Taking a Mickey Mouse Model to the Data

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1 Introduction

Here I present a simple model that can potentially be taken to the data in a likelihood-based estimation exercise. Even though the model has effectively only three structural equations, it includes some ingredients that one typically encounters in empirical DSGE models. This includes a stochastic growth process and backward-looking features such as habit-formation and price-indexation that help model in capturing the persistence in the data. The model and the accompanying DYNARE code can easily be complicated to accomodate more bells and whistles. However, the same principles will apply in the transformation of the theoretical model to bring to the data using DYNARE.

I first set up the optimization problems and get the primitive first order conditions. I will then derive the stationary, log-linearized version of the model which is taken to the data. Finally, I will establish the link of the model variables to their analogs in the data. Production of output (Y) is linear in hours. As in Justiniano, Primiceri and Tambalotti (2011), the production function is stimulated by a non-stationary AR(1) technology shock ε^{GROW}. Demand is determined only by consumption (C). The utility function is logarithmic in consumption so that a balanced growth path is feasible, without resorting to complications such as non-separability with leisure. A habit term determines the effect of lagged average consumption on current utility. Utility depends linearly in hours worked (H). Finally, a shock to the discount factor, ε^{DEM} captures random deviations in the consumer's patience to consume. P is the price of the consumption good and W is the real wage that accrues to labor. The consumer can smooth consumption by buying one period private nominal bonds (B) which are in zero net-supply. The return on the bond (R) is set by the central bank according to an empirical rule which is affected by an AR(1) shock ε^{MON}. The production side is encapsulated by the Rotemberg-style Phillips curve augmented with price indexation, which determines the behavior of price inflation (π). Steady-state variables are denoted without time-subscripts. Variables that have been stationarized are denoted by letters in smaller case and the stationary variables that have been log-linearized are indicated by a superscript ˆ. The model will be estimated using three quarterly time series: GDP growth, GDP deflator inflation and the Federal Funds rate.

2 Model Derivation

The consumer faces the following problem in every period. The (relevant pieces of the) Lagrangrean is given by

$$\begin{aligned}&\max_{C_t,B_t,\,H_t}\ \beta^t\varepsilon_t^{DEM}U(C_t,H_t)+\beta^{t+1}E_t\varepsilon_{t+1}^{DEM}U(C_{t+1},H_{t+1})\\&+\beta^t\lambda_t\left(W_tH_t+\frac{R_{t-1}B_{t-1}}{P_t}-C_t-\frac{B_t}{P_t}\right)\\&+\beta^{t+1}E_t\lambda_{t+1}\left(W_{t+1}H_{t+1}+\frac{R_tB_t}{P_{t+1}}-C_{t+1}-\frac{B_{t+1}}{P_{t+1}}\right)\end{aligned}$$

FOC for Consumption:

$$\varepsilon_t^{DEM}U_C(C_t,H_t)=\lambda_t \tag{1}$$

FOC for Hours:

$$U_H(C_t,H_t)=-U_C(C_t,H_t)W_t \tag{2}$$

FOC for Bonds

$$\lambda_t=\beta E_t\lambda_{t+1}\frac{R_t}{\pi_{t+1}} \tag{3}$$

We get the consumption euler equation by plugging in $\lambda_t=\varepsilon_t^{DEM}U_C(C_t,H_t)$

$$\varepsilon_t^{DEM}U_C(C_t,H_t)=\beta\mathbf{E}_t\varepsilon_{t+1}^{DEM}U_C(C_{t+1},H_{t+1})\frac{R_t}{\pi_{t+1}} \tag{4}$$

2.1 Utility Function

Utility depends on lagged average consumption $\tilde{C}$

$$U(C_t,H_t)=\log\left(C_t-h\tilde{C}_{t-1}\right)-H_t,h\in[0,1)$$

In a symmetric equilibrium, the marginal utility of consumption is given by

$$U_{Ct}=\frac{1}{C_t-hC_{t-1}} \tag{5}$$

Marginal utility of Labor:

$$U_{Ht} = -1 \tag{6}$$

Intra-temporal condition:

$$C_t - hC_{t-1} = W_t \tag{7}$$

The consumption euler equation is given by

$$\frac{\varepsilon_t^{DEM}}{C_t - hC_{t-1}} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{C_{t+1} - hC_t} \frac{R_t}{\pi_{t+1}} \tag{8}$$

