):

$$\hat{w}_t = \tau_b\left(\hat{w}_{t-1} + \hat{\pi}_{t-1}^c - \hat{\pi}_t^d\right) + \tau_f\left(E_t\left[\hat{w}_{t+1}\right] - \hat{\pi}_t^c + E_t\left[\hat{\pi}_{t+1}^d\right]\right) + \tau_c\left(\hat{\zeta}_t^N + \sigma_L \hat{N}_t - \hat{\psi}_t^z + \hat{\lambda}_t^w\right) \tag{1.31}$$

where:

$$\tau \equiv \frac{\bar{\lambda}^w \sigma_L (1-\beta\xi_w)}{\bar{\lambda}^w - 1} \quad \tau_b \equiv \frac{(1+\tau)\xi_w}{1+\xi_w\tau+\beta\xi_w^2} \quad \tau_f \equiv \frac{\beta\xi_w}{1+\xi_w\tau+\beta\xi_w^2} \quad \tau_c \equiv \frac{(1-\beta\xi_w)(1-\xi_w)}{1+\xi_w\tau+\beta\xi_w^2}$$

1.3.2 Real wage rigidity

Linearizing the equilibrium Nash bargaining wage (**??**) gives

$$\begin{aligned}\hat{w}_t^* &= \tau_1\left(\hat{\epsilon}_t + \vartheta\hat{\Omega}_t + \hat{\zeta}_t^\Omega\right) + \tau_2\left(\hat{\zeta}_t^N + \sigma_L\hat{N}_t - \hat{\psi}_t^z\right) \\ &\quad - \tau_3 E_t\left[\hat{\epsilon}_{t+1} + \hat{\zeta}_{t+1}^\Omega + \hat{\psi}_{t+1}^z - \hat{\psi}_t^z - \hat{\mu}_{t+1}^z - \hat{\pi}_{t+1}^d\right] + \tau_4 E_t\left[\hat{\Omega}_{t+1}\right]\end{aligned} \tag{1.32}$$

where:

$$\tau_1 \equiv \frac{B\bar{\Omega}^\vartheta}{\bar{w}^*} \quad \tau_2 \equiv \frac{A_L \bar{N}^{\sigma_L}}{\bar{\psi}^z \bar{w}^*} \quad \tau_3 \equiv \frac{\beta(1-\chi)B\bar{\Omega}^\vartheta(1-\bar{\Omega})}{\bar{\mu}^z\bar{\pi}^d\bar{w}^*} \quad \tau_4 \equiv \frac{\beta(1-\chi)B\bar{\Omega}^\vartheta\left(\bar{\Omega}(1+\vartheta)-\vartheta\right)}{\bar{\mu}^z\bar{\pi}^d\bar{w}^*}$$

which (**??**) implies the the real wage follows:

$$\hat{w}_t = f\hat{w}_{t-1} + (1-f)\hat{w}_t^* \tag{1.33}$$

1.4 Relative prices

linearizing (??) and (??) gives:

$$\hat{\gamma}_t^{c,d} = \left(1 - \frac{(1-\omega_c)}{(\bar{\gamma}^{c,d})^{1-\eta_c}}\right)\hat{\gamma}_t^{mc,d} \tag{1.34}$$

$$\hat{\gamma}_t^{i,d} = \left(1 - \frac{(1-\omega_i)}{(\bar{\gamma}^{i,d})^{1-\eta_i}}\right)\hat{\gamma}_t^{mi,d} \tag{1.35}$$

Linearizing the relative price between imported consumption and the domestic price (??), between imported investment and the domestic price (??) and between export price and the foreign price level (??), lagging one period, then subtracting from the original equation gives:[8]

$$\hat{\gamma}_t^{mc,d} = \hat{\gamma}_{t-1}^{mc,d} + \hat{\pi}_t^{m,c} - \hat{\pi}_t^d \tag{1.36}$$

$$\hat{\gamma}_t^{mi,d} = \hat{\gamma}_{t-1}^{mi,d} + \hat{\pi}_t^{m,i} - \hat{\pi}_t^d \tag{1.37}$$

$$\hat{\gamma}_t^{x,*} = \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \tag{1.38}$$

According to (??):

$$\hat{\gamma}_t^f = \widehat{mc}_t^x + \hat{\gamma}_t^{x,*} \tag{1.39}$$

1.5 Monetary policy

Monetary policy (??) is given by:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)[r_\pi \hat{\pi}_{t-1}^c + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1}] + r_\Delta^\pi \Delta\hat{\pi}_t^c + r_\Delta^y \Delta\hat{y}_t + \epsilon_t^R \tag{1.40}$$

