Notes on the Model with SPV in the Gerali et al. (2010) Framework

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1 Introduction

The model we propose is essentially the same of Gerali et al. (2010). We have decided to change the banking sector which is enriched with a Special Purpose Vehicle (SPV) involved in a securitization process.

More specifically the bank transfers a part η of its portfolio loans to the SPV against a payment of a cost for servicing c. The SPV finances through the creation of a CDO which is tranched into senior, mezzanine and junior. The first one is purchased by the impatient household, the second is purchased by entrepreneurs, while the last one is in the wholesale bank's balance sheet. According to the literature, this measure mitigates the moral hazard problem of the securitization process. We also consider a credit enhancement in the CDO structure as in Fabozzi and Kothari (2008) and in Ashcraft et al. (2009), among others. We use Bayesian methods and the observed variables are the same as in Gerali et al, but with a 1998Q1 - 2013Q1 sample period. Below we only describe the model for the modified parts.

2 The model

2.1 Impatient Household

$$\max_{\{c_t^I, h_t^I, l_t^I\}} E_0 \sum_{t=0}^{\infty} \beta_I^t \left[\left(1 - a^I\right) \epsilon_t^z \log\left(c_t^I(i) - a^I c_{t-1}^I\right) + \epsilon_t^h \log h_t^I(i) - \frac{l_t^I(i)^{1+\phi}}{1+\phi} \right] \tag{1}$$

st.

$$c_t^I(i) + q_t^h \Delta h_t^I(i) + \frac{\left(1 + r_{t-1}^{BH}\right) b_{t-1}^I(i)}{\pi_t} + Senior_t = w_t^I l_t^I(i) + b_t^I(i) + t_t^I + \frac{\left(1 + r_{t-1}^{senior}\right) Senior_{t-1}}{\pi_t} \tag{2}$$

$$\left(1 + r_t^{BH}\right) b_t^I(i) \leq m_t^I E_t \left[q_{t+1}^h h_t^I(i) \pi_{t+1} \right] \tag{3}$$

Impatient households choose c_t^I, h_t^I, b_t^I and $Senior_t$.

2.2 Entrepreneurs

$$\max_{\{c_t^E, b_t^E, l_t^E, u_t\}} E_0 \sum_{t=0}^{\infty} \beta_E^t \log\left(c_t^E(i) - a^E c_{t-1}^E\right) \tag{4}$$

st.

$$\begin{aligned} & c_t^E + w_t^P l_t^{E,P}(i) + w_t^I l_t^{E,I}(i) + \frac{\left(1 + r_{t-1}^{BE}\right) b_{t-1}^E(i)}{\pi_t} + q_t^k k_t^E(i) + \psi\left(u_t(i)\right) k_{t-1}^E(i) + \\ & Mezz_t + \frac{\left(1 + r_{t-1}^{bond}\right) b_{t-1}^E(i)}{\pi_t} bond_{t-1} = \frac{y_t^E(i)}{x_t} + b_t^E(i) + \\ & q_t^k\left(1-\delta\right) k_{t-1}^E + \frac{\left(1 + r_{t-1}^{Mezz}\right) Mezz_{t-1}}{\pi_t} Mezz_{t-1} + bond_t \end{aligned} \tag{5}$$

$$\left(1 + r_t^{BE}\right) b_t^E(i) \leq m_t^E E_t\left[q_{t+1}^k k_t^E(i)\left(1-\delta\right)\pi_{t+1}\right] \tag{6}$$

$$\left(1 + r_t^{bond}\right) bond_t \leq \left(1 - m_t^E\right) E_t\left[q_{t+1}^k k_t^E(i)\left(1-\delta\right)\pi_{t+1}\right] \tag{7}$$

Entrepreneurs choose c_t^E, b_t^E, h_t^E, u_t, $l_t^{E,P}$, $l_t^{E,I}$, r_t^{bond} and $Mezz_t$.

2.3 Banks and SPV

The introduction of an SPV complicates the model. We assume that: 1) the wholesale unit moves all resources to the lending unit; 2) the lending unit differentiates them at no cost and resells them to impatient households and entrepreneurs, as in Gerali et al. (2010); 3) the lending unit passes a part η of loans to the wholesale, which transfers them to the SPV; 4) the lending unit obtains a cost (for the SPV) c for servicing. All these actions are assumed to occur in the same time.

The CDO tranches (Senior, Mezzanine and Junior) are defined exogenously as a percentage of loans transfered to the SPV: i.e. Senior tranche $= perc_{senior} \times \eta \times B_t$.

