Welfare Costs and Long-Run Consumption Risk in a Production Economy*

Mariano Massimiliano Croce†

(Job Market Paper)

Abstract

I measure and characterize the welfare costs of consumption uncertainty when a representative agent has recursive preferences and is exposed to long-run consumption risk. In an endowment economy, I show that: 1) welfare costs can be high even when risk aversion is moderate and the short-run volatility of consumption growth is low; and 2) welfare costs are far more sensitive to the intertemporal elasticity of substitution in consumption (IES) than to risk aversion. In particular, welfare costs are sharply increasing in the IES. I show that these results are preserved in a general equilibrium production economy even though agents have access to a saving technology that allows them to smooth consumption over time. The welfare costs are significantly higher than those estimated by Lucas(1987). In order to compensate the agent for aggregate uncertainty, her lifetime consumption should be increased by 200%. (JEL classification: E20, E32, G12, D81)

Keywords: Production, Long-Run Risk, Asset Pricing, Recursive Utility
First Draft: July 15, 2005. *This Draft:* December 11, 2006

---

[*] I would like to thank Dave Backus, Gian Luca Clementi, Chris Edmond, Chris Hellwig, Kenneth Judd, Dirk Krueger, Martin Lettau, Sydney Ludvigson, Fabrizio Perri, Tom Sargent, Thomas Tallarini, Pietro Veronesi, S. Zin along with seminar participants at the European Central Bank, the Board of Governors of the Federal Reserve System, the SED(2006) meetings, STERN-NYU.

[†] Department of Economics, New York University; Email: massimiliano.croce@nyu.edu; http://homepages.nyu.edu/∼mmc287

1 Introduction

A chief concern of US policymakers is the stabilization of economic growth. Such policy goals are grounded in standard consumption-smoothing theory, which implies that in a society averse to risk and with a preference for stable consumption over time, aggregate consumption fluctuations produce welfare losses. Convinced by this theory, in 1978 the US Congress ratified the Full Employment and Balanced Growth Act which mandates that the Federal Reserve maintain both a low inflation rate and stabile economic activity. Yet, academics have been more skeptical of such policy prescriptions. For example, Lucas(1987) argued that a further stabilization of economic activity around its long run trend should represent only a minor concern since it could produce only small benefits. In particular, in an endowment economy in which the representative agent has time-additive log preferences and serially uncorrelated consumption fluctuations, Lucas(1987) calculated that individuals would be willing to sacrifice at most .1% of their lifetime consumption for policies devoted to removing the residual amount of aggregate risk. On the other hand, others, as for example Tallarini(2000), have argued that once one takes into account the information from financial markets, consumption theory can imply large welfare costs, since the high risk premium observed in equity returns suggests that households can be extremely averse to even small fluctuations in consumption. Because the literature has yet to reach a consensus on how risk averse individuals are, it has also not reached a consensus on how large are the welfare costs of stochastic fluctuations in consumption.

In this paper, I argue that the emphasis on risk aversion is misplaced. To do so, I study the welfare costs of stochastic fluctuations in consumption in a model that can explain key features of both quantities and asset prices with a moderate degree of risk aversion. In order to construct this model, I refer to a body of recent finance literature that combines recursive preferences of the type advocated by Epstein and Zin (1989) and Weil(1989) - which allows relative risk aversion to differ from the inverse of the intertemporal elasticity of substitution in consumption - with a consumption process that is simultaneously affected by two different sources of uncertainty. The first, 'Short Run Risk', generates i.i.d. shocks to consumption growth. The second source of uncertainty, in contrast, generates small but very persistent deviations of the consumption drift from its unconditional mean. This latter source of uncertainty is known as 'Long Run Risk' since it produces low frequency fluctuations whose volatility is almost negligible over a short time-horizon but is larger over longer horizons. Several recent papers show that such a model explains high risk premia observed in financial market data with moderate risk aversion.[1]