The real exchange rate (??) can be expressed as:[9]

$$\hat{x}_t = -\widehat{mc}_t^x - \hat{\gamma}_t^{x,*} - \omega_c \left(\frac{\bar{\lambda}^{m,c}}{\bar{\gamma}^{c,d}}\right)^{1-\eta_c} \hat{\gamma}_t^{mc,d} \tag{1.41}$$

[8] Equation (??) implies $\hat{\gamma}_t^{mc,d} = \hat{P}_t^{m,c} - \hat{P}_t^d$; (??) implies $\hat{\gamma}_t^{mi,d} = \hat{P}_t^{m,i} - \hat{P}_t^d$; and (??) implies $\hat{\gamma}_t^{x,*} = \hat{P}_t^x - \hat{P}_t^*$.

[9] Equation (??) can be rewritten as:

$$\hat{x}_t = (\hat{S}_t + \hat{P}_t^* - \hat{P}_t^{m,c}) + (\hat{P}_t^{m,c} - \hat{P}_t^d) - (\hat{P}_t^c - \hat{P}_t^d)$$

Applying (1.12), (1.34) and the definition of the relative price between imported consumption goods and intermediate goods (??):

$$\hat{x}_t = -(\widehat{mc}_t^x + \hat{\gamma}_t^{x,*} + \hat{\gamma}_t^{mc,d}) + (\hat{\gamma}_t^{mc,d}) - \left(1 - \frac{(1-\omega_c)}{(\bar{\gamma}^{c,d})^{1-\eta_c}}\right)\hat{\gamma}_t^{mc,d}$$

Collecting common terms, then applying (??) gives (1.41).

1.6 Goods and financial market-clearing

1.6.1 Goods market-clearing

The asymmetric technology shock (**??**) is:

$$\hat{\tilde{z}}_t^* = \rho_{\tilde{z}^*}\hat{\tilde{z}}_{t-1}^* + \epsilon_t^{\tilde{z}^*} \tag{1.42}$$

Linearizing (**??**), assuming equality of the aggregate resource constraint, applying the steady state condition of aggregate hiring (**??**), (**??**), (**??**) and dividing both sides by the steady state value of output $\bar{y}$ gives:[10]

$$\begin{aligned}
&(1-\omega_c)\left[\bar{\gamma}^{c,d}\right]^{\eta_c}\frac{\bar{c}}{\bar{y}}\{\eta_c(\hat{\gamma}_t^{c,d})+\hat{c}_t\}+(1-\omega_i)\left[\bar{\gamma}^{i,d}\right]^{\eta_i}\frac{\bar{i}}{\bar{y}}\{\eta_i(\hat{\gamma}_t^{i,d})+\hat{i}_t\}+\frac{\bar{y}^*}{\bar{y}}\{\hat{y}_t^*-\eta_f(\hat{\gamma}_t^{x,*})+\hat{\tilde{z}}_t^*\}\\
&=\hat{y}_t-\alpha\{\hat{k}_t-\hat{\bar{k}}_t\}-\frac{B\chi\bar{N}\bar{\Omega}^{\vartheta}}{\bar{y}}\{\hat{\epsilon}_t+\vartheta\hat{\Omega}_t+\hat{\zeta}_t^{\Omega}+\hat{H}_t\}
\end{aligned} \tag{1.43}$$

1.6.2 Evolution of net foreign assets

Linearizing (**??**) gives[11]

$$\begin{aligned}
\hat{a}_t=&\left(\frac{\bar{y}^*\bar{\tilde{z}}^*}{\overline{mc}^x\left(\bar{\gamma}^{x,*}\right)^{\eta_f}}\right)\left(1+\hat{y}_t^*+\hat{\tilde{z}}_t^*-\widehat{mc}_t^x-\eta_f\hat{\gamma}_t^{x,*}\right)-\frac{\omega_c\bar{c}(\bar{\gamma}^{c,d})^{\eta_c}}{\bar{\gamma}^f\left(\bar{\gamma}^{mc,d}\right)^{\eta_c}}\left(1+\eta_c\hat{\gamma}_t^{c,d}+\hat{c}_t-\hat{\gamma}_t^f-\eta_c\hat{\gamma}_t^{mc,d}\right)\\
&-\frac{\omega_i\bar{i}(\bar{\gamma}^{i,d})^{\eta_i}}{\bar{\gamma}^f\left(\bar{\gamma}^{mi,d}\right)^{\eta_i}}\left(1+\eta_i\hat{\gamma}_t^{i,d}+\hat{i}_t-\hat{\gamma}_t^f-\eta_i\hat{\gamma}_t^{mi,d}\right)+\frac{\bar{R}^*\Phi(\bar{a},\bar{\tilde{\phi}})}{\bar{\pi}^d\bar{\mu}^z}\hat{a}_{t-1}
\end{aligned}$$

