

Fiscal Devaluation in Countries with Large Underground Sectors

Abstract

Fahri et al. (2013) have shown how it is possible to mimic the effect of a devaluation of the exchange rate in countries that have a fixed exchange rate. This is done by an appropriate mix of conventional tax and subsidy. I study a fiscal devaluation in a small open economy/ where underground sectors and tax evasion are relevant. Domestic firms are able to hire a share of their labor force from the underground market and to evade the VAT tax.

1 Introduction

Fiscal devaluations have been proposed since the time of the Gold Standard as a way of restoring competitiveness of countries unable to devalue their exchange rate. Recently, fiscal devaluation have been proposed as a way to overcome the fixed exchange rate of the European currency union.

Fiscal devaluation is defined as an increase in the value added tax timed with a decrease of the labor fiscal wedge (i.e. a reduction in social contributions). Due to the fact that the VAT tax is rebated to exporter and imposed on importers, a raise alters the relative prices of foreign and domestic goods, boosting internal demand. To avoid a change of domestic price,

An empirical analysis of the effects of past fiscal devaluation is presented by [de Mooji and Keen, 2011]. The authors find that fiscal devaluation do positively impact trade balances with significant short run effects, but that over time effects tends to wear off.

Fahri et al. (2013) showed how a small set of fiscal instruments can be used to robustly replicate the real allocation achieved under a depreciation of the nominal exchange rate. Their setup is that of a two country model Dynamic Stochastic General Equilibrium model. The authors note that their analysis should be expanded in order to cover the effect tax evasion.

Busato and Chiarini (2004) provide a DSGE setup where firm employ labor in a market and underground sector where no taxation is due. They show that such a model provides a stronger amplification mechanism and matches the data better.

The aim of this work is to provide a study of fiscal devaluation in a context where tax evasion is available. The main structure of the model borrows from Fahri et al. in that we have a two country, symmetric model with incomplete market. To this I add firms and consumers who allocates labor between a market sector and a tax free underground sector in a similar way to what happens in Busato and Chiarini.

Furthermore, I allow for the evasion of the VAT tax. As an extension, I allow for a different distribution of the gains from the reduction of the labor fiscal wedge.

The model is calibrated to match salient aspects of the Italian economy.

2 The model

2.1 Domestic household

There is an infinitely lived representative domestic household with preferences over sequences of consumption C_t , money holding M_t and labor N_t . To represent the underground labor market the household also allocates a share $\theta_t \in (0,1)$ of its labor to the market sector and $1 - \theta_t$ to the underground sector.

Consumption is a CES aggregator of domestic and foreign goods:

$$C_t = \left[\gamma_D^\sigma C_{Dt}^{\frac{\sigma-1}{\sigma}} + \gamma_F^\sigma C_{Ft}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

with σ being the elasticity of substitution and $\gamma_\ell \in [0,1]$ with $\ell = \{D, F\}$ allowing for domestic/foreign bias.

The household problem is to maximize the expected lifetime utility function $\mathbb{U}_0 = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, N_t, \theta_t, m_t)$. The utility function is a modified version of the one presented in [Busato and Chiarini, 2004] with the following functional form:

$$u(C_t, N_t, \theta_t, m_t) = \frac{C_t^{1-q} - 1}{1-q} - h \frac{N_t^{1+\gamma}}{1+\gamma} - f \frac{(1-\theta_t)^{1-v}}{1-v} + \chi \frac{m_t^{1-\varphi}}{1-\varphi}$$

The term parametrized by h represents total disutility from labor. The term in f disutility from working in the underground sector which might arise due to the lack of social security or health coverage and/or from social stigma.