I show that in long-run risk models with moderate risk aversion and stable short run consumption growth, the welfare costs of stochastic fluctuation in consumption are high. In an exchange economy, to compensate the agent for aggregate consumption uncertainty requires an increase in his lifetime consumption ranging from 120% to 1500%. Welfare costs are high for two reasons. First, the long-run component generates significant uncertainty about long-run consumption growth. This feature is particularly costly for agents with recursive preferences in which the intertemporal composition of risk matters. Second, I show that the welfare costs are much more sensitive to the degree of patience of the agent than to his risk aversion. The degree of patience depends both on the subjective discount factor and the intertemporal elasticity of substitution. In particular, welfare costs increase sharply in the intertemporal elasticity of substitution when this parameter exceeds unity. Values for the intertemporal elasticity of substitution in excess of unity are required in these models in order to match the observed low level of the risk free rate.

After having studied and characterized the costs in an endowment economy, I measure the welfare costs in a general equilibrium production economy in which the long-run risk is embedded in total factor productivity. I want to answer to two important questions. The first question is whether long-run risk in primitive shocks get translated into long-run risk in consumption, once the latter is endogenously determined from the former. The second question is whether it is possible to produce smooth consumption and reasonable co-movements in investment, output and consumption when the intertemporal elasticity of substitution is bigger than unity. I find that the long-run risk in productivity might propagate into consumption even when investment is free to adjust and that the model produces reasonable results for the volatilities and the correlations of the quantities. The implied welfare costs of aggregate consumption fluctuations are on the order of 200%. This number is 2000 times larger than that obtained by Lucas (1987). At the same time, this estimate is much smaller than the corresponding figure for an exchange economy. One of the main reasons for this is that, in a production economy, a lower subjective discount factor is required in order to match the risk free rate, in turn implying that household utility is less sensitive to consumption risk occurring in the distant future.

The next section reviews the related literature. Section 3 characterizes welfare costs in an endowment economy. Section 4 presents the results for the general equilibrium

---

[1] Bansal and Yaron(2004) were the first to draw out the potential importance of long-run risk for asset pricing phenomena. See also Bansal-Gallant-Tuchen(2004), Hansen-Heaton-Li (2005), Bansal-Dittmar-Lundblad(2005), Parker-Juillard (2005), Kiku (2005), Bansal-Dittmar-Kiku (2005), Croce-Lettau-Ludvigson (2005), Colacito-Croce (2005).

production economy. Finally, section 5 concludes.

2 Related Literature

This paper is intended as a contribution to both the welfare costs literature and the long-run risk literature. The most important papers related to this work are Tallarini (2000) and Bansal and Yaron (2004).

Tallarini (2000) argued that a more reliable measure of welfare costs can be obtained only through models that consider the information contained in asset prices. Such information, in fact, implicitly characterizes the intertemporal marginal rate of substitution of the representative agent, and for this reason, should be taken into account in every model concerning intertemporal decisions. Tallarini (2000) adopts a model designed to match, simultaneously, key features of asset prices and quantities. He is able to reverse Lucas' results finding that a further stabilization of US consumption could produce very high welfare benefits, ranging from 10% to 3572% of lifetime consumption. Tallarini obtains these results by introducing three main modifications to the Lucas(1987) model. First, to better reproduce the properties of consumption, he focuses on an exchange economy in which the endowment is subject to permanent shocks.[2] Second, to have a volatile intertemporal marginal rate of substitution, he increases the relative risk aversion (RRA).[3] Finally, to keep the risk free rate at a low level, he changes the utility function and adopts Epstein, Zin and Weil preferences which disentangle the RRA from the inverse of the intertemporal elasticity of substitution (IES). Tallarini (2000) focuses on the special case in which the IES is equal to 1 and he stresses the effect that a higher RRA can have on welfare costs. Tallarini's model does not solve the equity premium puzzle, meaning that extremely high values of RRA (ranging from 44 to 100) are required in order to justify the high stock market returns observed in the data. I extend Tallarini's (2000) welfare calculations by introducing adjustment costs and long-run fluctuations to a production economy in which the agent has an IES larger than 1 and moderate risk aversion. I choose to introduce these elements on the base of what found by Basal and Yaron(2004).

