1 The non-linear equilibrium system

$$e_t\left[\frac{C_t^w}{\left(C_{t-1}^w\right)^{\varphi}}\right]^{1-\gamma_w}-\beta_w\varphi E_t\left\{e_{t+1}\left[\frac{C_{t+1}^w}{\left(C_t^w\right)^{\varphi}}\right]^{1-\gamma_w}\right\}=C_t^w\lambda_t^w \tag{1.1}$$

$$\frac{\eta}{\left(1-H_t\right)^{\varsigma}}=\lambda_t^w W_t \tag{1.2}$$

$$\frac{\lambda_t^w}{R_t^D}=\beta_w E_t\left(\frac{\lambda_{t+1}^w}{\pi_{t+1}}\right) \tag{1.3}$$

$$e_t\left[\frac{C_t^b}{\left(C_{t-1}^b\right)^{\varphi}}\right]^{1-\gamma_b}-\beta_b\varphi E_t\left\{e_{t+1}\left[\frac{C_{t+1}^b}{\left(C_t^b\right)^{\varphi}}\right]^{1-\gamma_b}\right\}=C_t^b\lambda_t^b \tag{1.4}$$

$$\frac{\lambda_t^b}{R_t}=\beta_b E_t\left[\frac{\lambda_{t+1}^b}{\pi_{t+1}}\right] \tag{1.5}$$

$$E_t R_{t+1}^Z = E_t\left\{\frac{1}{1-\delta_t^Z}\left[\frac{R_t Q_t^Z}{Q_{t+1}^Z}\left(1+\chi z\left(\frac{Z_t}{Z_{t-1}}-1\right)\frac{Z_t}{Z_{t-1}}\right)-\chi z\left(\frac{Z_{t+1}}{Z_t}-1\right)\left(\frac{Z_{t+1}}{Z_t}\right)^2\pi_{t+1}\right]\right\} \tag{1.6}$$

$$\frac{1+\vartheta_D}{\vartheta_D}\left(R_t^D-1\right)=R_t-1-\frac{\phi_R}{\vartheta_D}\left(\frac{R_t^D}{R_{t-1}^D}-1\right)\frac{R_t^D}{R_{t-1}^D}+\frac{\beta_b\phi_R}{\vartheta_D}\left(\frac{R_{t+1}^D}{R_t^D}-1\right)\frac{R_{t+1}^D}{R_t^D} \tag{1.7}$$

$$B_t^L=(1-s_t)D_t+Q_t^Z Z_t \tag{1.8}$$

$$B_t^L+L_t=Q_t^Z Z_t+D_t+m_t+\left(\Gamma_t-1\right)\left(s_t D_t+m_t\right) \tag{1.9}$$

$$rp_t^B=\left(\frac{D_t}{(1-\mu_t)L_t+B_t^L}\right)^{\upsilon} \tag{1.10}$$

$$rp_t^{\kappa}=\left(\frac{\bar{\kappa}-\kappa_t}{\bar{\kappa}}Q_t^Z Z_t\right)^{-\xi} \tag{1.11}$$

$$R_t^L=R_t+R_t\nu\left(\frac{D_t}{(1-\mu_t)L_t+B_t^L}\right)^{1+\upsilon} \tag{1.12}$$

$$R_t^L-R_t rp_t^B=\xi\left(\frac{\bar{\kappa}-\kappa_t}{\bar{\kappa}}Q_t^Z Z_t\right)^{-\xi}\frac{E_t R_{t+1}^Z}{\bar{\kappa}-\kappa_t} \tag{1.13}$$

$$\delta_t^Z=E_t\left[\frac{\pi_{t+1}}{\chi_{\delta^Z}Q_t^Z Z_t}\right] \tag{1.14}$$

$$L_t=(s_t D_t+m_t)\Gamma_t \tag{1.15}$$

$$L_t=\kappa_t Q_t^Z Z_t\Gamma_t \tag{1.16}$$

$$R_t^L=\Gamma_t^{-1}\left[R_t rp_t^B+\kappa_t^{-1}(R_{t+1}^Z rp_t^{\kappa}-R_t)Q_t^Z\right] \tag{1.17}$$

$$E_t F_{t+1}=E_t\left[\frac{r_{t+1}^K+(1-\delta)Q_{t+1}^K}{Q_t^K}\right] \tag{1.18}$$

$$E_t F_{t+1} = E_t \left[\frac{R_t^L}{\pi_{t+1}} rp_t^E \right] \tag{1.19}$$

$$L_t = Q_t^K K_{t+1} - N_t \tag{1.20}$$

$$rp_t^E = \left(\frac{Q_t^K K_{t+1}}{N_t} \right)^{\psi_t} \tag{1.21}$$

$$N_t = \nu \left[F_t Q_{t-1}^K K_t - E_{t-1} F_t (Q_{t-1}^K K_t - N_{t-1} \right] + (1-\nu) g_t \tag{1.22}$$

