EF 9906: Theory and Methods in International Finance

Nken Moïse
Assignment 1

September 24, 2014

The model[1]

Households maximization problem:

Defining the households utility function,

$$E_0 \sum_{t=0}^{+\infty} \beta^t U(c_t, (1-l_t)) = \frac{(c_t^{\phi}.(1-l_t)^{(1-\phi)})^{(1-\alpha)}}{(1-\alpha)}$$

Therefore, the problem is:

$$\underset{c_t,k_t,u_t,l_t}{max} E_0 \sum_{t=0}^{+\infty} \beta^t \frac{(c_t^{\phi}.(1-l_t)^{(1-\phi)})^{(1-\alpha)}}{(1-\alpha)}$$

subject to:

$$k_t = (1-\delta(u_t)).k_{t-1} + i_t, \ where\, \delta(u_t) = \frac{\omega_0.u_t^{\omega_1}}{\omega_1} \tag{1}$$

$$c_t + i_t + p_t.e_t = w_t.l_t + r_t.u_t.k_{t-1} \tag{2}$$

$$e_t = a_t(u_t).k_{t-1}, \ where\, a(u_t) = \frac{\upsilon_0.u_t^{\upsilon_1}}{\upsilon_1} \tag{3}$$

$$logz_t = \rho logz_{t-1} + \varepsilon_t, \ where\, \varepsilon_t \sim iid(0, \sigma_\varepsilon^2) \tag{4}$$

The lagrangian is:

$$L(.) = E_0 \sum_{t=0}^{+\infty} \beta^t \{U(c_t,(1-l_t)) + \lambda_t(w_t.l_t + r_t.k_{t-1}.u_t - k_t + (1-\delta(u_t)).k_{t-1} - c_t - p_t.a(u_t).k_{t-1})\} \tag{5}$$

First order conditions (F.O.C):

$$c_t : \lambda_t = U_1(c_t, (1-l_t)) \tag{6}$$

$$l_t : U_2(c_t, (1-l_t)) = \lambda_t.w_t \tag{7}$$

$$k_t : \beta^t \lambda_t = E_t \beta^{t+1} \lambda_{t+1}(r_{t+1}.u_{t+1} - p_{t+1}.a(u_{t+1}) + (1-\delta(u_{t+1})) \tag{8}$$

$$u_t : \delta^{'}(u_t).k_{t-1} + p_t.a_t^{'}(u_t).k_{t-1} = r_t.k_{t-1} \tag{9}$$

$$\lambda_t : c_t + k_t - (1-\delta(u_t))k_{t-1} + p_t.a(u_t).k_{t-1} = w_t.l_t + r_t.u_t.k_{t-1} \tag{10}$$

[1]The model is written in the Dynare syntax.

Let define,

$$U_1(c_t,(1-l_t)) = \frac{\partial U(c_t,(1-l_t))}{\partial c_t} = \frac{\phi.(c_t^{\phi}.(1-l_t)^{(1-\phi)}))^{(1-\alpha)}}{c_t} \tag{11}$$

$$U_2(c_t,(1-l_t)) = \frac{\partial U(c_t,(1-l_t))}{\partial l_t} = -\frac{(1-\phi).(c_t^{\phi}.(1-l_t)^{(1-\phi)}))^{(1-\alpha)}}{(1-l_t)} \tag{12}$$

$$\delta^{'}(u_t) = \frac{\partial \delta(u_t)}{\partial u_t} = \omega_0.u^{(\omega_1-1)} \tag{13}$$

$$a^{'}(u_t) = \frac{\partial a(u_t)}{\partial u_t} = \upsilon_0.u^{(\upsilon_1-1)} \tag{14}$$

Combining F.O.C's c_t and l_t give us:

$$w_t.U_1(c_t,(1-l_t)) = U_2(c_t,(1-l_t))$$

$\Rightarrow$ Using definition of $U_i(c_t,(1-l_t))$, with $i=1,2$ yields :

$$w_t = \frac{c_t.(1-\phi)}{\phi.(1-l_t)} \tag{15}$$

Combining c_t and k_t's F.O.C give us:

$$U_1(c_t,(1-l_t)) = E_t\beta U_1(c_{t+1},(1-l_{t+1}))(r_{t+1}.u_{t+1} - p_{t+1}.a(u_{t+1}) + (1-\delta(u_{t+1}))$$