Bansal and Yaron, in fact, show that in an exchange economy with long-run risk and Epstein-Zin-Weil preferences, it is possible to reconcile consumption and asset prices

---

[2] Tallarini(2000) also studies the case in which consumption is very persistent but the trend is stationary. The implied welfare costs are smaller in this case. These findings are confirmed also in Reis(2005).

[3] See Tallarini (2000), panel (a) of Table 2.

properties with a moderate risk aversion and an elasticity of intertemporal substitution slightly larger than 1. I extend their results to a production economy in which the agent is free to adjust quantities and accumulate capital. By doing so, I bridge part of the gap between the current long-run risk literature, that takes consumption as given, and the business cycle literature.

Reconciling asset market factors with aggregate quantities behavior has proved a challenge for modern stochastic dynamic general equilibrium models. Jerman (1998), Lettau-Uhlig (2000), and Boldrin-Christiano-Fisher (2001) have proposed models based on preferences with habit formation. In particular, Jerman (1998) is able to produce low risk free rate, high equity premium, and high volatility for excess returns and relative volatilities for consumption, investment, and output on the order of what is observed in the data by introducing capital adjustment costs. However, as pointed out in Boldrin-Christiano-Fisher (2001) and Lettau-Uhlig(2000), Jerman's model produces a countercyclical response of labor to a persistent shock to productivity. Boldrin-Christiano-Fisher (2001) propose a two-sector economy that does not generate this counterfactual behavior but, unfortunately, that predicts a negative serial correlation for consumption and an excessively volatile risk free rate. By contrast, in Tallarini (2000), the interest rate and the excess returns are overly smooth. Even if the Sharpe-ratios are close to those observed in the asset markets, the highest annualized equity premium produced by Tallarini is .44%. In respect of the welfare costs literature, this paper extends the analysis of Obstfeld(1994), by examining the role of the IES in economies with long-run consumption fluctuations, and of Reis(2005), by formally characterizing the market return mean-variance frontier. This paper refers also to Alvarez-Jerman (2004), which adopts an approach that does not require the specification of preferences and that instead uses asset prices alone. These authors show that low frequency consumption fluctuations can be much more costly than fluctuations corresponding to business cycle frequencies. Their findings appear consistent with my results.

3 The exchange economy

In this section I solve a model with exogenous long-run risk in consumption in order to find analytical formulas that relate welfare costs to the restrictions implied by asset prices. In particular, I show: (1) the interaction between welfare costs and the risk free rate (through the IES and the subjective discount factor); and (2) the link between welfare costs and the equity premium (through the RRA).

I use an exchange economy to explore analytically the sensitivity of welfare costs to the preference parameters and the long-run component. This is going to be helpful in interpreting my numerical results, in the economy with endogenous production.[4]

3.1 Economy setup

I study an economy in which time is discrete and there is a representative agent who has Epstein-Zin-Weil(1989) recursive preferences that take the following form:

$$U_t = \left[(1-\delta)C_t^{1-\frac{1}{\Psi}} + \delta E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right]^{\frac{1}{1-\frac{1}{\Psi}}} \tag{1}$$

where γ is the coefficient of risk aversion, Ψ is the intertemporal elasticity of substitution (IES) and $\theta \equiv \frac{1-\gamma}{1-1/\Psi}$.
The intertemporal marginal rate of substitution of this agent is:

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\Psi}}\left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma} \tag{2}$$

In order to study the properties of this economy I specify the consumption process. I assume that there is no storage technology and that the aggregate endowment has no durable component and I model the consumption growth rate[5] similarly in Bansal-Yaron(2004):[6]

$$\Delta c_{t+1} = \mu + x_t + \sigma\epsilon_{t+1}^c \tag{3}$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_t^x \tag{4}$$

$$\begin{bmatrix}\epsilon^c \\ \epsilon^x\end{bmatrix} \sim iidN\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) \tag{5}$$

The parameter ρ is calibrated to be close to 1 while the ratio $\frac{\sigma_x}{\sigma}$ is calibrated in order

---

[4] In order to compute welfare costs, it is necessary to have an accurate computation of the *level* of the utility-consumption ratio at the stochastic steady state. I choose to solve the model numerically in order to take into account possible departures from the log-normal environment and to better determine the values of unconditional means.

