Research Inquiry

Ningru Zhao

August 4, 2014

Outline

Utility Function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [\frac{(c^\gamma (1-N)^{1-\gamma})^{1-\sigma} - 1}{1-\sigma}] \quad (1)$$

Budget Constrain

$$S_t + C_t + B_t \leq (1 + r_t^e)S_{t-1} + (1 + r_{t-1}^f)B_{t-1} + W_t N_t \quad (2)$$

- S_t is the stock shares that the representative household purchases from firms in the economy. B_t is the bonds that the representative household issues in the economy.

Optimization

$$1 + r_t^e = \frac{U_{c,t}}{\beta U_{c,t+1}} \tag{3}$$

Production Function

$$Y_t = A_t(u_t K_t)^{1-\alpha}(N_t)^{\alpha} \tag{4}$$

- u_t is capital utilization rate.
- A_t is the aggregate technological shocks that have the following process.

$$log(A_t) = \rho_A log(A_{t-1}) + \epsilon_{A,t} \tag{5}$$

- $\epsilon_{A,t}$ is i.i.d normal distribution, $N(0, \sigma_A^2)$.

Capital Accumulation Process

$$K_{t+1} = [1 - \Phi(\frac{I_t}{I_{t-1}})]I_t + [1 - \phi(u_t)]K_t \tag{6}$$

- $\Phi(\frac{I_t}{I_{t-1}})$ is the adjustment cost for investment.

$$\Phi(\frac{I_t}{I_{t-1}}) = (\frac{I_t}{I_{t-1}} - 1)^2 \tag{7}$$

- $\phi(u_t)$ denotes the function of capital utilization rate in production process.

$$\phi(u_t) = \omega_0 + \omega_1(u_t - 1) + \frac{\omega_2}{2}(u_t - 1)^2 \tag{8}$$

Model Completion

$$Y_t = C_t + I_t \tag{9}$$

$$A_t(u_t K_t)^{1-\alpha}(N_t)^{\alpha} = C_t + I_t \tag{10}$$

Optimization

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - N_t)$$

$$s.t \quad A_t(u_t K_t)^{1-\alpha}(N_t)^{\alpha} = C_t + I_t$$

$$K_{t+1} = [1 - \Phi(\frac{I_t}{I_{t-1}})]I_t + [1 - \phi(u_t)]K_t$$

First Oder Conditions

$$U_c = \mu_t \tag{11}$$

$$U_N = \mu_t \alpha A_t (u_t K_t)^{1-\alpha} (N_t)^{\alpha-1} \tag{12}$$

$$\mu_t (1-\alpha) A_t (u_t)^{-\alpha} K_t^{1-\alpha} N_t^{\alpha} = q_t \phi_u(u_t) K_t \tag{13}$$

$$\mu_t = q_t [1 - \Phi(\frac{I_t}{I_{t-1}}) - \Phi_I(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}}] + E_t[\beta q_{t+1} \Phi_I(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2] \tag{14}$$

$$q_t = \beta E_t[\mu_{t+1}(1-\alpha) A_{t+1} (u_{t+1})^{1-\alpha} (K_{t+1})^{-\alpha} N_{t+1}^{\alpha} + q_{t+1}(1 - \phi(u_{t+1}))] \tag{15}$$

$$1 + r_{t+1}^e = E_t \frac{1}{\beta} [\frac{q_t}{q_{t+1}} \frac{\phi_u(u_t)}{\phi_u(u_{t+1})} \frac{A_{t+1}}{A_t} (\frac{N_{t+1} u_t K_t}{N_t u_{t+1} K_{t+1}})^{\alpha}] \tag{16}$$

Model Solution

- μ_t and q_t are Lagrangian multiplier associated with equation(10) and equation (6), respectively.
- Combining all the first order conditions for consumer and firm, and all auxiliary conditions, I can compute the sequence of C_t, Y_t, I_t, K_t, N_t, μ_t, q_t, u_t and r_t^e.
- Note: consider the capacity utilization rate u_t is unit in equilibrium and the value of Φ function should be equal to discount rate 0.025, get $\omega_0 = 0.025$, $\omega_1 = 0.0454$ and ω_2 is free to calibrate.