$\Rightarrow$ Using definition of $U_{1,t}(c_t,(1-l_t))$, and $U_{1,t+1}(c_{t+1},(1-l_{t+1}))$ yields :

$$1 = E_t\beta\frac{c_{t+1}}{c_t}.(r_{t+1}.u_{t+1} - p_{t+1}.a(u_{t+1}) + (1-\delta(u_{t+1})) \tag{16}$$

By combining u_t's F.O.C and definition of $\delta^{'}(u_t)$ and $a^{'}(u_t)$, we obtain:

$$r_t.k_{t-1} = \omega_0.u^{(\omega_1-1)}.k_{t-1} + p_t.k_{t-1}.\upsilon_0.u^{(\upsilon_1-1)} \tag{17}$$

Firms maximization problem:

$$\underset{l_t,k_{t-1}}{max}\, y_t - w_t.l_t + r_t.u_t.k_{t-1} \tag{18}$$

subject to:

$$y_t = (z_t.l_t)^{\theta}.(k_{t-1}.u_t)^{(1-\theta)} \tag{19}$$

F.O.C:

$$l_t : w_t = \frac{\theta.y_t}{l_t} \tag{20}$$

$$k_{t-1} : r_t.u_t = \frac{(1-\theta).y_t}{k_{t-1}} \tag{21}$$

Using competitive prices, $w_t \, and \, r_t.u_t$, we define equations that characterize the equilibrium:

$$\frac{c_{t+1}}{c_t} = E_t\beta((\frac{(1-\theta).(z_{t+1}.l_{t+1})^{\theta}.(k_t.u_{t+1})^{(1-\theta)}}{k_t}) + (1-\delta(u_t)) - p_{t+1}.a(u_{t+1})) \quad (22)$$

$$\frac{\theta.(z_t.l_t)^{\theta}.(k_{t-1}.u_t)^{(1-\theta)}}{l_t} = \frac{c_t.(1-\phi)}{\phi.(1-l_t)} \quad (23)$$

$$log z_t = \rho log z_{t-1} + \varepsilon_t \quad (24)$$

$$c_t + k_t - (1-(\delta(u_t))k_{t-1} + p_t.(a(u_t)).k_{t-1} = y_t \quad (25)$$

$$r_t = \omega_0.u^{(\omega_1-1)}.k_{t-1} + p_t.k_{t-1}.\upsilon_0.u^{(\upsilon_1-1)} \quad (26)$$

$$y_t = (z_t.l_t)^{\theta}.(k_{t-1}.u_t)^{(1-\theta)} \quad (27)$$

$$\delta(u_t) = \frac{\omega_0.u_t^{\omega_1}}{\omega_1} \quad (28)$$

$$a(u_t) = \frac{\upsilon_0.u_t^{\upsilon_1}}{\upsilon_1} \quad (29)$$

By simplifying we obtain the equations system that solve variables $\{c_t, k_t, u_t, l_t, y_t, z_t\}$

$$\frac{c_{t+1}}{c_t} = E_t\beta((\frac{(1-\theta).(z_{t+1}.l_{t+1})^{\theta}.(k_t.u_{t+1})^{(1-\theta)}}{k_t})+(1-(\frac{\omega_0.u_t^{\omega_1}}{\omega_1}))-p_{t+1}.(\frac{\upsilon_0.u_t^{\upsilon_1}}{\upsilon_1})) \quad (30)$$

$$\frac{\theta.(z_t.l_t)^{\theta}.(k_{t-1}.u_t)^{(1-\theta)}}{l_t} \quad (31)$$

$$log z_t = \rho log z_{t-1} + \varepsilon_t \quad (32)$$

$$c_t + k_t - (1 - \frac{\omega_0.u_t^{\omega_1}}{\omega_1}).k_{t-1} + p_t.(\frac{\upsilon_0.u_t^{\upsilon_1}}{\upsilon_1}).k_{t-1} = y_t \quad (33)$$

$$r_t = \omega_0.u_t^{(\omega_1-1)} + p_t.\upsilon_0.u_t^{(\upsilon_1-1)} \quad (34)$$

$$y_t = (z_t.l_t)^{\theta}.(k_{t-1}.u_t)^{(1-\theta)} \quad (35)$$