[5] From here on, I adopt the convention of denoting log-variables in small letters. For example, $c_{t+1} = \log C_{t+1}$, $m_{t+1} = \log M_{t+1}$,...

[6] For a more general specification see Hansen-Heaton-Li (2005)

to be small enough that consumption growth is not highly serially correlated. On the basis of this calibration strategy, the long-run component x_t is a small but persistent deviation of the consumption drift from its unconditional mean, μ.

To account for market return data, I calibrate the preferences in order to match the mean and the volatility of the risk free rate and the excess returns in post-war US data. While the model specified in (1)-(5) gives precise implications for the risk free rate and the asset paying consumption, it does not reveal how an asset that pays dividends should be priced. In order to account for these elements, as in to Croce, Lettau and Ludvigson (2006), I assume that the growth rate of dividends evolves in the following way:[7]

$$\Delta d_{t+1} = \mu_d + \phi x_t + \phi_c \sigma \epsilon^c_{t+1} + \sigma_d \epsilon^d_{t+1} \tag{6}$$

$$\begin{bmatrix} \epsilon^c \\ \epsilon^x \\ \epsilon^d \end{bmatrix} \sim iidN \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \tag{7}$$

The parameters (σ_d, ϕ, ϕ_c) allow calibration of the overall volatility of dividends and their correlation with consumption. The parameters (ϕ, ϕ_c) determine the relative importance of the idiosyncratic shock ϵ^c_{t+1} and the long-run component.

I assume that the securities markets are complete, in order to have a simple asset pricing model. Let V^d_t denote the ex-dividend price-dividend ratio of a claim to an asset that pays a dividend stream that grows as follows (6)-(7). From the first order condition for optimal consumption choice and the definition of returns:

$$1 = E_t \left[M_{t+1} R^d_{t+1} \right], \; R^d_{t+1} \equiv \frac{V^d_{t+1} + 1}{V^d_t} e^{\Delta d_{t+1}}$$

Finally, in order to specify the information set of the agent, for simplicity, I focus on the benchmark case in which the agent has full information:

$$E_t[\cdot] \equiv E[\cdot \mid \{x_k, \epsilon^x_k, \epsilon^c_k, \epsilon^d_k\}^t_{k=-\infty}]$$

The agent observes dividend and consumption growth and their specific components.[8]

---

[7] Dividends are a sub-component of the total endowment; the residual part corresponds to labor income.

[8] A different information structure is analyzed in Croce-Lettau-Ludvigson (2005)

3.2 The costs of uncertainty: definitions

I define the cost of uncertainty as the percentage increase of consumption $\Lambda > 0$ that one must give to the agent in every period and along every history in order to make him indifferent between the consumption process $\{C^i\}$ and a less risky consumption process $\{C^j\}$:

$$U(\{(1+\Lambda)C^i\}) \quad = \quad U(\{C^j\})$$

Let u^j denote the log of the utility-consumption ratio for the generic consumption process $\{C^j\}$ and the following holds:[9]

$$\lambda \equiv \log(1+\Lambda) \quad = \quad u^j - u^i \tag{8}$$

In order to have a measure of the cost, I simply compute the value of the utility-consumption ratio in log units for the two different consumption processes and calculate their difference. In the economy described above, given any calibration for $\{\mu, \sigma, \sigma_x\}$ there are three different consumption processes to look at:

$$\begin{aligned} C^* &: \quad \Delta c^*_{t+1} = \mu^* \\ C^{**} &: \quad \Delta c^{**}_{t+1} = \mu^{**} + \sigma\epsilon^c_{t+1} \\ C &: \quad \Delta c_{t+1} = \mu + \sigma\epsilon^c_{t+1} + x_t \end{aligned}$$

The unconditional drift of these processes, μ^*, μ^{**} and μ, differ from each other because they are adjusted for a Jensen inequality term in order to keep constant the unconditional growth rate of consumption in raw units.