2.2 Production Function and Cost Minimization

The production function is linear in labor.

$$Y_t = \varepsilon_t^{GROW} H_t \tag{9}$$

The producer faces the problem

$$\min_{H_t} W_t H_t + MC_t(Y_t - \varepsilon_t^{GROW} H_t)$$

$$W_t + MC_t(-\varepsilon_t^{GROW}) = 0$$

$$\frac{W_t}{\varepsilon_t^{GROW}} = MC_t \tag{10}$$

2.3 Goods Market Clearing

What is produced is consumed.

$$Y_t = C_t \tag{11}$$

3 Stationarizing and Log-Linearizing Model Equations

3.1 The Non-stationary Technology Shock

In the model, growth is determined by a technology shock ε^{GROW} which follows the process

$$\log \varepsilon_t^{GROW} - \log \varepsilon_{t-1}^{GROW} = (1 - \rho_{\Delta GROW}) \log \gamma + \rho_{\Delta GROW} \left(\log \varepsilon_{t-1}^{GROW} - \log \varepsilon_{t-2}^{GROW} \right) + \eta_t^{\Delta GROW} \tag{12}$$

such that γ is the gross steady-state growth rate. The growth rate can vary over time so that

$$\frac{\varepsilon_t^{GROW}}{\varepsilon_{t-1}^{GROW}} = \gamma_t$$

We can rewrite Equation 12 as

$$\log \frac{\varepsilon_t^{GROW}}{\varepsilon_{t-1}^{GROW}} = (1-\rho_{\Delta GROW}) \log \gamma + \rho_{\Delta GROW} \log \frac{\varepsilon_{t-1}^{GROW}}{\varepsilon_{t-2}^{GROW}} + \eta_t^{\Delta GROW}$$

$$\rightarrow \log \gamma_t = (1-\rho_{\Delta GROW}) \log \gamma + \rho_{\Delta GROW} \log \gamma_{t-1} + \eta_t^{\Delta GROW}$$

Define

$$\log \gamma_t - \log \gamma = \hat{\gamma}_t$$

$$\hat{\gamma}_t + \log \gamma = (1-\rho_{\Delta GROW}) \log \gamma + \rho_{\Delta GROW} \left(\hat{\gamma}_{t-1} + \log \gamma\right) + \eta_t^{\Delta GROW}$$

Simplifying, we get

$$\hat{\gamma}_t = \rho_{\Delta GROW} \hat{\gamma}_{t-1} + \eta_t^{\Delta GROW}$$

The reasons why we need the non-stationary technology process in the model are bifold. One, we do not have to pre-filter or detrend the data we use in the estimation because many model variables will inherit the trend of the technology shock and can be easily linked to the unfiltered data analogs. Secondly, the stochastic component in the growth process ensures that the growth rate can potentially vary over time and the average spread in the time-varying growth rates is captured by the standard deviation of the shock.

3.2 Production Function and Marginal Cost

Express the variables in terms of their stationary analogs (in small case). For example, $Y_t = \varepsilon_t^{GROW} y_t$. Note that hours are always stationary and hence do not have be detrended. Then take the total derivative evaluated at the steady-state to linearize.

$$y_t = H_t$$

$$\rightarrow \hat{y}_t = \hat{H}_t \tag{13}$$

The real marginal cost is given by

$$\frac{w_t \varepsilon_t^{GROW}}{\varepsilon_t^{GROW}} = \frac{MC_t}{\varepsilon_t^{GROW}}$$

$$\rightarrow w_t = mc_t$$

$$\rightarrow \hat{w}_t = \widehat{mc}_t$$

3.3 Goods Market Clearing

$$y_t = c_t$$

$$\rightarrow \hat{y}_t = \hat{c}_t \tag{14}$$

3.4 Consumption Euler

Begin with

$$\frac{\varepsilon_t^{DEM}}{C_t - hC_{t-1}} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{C_{t+1} - hC_t} \frac{R_t}{\pi_{t+1}}$$

Express in terms of stationary variables. For example, $C_t = c_t \varepsilon_t^{GROW}$

$$\frac{\varepsilon_t^{DEM}}{c_t \varepsilon_t^{GROW} - hc_{t-1}\varepsilon_{t-1}^{GROW}} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{c_{t+1}\varepsilon_{t+1}^{GROW} - hc_t \varepsilon_t^{GROW}} \frac{R_t}{\pi_{t+1}}$$

