

Research Inquiry

Ningru Zhao

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1 model

$$U = (1 - \lambda_h - \lambda_w)\log(c_t^f) + \lambda_h(\log c_t^h + v\log l_t^h) + \lambda_w(\log c_t^w + \phi\log l_t^w) \quad (1)$$

$$c_t^f = a_t^f (k_t^f)^\theta (n_t^{hf} + n_t^{wf})^{1-\theta} = y_t^f \quad (2)$$

$$y_t^m = a_t^m (k_t^m)^\alpha (n_t^{hm} + n_t^{wm})^{1-\alpha} \quad (3)$$

$$k_{t+1} = y_t^m - c_t^h - c_t^w + (1 - \delta)k_t \quad (4)$$

$$k_t = k_t^m + k_t^f \quad (5)$$

$$\begin{cases} l_t^h + n_t^{hf} + n_t^{hm} = 1 \\ l_t^w + n_t^{wf} + n_t^{wm} = 1 \end{cases} \quad (6)$$

$$i_t = y_t^m - c_t^h - c_t^w \quad (7)$$

$$\begin{cases} r_t^{kf} = \theta a_t^f (k_t^f)^{\theta-1} (n_t^{hf} + n_t^{wf})^{1-\theta} \\ r_t^{km} = \alpha a_t^m (k_t^m)^{\alpha-1} (n_t^{hm} + n_t^{wm})^{1-\alpha} \end{cases} \quad (8)$$

$$\begin{cases} w_t^{wf} = w_t^{hf} = (1 - \theta) a_t^f (k_t^f)^\theta (n_t^{hf} + n_t^{wf})^{-\theta} \\ w_t^{wm} = w_t^{hm} = (1 - \alpha) a_t^m (k_t^m)^\alpha (n_t^{hm} + n_t^{wm})^{-\alpha} \end{cases} \quad (9)$$

2 solution

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t (1 - \lambda_h - \lambda_w) \log(c_t^f) + \lambda_h (\log c_t^h + v \log l_t^h) + \lambda_w (\log c_t^w + \phi \log l_t^w)$$

$$s.t \quad c_t^f = a_t^f (k_t^f)^\theta (n_t^{hf} + n_t^{wf})^{1-\theta}$$

$$k_{t+1} = y_t^m - c_t^h - c_t^w + (1 - \delta)k_t$$

$$L = (1 - \lambda_h - \lambda_w) \log(c_t^f) + \lambda_h (\log c_t^h + v \log l_t^h) + \lambda_w (\log c_t^w + \phi \log l_t^w) + \mu_t (a_t^f (k_t^f)^\theta (n_t^{hf} + n_t^{wf})^{1-\theta} - c_t^f) + \eta_t (y_t^m - c_t^h - c_t^w + (1 - \delta)k_t - k_{t+1}) \quad (10)$$

2.1 F.O.C

$$\frac{1 - \lambda_h - \lambda_w}{c_t^f} = \mu_t \quad (11)$$

$$\frac{\lambda_h}{c_t^h} = \eta_t \quad (12)$$

$$\frac{\lambda_w}{c_t^w} = \eta_t \quad (13)$$

$$\frac{\lambda_h v}{l_t^h} = \mu_t w_t^{hf} \quad (14)$$

$$\frac{\lambda_w \phi}{l_t^w} = \mu_t w_t^{wf} \quad (15)$$

$$\frac{\lambda_h v}{l_t^h} = \eta_t w_t^{hm} \quad (16)$$

$$\frac{\lambda_w \phi}{l_t^w} = \eta_t w_t^{wm} \quad (17)$$

$$\eta_t = \beta E_t \mu_{t+1} r_t^{kf} + \beta E_t \eta_{t+1} (1 - \delta) \quad (18)$$

$$\eta_t = \beta E_t \eta_{t+1} (r_t^{km} + 1 - \delta) \quad (19)$$