The total cost of uncertainty can be computed by comparing the utility that the agent would have in an economy with consumption $\{C^*\}$ with that one associated with the process $\{C\}$:

$$\lambda^{tot} \quad = \quad u(\{C^*\}) - u(\{C\}) \tag{9}$$

This cost can be decomposed into two parts. The cost of the idiosyncratic shock (that I

---

[9] I impose $C^j_0 = C^i_0$ in order to focus only on differences in the growth rates and neglect differences in the initial levels. I assume that U is homogeneous of degree 1.

also denote as short-run risk) is:

$$\lambda^{srr} \quad = \quad u(\{C^{*}\}) - u(\{C^{**}\}) \tag{10}$$

and, simply by calculating the difference, the cost of the long-run risk is $\lambda^{tot} - \lambda^{srr}$, equivalent to:

$$\lambda^{lrr} \quad = \quad u(\{C^{**}\}) - u(\{C\}) \tag{11}$$

3.3 Welfare Costs and Asset Prices

In this section the model is solved and I show the implied expressions for welfare costs, the risk free rate and the equity premium. When the IES is different from 1 and there is a long-run component in consumption growth, it is impossible to find an exact solution for the utility-consumption ratios and the welfare costs. In order to study and compute the latter, I approximate the utility-consumption ratio. In Appendix B and Appendix C, I detail two alternative approximation schemes. One is based on a log-linearization of the model, the other adopts numerical integration methods. Since they produce similar results, I focus on the log-linear approximation in order to provide simple formulas.[10]

Let μ^L denote the unconditional mean of the consumption growth rate $E[C_{t+1}/C_t]$; in order to preserve this mean I impose:

$$\mu = \mu^L - \frac{1}{2}\left(\sigma^2 + \frac{\sigma_x^2}{1-\rho^2}\right) \tag{12}$$

The log-linearization of the model produces the following result for the utility-consumption ratio when there are both short and long-run risk:

$$\frac{\overline{\overline{U}}^{lrr}}{C} \quad \approx \quad \left(\frac{1-\delta}{1-\delta^{lrr}}\right)^{\frac{1}{1-\frac{1}{\Psi}}} \tag{13}$$

where

$$\delta^{lrr} \quad \equiv \quad \delta e^{(1-\frac{1}{\Psi})\left[\mu^L - .5\gamma\sigma^2 - .5\left(\frac{1}{1-\rho^2} + (\gamma-1)\frac{(\delta^{lrr})^2}{(1+\rho\delta^{lrr})^2}\right)\sigma_x^2\right]} \tag{14}$$

In order to find the utility consumption ratio when there is only short-run risk, it is enough

---

[10] All figures related to welfare costs are based on the log-linear approximation. All values reported in tables are produced by solving the model numerically. All results concerning welfare costs are consistent with the analytical methods in Hansen-Heaton-Li(2005).

to simply impose $\rho = \sigma_x = 0$ in (12) - (14). In order to compute the utility consumption ratio when there is no uncertainty at all, simply impose $\sigma = \sigma_x = \rho = 0$.[11] The implied welfare costs are:

$$\begin{aligned}
\lambda^{srr} &\approx .5\gamma\sigma^2 \frac{\delta^{srr}}{1-\delta^{srr}} && (15)\\
\delta^{srr} &= \delta e^{(1-\frac{1}{\Psi})\left[\mu^L - .5\gamma\sigma^2\right]} \\
\lambda^{lrr} &\approx .5\left(\frac{1}{1-\rho^2} + (\gamma-1)\frac{(\delta^{lrr})^2}{(1+\rho\delta^{lrr})^2}\right)\sigma_x^2 \frac{\delta^{lrr}}{1-\delta^{lrr}} && (16)
\end{aligned}$$