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \tag{1.23}$$

$$\frac{1}{Q_t^K} = x_t - \chi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta_w \chi_I E_t \left[\left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \frac{Q_{t+1}^K}{Q_t^K} \frac{\lambda_{t+1}^w}{\lambda_t^w} \right] \tag{1.24}$$

$$K_{t+1} = (1-\delta) K_t + x_t I_t - \frac{\chi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \tag{1.25}$$

$$\log \left({}^{R_t}/_{R} \right) = \varrho_\pi \log \left({}^{\pi_t}/_{\pi} \right) + \varrho_Y \log \left({}^{Y_t}/_{Y} \right) + \varepsilon_{R_t} \tag{1.26}$$

$$Y_t = C_t^w + C_t^b + I_t + G_t \tag{1.27}$$

$$C_t = C_t^w + C_t^b \tag{1.28}$$

$$\ln (A_t) = (1-\rho_A) \ln (A) + \rho_A \ln (A_{t-1}) + \varepsilon_{A_t} \tag{1.29}$$

$$\ln (x_t) = (1-\rho_x) \ln (x) + \rho_x \ln (x_{t-1}) + \varepsilon_{x_t} \tag{1.30}$$

$$\ln (e_t) = (1-\rho_e) \ln (e) + \rho_e \ln (e_{t-1}) + \varepsilon_{e_t} \tag{1.31}$$

$$\ln (G_t) = (1-\rho_G) \ln (G) + \rho_G \ln (G_{t-1}) + \varepsilon_{G_t} \tag{1.32}$$

$$\ln (\psi_t) = (1-\rho_\psi) \ln (\psi) + \rho_\psi \ln (\psi_{t-1}) + \varepsilon_{\psi_t} \tag{1.33}$$

$$\ln (\Gamma_t) = (1-\rho_\Gamma) \ln (\Gamma) + \rho_\Gamma \ln (\Gamma_{t-1}) + \varepsilon_{\Gamma_t} \tag{1.34}$$

$$\ln (\mu_t) = (1-\rho_\mu) \ln (\mu) + \rho_\mu \ln (\mu_{t-1}) + \varepsilon_{\mu_t} \tag{1.35}$$

$$\ln (m_t) = (1-\rho_m) \ln (m) + \rho_m \ln (m_{t-1}) + \varepsilon_{m_t} \tag{1.36}$$

$$r_t^K = \alpha m c_t \frac{Y_t}{K_t} \tag{1.37}$$

$$W_t = (1-\alpha) m c_t \frac{Y_t}{H_t} \tag{1.38}$$

$$\widetilde{P}_t (j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta_w \phi_p)^l \lambda_{t+l}^w Y_{t+l} (j) \, mc_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta_w \phi_p)^l \lambda_{t+l}^w Y_{t+l} (j) \, {}^{\pi^l}/_{P_{t+l}}} \tag{1.39}$$

2 Steady-States

Letters without subscript represent the steady-state of the variable. So, for instance, Y is the steady-state of output.

$$\frac{1}{Q^K} = x = 1 \tag{2.1}$$

$$mc = \frac{\theta - 1}{\theta} \tag{2.2}$$

$$R^Z = \frac{R}{(1-\delta^Z)} \tag{2.3}$$

$$r^K = \alpha mc \frac{1}{\frac{K}{Y}} \tag{2.4}$$

$$F = r^K + 1 - \delta \tag{2.5}$$

$$R^L = \left(\frac{F\pi}{rp^E}\right) \tag{2.6}$$

$$Q^Z Z = \left[\frac{\pi}{\chi_{\delta^Z}\delta^Z}\right] \tag{2.7}$$

$$L = \kappa Q^Z Z \Gamma \tag{2.8}$$

$$D = \frac{L}{s\Gamma}\ since\ m = 0 \tag{2.9}$$

$$B^L = (1-s)D + Q^Z Z \tag{2.10}$$

$$N = L \tag{2.11}$$

$$K = 2N \tag{2.12}$$

$$I = \delta K \tag{2.13}$$

$$Y = K * 6.753 \tag{2.14}$$

$$G = Y * 0.17 \tag{2.15}$$

$$H = \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{K}\right)^{\frac{\alpha}{1-\alpha}} \tag{2.16}$$

