

1 Dynamic Model

$$\psi_t \equiv \nu_t^{1-\gamma} \quad (1)$$

$$\theta_{t+1} \doteq \frac{1}{\gamma} \left(\frac{\mathbb{E}_t [\psi_{t+1} R_{k_{t+1}}^{-\gamma} \mathbb{Y}_{t+1}]}{\mathbb{E}_t [\psi_{t+1} R_{k_{t+1}}^{-\gamma-1} (\mathbb{Y}_{t+1})^2]} \right) \quad (2)$$

$$\varrho_{t+1} \doteq \left(\mathbb{E}_t \left[\psi_{t+1} \left[R_{k_{t+1}}^{1-\gamma} + (1-\gamma) R_{k_{t+1}}^{-\gamma} \theta_{t+1} \mathbb{Y}_{t+1} - \frac{1}{2} \gamma (1-\gamma) R_{k_{t+1}}^{-\gamma-1} (\theta_{t+1} \mathbb{Y}_{t+1})^2 \right] \right] \right)^{\frac{1}{1-\gamma}} \quad (3)$$

$$\mathbb{Y}_t \equiv R_{h_{t+1}} - R_{k_{t+1}} + \eta_t \quad (4)$$

$$\nu_t = \left[1 + \beta^{\frac{1}{\mu}} \varrho_{t+1}^{\frac{1-\mu}{\mu}} \right]^{\frac{1}{\mu-1}} \quad (5)$$

$$u_t = \nu_t^{1-\frac{1}{\mu}} \quad (6)$$

$$\theta_t = \frac{1}{1+k_t} \quad (7)$$

$$R_t^k \equiv 1 + r_{k_t} - \delta \quad (8)$$

$$R_t^h \equiv 1 + r_{h_t} - \delta \quad (9)$$

$$r_{k_t} = A_t \alpha (k)^{\alpha-1} \quad (10)$$

$$r_{h_t} = A_t (1-\alpha) k^\alpha \quad (11)$$

$$R_t = R_{k_t} + \theta_t [R_{h_t} - R_{k_t}] \quad (12)$$

$$1 + g_{r_{t+1}} = \mathbb{E}_t [R_{t+1}] (1 - u_t) \quad (13)$$

Exogenous process

$$\ln A_t = \rho \ln A_{t-1} + e_t \quad (14)$$

$$\eta_t \sim N(0, \sigma_t) \quad (15)$$

$$\sigma_t = \sigma([1 - \lambda \ln A_t])^2 \quad (16)$$

For given parameter values and exogenous processes, there are 13 unknowns

$$\{\psi_t, \nu_t, u_t, \theta_t, k_t, R_t, R_{k_t}, R_{h_t}, r_{k_t}, r_{h_t}, \mathbb{Y}_t, \varrho_t, g_{r_t}\}$$

with 13 equations so a solution exist.

2 Steady State Equations

$$\nu = \left[\frac{1}{1 - \beta^\epsilon \varrho^{\epsilon-1}} \right]^{\frac{1}{\epsilon-1}} \quad (17)$$

$$\varrho \doteq [\mathbb{R}]^{\frac{1}{1-\gamma}} \quad (18)$$

$$\mathbb{R} \doteq R_k^{1-\gamma} + (1-\gamma)R_k^{-\gamma}\theta(R_h - R_k) - 0.5\gamma(1-\gamma)R_k^{-\gamma-1}\theta^2 \left[(R_h - R_k)^2 + \sigma^2 \right] \quad (19)$$

$$\theta \doteq \frac{1}{\gamma} \left(\frac{R_k(R_h - R_k)}{(R_h - R_k)^2 + \sigma^2} \right) \quad (20)$$

$$u = \nu^{1-\epsilon} \quad (21)$$

$$\theta = \frac{1}{1+k} \quad (22)$$

$$R^k \equiv 1 + r_k - \delta \quad (23)$$

$$R^h \equiv 1 + r_{h_t} - \delta \quad (24)$$

$$r_k = A\alpha(k)^{\alpha-1} \quad (25)$$

$$r_h = A(1-\alpha)(k)^\alpha \quad (26)$$

$$R = R_k + \theta[R_h - R_k] \quad (27)$$

$$1 + g = R(1 - u) \quad (28)$$

For given parameter values and $A (\equiv 1)$, σ the SS system include 12 unknowns $\{\nu, u, \varrho, \theta, R, \mathbb{R}, Rk, Rh, rk, rh, gr, k\}$ with 12 equations thus a solution exist.