Simplify

$$\frac{\varepsilon_t^{DEM}}{\varepsilon_t^{GROW}\left(c_t - hc_{t-1}\frac{\varepsilon_{t-1}^{GROW}}{\varepsilon_t^{GROW}}\right)} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{\varepsilon_t^{GROW}\left(c_{t+1}\frac{\varepsilon_{t+1}^{GROW}}{\varepsilon_t^{GROW}} - hc_t\right)} \frac{R_t}{\pi_{t+1}}$$

Cancel alike terms and define the stochastic gross growth term $\frac{\varepsilon_t^{GROW}}{\varepsilon_{t-1}^{GROW}} = \gamma_t$ to express

$$\frac{\varepsilon_t^{DEM}}{c_t - h\frac{c_{t-1}}{\gamma_t}} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{c_{t+1}\gamma_{t+1} - hc_t} \frac{R_t}{\pi_{t+1}}$$

Since the log-linearization of this equation is a bit more complex than that of the others, I relegate it to the Appendix. At the end of the algebraic manipulations, we get

$$\hat{c}_t = \frac{\gamma}{\gamma+h} E_t\left(\hat{c}_{t+1} + \hat{\gamma}_{t+1}\right) + \frac{h}{\gamma+h}\left(\hat{c}_{t-1} - \hat{\gamma}_t\right) - \frac{\gamma-h}{\gamma+h}\left(\hat{R}_t - E_t\hat{\pi}_{t+1} + \hat{\varepsilon}_{t+1}^{DEM} - \hat{\varepsilon}_t^{DEM}\right)$$

Using the fact that $\hat{y}_t = \hat{c}_t$

$$\hat{y}_t = \frac{\gamma}{\gamma+h} E_t\left(\hat{y}_{t+1} + \hat{\gamma}_{t+1}\right) + \frac{h}{\gamma+h}\left(\hat{y}_{t-1} - \hat{\gamma}_t\right) - \frac{\gamma-h}{\gamma+h}\left(\hat{R}_t - E_t\hat{\pi}_{t+1} + \hat{\varepsilon}_{t+1}^{DEM} - \hat{\varepsilon}_t^{DEM}\right)$$

I will now create an 'auxilliary' shock which is a linear transformation of the demand shock

$$\widehat{\hat{\varepsilon}}_t^{DEM} = \frac{\gamma-h}{\gamma+h}\left(\hat{\varepsilon}_t^{DEM} - E_t\hat{\varepsilon}_{t+1}^{DEM}\right)$$

The rescaling of the shock ensures that it has a unit coefficient when it is linked to the data series during the estimation. This makes it much easier to set a prior for the shock variance and has been shown to improve convergence properties of the estimation. This strategy is commonly used in empirical DSGE studies, for example Smets and Wouters (2007).

$$\hat{y}_t = \frac{\gamma}{\gamma+h} E_t\left(\hat{y}_{t+1} + \hat{\gamma}_{t+1}\right) + \frac{h}{\gamma+h}\left(\hat{y}_{t-1} - \hat{\gamma}_t\right) - \frac{\gamma-h}{\gamma+h}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \widehat{\hat{\varepsilon}}_t^{DEM}$$

Observe that by setting $\hat{\gamma}_t = \hat{\gamma}_{t+1} = 0$, we are in the deterministic growth scenario of Smets and Wouters (2007) or Jacob and Peersman (2008).

3.5 Intratemporal Condition

$$C_t - hC_{t-1} = W_t$$

Stationarizing

$$c_t - h\frac{c_{t-1}}{\gamma_t} = w_t$$

The log-linearized version is given by

$$\frac{\gamma}{\gamma - h}\hat{c}_t - \frac{h}{\gamma - h}\left(\hat{c}_{t-1} - \hat{\gamma}_t\right) = \hat{w}_t$$

Using the fact that $\hat{y}_t = \hat{c}_t$

$$\frac{\gamma}{\gamma - h}\hat{y}_t - \frac{h}{\gamma - h}\left(\hat{y}_{t-1} - \hat{\gamma}_t\right) = \hat{w}_t \tag{15}$$

3.6 Phillips Curve

I will not derive the Phillips Curve here. It is a Rotemberg-type cost based Phillips curve augmented with price indexation. The adjustment cost parameter $\phi > 0$, the price elasticity of demand $\eta > 1$ and the degree of price indexation $\iota \in [0, 1]$.