$$W = (1-\alpha) mc \frac{Y}{H} \tag{2.17}$$

$$\lambda = \frac{\eta}{(1-H)^{\varsigma} W} \tag{2.18}$$

$$Z = Y * 0.294 \tag{2.19}$$

$$Q^Z = \frac{Q^Z Z}{Z} \tag{2.20}$$

$$C^w = Y * 0.624 \tag{2.21}$$

$$C^b = Y * 0.037 \tag{2.22}$$

3 The log-linearized equilibrium system

$\widetilde{x}_t$ is the log-deviation of the variable x_t from its steady state $x.\widetilde{x}_t = \log\left(\frac{x_t}{x}\right)$ so $\widetilde{x}_t \approx \frac{x_t - x}{x}$.

$$\begin{aligned} & e\left[\frac{C^w}{(C^w)^\varphi}\right]^{(1-\gamma_w)}\left\{\widetilde{e}_t + (1-\gamma_w)\widetilde{C}_t^w - \varphi(1-\gamma_w)\widetilde{C}_{t-1}^w\right\} \\ -\beta_w\varphi e\left[\frac{C^w}{(C^w)^\varphi}\right]^{(1-\gamma_w)} & E_t\left\{\widetilde{e}_{t+1} + (1-\gamma_w)\widetilde{C}_{t+1}^w - \varphi(1-\gamma_w)\widetilde{C}_t^w\right\} = C^w\lambda^w(\widetilde{C}_t^w + \widetilde{\lambda}_t^w) \end{aligned} \tag{3.1}$$

$$\left[-\varsigma\frac{H}{(1-H)}\right]\widetilde{H}_t = \widetilde{\lambda}_t^w + \widetilde{W}_t \tag{3.2}$$

$$\widetilde{\lambda}_t^w - \widetilde{R}_t^D = E_t\left[\widetilde{\lambda}_{t+1}^w - \widetilde{\pi}_{t+1}\right] \tag{3.3}$$

$$\begin{aligned} & e\left[\frac{C^b}{(C^b)^\varphi}\right]^{(1-\gamma_b)}\left\{\widetilde{e}_t + (1-\gamma_b)\widetilde{C}_t^b - \varphi(1-\gamma_b)\widetilde{C}_{t-1}^b\right\} \\ -\beta_b\varphi e\left[\frac{C^b}{(C^b)^\varphi}\right]^{(1-\gamma_b)} & \left\{\widetilde{e}_{t+1} + (1-\gamma_b)\widetilde{C}_{t+1}^b - \varphi(1-\gamma_b)\widetilde{C}_t^b\right\} = C^b\lambda^b(\widetilde{C}_t^b + \widetilde{\lambda}_t^b) \end{aligned} \tag{3.4}$$

$$\widetilde{\lambda}_t^b - \widetilde{R}_t = E_t\left[\widetilde{\lambda}_{t+1}^b - \widetilde{\pi}_{t+1}\right] \tag{3.5}$$

$$R^Z\widetilde{R}_t^Z = -R^Z + \left[\frac{R}{(1-\delta^Z)\left(1 - \frac{\delta^Z}{1-\delta^Z}\widetilde{\delta}_{t-1}^Z\right)}\right]\left(1 + \widetilde{Q}_{t-1}^Z - \widetilde{Q}_t^Z + \widetilde{R}_{t-1} + \chi z\left(\widetilde{Z}_{t-1} - \widetilde{Z}_{t-2}\right) + \chi z\,(1-\pi)\left(\widetilde{Z}_t - \widetilde{Z}_{t-1}\right)\right) \tag{3.6}$$

$$\left[\left(\frac{1+\vartheta_D}{\vartheta_D}\right)R^D\right]\widetilde{R}_t^D = R\widetilde{R}_t + \left[(\beta_b - 1)\frac{\phi_R}{\vartheta_D}\right](\widetilde{R}_t^D - \widetilde{R}_{t-1}^D) \tag{3.7}$$