Each period the households receives nominal wages W_t and the rental rate of capital R_t . Furthermore, households earns the profits from the final good and intermediate good producers, Π_t and ϖ_t respectively. Each period the household saves asset B_{t+1} at price Q_t and payoff D_t , stocks money M_t , consumes and invests in a CES aggregator I_t , identical to the consumption one. Hence, the budget constraint is as follow:

$$\begin{aligned}
P_t & \left((1 - \zeta_t^c) C_t + (1 - \zeta_t^I) I_t \right) + Q_t B_{t+1} + M_t \\
& \leq (1 - \tau_t^d) (\Pi_t + \varpi_t) + W_t N_t [(1 - \tau_t^n) \theta_t + (1 - \theta_t)] + (1 - \tau_t^k) R_t K_t + M_{t-1} \\
& \quad + Q_t (B_t + D_t) + T_t
\end{aligned}$$

where ζ_t^c, ζ_t^I are consumption and investment subsidy; $\tau_t^d, \tau_t^n, \tau_t^k$ are respectively the dividend, income and capital tax rates.

I_t is the CES investment aggregator of domestic and foreign goods, and P_t is the price aggregator of said goods, assuming that prices are inclusive of VAT taxes:

$$\begin{aligned}
I_t & = \left[\gamma_D^\sigma I_{Dt}^{\frac{\sigma-1}{\sigma}} + \gamma_F^\sigma I_{Ft}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
P_t & = [\gamma_D P_{Dt}^{1-\sigma} + \gamma_F P_{Ft}^{1-\sigma}]^{\frac{1}{1-\sigma}}
\end{aligned}$$

The law of motion for capital has, for simplicity, the standard form

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Define $m_t = \frac{M_t}{P_t(1-\zeta_t^c)}$ to be the real money balance, then the Lagrangian for the household problem is the following

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t & \left\{ \frac{C_t^{1-q} - 1}{1-q} - h \frac{N_t^{1+\gamma}}{1+\gamma} - f \frac{(1-\theta_t)^{1-v}}{1-v} \right. \\
& \quad + \frac{\chi m_t^{1-\varphi}}{1-\varphi} - \lambda_t (P_t ((1 - \zeta_t^c) C_t + (1 - \zeta_t^I) (K_{t+1} - (1 - \delta)K_t)) + Q_t B_{t+1} + M_t \\
& \quad - (1 - \tau_t^d) (\varpi_t + \Pi_t) - (Q_t + D_t) B_t - W_t N_t [(1 - \tau_t^n) \theta_t + (1 - \theta_t)] \\
& \quad \left. - (1 - \tau_t^k) R_t K_t - M_{t-1} - T_t \right\}
\end{aligned}$$

yielding the following first order conditions

$$\mathcal{L}_{C_t}: C_t^{-q} = \lambda_t P_t (1 - \zeta_t^c) \quad (1)$$

$$\mathcal{L}_{N_t}: h N_t^\gamma = \lambda_t W_t (1 - \tau_t^n \theta_t) \quad (2)$$

$$\mathcal{L}_{\theta_t}: f (1 - \theta_t)^{-v} = \lambda_t W_t N_t \tau_t^n \quad (3)$$

$$\mathcal{L}_{K_{t+1}}: \lambda_t P_t (1 - \zeta_t^I) = \beta \mathbb{E}_t \{ \lambda_{t+1} P_{t+1} [(1 - \delta) (1 - \zeta_{t+1}^I) + (1 - \tau_{t+1}^k) r_{t+1}] \} \quad (4)$$

$$\mathcal{L}_{B_{t+1}}: \lambda_t Q_t = \beta \mathbb{E}_t[\lambda_{t+1}(Q_{t+1} + D_{t+1})] \quad (5)$$

$$\mathcal{L}_{B_{t+1}}: \frac{\chi M_t^{-\varphi}}{(P_t(1-\zeta_t^c))^{1-\varphi}} = \lambda_t - \beta \mathbb{E}_t[\lambda_{t+1}] \quad (6)$$

Plugging (1) into (4) we can immediately derive the usual Euler equation for the intratemporal condition on consumption:

$$C_t^{-q} \frac{(1-\zeta_t^l)}{(1-\zeta_t^c)} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-q} \left[(1-\delta) \frac{(1-\zeta_{t+1}^l)}{(1-\zeta_{t+1}^c)} + (1-\tau_{t+1}^k) r_{t+1} \right] \right\} \quad (7)$$

