Capítulo 1

Model - Endowment Economy

1.1 The model

- Endowment

$$Y_t = (\Lambda_y)e^{\epsilon_y} \tag{1.1}$$

- Household

We denote the unrestricted agent by "u" and the restricted ones by "r1", "r2" and "r3".

→ Utility Function

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_j^s ln(C_{t+s}^j) \tag{1.2}$$

for j=u,r1,...,r3.

→ Budget Constraint: unrestricted[1]

$$P_t C_t^u + B_t^u + \sum_i (1+\zeta_{i,t}) P_{i,t} B_t^{i,u} \leq R_{t-1} B_{t-1}^u + \sum_i \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{i,u} + P_t Y_t^i + \mathcal{P}_t^{ff,u} - T_t^u. \tag{1.3}$$

or

$$P_t C_t^u + B_t^u + \sum_i (1+\zeta_{i,t}) P_{i,t} B_t^{i,u} \leq R_{t-1} B_{t-1}^u + \sum_i P_{i,t} R_{i,t} B_{t-s}^{i,u} + P_t Y_t^i + \mathcal{P}_t^{ff,u} - T_t^u. \tag{1.4}$$

where $i = r1, ..., r3$.

→ Budget Constraint: restricted (i)

$$P_t C_t^i + P_{i,t} B_t^{i,i} \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{i,i} + P_t Y_t + \mathcal{P}_t^{ff,i} - T_t^i. \tag{1.5}$$

[1] Note that $P_{i,t} = \frac{1}{R_{i,t}-\kappa_i}$ and $P_{i,t}(s) = \kappa^s P_{i,t}$, where $P_{i,t}(s)$ is the today's price of a long-term bond issued s periods ago.

or

$$P_tC_t^i + P_{i,t}B_t^{i,i} \leq P_{i,t}R_{i,t}B_{t-s}^{i,i} + P_tY_t + \mathcal{P}_t^{ff,i} - T_t^i. \tag{1.6}$$

for $i = r1, ..., r3$.

- Financial Firms Profits

$$\mathcal{P}_t^{ff} = \omega_u \left(\sum_i \zeta_{i,t} P_{i,t} B_t^{i,u} \right) \tag{1.7}$$

- Government Policies

→ Central Bank rule:

$$\frac{R_t}{R} = \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \right] e^{\epsilon_{m,t}}. \tag{1.8}$$

→ Government budget constraint:

$$B_t + \sum_i P_{i,t}B_t^i = R_{t-1,t}B_{t-1} + \sum_i (1 + \kappa P_{i,t})B_{t-1}^i + P_tG_t - T_t. \tag{1.9}$$

where $i = r1, ..., r3$

→ Supply of long-term bonds:

$$\frac{P_{i,t}B_t^{L,i}}{P_t} = \Gamma e^{\epsilon_{B,i,t}} \tag{1.10}$$

for $i = r1, ..., r3$

→ Real primary fiscal surplus:

$$\frac{T_t}{P_t} - G_t = \Phi \left(B_t + \sum_i \frac{P_{i,t-1}B_{t-1}^i}{P_{t-1}} \right)^{\phi_T} e^{\epsilon_{T,t}} \tag{1.11}$$

→ Government spending:

$$ln(\frac{G_{z,t}}{\overline{G}_z}) = \epsilon_{G,t} \tag{1.12}$$

- Transaction Costs

Transaction costs are function of the detrended real value of long-term bonds plus an error:

$$\zeta_{t,i} \equiv \Psi \epsilon_{\zeta,t,i} \tag{1.13}$$

1.2 Solving the model

- CPO's

$$[\partial C_t^u] : \frac{1}{C_t^u} = \lambda_t^u P_t \tag{1.14}$$

$$[\partial B_t] : \lambda_t^u = \beta_u \mathbb{E}_t[\lambda_{t+1}^u R_t] \tag{1.15}$$

$$[\partial B_t^{i,u}] : \lambda_t^u (1 + \zeta_{i,t}) P_{L,i,t} = \beta_u \mathbb{E}_t \left[\frac{R_{L,i,t+1}}{R_{L,i,t+1} - \kappa_i} \lambda_{t+1}^u \right] \tag{1.16}$$

$$[\partial C_t^i] : \frac{1}{C_t^i} = \lambda_t^i P_t \tag{1.17}$$

$$[\partial B_t^{i,i}] : \lambda_t^i = \beta_i \mathbb{E}_t \left[\frac{R_{L,i,t+1}}{R_{L,i,t+1} - \kappa_i} \lambda_{t+1}^i \right] \tag{1.18}$$