Both the long- and the short-run risks have a permanent effect on consumption. For this reason, the agent discounts the welfare losses over an infinite time horizon according to the *effective* discount factors δ^{srr} (in an economy with only short-run risk) and δ^{lrr} (in an economy with long-run risk).[12] The welfare costs associated with the short-run risk depend, then, on the effective discount factor δ^{srr}, the degree of risk aversion, γ, and the volatility of the short-run news, σ. The welfare costs associated with long-run risk have a more complicated expression, but the intuition behind the formula is simple. The long-run component, in fact, produces additional welfare losses by introducing fluctuations in the future utility-consumption ratio. The elasticity of the latter to the long-run risk is measured by $\frac{\delta^{lrr}}{1+\rho\delta^{lrr}}$. The welfare costs generated by the long-run component depend, then, on the degree of risk aversion, γ, the volatility of long-run news, σ_x, and the sensitivity of the utility function to long-run news, which is a function of both the persistence of long-run risk, ρ, and the effective degree of patience of the agent, δ^{lrr}. When measuring welfare costs, I calibrate the preference parameters in order to match, firstly, the mean of the risk free rate and of the excess returns. Consistent with Basal and Yaron(2004), the following expression for the mean of the risk free rate and the equity premium is obtained:[13]

$$E[r_t^f] + .5V[r_t^f] \approx -\log(\delta) + \frac{1}{\Psi}\mu^e \qquad (17)$$

$$\begin{aligned}
\mu^e &\equiv \mu - \frac{1}{2}(\Psi-1)(\gamma-\frac{1}{\Psi})\frac{(\delta^{lrr})^2}{(1-\rho\delta^{lrr})^2}\sigma_x^2 \\
&- \frac{1}{2}\Psi\left[\gamma^2 - \left(\frac{1}{\Psi}-\gamma\right)(1-\gamma)\right]\sigma^2
\end{aligned}$$

---

[11] In these last two cases, the solution for the utility-consumption ratio is exact.

[12] This is consistent with Obstfeld's (1994) terminology.

[13] See Appendix B.2 for the computations.

$$
\begin{aligned}
E[r^d_{t+1} - r^f_t] + \frac{1}{2} V_t(r^d_{t+1} - r^f_t) &= \gamma\phi_c\sigma^2 + (\gamma - 1)\kappa_d \frac{\delta^{lrr}}{1 - \rho\delta^{lrr}} \frac{\phi_x - \frac{1}{\Psi}}{1 - \rho\kappa_d} \sigma^2_x \qquad (18) \\
\kappa_d &= \frac{\overline{V^d}}{1 + \overline{V^d}}
\end{aligned}
$$

The economic intuition behind this asset pricing model has largely been explained by Bansal-Yaron(2004). I focus on the implications for welfare costs. In particular, I show how to calibrate the consumption process and then examine the connection between asset prices and welfare costs.

3.4 Calibrating consumption

The choice of the parameters for the long-run component is very important for the measure of welfare costs. I start by calibrating $\rho = .98$, a benchmark value in the long-run risk literature.[14] As shown in Appendix B.2, this helps to keep the persistence of the risk free rate and the price-dividend ratio close to the values observed in the data.

I calibrate $\{\sigma, \sigma_x\}$ in order to determine a quarterly volatility of consumption growth of about 1.4% and an autocorrelation of about .30. These values are in the range of those found by Bansal-Yaron (2004), Bansal- Dittmar-Lundblan (2005), Lettau-Ludvigson (2005), Colacite-Croce (2005) and others.

In Table 2, I report my benchmark calibration and the statistics obtained from the simulations of the model for consumption. As in Bansal-Yaron (2004), I assume that the agent decision horizon is monthly. I then calibrate my model at a monthly frequency, but target quarterly statistics, as is typically done in business cycle literature. For this reason, I simulate the model at a monthly frequency and time-aggregate the quantities to a quarterly frequency. In order to keep the autocorrelation of the quarterly consumption growth low, the volatility of the long-run component must be small. This explains, in this calibration, why the volatility of the innovation to the long-run component, σ_x, is just 4.4% of the volatility of the short-run shock, σ.