Also, note that by the properties of the CES aggregator of consumption we can derive demand for the Domestic and Foreign consumption good given the relative prices as

$$C_{\ell t} = \gamma_\ell \left(\frac{P_{\ell t}}{P_t} \right)^{-\sigma} C_t, \quad \ell = \{D, F\}$$

From (2) and (3) we are able to derive the intratemporal condition for total labor supplied and share allocated to the underground sector which takes the form:

$$N_t = \left[\left(\frac{f}{h\tau_t^n} \right) \frac{1-\tau_t^n \theta_t}{(1-\theta_t)^\nu} \right]^{\frac{1}{1+\gamma}} \quad (8)$$

from this we obtain the Euler equation by expressing the Lagrange multiplier in terms of share of the underground sector

$$\lambda_t = \theta_t = \frac{h}{w_t} \left(\frac{f}{h\tau_t^n} \right)^{\frac{\gamma}{1+\gamma}} \left[\frac{(1-\theta_t)^{-\nu\gamma}}{1-\tau_t^n \theta_t} \right]^{\frac{1}{1+\gamma}} \quad (9)$$

and then plugging (9) into (4)

$$\theta_t P_t (1-\zeta_t^l) = \beta \mathbb{E}_t \{ \theta_{t+1} [(1-\delta)(1-\zeta_{t+1}^l) P_{t+1} + (1-\tau_{t+1}^k) R_{t+1}] \} \quad (10)$$

By (5) we define the stochastic discount factor of the representative household to be, generalizing to any number s of period ahead,

$$S_{t,s} = \beta^{s-t} \frac{\lambda_{t+s}}{\lambda_t} = \beta^{s-t} \left(\frac{C_{t+s}}{C_t} \right)^{-q} \frac{P_t(1-\zeta_t^c)}{P_{t+s}(1-\zeta_{t+s}^c)} \quad (11)$$

pricing the **available asset in terms of consumption**

$$Q_t = \mathbb{E}_t[\mathbb{S}_{t+1}(Q_{t+1} + D_{t+1})] \quad (12)$$

Finally, by making use of (6) and the definition in (11) we derive the money demand equation

$$\chi C_t^q \left(\frac{M_t}{P_t(1-\zeta_t^c)} \right)^{-\varphi} = 1 - \beta \mathbb{E}_t[\mathbb{S}_{t+1}] \quad (13)$$

where the right-hand side is an increasing function of the nominal interest rate i_t and satisfying $1 + i_t = 1/\mathbb{E}_t[\mathbb{S}_{t+1}]$.

This concludes the specification of the domestic household problem. The setup closely mirrors that of [Fahri et al. , 2013]. Foreign sector of the economy will be specified later on.

2.2 Domestic Firms

There are two types of firms: a continuum of monopolistically competitive intermediate good producers; a final good producer who aggregates all varieties into a single homogenous good.

Intermediate producers rent capital and labor from the household, also allocating labor between the market and the underground sectors in order to avoid the cost of the payroll tax ζ_t^p . Goods pricing is subject to a Calvo friction.

The final good producers sells to both domestic and foreign costumers and in doing so also sets $\eta_t \in (0,1)$ which is the share of VAT tax rate evaded.

I discuss first the simpler problem of the final good producer and turn to the intermediate good producers later, the $j \in [0,1]$ subscripts denoting a generic intermediate good producer.

2.2.1 Final Good Producer

A competitive firm aggregates all varieties produced by domestic firms into an homogeneous good which is later sold both at home and on the foreing market. The production function is as follows

$$Y_t = \left[\int_0^1 Y_{jt}^{\lambda-1} dj \right]^{\frac{\lambda}{\lambda-1}}$$

so that by the CES aggregator properties we obtain demand for each variety as

$$Y_{it} = \left(\frac{\mathcal{P}_{jt}}{\mathcal{P}_t} \right)^{-\lambda} Y_t$$

with

$$\mathcal{P}_t = \left(\int_0^1 \mathcal{P}_{jt}^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}}$$