- Aggregation:

→ Sum the Budget Constraints:

$$\omega_u P_t C_t^u + \textstyle\sum_i \omega_i P_t C_t^i + B_t + \omega_u \sum_i \zeta_{i,t} P_{L,i,t} B_t^{L,i,u} + \sum_i P_{L,i,t} \left(\omega_u B_t^{L,i,u} + \omega_i B_t^{L,i,i} \right) =$$

$$R_{t-1} B_{t-1} + \textstyle\sum_i P_{L,i,t} \left(\omega_u B_{t-1}^{L,i,u} + \omega_i B_{t-1}^{L,i,i} \right) + P_t \left(\omega_u Y_u + \sum_i \omega_i Y_i \right) + \mathcal{P}_t^{ff} - T_t$$

Substituting (1.9) and (1.7) above we have:

$$\omega_u C_t^u + \sum_i \omega_i C_t^i = Y_t - G_t \tag{1.19}$$

1.3 Normalizing

Consider the following normalization:

- $\Xi_t^j \equiv \lambda_t^j P_t, \forall j$
- $B_{z,t} \equiv B_t / P_t$
- $B_{z,t}^{L,i} \equiv B_t^{l,i} / P_t, \forall i$
- $T_{z,t} \equiv T_t / P_t$
- other x_t: $x_{z,t} = x_t$.

$$Y_{z,t} = (\Lambda_y)^{\phi_y} e^{\epsilon_y} \tag{1.20}$$

$$\frac{R_t}{R} = \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \right] e^{\epsilon_{m,t}}. \tag{1.21}$$

$$B_{z,t} + \sum_i P_{i,t} B_{z,t}^{L,i} = \frac{R_{t-1,t} B_{z,t-1}}{\Pi_t} + \sum_i \frac{(1+\kappa P_{i,t}) B_{z,t-1}^{L,i}}{\Pi_t} + G_{z,t} - T_{z,t}. \tag{1.22}$$

$$P_{i,t} B_{z,t}^{L,i} = \Gamma e^{\epsilon_{B,i,t}} \tag{1.23}$$

for $i = r1, ..., r3$

$$T_{z,t} - G_{z,t} = \Phi \left(B_{z,t} + \sum_i P_{i,t-1} B_{z,t-1}^{L,i} \right)^{\phi_T} e^{\epsilon_{T,t}} \tag{1.24}$$

$$ln(\frac{G_{z,t}}{G_z}) = \epsilon_{G,t} \tag{1.25}$$

$$\zeta_t \equiv \Psi \epsilon_{\zeta,t} \tag{1.26}$$

$$\frac{1}{C_{z,t}^u} = \Xi_t^u \tag{1.27}$$

$$\Xi_t^u = \beta_u \mathbb{E}_t [\frac{\Xi_{t+1}^u R_t}{\Pi_{t+1}}] \tag{1.28}$$

$$\Xi_t^u (1 + \zeta_{i,t}) P_{L,i,t} = \beta_u \mathbb{E}_t \left[\frac{R_{L,i,t+1}}{R_{L,i,t+1} - \kappa_i} \frac{\Xi_{t+1}^u}{\Pi_{t+1}} \right] \tag{1.29}$$

$$\frac{1}{C_{z,t}^i} = \Xi_t^i \tag{1.30}$$

for $i = r1, ..., r3$

$$\Xi_t^i P_{L,i,t} = \beta_i \mathbb{E}_t \left[\frac{R_{L,i,t+1}}{R_{L,i,t+1} - \kappa_i} \frac{\Xi_{t+1}^i}{\Pi_t} \right] \tag{1.31}$$

for $i = r1, ..., r3$

$$\omega_u C_{z,t}^u + \sum_i \omega_i C_{z,t}^i = Y_{z,t} - G_{z,t} \tag{1.32}$$