$$\hat{\pi}_t = \frac{\iota}{1 + \beta\iota}\hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\iota}E_t\hat{\pi}_{t+1} + \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\widehat{mc}_t$$

Plug in $\widehat{mc}_t = \hat{w}_t = \frac{\gamma}{\gamma - h}\hat{y}_t - \frac{h}{\gamma - h}\left(\hat{y}_{t-1} - \hat{\gamma}_t\right)$

$$\hat{\pi}_t = \frac{\iota}{1 + \beta\iota}\hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\iota}E_t\hat{\pi}_{t+1} + \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\left[\frac{\gamma}{\gamma - h}\hat{y}_t - \frac{h}{\gamma - h}\left(\hat{y}_{t-1} - \hat{\gamma}_t\right)\right] \tag{16}$$

Now I will use a rescaling of the growth shock $\hat{\gamma}_t$. Let us first simplify.

$$\begin{aligned}\hat{\pi}_t \quad = \quad & \frac{\iota}{1 + \beta\iota}\hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\iota}E_t\hat{\pi}_{t+1} + \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\left[\frac{\gamma}{\gamma - h}\hat{y}_t - \frac{h}{\gamma - h}\hat{y}_{t-1}\right] \\ & + \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\frac{h}{\gamma - h}\hat{\gamma}_t \end{aligned} \tag{17}$$

Create another auxilliary shock

$$\widehat{\tilde{\gamma}}_t = \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\frac{h}{\gamma - h}\hat{\gamma}_t = \chi\hat{\gamma}_t$$

Use this definition in the Phillips curve

$$\hat{\pi}_t = \frac{\iota}{1 + \beta\iota}\hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\iota}E_t\hat{\pi}_{t+1} + \frac{1}{1 + \beta\iota}\frac{\eta - 1}{\phi}\left[\frac{\gamma}{\gamma - h}\hat{y}_t - \frac{h}{\gamma - h}\hat{y}_{t-1}\right] + \widehat{\tilde{\gamma}}_t \tag{18}$$

3.7 Modified Consumption Euler

Given the definition of the rescaled growth rate shock in the Phillips curve, it is necessary to appropriately adjust the other equations where the shock appears. We start with the Euler

$$\hat{y}_t = \frac{\gamma}{\gamma+h}E_t\left(\hat{y}_{t+1}+\hat{\gamma}_{t+1}\right)+\frac{h}{\gamma+h}\left(\hat{y}_{t-1}-\hat{\gamma}_t\right)-\frac{\gamma-h}{\gamma+h}\left(\hat{R}_t-E_t\hat{\pi}_{t+1}\right)+\hat{\tilde{\varepsilon}}_t^{DEM}$$

After rescaling the shock, we get

$$\hat{y}_t = \frac{\gamma}{\gamma+h}E_t\left(\hat{y}_{t+1}+\frac{\hat{\tilde{\gamma}}_{t+1}}{\chi}\right)+\frac{h}{\gamma+h}\left(\hat{y}_{t-1}-\frac{\hat{\tilde{\gamma}}_t}{\chi}\right)-\frac{\gamma-h}{\gamma+h}\left(\hat{R}_t-E_t\hat{\pi}_{t+1}\right)+\hat{\tilde{\varepsilon}}_t^{DEM} \tag{19}$$

Remember that $\hat{\tilde{\varepsilon}}_t^{DEM}$ is also a rescaled shock. However, unlike the growth rate shock, it does not appear in other equations in the model and hence no further manipulations are required.

4 From the Theory to the Data

4.1 Structural Equations

At this juncture, I introduce an empirical monetary policy rule

$$\hat{R}_t = r_{-1}\hat{R}_{t-1}+(1-\rho_R)\left[r_\pi\hat{\pi}_t+r_y\hat{y}_t\right]+r_{\Delta y}\left(\hat{y}_t+\frac{\hat{\tilde{\gamma}}_t}{\chi}-\hat{y}_{t-1}\right)+\hat{\varepsilon}_t^{MON} \tag{20}$$

Observe that the (rescaled) stochastic growth term affects the output growth component.