The firm is competitive, therefore taking prices as given and maximizes the following profit function:

$$\Pi_t = [(1 - \tau_t^v)\eta_t + (1 - \eta_t)(\gamma + (1 - \gamma)(1 - s_t^v \tau_t^v))] P_{Dt} C_{Dt} + (1 + \zeta_t^{ex}) \mathcal{E}_t P_{Dt}^* C_{Dt}^* - \rho \frac{\eta_t^2}{2} - Y_t \mathcal{P}_t \quad (14)$$

where τ_t^v is the VAT tax rate, s_t^v is the surcharge on the VAT if evasion is discovered which happens with probability given by the Jewish letter dalet, γ . Hence, the term in bracket represents the expected value of evading the VAT taxes for any given η_t , which is the amount of VAT evaded. The squared term is a cost function of evasion which can be interpreted as costs of audit, double book-keeping and social stigma attached to evasion.

Finally, \mathcal{E}_t is the nominal exchange rate, P_{Dt}^* is price in foreign currency of exported goods, C_{Dt}^* is foreign demand of domestic goods and ζ_t^{ex} is an export subsidy from the government. Therefore, we stick to the convention in [Fahri et al., 2013] defining the nominal exchange rate as the price of one unit of foreign currency in terms of units of home currency, implying immediately that higher values of \mathcal{E}_t entail home currency depreciation.

I will assume for simplicity that the firm operates under producer currency pricing, i.e. that the domestic price is fixed and foreign price satisfies the law of one price. Furthermore VAT evasion is negligible with respect to the size of export so that $P_{Dt} = P_{Dt} = \mathcal{E}_t \frac{1 + \zeta_t^{ex}}{\phi(\tau_t^v, \eta_t)} P_{Dt}^*$

Given that firm is competitive, $P_{Dt} = \mathcal{P}_t$ so that the only choice variable for the final good producer is η_t leading to the following first order condition:

$$\frac{\partial \Pi_t}{\partial \eta_t}: [(1 - \tau_t^v) - (\gamma + (1 - \gamma)(1 - s_t^v \tau_t^v))] P_{Dt} C_{Dt} = \rho \eta_t \quad (15)$$

2.2.2 Intermediate good producers

There is a continuum of firms $j \in [0,1]$ producing a differentiated good Y_{jt} according to the following production function:

$$Y_{jt} = A_t K_{jt}^\alpha [N_{jt} \theta_{jt}]^{1-\alpha} + Z_t (1 - \theta_{jt}) N_{jt}$$

Where K_{jt} is the capital rented for production and N_{jt} is the total labor input $\theta_{jt} \in (0,1)$ represents the share of labor allocated in the market sector and conversely $1 - \theta_{jt}$ is the share of labor in the underground sector. A_t and Z_t are two shocks to productivity in the two sectors following an AR(1) process

$$A_{t+1} = \rho_A A_t + \epsilon_t^A, \quad \text{with } \epsilon_t^A \sim \mathcal{N}(0, \sigma_A)$$

$$Z_{t+1} = \rho_Z Z_t + \epsilon_t^Z, \quad \text{with } \epsilon_t^Z \sim \mathcal{N}(0, \sigma_Z)$$

Notice that such processes entail a zero growth rate of technology.

Real profit for the j -th firm are given by

$$\varpi_{jt} = Y_{jt} - r_t K_{jt} - w_t N_{jt} [(1 + \zeta_t^p) \theta_{jt} + \wp (1 - \theta_{jt})]$$

where $\wp = [\gamma + (1 - \gamma)(1 + s_t^p \zeta_t^p)]$ is the expected value of evading the payroll tax (i.e. hiring labor in the underground sector), with γ the probability of a firm being caught evading and made to pay a surcharge equal to s_t^p . γ always represent the probability of a successful audit and is therefore a general measure of government efficiency.