1.4 Steady-State

$$Y_z = (\Lambda_y)^{\phi_y} \tag{1.33}$$

$$T_z = -B_z - \sum_i \frac{1}{R_{L,i} - \kappa_i} B_z^{L,i} + \frac{R_t B_z}{\Pi} + \sum_i \frac{(1+\kappa P_{L,i}) B_z^{L,i}}{\Pi} + G_z \tag{1.34}$$

$$P_i B_z^{L,i} = \Gamma \tag{1.35}$$

for $i = r1, ..., r3$

$$T_z - G_z = \Phi \left(B_z + \sum_i P_i B_z^{L,i} \right)^{\phi_T} \tag{1.36}$$

$$\frac{1}{C_z^u} = \Xi^u \tag{1.37}$$

$$R_t = \frac{\Pi}{\beta_u} \tag{1.38}$$

$$R_{L,i} = \frac{(1 + \zeta_{i,t})\Pi}{\beta_u} \tag{1.39}$$

for $i = r1, ..., r3$

$$\frac{1}{C_z^i} = \Xi_t^i \tag{1.40}$$

for $i = r1, ..., r3$

$$\beta_i = \frac{\beta_u}{(1 + \zeta)} \tag{1.41}$$

for $i = r1, ..., r3$

$$\omega_u C_z^u + \sum_i \omega_i C_z^i = Y_z - G_z \tag{1.42}$$

1.5 Log-linearization

$$\hat{Y}_{z,t} = \epsilon_{y,t} \tag{1.43}$$

$$\hat{\Xi}_t^j = -\hat{C}_t^j \tag{1.44}$$

for $j = u, r_1, ..., r_3$.

$$\hat{\Xi}_t^u = r_t + \mathbb{E}_t \left(\hat{\Xi}_{t+1}^u - \pi_{t+1} \right) \tag{1.45}$$

$$\hat{\zeta}_{i,t} + \hat{\Xi}_t^u = \frac{R_i}{R_i - \kappa} r_{i,t} + \mathbb{E}_t \left(\hat{\Xi}_{t+1}^u - \pi_{t+1} - \frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1} \right) \tag{1.46}$$

for $i = u, r_1, ..., r_3$

$$\hat{\Xi}_t^i = \frac{R_i}{R_i - \kappa_i} r_{i,t} + \mathbb{E}_t \left(\hat{\Xi}_{t+1}^i - \pi_{t+1} - \frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1} \right) \tag{1.47}$$

for $i = r_1, ..., r_3$

$$\hat{B}_{z,t}+\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\hat{B}_{z,t}^i = \beta_u^{-1}\left(\hat{B}_{z,t-1}+r_{t-1}\right)+\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\beta_i^{-1}\hat{B}_{z,t-1}^i$$
$$+\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\frac{(1-\Pi^{-1}\kappa_i)R_i}{R_i-\kappa_i}r_{i,t}+\frac{G_z}{B_z}\hat{G}_{z,t}$$
$$-\frac{Y_z}{B_z}\hat{T}_{z,t}-\left(\beta_u^{-1}+\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\beta_i^{-1}\right)(\pi_t).$$

with

$$T_{z,t}\equiv T_z+Y_z\hat{T}_{z,t}\Rightarrow \hat{T}_{z,t}=\frac{T_{z,t}}{Y_z}-\frac{T_z}{Y_z}$$

and $i=r_1,...,r_3$

$$\hat{B}_{z,t}^i-\frac{R_i}{R_i-\kappa_i}r_{i,t}=\epsilon_{B,i,t} \tag{1.48}$$

for $i=r_1,...,r_3$ and $j=r_1,...,r_3$

$$\frac{\hat{T}_{z,t}-G_z\hat{G}_{z,t}}{T_z-G_z}=\frac{\phi_T}{B_z+\sum_i \frac{B_z^i}{R_i-\kappa_i}}\left(B_z\hat{B}_{z,t-1}+\sum_j \frac{B_z^j}{R_j-\kappa_j}\left(\hat{B}_{z,t-1}^j-\frac{R_j}{R_j-\kappa_j}r_{j,t-1}\right)\right)+\epsilon_{T,t} \tag{1.49}$$

