For the model feature is same as Finn(2000) , except the household utility function it is refer as Finn(1995).

1 Household First order condition

Household MAX:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \{logC_t + \varphi(log(1 - N_t))\} \quad \text{As Finn(1995)} \tag{1}$$

Household Budget constraint:

$$C_t + I_t + P_{O,t}O_t = W_t N_t + R_t K_t u_t \tag{2}$$
$$K_{t+1} = I_t + (1 - \delta(u_t))K_t \tag{3}$$

其中, $0 < \beta < 1$, $\varphi < 0$, $Q_t = \frac{1}{r_t}$.

Utilization Rate:

$$\frac{O_t}{K_t} = a(u_t), \quad a(u_t) = \frac{\upsilon_0}{\upsilon_1} u_t^{\upsilon_1} \tag{4}$$
$$\Rightarrow u_t = \left(\frac{O_t}{K_t}\right)^{\frac{1}{\upsilon_1}} \left(\frac{\upsilon_1}{\upsilon_0}\right)^{\frac{1}{\upsilon_1}} \tag{5}$$

Depreciation Rate:

$$\delta(u_t) = \frac{\omega_0}{\omega_1} u_t^{\omega_1} \tag{6}$$

$0 < \delta(.) < 1, \omega_0 > 0, \omega_1 > 1$, $\upsilon_0 > 0$, $\upsilon_1 > 1$.
By Lagrange we can obtain the optimal choose of C_t , N_t , K_{t+1} , u_t First order condition:

$$\begin{aligned}\mathcal{L} : & E_t \sum_{t=0}^{\infty} \beta^t \{logC_t + \varphi(log(1 - N_t)) + \lambda_t(W_t N_t + R_t K_t u_t - C_t \\ & - (K_{t+1} - (1 - \delta(u_t))K_t) - P_{O,t} a(u_t) k_t\}\end{aligned} \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \quad \Rightarrow \quad \lambda_t = \frac{1}{C_t} \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \quad \Rightarrow \quad \lambda_t = \frac{\varphi}{(1-N_t)W_t} \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \quad \Rightarrow \quad \lambda_t = E_t \beta \lambda_{t+1} \left(R_{t+1} u_{t+1} + (1 - \delta(u_{t+1})) - P_{O,t+1} a(u_{t+1}) \right) \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial u_t} = 0 \quad \Rightarrow \quad R_t K_t = \delta'(u_t) K_t + P_{O,t} a'(u_t) K_t \tag{11}$$

上式可整理爲:

FOC Labor supply:

$$\frac{1}{C_t} = \frac{\varphi}{(1-N_t)W_t} \quad \Rightarrow \quad (1-N_t) = \frac{\varphi C_t}{W_t} \tag{12}$$

FOC Capital:

$$\Rightarrow E_t \beta \frac{C_t}{C_{t+1}} \left(R_{t+1} u_{t+1} + (1 - \delta(u_{t+1})) - P_{O,t+1} a(u_{t+1}) \right) = 1 \tag{13}$$

Utilization Rate:

$$\Rightarrow R_t K_t = \omega_0 u_t^{(\omega_1 - 1)} K_t + P_{O,t} \upsilon_0 u_t^{(\upsilon_1 - 1)} K_t \tag{14}$$

2 Firm First order condition

Firm MAX profits:

$$\pi_t = Y_{H,t} - R_t K_t u_t - W_t N_t \tag{15}$$

Firm production function:

$$Y_{H,t} = (A_t N_t)^{\alpha} (u_t K_t)^{(1-\alpha)} \tag{16}$$

By Lagrange we can obtain the optimal choose of N_t , K_t First order condition:

$$\mathcal{L} : (A_t N_t)^{\alpha}(u_t K_t)^{(1-\alpha)} - R_t K_t u_t - W_t N_t \tag{17}$$

FOC capital:

$$\frac{\partial \mathcal{L}}{\partial K_t} = 0 \quad \Rightarrow \quad R_t u_t = \frac{Y_{H,t}(1-\alpha)}{K_t} \tag{18}$$

FOC labor:

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \quad \Rightarrow \quad W_t = \frac{Y_{H,t}\alpha}{N_t} \tag{19}$$

3 Household Log linearize

Utilization Rate

$$\frac{O_t}{K_t} = a(u_t), \quad a(u_t) = \frac{\upsilon_0}{\upsilon_1} u_t^{\upsilon_1} \quad \Rightarrow \quad \hat{O}_t - \hat{K}_t = \hat{a}(\hat{u}_t), \quad \hat{a}(\hat{u}_t) = \upsilon_1 \hat{u}_t \tag{20}$$

$$u_t = \left(\frac{O_t}{K_t}\right)^{\frac{1}{\upsilon_1}} \left(\frac{\upsilon_1}{\upsilon_0}\right)^{\frac{1}{\upsilon_1}} \quad \Rightarrow \quad \hat{O}_t - \hat{K}_t = \upsilon_1 \hat{u}_t \tag{21}$$