$$\widetilde{B}_t^L = \left[\frac{(1-s)D}{B^L}\right]\widetilde{D}_t - \left(\frac{Ds}{B^L}\right)\widetilde{s}_t + \left(\frac{Q^Z Z}{B^L}\right)\left(\widetilde{Z}_t + \widetilde{Q}_t^Z\right) \tag{3.8}$$

$$B^L\widetilde{B}_t^L + L\widetilde{L}_t = Q^Z Z\left(\widetilde{Q}_t^Z + \widetilde{Z}_t\right) + D\widetilde{D}_t$$
$$+\Gamma sD\left(\widetilde{\Gamma}_t + \widetilde{s}_t + \widetilde{D}_t\right) + \Gamma m\left(\widetilde{\Gamma}_t + \widetilde{m}_t\right) - sD\left(\widetilde{s}_t + \widetilde{D}_t\right) \tag{3.9}$$

$$\widetilde{rp}_t^B = \upsilon\left[\widetilde{D}_t + \frac{L\mu}{(1-\mu)L + B^L}\widetilde{\mu}_t - \frac{(1-\mu)L}{(1-\mu)L+B^L}\widetilde{L}_t - \frac{B^L}{(1-\mu)L+B^L}\widetilde{B}_t^L\right] \tag{3.10}$$

$$\widetilde{rp}_t^\kappa = -\xi\left[\widetilde{Q}_t^Z + \widetilde{Z}_t - \frac{\kappa}{\bar{\kappa}-\kappa}\widetilde{\kappa}_t\right] \tag{3.11}$$

$$\left(\frac{R^L}{R^L - R}\right)\widetilde{R}_t^L = \left(\frac{R^L}{R^L - R}\right)\widetilde{R}_t$$
$$+(1+\upsilon)\left[\widetilde{D}_t + \frac{L\mu}{(1-\mu)L + B^L}\widetilde{\mu}_t - \frac{(1-\mu)L}{(1-\mu)L+B^L}\widetilde{L}_t - \frac{B^L}{(1-\mu)L+B^L}\widetilde{B}_t^L\right] \tag{3.12}$$

$$\left(\frac{R^L}{R^L - Rrp^B}\right)\widetilde{R}_t^L = \left(\frac{Rrp^B}{R^L - Rrp^B}\right)\left(\widetilde{R}_t + \widetilde{rp}_t^B\right)$$
$$+\xi\left[\left(\frac{\kappa}{\bar{\kappa}-\kappa}\right)\widetilde{\kappa}_t - \widetilde{Q}_t^Z - \widetilde{Z}_t\right] + \widetilde{R}_{t+1}^Z + \left(\frac{\kappa}{\bar{\kappa}-\kappa}\right)\widetilde{\kappa}_t \tag{3.13}$$

$$\widetilde{\delta}_t^Z = E_t\left[\widetilde{\pi}_{t+1}\right] - \widetilde{Q}_t^Z - \widetilde{Z}_t \tag{3.14}$$

$$\widetilde{L}_t = \widetilde{\Gamma}_t + \left(\frac{sD\Gamma}{sD\Gamma + m\Gamma}\right)\left(\widetilde{s}_t + \widetilde{D}_t\right) + \left(\frac{m\Gamma}{sD\Gamma + m\Gamma}\right)\widetilde{m}_t \tag{3.15}$$

$$\widetilde{L}_t = \widetilde{\kappa}_t + \widetilde{Q}_t^Z + \widetilde{Z}_t + \widetilde{\Gamma}_t \tag{3.16}$$

$$R^L\widetilde{R}_t^L = \left(\frac{Rrp^B}{\Gamma}\right)\left(\widetilde{R}_t + \widetilde{rp}_t^B - \widetilde{\Gamma}_t\right)$$
$$+\left(\frac{R^Z rp^\kappa Q^Z}{\Gamma\kappa}\right)\left(\widetilde{R}_{t+1}^Z + \widetilde{rp}_t^\kappa + \widetilde{Q}_t^Z - \widetilde{\Gamma}_t - \widetilde{\kappa}_t\right) - \left(\frac{RQ^Z}{\Gamma\kappa}\right)\left(\widetilde{R}_t + \widetilde{Q}_t^Z - \widetilde{\Gamma}_t - \widetilde{\kappa}_t\right) \tag{3.17}$$

$$\widetilde{F}_t = -\widetilde{Q}_{t-1}^K + \left(\frac{r^K}{FQ^K}\right)\widetilde{r}_t^K + \left(\frac{1-\delta}{F}\right)\widetilde{Q}_t^K \tag{3.18}$$