where $j=r_1,...,r_3$

$$\hat{G}_{z,t}=\epsilon_{G,t} \tag{1.50}$$

$$r_t=\phi_\pi\pi_t+\epsilon_{m,t} \tag{1.51}$$

$$\hat{\zeta}_{i,t}=\epsilon_{\zeta,i,t} \tag{1.52}$$

for $i=u,r_1,...,r_3$

$$\hat{Y}_{z,t}=\sum_i \frac{\omega_i C_z^i}{Y_z}\hat{C}_{z,t}^i+\frac{G_z}{Y_z}\hat{G}_{z,t} \tag{1.53}$$

where $i=u,r_1,...,r_3$

1.6 Simplifications Log-Lin Model

Substituting 1.43, 1.44, 1.51, 1.50 and 1.52:

$$-\hat{C}_t^u = \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t\left(-\hat{C}_{t+1}^u - \pi_{t+1}\right) \tag{1.54}$$

$$\epsilon_{\zeta,i,t} - \hat{C}_t^u = \frac{R_i}{R_i - \kappa} r_{i,t} + \mathbb{E}_t\left(-\hat{C}_{t+1}^u - \pi_{t+1} - \frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1}\right) \tag{1.55}$$

for $i = u, r_1, ..., r_3$

$$-\hat{C}_t^i = \frac{R_i}{R_i - \kappa_i} r_{i,t} + \mathbb{E}_t\left(-\hat{C}_{t+1}^i - \pi_{t+1} - \frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1}\right) \tag{1.56}$$

for $i = r_1, ..., r_3$

$$\begin{aligned}\hat{B}_{z,t} + \textstyle\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\hat{B}_{z,t}^i = \quad & \beta_u^{-1}\left(\hat{B}_{z,t-1} + (\phi_\pi \pi_{t-1} + \epsilon_{m,t-1})\right) + \textstyle\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\beta_i^{-1}\hat{B}_{z,t-1}^i \\ & + \textstyle\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\frac{(1-\Pi^{-1}\kappa_i)R_i}{R_i-\kappa_i} r_{i,t} + \frac{G_z}{B_z}\epsilon_{G,t} \\ & - \frac{Y_z}{B_z}\hat{T}_{z,t} - \left(\beta_u^{-1} + \textstyle\sum_i \frac{B_z^i/B_z}{R_i-\kappa_i}\beta_i^{-1}\right)(\pi_t).\end{aligned}$$

with

$$T_{z,t} \equiv T_z + Y_z \hat{T}_{z,t} \Rightarrow \hat{T}_{z,t} = \frac{T_{z,t}}{Y_z} - \frac{T_z}{Y_z}$$

and $i = r_1, ..., r_3$

$$\hat{B}_{z,t}^i - \frac{R_i}{R_i - \kappa_i} r_{i,t} = \epsilon_{B,i,t} \tag{1.57}$$

for $i = r_1, ..., r_3$ and $j = r_1, ..., r_3$

$$\frac{\hat{T}_{z,t} - G_z \epsilon_{G,t}}{T_z - G_z} = \frac{\phi_T}{B_z + \sum_i \frac{B_z^i}{R_i - \kappa_i}}\left(B_z \hat{B}_{z,t-1} + \sum_j \frac{B_z^j}{R_j - \kappa_j}\left(\hat{B}_{z,t-1}^j - \frac{R_j}{R_j - \kappa_j} r_{j,t-1}\right)\right) + \epsilon_{T,t} \tag{1.58}$$

where $j = r_1, ..., r_3$

$$\epsilon_{y,t} = \sum_i \frac{\omega_i C_z^i}{Y_z}\hat{C}_{z,t}^i + \frac{G_z}{Y_z}\epsilon_{G,t} \tag{1.59}$$

where $i = u, r_1, ..., r_3$

1.6.1 More simplifications

Substituting 1.57 in the system and 1.54 in 1.55

$$-\hat{C}_t^u = \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t \left(-\hat{C}_{t+1}^u - \pi_{t+1} \right) \tag{1.60}$$

$$\epsilon_{\zeta,i,t} + \phi_\pi \pi_t + \epsilon_{m,t} = \frac{R_i}{R_i - \kappa} r_{i,t} + \mathbb{E}_t \left(-\frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1} \right) \tag{1.61}$$