From the steady state property for imported consumption and investment, (**??**) and (**??**), we know that:

[10] Applying a Taylor expansion up to the first order to the stationary capital utilization cost, $\tilde{a}(u_t)\frac{\tilde{k}_t}{\mu_t^z}$, and ignoring the cross-product gives:

$$\tilde{a}(u_t)\frac{\tilde{k}_t}{\mu_t^z}\approx\tilde{a}(\bar{u})\frac{\bar{\tilde{k}}}{\bar{\mu}^z}+\tilde{a}'(\bar{u})\frac{\bar{\tilde{k}}}{\bar{\mu}^z\bar{\tilde{k}}}(k_t-\bar{k})+\left(-\tilde{a}'(\bar{u})\frac{\bar{k}}{\bar{\tilde{k}}^2}\frac{\bar{\tilde{k}}}{\bar{\mu}^z}+\frac{\tilde{a}(\bar{u})}{\bar{\mu}^z}\right)(\tilde{k}_t-\bar{\tilde{k}})$$

Recalling $\tilde{a}(\bar{u})=0$, $\tilde{a}'(\bar{u})=\bar{r}^k$ and at steady state $\bar{\tilde{k}}=\bar{k}$, the above equation can be simplified to:

$$\tilde{a}(u_t)\frac{\tilde{k}_t}{\mu_t^z}\approx 0+\bar{r}^k\frac{\bar{\tilde{k}}}{\bar{\mu}^z}\frac{(k_t-\bar{k})}{\bar{k}}+\left(-\bar{r}^k\frac{\bar{\tilde{k}}}{\bar{\mu}^z}+0\right)\frac{(\tilde{k}_t-\bar{\tilde{k}})}{\bar{\tilde{k}}}=\frac{\bar{r}^k\bar{\tilde{k}}}{\bar{\mu}^z}(\hat{k}_t-\hat{\tilde{k}}_t)$$

[11] Note $\bar{a}=0$ so that $\hat{a}_t\approx\bar{a}+1\times(a_t-\bar{a})=a_t$ and:

$$\begin{aligned}
&R_{t-1}^*\Phi(a_{t-1},\tilde{\phi}_{t-1})\left(\frac{a_{t-1}}{\pi_t^d\mu_t^z}\right)\left(\frac{S_t}{S_{t-1}}\right)\approx\frac{\bar{R}^*\bar{\Phi}\bar{a}\bar{S}}{\bar{\pi}^d\bar{\mu}^z\bar{S}}+\frac{\bar{\Phi}\bar{a}\bar{S}}{\bar{\pi}^d\bar{\mu}^z\bar{S}}(R_{t-1}^*-\bar{R}^*)+\frac{\bar{R}^*\bar{a}\bar{S}}{\bar{\pi}^d\bar{\mu}^z\bar{S}}(\Phi_{t-1}-\bar{\Phi})\\
&+\frac{\bar{R}^*\bar{\Phi}\bar{S}}{\bar{\pi}^d\bar{\mu}^z\bar{S}}(a_t-\bar{a})-\frac{\bar{R}^*\bar{\Phi}\bar{a}\bar{S}}{\left(\bar{\pi}^d\right)^2\bar{\mu}^z\bar{S}}(\pi_t^d-\bar{\pi}^d)-\frac{\bar{R}^*\bar{\Phi}\bar{a}\bar{S}}{\bar{\pi}^d\left(\bar{\mu}^z\right)^2\bar{S}}(\mu_t^z-\bar{\mu}^z)+\frac{\bar{R}^*\bar{\Phi}\bar{a}}{\bar{\pi}^d\bar{\mu}^z\bar{S}}(S_t-\bar{S})-\frac{\bar{R}^*\bar{\Phi}\bar{a}\bar{S}}{\bar{\pi}^d\bar{\mu}^z\left(\bar{S}\right)^2}(S_{t-1}-\bar{S})\\
&=\frac{\bar{R}^*\bar{\Phi}}{\bar{\pi}^d\bar{\mu}^z}\hat{a}_t
\end{aligned}$$