Demand:

$$\begin{aligned}\hat{y}_t \quad = \quad & \frac{\gamma}{\gamma+h}E_t\left(\hat{y}_{t+1}+\frac{\hat{\tilde{\gamma}}_{t+1}}{\chi}\right)+\frac{h}{\gamma+h}\left(\hat{y}_{t-1}-\frac{\hat{\tilde{\gamma}}_t}{\chi}\right)\\ & -\frac{\gamma-h}{\gamma+h}\left(\hat{R}_t-E_t\hat{\pi}_{t+1}\right)+\hat{\tilde{\varepsilon}}_t^{DEM}\end{aligned} \tag{21}$$

Phillips curve:

$$\hat{\pi}_t = \frac{\iota}{1+\beta\iota}\hat{\pi}_{t-1}+\frac{\beta}{1+\beta\iota}E_t\hat{\pi}_{t+1}+\frac{1}{1+\beta\iota}\frac{\eta-1}{\phi}\left[\frac{\gamma}{\gamma-h}\hat{y}_t-\frac{h}{\gamma-h}\hat{y}_{t-1}\right]+\hat{\tilde{\gamma}}_t \tag{22}$$

Shocks:

$$\hat{\tilde{\varepsilon}}_t^{DEM} = \rho_{DEM}\hat{\tilde{\varepsilon}}_{t-1}^{DEM}+\eta_t^{DEM} \tag{23}$$

$$\hat{\tilde{\gamma}}_t = \rho_{\Delta GROW}\hat{\tilde{\gamma}}_{t-1}+\eta_t^{\Delta GROW} \tag{24}$$

$$\hat{\varepsilon}_t^{MON} = \rho_{MON}\hat{\varepsilon}_{t-1}^{MON}+\eta_t^{MON} \tag{25}$$

4.2 Observation Equations: To the Data

The model will be estimated with three series: GDP growth, GDP deflator inflation and the Federal Funds rate. So we have to find ways to link the model variables to these three series. Let us first look at the growth rate of output in the data

$$\log Y_t^{DATA} - \log Y_{t-1}^{DATA}$$

Similarly, in the model

$$\log Y_t^{MODEL} - \log Y_{t-1}^{MODEL} = \log \frac{Y_t^{MODEL}}{Y_{t-1}^{MODEL}}$$

In terms of detrended model variables

$$\begin{aligned} \log \frac{Y_t^{MODEL}}{Y_{t-1}^{MODEL}} &= \log \frac{y_t^{MODEL}\varepsilon_t^{GROW}}{y_{t-1}^{MODEL}\varepsilon_{t-1}^{GROW}} = \log \frac{y_t^{MODEL}\gamma_t}{y_{t-1}^{MODEL}} \\ &= \log y_t^{MODEL} - \log y_{t-1}^{MODEL} + \log \gamma_t \end{aligned}$$

We know that

$$\begin{aligned} \hat{y}_t^{MODEL} &= \log y_t^{MODEL} - \log y \\ &\rightarrow \log y_t^{MODEL} - \log y_{t-1}^{MODEL} = \hat{y}_t^{MODEL} - \hat{y}_t^{MODEL} \end{aligned}$$

Similarly

$$\log \gamma_t - \log \gamma = \hat{\gamma}_t$$

So we can finally equate the model variable to the observed data series

$$\log Y_t^{DATA} - \log Y_{t-1}^{DATA} = \log Y_t^{MODEL} - \log Y_{t-1}^{MODEL} = \hat{y}_t^{MODEL} - \hat{y}_t^{MODEL} + \hat{\gamma}_t + \log \gamma$$

Defining $\log \gamma = \gamma^{DATA}$, finally we have

$$\log Y_t^{DATA} - \log Y_{t-1}^{DATA} = \hat{y}_t^{MODEL} - \hat{y}_t^{MODEL} + \hat{\gamma}_t + \gamma^{DATA}$$

Since we will estimate γ^{DATA}, the steady-state gross growth rate in the model is given by

$$\gamma^{MODEL} = 1 + \frac{\gamma^{DATA}}{100}$$

The division by 100 is because the data, I typically use, is measured in percentages. Now let us consider the interest rate and inflation.

$$R_t^{DATA} = R^{DATA} + \hat{R}_t$$

$$\pi_t^{DATA} = \pi^{DATA} + \hat{\pi}_t$$

Since what we measure in the data is *net* inflation, the steady-state model gross inflation rate is given by

$$\pi^{MODEL} = 1 + \frac{\pi^{DATA}}{100}$$

Similarly, the steady-state nominal interest rate in the model is connected to the data analog in the following manner.