for $i = u, r_1, ..., r_3$

$$-\hat{C}_t^i = \frac{R_i}{R_i - \kappa_i} r_{i,t} + \mathbb{E}_t \left(-\hat{C}_{t+1}^i - \pi_{t+1} - \frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1} \right) \tag{1.62}$$

for $i = r_1, ..., r_3$

$$\begin{aligned} \hat{B}_{z,t} + \textstyle\sum_i \frac{B_z^i/B_z}{R_i - \kappa_i} \left(\frac{R_i}{R_i - \kappa_i} r_{i,t} + \epsilon_{B,i,t} \right) = \quad & \beta_u^{-1} \left(\hat{B}_{z,t-1} + (\phi_\pi \pi_{t-1} + \epsilon_{m,t-1}) \right) + \textstyle\sum_i \frac{B_z^i/B_z}{R_i - \kappa_i} \beta_i^{-1} \left(\frac{R_i}{R_i - \kappa_i} r_{i,t-1} + \epsilon_{B,i,t-1} \right) \\ & + \textstyle\sum_i \frac{B_z^i/B_z}{R_i - \kappa_i} \frac{(1 - \Pi^{-1}\kappa_i) R_i}{R_i - \kappa_i} r_{i,t} + \frac{G_z}{B_z} \epsilon_{G,t} \\ & - \frac{Y_z}{B_z} \hat{T}_{z,t} - \left(\beta_u^{-1} + \textstyle\sum_i \frac{B_z^i/B_z}{R_i - \kappa_i} \beta_i^{-1} \right) (\pi_t). \end{aligned}$$

with

$$T_{z,t} \equiv T_z + Y_z \hat{T}_{z,t} \Rightarrow \hat{T}_{z,t} = \frac{T_{z,t}}{Y_z} - \frac{T_z}{Y_z}$$

and $i = r_1, ..., r_3$

$$\frac{\hat{T}_{z,t} - G_z \epsilon_{G,t}}{T_z - G_z} = \frac{\phi_T}{B_z + \sum_i \frac{B_z^i}{R_i - \kappa_i}} \left(B_z \hat{B}_{z,t-1} + \sum_j \frac{B_z^j}{R_j - \kappa_j} (\epsilon_{B,j,t-1}) \right) + \epsilon_{T,t} \tag{1.63}$$

where $j = r_1, ..., r_3$

$$\epsilon_{y,t} = \sum_i \frac{\omega_i C_z^i}{Y_z} \hat{C}_{z,t}^i + \frac{G_z}{Y_z} \epsilon_{G,t} \tag{1.64}$$

where $i = u, r_1, ..., r_3$

1.6.2 And more simplifications

Substituting 1.61 in 1.62

$$-\hat{C}^u_t = \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t\left(-\hat{C}^u_{t+1} - \pi_{t+1}\right) \tag{1.65}$$

$$\epsilon_{\zeta,i,t} + \phi_\pi \pi_t + \epsilon_{m,t} = \frac{R_i}{R_i - \kappa} r_{i,t} + \mathbb{E}_t\left(-\frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1}\right) \tag{1.66}$$

for $i = u, r_1, ..., r_3$

$$-\hat{C}^i_t + \mathbb{E}_t \hat{C}^i_{t+1} = \epsilon_{\zeta,i,t} + \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t\left(-\pi_{t+1}\right) \tag{1.67}$$

for $i = r_1, ..., r_3$

$$\begin{aligned} \hat{B}_{z,t} + \textstyle\sum_i \frac{B^i_z/B_z}{R_i-\kappa_i}\left(\frac{R_i}{R_i-\kappa_i} r_{i,t} + \epsilon_{B,i,t}\right) = \;& \beta_u^{-1}\left(\hat{B}_{z,t-1} + (\phi_\pi \pi_{t-1} + \epsilon_{m,t-1})\right) + \textstyle\sum_i \frac{B^i_z/B_z}{R_i-\kappa_i}\beta_i^{-1}\left(\frac{R_i}{R_i-\kappa_i} r_{i,t-1} + \epsilon_{B,i,t-1}\right) \\ & + \textstyle\sum_i \frac{B^i_z/B_z}{R_i-\kappa_i}\frac{(1-\Pi^{-1}\kappa_i)R_i}{R_i-\kappa_i} r_{i,t} + \frac{G_z}{B_z}\epsilon_{G,t} \\ & - \frac{Y_z}{B_z}\hat{T}_{z,t} - \left(\beta_u^{-1} + \textstyle\sum_i \frac{B^i_z/B_z}{R_i-\kappa_i}\beta_i^{-1}\right)(\pi_t). \end{aligned}$$