2.3.1 Wholesale

$$\begin{aligned} & \max_{\{B_t, D_t\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P [\left(1-\eta\right) R_t^b B_t + B_t - B_{t+1} + \left(1 + r_t^{junior}\right) Junior_t - Junior_{t+1} + \\ & + \left(1 + r_t^{bond}\right) bond_t - bond_{t+1} + D_{t+1} - \left(1 + R_t^d\right) D_t + \Delta K_{t+1}^b - \\ & - \frac{k_{kb}}{2}\left\{\frac{K_t^b}{\left[rw_{house}\left(B_t - b_t^E\right)\right] + \left[rw_{entr}\left(B_t - b_t^I\right)\right] + \left[rw_{junior}\left(Junior_t\right)\right]} - v^b\right\}^2 K_t^b] \end{aligned} \tag{8}$$

st.

$$(1-\eta)\, B_t + bond_t + Junior_t = K_t^b + D_t. \tag{9}$$

In equation (8) rw_{house}, rw_{entr} and rw_{junior} are the regulatory weights from Basel II capital requirements directive.
Wholesale unit chooses B_t.

2.3.2 Lending

$$\max_{\{r_t^{BH}(j), r_t^{BE}(j)\}} -(1-\eta)\, R_t^b B_t(j) - \frac{k_{BH}}{2}\left(\frac{r_t^{BH}(j)}{r_{t-1}^{BH}(j)} - 1\right)^2 r_t^{BH} b_t^I - \frac{k_{BE}}{2}\left(\frac{r_t^{BE}(j)}{r_{t-1}^{BE}(j)} - 1\right)^2 r_t^{BE} b_t^E +$$
$$+ c \times \eta \times B_t(j) + (1-\eta) \times \left[r_t^{BH}(j) b_t^I(j) + r_t^{BE}(j) b_t^E(j)\right], \tag{10}$$

where c is the cost of servicing received by the lending unit and η is the percentage of loans portfolio transferred to the SPV. Both η and c are fixed parameters.
Lending unit chooses $r_t^{BH}(j)$ and $r_t^{BE}(j)$.

2.3.3 SPV

$$\max_{\{r_t^{Senior}, r_t^{Mezz}, r_t^{Junior}\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \{\eta \left[r_t^{BH}(j) b_t^I(j) + r_t^{BE}(j) b_t^E(j)\right] + \eta \Delta B_{t+1} -$$
$$- c \times \eta \times B_t(j) - r_t^{Senior} Senior_t - r_t^{Mezz} Mezz_t - r_t^{Junior} Junior_t - \frac{k_{SPV}}{2}\left(\frac{K_t^{SPV}}{Junior_t} - 1\right)^2 K_t^{SPV}\} \tag{11}$$

st.

$$\eta \left[b_t^I(j) + b_t^E(j)\right] = Senior_t + Mezz_t + Junior_t + K_t^{SPV} \tag{12}$$

$$\left(1 + r_t^{CDO}\right) CDO_t \leq \left[\left(1 + r_t^{BH}(j)\right) b_t^I(j) + \left(1 + r_t^{BE}(j)\right) b_t^E(j)\right] \eta + \Delta K_t^{SPV} + CE_t \tag{13}$$

$$CDO_t = CDO_{t-1}\left(1 + r_{t-1}^{CDO}\right) + \eta \times \left[B_t(j) - B_{t-1}(j)\right], \tag{14}$$

where $CDO_t = Senior_t + Mezz_t + Junior_t$, r_t^{CDO} considers every rate for every tranche, while CE_t is the amount of credit enhancement.

2.3.4 Overall Profits

By aggregating profits from all units we can obtain:

$$
\begin{aligned}
& r_t^{BH}(j) b_t^I(j) + r_t^{BE}(j) b_t^E(j) + r_t^{bond} bond_t - r_t^d d_t - r_t^{Senior} Senior_t - r_t^{Mezz} Mezz_t - \\
& - \frac{k_{kb}}{2} \left\{ \frac{K_t^b}{\left[rw_{house} \left(B_t - b_t^E \right) \right] + \left[rw_{entr} \left(B_t - b_t^I \right) \right] + \left[rw_{junior} \left(Junior \right) \right]} - v^b \right\}^2 K_t^b - \\
& - \frac{k_{SPV}}{2} \left(\frac{K_t^{SPV}}{Junior_t} - 1 \right)^2 K_t^{SPV} - \\
& - \frac{k_{BH}}{2} \left(\frac{r_t^{BH}(j)}{r_{t-1}^{BH}(j)} - 1 \right)^2 r_t^{BH} b_t^I - \frac{k_{BE}}{2} \left(\frac{r_t^{BE}(j)}{r_{t-1}^{BE}(j)} - 1 \right)^2 r_t^{BE} b_t^E - \frac{k_d}{2} \left(\frac{r_t^d(j)}{r_{t-1}^d(j)} - 1 \right)^2 r_t^d d_t,
\end{aligned}
\tag{15}
$$

where the three final terms correspond to adjustment costs.

References

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