This is a basic 2 sector, 1 agent STATIC model.
Cobb-Douglas Utility
Cobb-Douglas Production for X, Y
The agent receives any TAX

Zero (Non positive) Profit Inequalities
(1)$P_X \leq \frac{w^{\alpha}(r)^{1-\alpha}(1+TX)}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$
(2)$P_Y \leq \frac{w^{1-\alpha}(r)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$

Goods Market Clear
(3)$X \geq \left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{P_Y}{P_X}\right)^{1-\beta}$
(4)$Y_t \geq \left(\frac{1-\beta}{\beta}\right)^{\beta}\left(\frac{P_X}{P_Y}\right)^{\beta}$

Factor Markets Clear
$L_X \geq \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{w}\right)^{1-\alpha}X$
$L_Y \geq \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{r}{w}\right)^{\alpha}Y$
$K_X \geq \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{w}{r}\right)^{\alpha}X$
$K_Y \geq \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{w}{r}\right)^{1-\alpha}Y$

(5)$L \geq L_X + L_Y$
(6)$K \geq K_X + K_Y$

Household Income
(7)$I = wL + rK + TX \cdot X \cdot w^{\alpha}r^{1-\alpha}$