First order conditions are:

$$\frac{\partial \varpi_{jt}}{\partial K_{jt}} : \alpha A_t \left[\frac{N_{jt} \theta_{jt}}{K_{jt}} \right]^{1-\alpha} = r_t \quad (16)$$

$$\frac{\partial \varpi_{jt}}{\partial N_{jt}} : (1 - \alpha) A_t \left(\frac{K_{jt}}{N_{jt} \theta_{jt}} \right)^\alpha \theta_{jt} + Z_t (1 - \theta_{jt}) = w_t [(1 + \zeta_t^p) \theta_{jt} + \wp (1 - \theta_{jt})] \quad (17)$$

$$\frac{\partial \varpi_{jt}}{\partial \theta_{jt}} : \left[(1 - \alpha) A_t \left(\frac{K_{jt}}{N_{jt} \theta_{jt}} \right)^\alpha - Z_t \right] N_{jt} = w_t N_{jt} \underbrace{[1 + \zeta_t^p - \wp]}_{\Sigma} \quad (18)$$

From (16) and (18) we can derive an expression for the optimal capital-labor ratio in the market sector

$$\Lambda_t = \frac{K_{jt}}{N_{jt} \theta_{jt}} = \frac{\alpha}{1 - \alpha} \frac{w_t \Sigma + Z_t}{r_t} \quad (19)$$

also from (17) we obtain the ratio between market and underground sector share

$$\Gamma_t = \frac{1 - \theta_{jt}}{\theta_{jt}} = \frac{(1 - \alpha) A_t \Lambda_t^\alpha - (1 + \zeta_t^p) w_t}{\wp w_t - Z_t} \quad (20)$$

then we derive an expression for the marginal cost. The function has constant return to scale for any given level of θ_t , therefore setting $Y_{jt} = 1$ and plugging in (19)

$$\Omega_t = \theta_{jt} N_{jt} = [A_t \Lambda_t + Z_t \Gamma_t]^{-1} \quad (21)$$

and then manipulating the real cost component of the profit function, again substituting the capital-labor market sector ratio and using (21) we obtain an expression for real marginal costs

$$mc_t = \left[\frac{\alpha}{1-\alpha} (w_t \Sigma + Z_t) + w_t (1 + \zeta_t^p + \rho \Gamma_t) \right] \Omega_t \quad (22)$$

Firms set prices according to a Calvo friction. Any firm is able to adjust its price with probability $1 - \zeta$ so that the current price of each firm is given by the following

$$\mathcal{P}_{jt} = \begin{cases} \mathcal{P}_{j,t-1} & \text{with probability } \zeta \\ \bar{\mathcal{P}}_{jt} & \text{with probability } 1 - \zeta \end{cases}$$

where $\bar{\mathcal{P}}_{jt}$ is the reset price. Each firm shall choose a reset price such that it maximizes expected value profit valued according to the household discount factor $\sum_{s \geq t} \zeta^{t-s} \mathbb{E}_t [\mathcal{S}_{t,s} (1 - \tau_s^d) \omega_{is}]$ which can also be written, making use of (22)

$$\begin{aligned} \max_{\bar{\mathcal{P}}_{it}} \mathbb{E}_t \sum_{s \geq t} (\beta \zeta)^{s-t} \frac{\lambda_s}{\lambda_t} \left[\left(\frac{\bar{\mathcal{P}}_{it}}{P_s} (1 - \tau_s^d) - mc_t \right) Y_{is} \right] \\ \text{s. to } Y_{is} = \left(\frac{\bar{\mathcal{P}}_{it}}{P_s} \right)^{-\lambda} Y_s \end{aligned}$$

that is

$$\max_{\bar{\mathcal{P}}_{it}} \mathbb{E}_t \sum_{s \geq t} (\beta \zeta)^{s-t} \frac{\lambda_s}{\lambda_t} \left[\left(\left(\frac{\bar{\mathcal{P}}_{it}}{P_s} \right)^{1-\lambda} (1 - \tau_s^d) - mc_s \left(\frac{\bar{\mathcal{P}}_{it}}{P_s} \right)^{-\lambda} \right) Y_s \right]$$