with

$$T_{z,t} \equiv T_z + Y_z \hat{T}_{z,t} \Rightarrow \hat{T}_{z,t} = \frac{T_{z,t}}{Y_z} - \frac{T_z}{Y_z}$$

and $i = r_1, ..., r_3$

$$\frac{\hat{T}_{z,t} - G_z \epsilon_{G,t}}{T_z - G_z} = \frac{\phi_T}{B_z + \sum_i \frac{B^i_z}{R_i-\kappa_i}}\left(B_z \hat{B}_{z,t-1} + \sum_j \frac{B^j_z}{R_j - \kappa_j}\left(\epsilon_{B,j,t-1}\right)\right) + \epsilon_{T,t} \tag{1.68}$$

where $j = r_1, ..., r_3$

$$\epsilon_{y,t} = \sum_i \frac{\omega_i C^i_z}{Y_z}\hat{C}^i_{z,t} + \frac{G_z}{Y_z}\epsilon_{G,t} \tag{1.69}$$

where $i = u, r_1, ..., r_3$

1.6.3 And more...

Substituting 1.68

$$-\hat{C}_t^u = \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t \left(-\hat{C}_{t+1}^u - \pi_{t+1} \right) \tag{1.70}$$

$$\epsilon_{\zeta,i,t} + \phi_\pi \pi_t + \epsilon_{m,t} = \frac{R_i}{R_i - \kappa} r_{i,t} + \mathbb{E}_t \left(-\frac{\kappa_i}{R_i - \kappa_i} r_{i,t+1} \right) \tag{1.71}$$

for $i = u, r_1, ..., r_3$

$$-\hat{C}_t^i + \mathbb{E}_t \hat{C}_{t+1}^i = \epsilon_{\zeta,i,t} + \phi_\pi \pi_t + \epsilon_{m,t} + \mathbb{E}_t \left(-\pi_{t+1} \right) \tag{1.72}$$

for $i = r_1, ..., r_3$

$$\begin{aligned} \hat{B}_{z,t} + \textstyle\sum_i \frac{B_z^i / B_z}{R_i - \kappa_i} \left(\frac{R_i}{R_i - \kappa_i} r_{i,t} + \epsilon_{B,i,t} \right) = \quad & \beta_u^{-1} \left(\hat{B}_{z,t-1} + (\phi_\pi \pi_{t-1} + \epsilon_{m,t-1}) \right) + \textstyle\sum_i \frac{B_z^i / B_z}{R_i - \kappa_i} \beta_i^{-1} \left(\frac{R_i}{R_i - \kappa_i} r_{i,t-1} + \epsilon_{B,i,t-1} \right) \\ & + \textstyle\sum_i \frac{B_z^i / B_z}{R_i - \kappa_i} \frac{(1 - \Pi^{-1} \kappa_i) R_i}{R_i - \kappa_i} r_{i,t} + \frac{G_z}{B_z} \epsilon_{G,t} \\ & - \frac{Y_z}{B_z} \left[(T_z - G_z) \left(\frac{\phi_T}{B_z + \sum_i \frac{B_z^i}{R_i - \kappa_i}} \left(B_z \hat{B}_{z,t-1} + \textstyle\sum_j \frac{B_z^j}{R_j - \kappa_j} \left(\epsilon_{B,j,t-1} \right) \right) + \epsilon_{T,t} \right) + G_z \epsilon_{G,t} \right] \\ & - \left(\beta_u^{-1} + \textstyle\sum_i \frac{B_z^i / B_z}{R_i - \kappa_i} \beta_i^{-1} \right) (\pi_t). \end{aligned}$$

where $i = r_1, ..., r_3$

$$\epsilon_{y,t} = \sum_i \frac{\omega_i C_z^i}{Y_z} \hat{C}_{z,t}^i + \frac{G_z}{Y_z} \epsilon_{G,t} \tag{1.73}$$

where $i = u, r_1, ..., r_3$