$$\widetilde{F}_t = \widetilde{R}_{t-1}^L - \widetilde{\pi}_t + \widetilde{rp}_{t-1}^E \tag{3.19}$$

$$L\widetilde{L}_t = \left(Q^K K\right)\left(\widetilde{Q}_t^K + \widetilde{K}_t\right) - N\widetilde{N}_t \tag{3.20}$$

$$\widetilde{rp}_t^E = (\psi)\left(\widetilde{Q}_t^K + \widetilde{K}_t - \widetilde{N}_t\right) \tag{3.21}$$

$$\frac{\widetilde{N}_t}{\nu F} = \left(\frac{Q^K K}{N}\right)\widetilde{F} - \left(\frac{Q^K K}{N} - 1\right)\left(\widetilde{R}^L_{t-1} - \pi_t\right)$$

$$-\left(\psi\right)\left(\frac{Q^K K}{N} - 1\right)\left(\widetilde{Q}^K_{t-1} + \widetilde{K}_{t-1}\right) + \left(\psi\right)\left(\left(\frac{Q^K K}{N} - 1\right) + 1\right)\widetilde{N}_{t-1} \tag{3.22}$$

$$\widetilde{Y}_t = \widetilde{A}_t + \alpha\widetilde{K}_{t-1} + \left(1-\alpha\right)\widetilde{H}_t \tag{3.23}$$

$$-\left(\frac{1}{Q^K}\right)\widetilde{Q}^K_t = x\widetilde{x}_t - \chi_I\left(\widetilde{I}_t - \widetilde{I}_{t-1}\right) + \beta_w\chi_I\left(\widetilde{I}_{t+1} - \widetilde{I}_t\right) \tag{3.24}$$

$$\widetilde{K}_t = \left(1-\delta\right)\widetilde{K}_{t-1} + \delta\left(\widetilde{x}_t + \widetilde{I}_t\right) \tag{3.25}$$

$$\widetilde{R}_t = \varrho_\pi\widetilde{\pi}_t + \varrho_Y\widetilde{Y}_t + \varepsilon_{R_t} \tag{3.26}$$

$$\widetilde{Y}_t = \frac{C^w}{Y}\widetilde{C}^w_t + \frac{C^b}{Y}\widetilde{C}^b_t + \frac{I}{Y}\widetilde{I}_t + \frac{G}{Y}\widetilde{G}_t \tag{3.27}$$

$$\widetilde{C}_t = \frac{C^w}{C}\widetilde{C}^w_t + \frac{C^b}{C}\widetilde{C}^b_t \tag{3.28}$$

$$\widetilde{A}_t = \rho_A\widetilde{A}_{t-1} + \varepsilon_{A_t} \tag{3.29}$$

$$\widetilde{x}_t = \rho_x\widetilde{x}_{t-1} + \varepsilon_{x_t} \tag{3.30}$$

$$\widetilde{e}_t = \rho_e\widetilde{e}_{t-1} + \varepsilon_{e_t} \tag{3.31}$$

$$\widetilde{G}_t = \rho_G\widetilde{G}_{t-1} + \varepsilon_{G_t} \tag{3.32}$$

$$\widetilde{\psi}_t = \rho_\psi\widetilde{\psi}_{t-1} + \epsilon_{\psi_t} \tag{3.33}$$

$$\widetilde{\Gamma}_t = \rho_\Gamma\widetilde{\Gamma}_{t-1} + \epsilon_{\Gamma_t} \tag{3.34}$$

$$\widetilde{\mu}_t = \rho_\mu\widetilde{\mu}_{t-1} + \varepsilon_{\mu_t} \tag{3.35}$$

$$\widetilde{m}_t = \rho_m \widetilde{m}_{t-1} + \varepsilon_{m_t} \tag{3.36}$$

$$\widetilde{r}_t^K = \widetilde{mc}_t + \widetilde{Y}_t - \widetilde{K}_{t-1} \tag{3.37}$$

$$\widetilde{W}_t = \widetilde{mc}_t + \widetilde{Y}_t - \widetilde{H}_t \tag{3.38}$$

$$\widetilde{\pi}_t = \beta_w \widetilde{\pi}_{t+1} + \frac{(1 - \beta_w \phi_p)(1 - \phi_p)}{\phi_p} \widetilde{mc}_t \tag{3.39}$$