Now, deriving in $\bar{\mathcal{P}}_{jt}$, removing all irrelevant constants we obtain the FOC defining the optimal reset price as

$$\mathbb{E}_t \sum_{s \geq t} (\beta \zeta)^{s-t} \lambda_s [(1 - \lambda)(1 - \tau_t^d) + \lambda \bar{\mathcal{P}}_t mc_s] \mathcal{P}_s^\lambda Y_s = 0 \quad (23)$$

where we made use of the fact that in a symmetric equilibrium $\bar{\mathcal{P}}_{jt} = \bar{\mathcal{P}}_t$ for all j . Hence, the reset price satisfies

$$\bar{\mathcal{P}}_t = \frac{\mathbb{E}_t \sum_{s \geq t} \lambda_s (\beta \zeta)^{s-t} [(1 - \lambda)(1 - \tau_s^d) \mathcal{P}_s^\lambda Y_s]}{\mathbb{E}_t \sum_{s \geq t} \lambda_s (\beta \zeta)^{s-t} \lambda mc_s \mathcal{P}_s^\lambda Y_s}$$

Notice, that (23) can also be recursively expressed as $\lambda g_t^1 = (\lambda - 1)g_t^2$ where

$$\begin{aligned} g_t^1 &= \{\lambda_t m c_t \mathcal{P}_t^\lambda Y_t + \beta \zeta \mathbb{E}_t[g_{t+1}^1]\} \\ g_t^2 &= \lambda_t (1 - \tau_t^d) \mathcal{P}_t^\lambda Y_t + \beta \zeta \mathbb{E}_t[g_{t+1}^2] \end{aligned}$$

Finally, we determine the behaviour of the aggregate pricing index under Calvo's rule as follows

$$\mathcal{P}_t = \left(\int_0^1 \mathcal{P}_{jt}^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}} = \left(\int_0^{1-\zeta} \bar{\mathcal{P}}_{jt}^{1-\lambda} dj + \int_{1-\zeta}^1 \mathcal{P}_{j,t-1}^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}} = \left[(1-\zeta) \bar{\mathcal{P}}_t^{1-\lambda} + \zeta \mathcal{P}_{t-1}^{1-\lambda} \right]^{\frac{1}{1-\lambda}}$$

and therefore dividing both sides by \mathcal{P}_{t-1} we obtain inflation dynamics as

$$\begin{aligned} \left(\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} \right)^{1-\lambda} &= \zeta + (1-\zeta) \left(\frac{\bar{\mathcal{P}}_t}{\mathcal{P}_{t-1}} \right)^{1-\lambda} \\ \mathcal{P}_t^{1-\lambda} &= \zeta \mathcal{P}_{t-1}^{1-\lambda} + (1-\zeta) \bar{\mathcal{P}}_t^{1-\lambda} \end{aligned}$$

For computational purpose it is also useful to set a price dispersion term given by

$$v_t^p = \int_0^1 \left(\frac{\mathcal{P}_{jt}}{\mathcal{P}_t} \right)^{-\lambda} dj$$

which using the above derivation behaves as

$$v_t^p = \zeta \left(\frac{\mathcal{P}_{t-1}}{\mathcal{P}_t} \right)^{-\lambda} v_{t-1}^p + (1-\zeta) \left(\frac{\bar{\mathcal{P}}_t}{\mathcal{P}_t} \right)^{-\lambda}$$

2.3 Government problem

We now specify the government problem. The budget constraint of the government is given by

$$M_t - M_{t-1} + TR_t = T_t$$

where $M_t - M_{t-1}$ is seignorage revenue from money supply, TR_t is the total of the tax revenue and T_t is a lump sum transfer to households.

$$\begin{aligned} TR_t &= [\zeta_t^l P_t I_t + \tau_t^d (\varpi_t + \Pi_t) + W_t N_t \tau_t^n \theta_t + \tau_t^k R_t K_t] \\ &\quad + [\tau_t^v \eta_t P_{Dt} C_{Dt} + (1-\eta_t)(1-\gamma) s_t^v \tau_t^v P_{Dt} C_{Dt} - \zeta_t^{ex} \mathcal{E}_t P_{Dt}^* C_{Dt}^*] + \\ &\quad + [\tau_t^v C_{Ft} P_{Ft} + \zeta_t^p W_t N_t \theta_t + (1-\gamma) s_t^p \zeta_t^p (1-\theta_t)] + \tau_t^v C_{Ft} P_{Ft} + \tau_t^v I_{Ft} \end{aligned}$$

