

Based on BGG(1999)

Gertler (2007) *External Constraints on Monetary Policy and the Financial Accelerator*,

1. Households

C_t be a composite of consumption goods., following CES index defines household preferences over home consumption, C_{Ht} , and foreign consumption, C_{Ft} :

$$C_t = [\gamma^{\frac{1}{\rho}} (C_t^h)^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} (C_t^f)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}$$

Solving the above problem, an expression linking the relative prices of domestic and foreign goods with their quantities as well as a consumer price index are obtained.:

$$\frac{C_t^h}{C_t^f} = \left(\frac{\gamma}{1-\gamma}\right) \left(\frac{P_t^h}{P_t^f}\right)^{-\rho}$$

The corresponding consumer price index (CPI), P_t , is given by:

$$P_t = [\gamma (P_t^h)^{1-\rho} + (1-\gamma) (P_t^f)^{1-\rho}]^{\frac{1}{1-\rho}}$$

Households maximize their discounted utility:

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k} - bC_{t+k-1}) + \zeta \ln\left(\frac{M_{t+k}}{P_{t+k}}\right) + \xi \ln(1 - H_{t+k})]$$

s.t.:

$$C_t = \frac{W_t}{P_t} H_t + \prod_t^R -T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{B_{t+1} - (1+rn_t)B_t}{P_t} - \frac{S_t B_{t+1}^* - (1+rn_t^*)B_t^*}{P_t}$$

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First Order Conditions:

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta \frac{b}{C_{t+1} - bC_t}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} (1+rn_{t+1}) \frac{P_t}{P_{t+1}} \right\}$$

$$\lambda_t \frac{W_t}{P_t} = \xi \frac{1}{1-H_t}$$

$$E_t \left\{ \lambda_{t+1} \frac{P_t}{P_{t+1}} [(1+rn_{t+1}) - (1+rn_{t+1}^*) \frac{S_{t+1}}{S_t}] \right\} = 0$$

$$\lambda_t \frac{M_t}{P_t} (1 - \frac{1}{1+rn_t}) = \zeta$$

Let $P_{FW,t}$ denote the wholesale price of foreign goods in domestic currency, and P_{F^*t} the foreign

currency price of such goods. The law of one price then implies :

$$P_{w,t}^f = S_t P_t^{f*}$$

Log-linearisation:

$$c_t = \gamma c_t^h + (1-\gamma)c_t^f \quad 1$$

$$c_t^h = c_t^f - \rho(p_t^h - p_t^f) \quad 2$$

$$p_t = \gamma p_t^h + (1-\gamma)p_t^f \quad 3$$

$$\hat{\lambda}_t = \frac{1}{(1-b)}[-\hat{c}_t + b\hat{c}_{t-1}] \quad 4$$

$$\hat{\lambda}_{t+1} + r_{t+1} - \pi_{t+1} = \hat{\lambda}_t \quad 5$$

$$w_t - p_t = -\hat{\lambda}_t + \eta^{-1}h_t \quad \eta^{-1} = \frac{H}{1-H} \quad 6$$

$$r_{t+1} = s_{t+1} - s_t + r_t^* \quad 7$$

$$m_t = -\hat{\lambda}_t - \frac{1}{1/\beta - 1} r_t \quad 8$$

$$P_{w,t}^f = s_t + p_t^{f*} \quad 9$$

foreign demand for the home tradable good, C_t^{h*}

$$C_t^{h*} = \left(\frac{P_t^{h*}}{P_t^*}\right)^{-\rho} Y_t^*$$

$$c_t^{h*} = -\rho(p_t^{h*} - p_t^*) + y_t^* \quad 10$$

2. Firms

2.1 capital producers:

the index of total investment combines goods produced domestically and imported from abroad.:

$$I_t = \left[\gamma_I \frac{1}{\rho_I} (I_t^h)^{\frac{\rho_I-1}{\rho_I}} + (1-\gamma_I) \frac{1}{\rho_I} (I_t^f)^{\frac{\rho_I-1}{\rho_I}} \right]^{\rho_I-1}$$

link between relative prices and quantities of domestic and imported goods is obtained

$$\frac{I_t^h}{I_t^f} = \left(\frac{\gamma_I}{1-\gamma_I}\right) \left(\frac{P_t^h}{P_t^f}\right)^{-\rho_I}$$

$$P_{I,t} = \left[\gamma_I (P_t^h)^{1-\rho_I} + (1-\gamma_I) (P_t^f)^{1-\rho_I} \right]^{\frac{1}{1-\rho_I}}$$

Capital production technology is described by the following expression:

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t$$

The profit realized by capital producers is:

$$Q_t \Phi\left(\frac{I_t}{K_t}\right)K_t - \left(\frac{P_t^h}{P_t}\right)I_t$$

FOC:

$$E_{t-1} \left\{ Q_t \Phi'\left(\frac{I_t}{K_t}\right) - \left(\frac{P_t^h}{P_t}\right) \right\} = 0$$

Log-linearisation:

$$i_t = \gamma_I i_t^h + (1-\gamma_I) i_t^f \quad 11$$

$$i_t^h = -\rho_I (p_t^h - p_t^f) + i_t^f \quad 12$$

$$p_{t,t} = \gamma_I p_t^h + (1-\gamma_I) p_t^f \quad 13$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \quad 14$$

$$q_t + p_t - p_{t,t} = \varphi(i_t - k_t) \quad \varphi = \frac{\Phi''(I/K) I}{\Phi'(I/K) K} \quad 15$$

2. *Entrepreneurs*

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

$$\ln A_t - \ln(\bar{A}) = \rho^A (\ln A_{t-1} - \ln(\bar{A})) + \varepsilon_t^A$$

$$L_t = H_t^\Omega (H_t^e)^{1-\Omega}$$

Entrepreneurs employ labour to equate the marginal productivity and the average wage. Accordingly, labor demand satisfies:

$$(1-\alpha)\Omega \frac{Y_t}{H_t} = \frac{W_t}{P_{w,t}}$$

$$(1-\alpha)(1-\Omega) \frac{Y_t}{H_t^e} = \frac{W_t^e}{P_{w,t}}$$

P_t^w is the wholesale price.

The budget constraint for capital accumulation:

$$Q_t K_{t+1} = N_{t+1} + B_{t+1}$$

The expected return on capital $\{1+r_{t+1}^k\}$

$$E_t(1+r_{t+1}^k) = E_t \left\{ \frac{\frac{1}{X_{t+1}^h} \frac{\alpha Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t} \right\}$$

The risk premium between the expected return and the economy-wide free interest rate depends on how heavily the firm is leveraged.

$$\frac{Q_t K_{t+1}}{N_{t+1}} = \varphi^r \left(\frac{E_t(1+r_{t+1}^k)}{1+r_{t+1}} \right)$$

Fisher equation:

$$1+r_{t+1} = (1+rn_{t+1}) \frac{P_t}{P_{t+1}}$$

net worth of the firm:

$$N_{t+1} = v_t^n [\gamma V_t] + \frac{W_t^e}{P_t}$$

$$\ln v_t^n = \rho^v \ln v_{t-1}^n + \varepsilon_t^{v^n}$$

$$N_{t+1} = v_t^n \gamma [R_t^k Q_{t-1} K_t - R_t (Q_{t-1} K_t - N_t)] + \frac{W_t^e}{P_t}$$

Log-linearisation:

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{16}$$

$$y_t - h_t = w_t - p_{w,t} \tag{17}$$

$$y_t - x_t^h = w_t^e - p_{w,t} \tag{18}$$

$$E_t \{ r_{t+1}^k \} = (1-\chi)(y_{t+1} - k_{t+1} - x_{t+1}) + \chi q_{t+1} - q_t, \chi = (1-\delta) / R^k \tag{19}$$

$$E_t \{ r_{t+1}^k \} - r_{t+1} = -\chi_r [n_{t+1} - (q_t + k_{t+1})] \quad \chi_r = \frac{\varphi_r(R^k / R)}{\varphi'_r(R^k / R)} \left(\frac{R^k}{R} \right)^{-1} \tag{20}$$

$$r_{t+1} = i_{t+1} - \pi_{t+1} \tag{21}$$

$$n_{t+1} = \frac{\gamma R^k K}{N} (r_t^k - r_t) + \gamma R (\hat{v}_t^n + r_t + n_t) \tag{22}$$

Retailers, price setting, and inflation

Calvo-style price setting

Let $Y_{Ht}(i)$ be the good sold by retailer z . Final domestic output is a CES composite of individual retail goods:

$$Y_t^h = \left[\int_0^1 Y_t^h(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$Y_t^h(z) = \left(\frac{P_i^h(i)}{P_t^h} \right)^{-\epsilon} Y_t^h$$

$$P_t^h = \left[\theta (P_{t-1}^h)^{1-\epsilon} + (1-\theta) (P_{t-1}^{ho})^{1-\epsilon} \right]^{1/(1-\epsilon)}$$

$$\max \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \frac{P_t^{ho} - P_{t+k}^w}{P_{t+k}} Y_{t+k}^{ho}(z) \right\}$$

s inflation rate for **domestically** produced goods

$$\pi_t^h = \beta E_t \{ \pi_{t+1}^h \} + \kappa (p_{w,t} - p_t^h) \quad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \quad 23$$

The inflation rate for **foreign** goods then satisfies:

$$\pi_t^f = \beta E_t \{ \pi_{t+1}^f \} + \kappa (s_t + p_t^{f*} - p_t^f),$$

We assume that the foreign price level p_t^* , the foreign output y_t^* and the foreign interest rate rm_t^* are exogenous and follow an AR(1) process given in log-linearized form:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Where $z_t = \{ p_t^*, y_t^*, rm_t^* \}$, ρ_z is an autoregressive coefficient vector and a vector of uncorrelated and normally distributed innovations with zero mean and standard deviation .

3. Resource Constraint Government Budget Constraint

$$Y_t^h = C_t^h + C_t^{eh} + C_t^{h*} + I_t^h + G_t^h$$

$$\ln G_t^h - \ln(\overline{G^h}) = \rho^G (\ln G_{t-1}^h - \ln(\overline{G^h})) + \varepsilon_t^g$$

$$\frac{P_t^h}{P_t} G_t^h = \frac{M_t - M_{t-1}}{P_t} + T_t$$

Log-linearisation:

$$y_t^h = \frac{C^h}{Y} c_t^h + \frac{C^{eh}}{Y} c_t^{eh} + \frac{C^{h*}}{Y} c_t^{h*} + \frac{I^h}{Y} i_t^h + \frac{G^h}{Y} g_t^h \quad 24$$

4. Exchange Rate Regimes

(i) a pure fixed exchange rate regime;

$$S_t = \bar{S} \quad \forall t.$$

Log-linearisation:

$$s_t = 0$$

(ii) a floating exchange rate regime, where the central bank manages the nominal interest rate according to a Taylor rule;

Log-linearisation:

$$\dot{i}_t = \rho_i \dot{i}_{t-1} + \rho_{i\pi} \pi_t + \rho_{iy} y_t$$

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Model Parametrization:

γ	0.7	γ_I	0.7	C^{h^*} / Y	0.12
C^h / Y	0.3	I / Y	0.42	G / Y	0.15
β	0.9936	R	1.0064	R^k	1.0264
b	0.65	δ	0.03	α	0.45
K / N	2.5	ϵ	11	ϵ_w	2
θ	0.75	χ_r	0.0215	φ	0.25
Ω	0.99	η	2		