Depreciation Rate

$$\delta(u_t) = \frac{\omega_0 u_t^{\omega_1}}{\omega_1} \quad \Rightarrow \quad \hat{\delta}(\hat{u}_t) = \omega_1 \hat{u}_t \tag{22}$$

Law of Motion:

$$\begin{aligned} & K_{t+1} = I_t + (1-\delta(u_t))K_t \\ \Rightarrow\; & \hat{K}_{t+1} = \bar{\delta}(\bar{u})\hat{I}_t + (1-\bar{\delta}(\bar{u}))\hat{K}_t - \bar{\delta}(\bar{u})\omega_1 \hat{u}_t \end{aligned} \tag{23}$$

Oil shocks

$$logP_{O,t} = (1-\rho^O)log\overline{P_O} + \rho^O(logP_{O,t-1}) + \varepsilon_t^O \quad \Rightarrow \quad \hat{P}_{O,t} = \rho^O \hat{P}_{O,t-1} + \varepsilon_t^O \tag{24}$$

labor supply:

$$(1-N_t) = \frac{\varphi C_t}{W_t} \Rightarrow \bar{N}\hat{N}_t + (1-\bar{N})(\hat{C}_t - \hat{W}_t) = 0 \tag{25}$$

capital:

$$E_t\beta\frac{C_t}{C_{t+1}}\left(R_{t+1}u_{t+1}+(1-\delta(u_{t+1}))-P_{O,t+1}a(u_{t+1})\right)=1$$

$$\bar{R}\bar{u}(\hat{R}_{t+1}+\hat{u}_{t+1})-\bar{\delta}(\bar{u})\omega_1\hat{u}_{t+1}-\bar{P_O}\frac{\upsilon_0}{\upsilon_1}\bar{u}^{\upsilon_1}(\hat{P}_{O,t+1}+\upsilon_1\hat{u}_{t+1})=\frac{1}{\beta}(\hat{C}_{t+1}-\hat{C}_t) \tag{26}$$

Utilization Rate:

$$R_t=\omega_0u_t^{(\omega_1-1)}+P_{O,t}\upsilon_0u_t^{(\upsilon_1-1)}$$

$$\Rightarrow\bar{R}\hat{R}_t=\omega_0\bar{u}^{(\omega_1-1)}(\omega_1-1)\hat{u}_t+\bar{P_O}\upsilon_0\bar{u}^{(\upsilon_1-1)}\left(\hat{P}_{O,t}+(\upsilon_1-1)\hat{u}_t\right) \tag{27}$$

$$\Rightarrow\hat{R}_t=\frac{1}{\bar{R}}\omega_0\bar{u}^{(\omega_1-1)}(\omega_1-1)\hat{u}_t+\frac{\bar{P_O}}{\bar{R}}\upsilon_0\bar{u}^{(\upsilon_1-1)}\left(\hat{P}_{O,t}+(\upsilon_1-1)\hat{u}_t\right) \tag{28}$$

4 Firm Log linearize

Firm production function:

$$Y_{H,t}=(A_tN_t)^{\alpha}(u_tK_t)^{(1-\alpha)}\quad\Rightarrow\quad\hat{Y}_{H,t}=\alpha\hat{A}_t+\alpha\hat{N}_t+(1-\alpha)\hat{u}_t+(1-\alpha)\hat{K}_t \tag{29}$$

technology shocks

$$logA_t=(1-\rho^A)log\overline{A}+\rho^A(logA_{t-1})+\varepsilon_t^A\quad\Rightarrow\quad\hat{A}_t=\rho^A\hat{A}_{t-1}+\varepsilon_t^A \tag{30}$$

capital:

$$R_tu_t=\frac{Y_{H,t}(1-\alpha)}{K_t}\quad\Rightarrow\quad\hat{R}_t=\hat{Y}_{H,t}-\hat{K}_t-\hat{u}_t \tag{31}$$

labor:

$$W_t=\frac{Y_{H,t}\alpha}{N_t}\quad\Rightarrow\quad\hat{W}_t=\hat{Y}_{H,t}-\hat{N}_t \tag{32}$$

5 good market equilibrium

Resource constraint

$$Y_{H,t}=C_t+I_t+P_{O,t}O_t\quad\Rightarrow\quad\hat{Y}_{H,t}=\frac{\bar{C}}{\bar{Y}}\hat{C}_t+\frac{\bar{I}}{\bar{Y}}\hat{I}_t+\frac{\bar{P}_O\bar{O}}{\bar{Y}}\hat{P}_{O,t}\hat{O}_t \tag{33}$$