2.4 Financial markets

Markets are incomplete and here exists only one bond denominated in foreign currency, with payoff $D_{t+1}^* = 1$ whose price in foreign currency is then given by

$$Q_{t+1}^* = \frac{1}{1+i_{t+1}^*}$$

The world interest rate depends on how much the country as a whole borrows

$$i_{t+1}^* = \bar{i} + (e^{B-B_{t+1}} - 1) + \epsilon_t^B$$

and $\epsilon_t^B \sim \mathcal{N}(0, \sigma_B)$ and B is debt steady state level, $\bar{i} = \frac{1}{\beta} - 1$

Since the domestic and foreign economies are symmetric the international risk sharing condition is given by

$$\mathbb{E}_t \left\{ \frac{q_{t+1}^* + d_{t+1}^*}{q_t^*} \left[S_{t+1} - S_{t+1}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] \right\} = 0$$

which can be rewritten, using the definition of the stochastic discount factors as,

$$\mathbb{E}_t \left\{ \frac{q_{t+1}^* + d_{t+1}^*}{q_t^*} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-q} \frac{Q_{t+1}}{Q_t} - \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-q} \right] \right\} = 0$$

where $Q_t = \frac{P_t^*}{P_t(1-\zeta_t^c)}$ is the real interest rate.

By summing up the budget constraint of the household, the firms profit and the government budget constraint we obtain the home country budget constraint as

$$Q_t B_{t+1} - B_t(Q_t + D_t) = \mathcal{E}_t P_{Dt}^* X_{Dt}^* - (1 - \tau_t^v) P_{Ft} C_{Ft} - (1 - \tau_t^v) P_{Ft} I_{Ft} - \frac{\eta_t^2}{2}$$

which can be rewritten, dividing by $P_t^* \mathcal{E}_t$,

$$q_t^* B_{t+1} - (q_t^* + d_t^*) B_t = \frac{P_{Dt}^*}{P_t^*} [X_{Dt}^* - \Delta_t X_{Ft}] - \frac{\eta_t^2}{2 P_t^* \mathcal{E}_t}$$

where $\Delta_t = \frac{(1-\tau_t^v) P_{Ft}}{\mathcal{E}_t P_{Dt}^*}$ are the terms of trade. The right hand side is the trade surplus, given $X_{Dt}^* = C_{Dt}^* + I_{Dt}^*$ is foreign demand and X_{Ft} is import.

2.5 Market Clearing and aggregation

We now list all market clearing conditions for the domestic side of the economy

$$\int_0^1 \theta_{jt} dj = \theta_t$$

$$\int_0^1 K_{jt} dj = K_t$$

$$\int_0^1 N_{jt} dj = N_t$$

$$Y_t = \int_0^1 Y_{jt} dj = A_t K_t^\alpha [N_t \theta_t]^{1-\alpha} + Z_t (1 - \theta_t) N_t$$

$$Y_t = C_{Dt} + C_{Dt}^* + I_{Dt} + I_{Dt}^* = X_{Dt} + X_{Dt}^*$$

$$B_t = -B_t^*$$

$$P_{Ft} (1 - \tau_t^v) = \varepsilon_t P_{Ft}^*$$

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$$\varepsilon_t = \varepsilon_0 \delta_t$$

4 Calibration

The model is calibrated to match relevant aspects of the Italian economy, borrowing parameters from the work of Busato and Chiarini (2004). The rest of the parameters are standard in the literature.

σ		
γ_D		
γ_F		
q		
h		
γ		
v		
χ		
φ		
ζ_t^c		
ζ_t^l		
τ_t^v		

τ_t^d		
τ_t^n		
τ_t^k		
δ		
β		
λ		
ϵ		
γ		
ζ_t^{ex}		
ρ		
α		
ρ^A		
ρ^Z		
ρ^{A*}		
ρ^{Z*}		
ρ^B		
s^v		
s^p		
ζ_t^p		
φ		Steady state
ζ		
\mathcal{E}_0		
B		SS

5 Simulation and results

6 Appendix A – Foreign Side Equations

7 Appendix B – Dynare Code

8 Bibliography