$$\frac{\omega_c \bar{c} (\bar{\gamma}^{c,d})^{\eta_c}}{\bar{\gamma}^f \left(\bar{\gamma}^{mc,d}\right)^{\eta_c}} = \frac{c^m}{\bar{\gamma}^f} \qquad \frac{\omega_i \bar{i} (\bar{\gamma}^{i,d})^{\eta_i}}{\bar{\gamma}^f \left(\bar{\gamma}^{mi,d}\right)^{\eta_i}} = \frac{i^m}{\bar{\gamma}^f}$$

Applying (1.34), (1.35), and the steady state properties: $\Phi(0,0) = 1$, $\bar{R}^* = \bar{R}$, (**??**), (**??**) and (**??**) give the evolution of net foreign assets:

$$\begin{aligned} \hat{a}_t =& \bar{y}^* \left(\hat{y}^*_t + \hat{\tilde{z}}^*_t - \widehat{mc}^x_t - \eta_f \hat{\gamma}^{x,*}_t \right) + (\bar{c}^m + \bar{i}^m) \hat{\gamma}^f_t - \bar{c}^m \left(-\eta_c (1-\omega_c) \left(\bar{\gamma}^{c,d}\right)^{-(1-\eta_c)} \hat{\gamma}^{mc,d}_t + \hat{c}_t \right) \\ &+ \bar{i}^m \left(-\eta_i (1-\omega_i) \left(\bar{\gamma}^{i,d}\right)^{-(1-\eta_i)} \hat{\gamma}^{mi,d}_t + \hat{i}_t \right) + \frac{\bar{R}}{\bar{\pi}^d \bar{\mu}^z} \hat{a}_{t-1} \end{aligned} \tag{1.44}$$

1.6.3 Loan market-clearing

Linearizing (**??**), applying the steady state condition (**??**) gives the loan market clearing condition:

$$\bar{w}\bar{N}(\hat{w}_t + \hat{N}_t) = \bar{m}\left(\hat{\mu}^m_t + \hat{m}_t - \hat{\pi}^d_t - \hat{\mu}^z_t\right) - \bar{q}\hat{q}_t \tag{1.45}$$

From the definition of money growth (**??**)

$$\hat{\mu}^m_{t-1} = \hat{m}_t + \hat{\pi}^d_{t-1} + \hat{\mu}^z_{t-1} - \hat{m}_{t-1} \tag{1.46}$$

1.6.4 Foreign economy

The foreign economy is modeled as:

$$\hat{y}^*_t = y^*_1 \hat{y}^*_{t-1} + y^*_2 \hat{\pi}^*_{t-1} + y^*_3 \hat{R}^*_{t-1} + \epsilon^{y^*}_t \tag{1.47}$$

$$\hat{\pi}^*_t = \pi^*_1 \hat{y}^*_{t-1} + \pi^*_2 \hat{\pi}^*_{t-1} + \pi^*_3 \hat{R}^*_{t-1} + \epsilon^{\pi^*}_t \tag{1.48}$$

$$\hat{R}^*_t + R^*_4 \hat{y}^*_t + R^*_5 \hat{\pi}^*_t = R^*_1 \hat{y}^*_{t-1} + R^*_2 \hat{\pi}^*_{t-1} + R^*_3 \hat{R}^*_{t-1} + \epsilon^{R^*}_t \tag{1.49}$$

1.6.5 Markups

The markups follow the following dynamics:

$$\hat{\lambda}^d_t = \frac{1}{\bar{\lambda}^d} \epsilon^{\lambda^d}_t \tag{1.50}$$

$$\hat{\lambda}^{mc}_t = \frac{1}{\bar{\lambda}^{mc}} \epsilon^{\lambda^{mc}}_t \tag{1.51}$$

$$\hat{\lambda}^{mi}_t = \frac{1}{\bar{\lambda}^{mi}} \epsilon^{\lambda^{mi}}_t \tag{1.52}$$

$$\hat{\lambda}^x_t = \frac{1}{\bar{\lambda}^x} \epsilon^{\lambda^x}_t \tag{1.53}$$

$$\hat{\lambda}^w_t = \frac{1}{\bar{\lambda}^w} \epsilon^{\lambda^w}_t \tag{1.54}$$