$$R^{MODEL} = 1 + \frac{R^{DATA}}{100}$$

In this estimation exercise, we will not estimate the mean of the interest rate from the data, but will instead allow it to be determined by the model-consistent interest rate that is the combination of the discount factor, trend output growth rate and the inflation. So I will rewrite

$$R^{DATA} = 100\left(R^{MODEL} - 1\right)$$

I will regroup the three observation equations here

$$\log Y_t^{DATA} - \log Y_{t-1}^{DATA} = \hat{y}_t^{MODEL} - \hat{y}_t^{MODEL} + \hat{\gamma}_t + \gamma^{DATA} \quad (26)$$

$$R_t^{DATA} = R^{DATA} + \hat{R}_t \quad (27)$$

$$\pi_t^{DATA} = \pi^{DATA} + \hat{\pi}_t \quad (28)$$

4.3 Steady-State Restrictions

Fix the discount factor $\beta = 0.99$. We will estimate π and γ from the data, together with other stuctural parameters. The steady-state parameters or variables are determined by the model restrictions. Importantly, the Euler equation in steady state is given by

$$\frac{1}{c - h\frac{c}{\gamma}} = \beta \frac{1}{c\gamma - hc} \frac{R}{\pi}$$

This can be simplified as

$$R^{MODEL} = \frac{\gamma^{MODEL}\pi^{MODEL}}{\beta} \quad (29)$$

This restriction implies that once we fix the discount factor and estimate the growth rate and mean inflation from the data, the nominal interest rate is automatically determined. Note that we do not need other steady-state levels of variables (for example y) for the linearized system. If required, these can be obtained with additional restrictions.

A Appendix

A.1 Log-linearizing the Consumption Euler

Let us begin with

$$\frac{\varepsilon_t^{DEM}}{c_t - h\frac{c_{t-1}}{\gamma_t}} = \beta \mathbf{E}_t \frac{\varepsilon_{t+1}^{DEM}}{c_{t+1}\gamma_{t+1} - hc_t} \frac{R_t}{\pi_{t+1}} \quad (30)$$

In steady-state

$$\frac{1}{c-h\frac{c}{\gamma}}=\beta\frac{1}{c\gamma-hc}\frac{R}{\pi}\rightarrow\frac{\gamma}{c\gamma-hc}=\beta\frac{1}{c\gamma-hc}\frac{R}{\pi}\rightarrow\frac{\gamma}{\beta}=\frac{R}{\pi}$$

This can be simplified as

$$\frac{1}{\beta\gamma^{-1}}=\frac{R}{\pi} \tag{31}$$

Log-linearize the left hand side

$\left(c_t-h\frac{c_{t-1}}{\gamma_t}\right)^{-1}\rightarrow -1\left(c-h\frac{c}{\gamma}\right)^{-1-1}\left[c\hat{c}_t-h\frac{c}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]$

$\rightarrow -1\left(\frac{c\gamma-hc}{\gamma}\right)^{-1-1}\left[c\hat{c}_t-h\frac{c}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]$

$\rightarrow -1\left(\frac{\gamma}{c\gamma-hc}\right)\left(\frac{\gamma}{c\gamma-hc}\right)\left[c\hat{c}_t-h\frac{c}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]$

Do the same to the right hand side

$\beta E_t\left[\frac{1}{c\gamma-hc}\frac{R}{\pi}\left(\hat{R}_t-\hat{\pi}_{t+1}\right)+\frac{R}{\pi}\left(-1\right)\left(c\gamma-hc\right)^{-1-1}\left[c\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-hc\hat{c}_t\right]\right]$

$\rightarrow \beta E_t\left[\frac{1}{c\gamma-hc}\frac{R}{\pi}\left(\hat{R}_t-\hat{\pi}_{t+1}\right)-\frac{R}{\pi}\left(c\gamma-hc\right)^{-1-1}\left[c\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-hc\hat{c}_t\right]\right]$

$\rightarrow \beta E_t\left[\frac{1}{c\gamma-hc}\frac{1}{\beta\gamma^{-1}}\left(\hat{R}_t-\hat{\pi}_{t+1}\right)-\frac{1}{\beta\gamma^{-1}}\left(c\gamma-hc\right)^{-1-1}\left[c\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-hc\hat{c}_t\right]\right]$

Use the steady-state condition to cancel out like terms

$\rightarrow \frac{\gamma}{c\gamma-hc}E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)-\gamma\left(c\gamma-hc\right)^{-1-1}cE_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$

