Model - Households

$$U(c_t, 1-h_t; S_t) = \frac{\left\{\mu\left[c_t^{\alpha}(1-h_t)^{1-\alpha}\right]^{\phi} + (1-\mu)(1/S_t)^{\phi}\right\}^{(1-\eta)/\phi} - 1}{1-\eta}$$

$$\max_{\{c_t, h_t, k_{t+1}\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t A_{ut} u(c_t, 1-h_t; S_t)$$

subject to

$$c_t(1+\tau_{ct}) + A_{xt}x_t(1+\tau_{xt}) = w_t h_t(1-\tau_{ht}) + (1-\tau_{kt})(r_t-\delta)k_t + \delta k_t + \psi_t$$

Model - Firms (Final)
Firms maximize profits

$$\max_{K_t, H_t, E_t} \Pi_1 = \{Y_t - w_t H_t - r_t K_t - p_t E_t\}$$

$$\text{s.t. } Y_t = (A_{kt}K_t)^{\theta_k}(A_{kt}H_t)^{\theta_h}(A_{et}E_t)^{\theta_e}$$

Solve for $p = \partial Y/\partial E$ and invert it for *energy demand*

$$E(p_t) \equiv E_t = p_t^{\theta_e - 1} A_{et}^{-1}\left[\theta_e A_{et}(A_{kt}K_t)^{\theta_k}(A_{kt}H_t)^{\theta_h}\right]^{\frac{1}{1-\theta_e}}$$

Model - Firms (Intermediate)

- Price takers in the input markets
- Monopolistically competitive in the output market

$$\max_{p_t} \Pi_2 = \{(p_t - \tau_{dt}\xi)E(p_t) - E(p_t)\}$$

Transform final goods into intermediate goods through their unique monopolistic blueprints, sold back as final goods.

$$\frac{\partial E(\cdot)}{\partial p}(p_t - \tau_{dt}\xi - 1) = -E(p_t)$$

$$\partial E(\cdot)/\partial p = (\theta_e - 1)A_{et}^{-1}p_t^{\theta_e - 2}\left[\theta_e A_{et}(A_{kt}K_t)^{\theta_k}(A_{kt}H_t)^{\theta_h}\right]^{\frac{1}{1-\theta_e}}$$

Model - Firms (Intermediate)

$$\frac{\partial E(\cdot)}{\partial p}(p_t - \tau_{dt}\xi - 1) = -E(p_t)$$

Solving this out, the *supply* of energy is given by

$$\mathcal{P}(\theta_e, \tau_d, \xi) \equiv \left(\frac{1}{1-\theta_e}\right)\left(\frac{1}{p_t}\right)^{\theta_e}(p_t - \tau_{dt}\xi - 1) + 1 = 0$$

Substituting this into the marginal product of energy, solved for energy, the equilibrium condition for energy is

$$E_t = \mathcal{P}(\theta_e, \tau_d, \xi)^{\theta_e - 1} A_{et}^{-1}\left[\theta_e A_{et}(A_{kt}K_t)^{\theta_k}(A_{kt}H_t)^{\theta_h}\right]^{\frac{1}{1-\theta_e}}$$

Equilibrium
We have $\{C_t, H_t, K_t, E_t, S_t\}$, so we need 5 equations

- Intertemporal Euler identifies C_t
- Intratemporal Euler identifies H_t

- Budget constraint identifies K_t
- Energy demand identifies E_t
- Pollution identity $S_t = \xi E_t$ identifies S_t