$\rightarrow \frac{\gamma}{c\gamma-hc}E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)-\frac{\gamma}{c\gamma-hc}\frac{1}{c\gamma-hc}cE_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$

$\rightarrow \frac{\gamma}{c\gamma-hc}E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)-\frac{\gamma}{c\gamma-hc}\frac{1}{\gamma-h}E_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$

$\rightarrow \frac{\gamma}{c\gamma-hc}E_t\left[\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t-\frac{1}{\gamma-h}\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]\right]$

Equate the two sides and simplify. Since the following steps only involve algebra, I will not explain each transformation.

$$\begin{aligned}&-1\left(\frac{\gamma}{c\gamma-hc}\right)\left(\frac{\gamma}{c\gamma-hc}\right)\left[c\hat{c}_t-h\frac{c}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]\\ =\;&\frac{\gamma}{c\gamma-hc}E_t\left[\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t-\frac{1}{\gamma-h}\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]\right]\end{aligned}$$

$$\rightarrow -1\left(\frac{\gamma}{c\gamma-hc}\right)\left[c\hat{c}_t-\frac{hc}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]=\hat{R}_t-E_t\hat{\pi}_{t+1}+E_t\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t-\frac{1}{\gamma-h}E_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$$

$$\rightarrow \left(\frac{\gamma}{c\gamma-hc}\right)\left[c\hat{c}_t-\frac{hc}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)\right]=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)+\frac{1}{\gamma-h}E_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$$

$$\rightarrow \left(\frac{\gamma}{c\gamma-hc}\right)c\hat{c}_t-\left(\frac{\gamma}{c\gamma-hc}\right)\frac{hc}{\gamma}\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)+\frac{1}{\gamma-h}E_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$$

$$\rightarrow \frac{\gamma}{\gamma-h}\hat{c}_t-\left(\frac{h}{\gamma-h}\right)\left(\hat{c}_{t-1}-\hat{\gamma}_t\right)=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}^{DEM}_{t+1}-\hat{\varepsilon}^{DEM}_t\right)+\frac{1}{\gamma-h}E_t\left[\gamma\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-h\hat{c}_t\right]$$

$$\rightarrow \frac{\gamma}{\gamma-h}\hat{c}_t-\left(\frac{h}{\gamma-h}\right)(\hat{c}_{t-1}-\hat{\gamma}_t)=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}_{t+1}^{DEM}-\hat{\varepsilon}_t^{DEM}\right)+\frac{\gamma}{\gamma-h}E_t\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)-\frac{h}{\gamma-h}\hat{c}_t$$

$$\rightarrow \frac{\gamma}{\gamma-h}\hat{c}_t+\frac{h}{\gamma-h}\hat{c}_t=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}_{t+1}^{DEM}-\hat{\varepsilon}_t^{DEM}\right)+\frac{\gamma}{\gamma-h}E_t\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)+\left(\frac{h}{\gamma-h}\right)(\hat{c}_{t-1}-\hat{\gamma}_t)$$

$$\rightarrow \frac{\gamma+h}{\gamma-h}\hat{c}_t=-E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}_{t+1}^{DEM}-\hat{\varepsilon}_t^{DEM}\right)+\frac{\gamma}{\gamma-h}E_t\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)+\left(\frac{h}{\gamma-h}\right)(\hat{c}_{t-1}-\hat{\gamma}_t)$$

$$\rightarrow \hat{c}_t=-\frac{\gamma-h}{\gamma+h}E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}_{t+1}^{DEM}-\hat{\varepsilon}_t^{DEM}\right)+\frac{\gamma-h}{\gamma+h}\frac{\gamma}{\gamma-h}E_t\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)+\frac{\gamma-h}{\gamma+h}\left(\frac{h}{\gamma-h}\right)(\hat{c}_{t-1}-\hat{\gamma}_t)$$

Finally we arrive at the consumption equation:

$$\hat{c}_t=-\frac{\gamma-h}{\gamma+h}E_t\left(\hat{R}_t-\hat{\pi}_{t+1}+\hat{\varepsilon}_{t+1}^{DEM}-\hat{\varepsilon}_t^{DEM}\right)+\frac{\gamma}{\gamma+h}E_t\left(\hat{c}_{t+1}+\hat{\gamma}_{t+1}\right)+\frac{h}{\gamma+h}(\hat{c}_{t-1}-\hat{\gamma}_t)$$

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