3.5 Results

Given the calibration for $\{\rho, \sigma, \sigma_x\}$, the role of the preference parameters $\{\gamma, \delta, \Psi\}$ is now considered with respect to welfare costs. First the special case $\Psi = 1$ is considered, then

---

[14] See Bansal-Yaron (2004), Hansen-Sargent (2005), Colacito-Croce (2005), Croce-Lettau-Ludvigson (2006), Colacito (2006)

I examine the role of $\Psi \neq 1$.

3.5.1 The special case $\Psi = 1$

The case in which $\Psi = 1$ is interesting for at least two reasons: (1) this is what Lucas(1987) and Tallarini (2000) assumed;[15] (2) it can be reinterpreted in terms of robust control.[16] In this case:

$$\lambda^{srr} = .5\gamma\sigma^2 \frac{\delta}{1-\delta} \tag{19}$$

$$\lambda^{lrr} = .5\left[\frac{1}{1-\rho^2} + (\gamma - 1)\frac{\delta^2}{(1-\rho\delta)^2}\right]\sigma_x^2 \frac{\delta}{1-\delta} \tag{20}$$

Tallarini(2000) has discussed (19). I examine (20). Two different forces, the persistence and the magnitude of the long-run component, are at work. The smaller this component is with respect to the idiosyncratic shock (low σ_x), the smaller is the long-run welfare cost. On the other hand, the closer ρ is to unity, the higher the costs are.

In Fig. 1, in the top panel, I plot the welfare cost of the short-run risk and the total welfare costs with respect to the risk aversion coefficient while keeping fixed the subjective discount factor. The vertical difference between the two curves measures the welfare cost of the long-run component. The latter represents about 80% of the total cost. Even though the long-run component has a small volatility, its persistence amplifies the impact on welfare costs.

In the bottom panel of Fig. 1, I graph welfare costs with respect to the subjective discount factor δ. Welfare costs are a hyperbolic function of δ - as shown in equation (19) and (20) - and move toward infinity when the discount δ is close to 1. The economic intuition behind this result is straightforward and has been explained by Tallarini (2000): when a shock has permanent effects on the levels of consumption, welfare losses induced by this source of uncertainty are discounted over an infinite time horizon at the rate δ. When the agent is very patient, $\delta \approx 1^-$, she gives a very high relative weight to future utility and experiences very high welfare costs because of the uncertainty of his continuation value.

In accordance with the usual practice in the long-run risk literature, I determine the preference parameters $\{\delta, \gamma\}$ with respect to their implications for the levels of equity premium and risk free rate.

Let us first focus on the restriction imposed by the risk free rate. More precisely, taking

---

[15] When $\Psi = 1$ the Epstein-Zin-Weil (1989) aggregator collapses into a log function.

[16] See Barillas-Hansen-Sargent(2006).

the RRA, γ, as given, I want to calibrate the subjective discount factor with respect to its implications for the mean of the risk free rate. In this economy:

$$E[r^f] \quad = \quad -\log(\delta) + \mu - .5(2\gamma - 1)\sigma^2 \tag{21}$$

Notice that *high risk aversion helps to keep the risk free rate at a low level.* The intuition behind this result is immediately apparent if we interpret these preferences in terms of robust control. The agent, concerned about model mis-specification, prices a riskless bond adopting a conservative evaluation of the probability densities. Because of his pessimism, the agent focuses on the worst-case scenario and, as a result, his expected growth is penalized and distorted downward according to σ and γ. The first parameter measures the degree of uncertainty of the economy,[17] the second parameter measures the degree of concern about mis-specification.[18] A lower expectation about future consumption growth mitigates the incentive to borrow and allows the market to clear with a lower interest rate.

Equation (21) gives expression to the subjective discount factor as a function of the relative risk aversion and the mean of the risk free rate:

$$\delta(\gamma, E[r^f]) \quad = \quad exp(-E[r^f] + \mu - .5(2\gamma - 1)\sigma^2) \tag{22}$$

This is probably one of the most important equations of this paper since it shows a key interaction between the subjective discount factor and the RRA. Welfare costs, in fact, are crucially driven by the negative trade-off between relative risk aversion and the degree of patience introduced by the the risk free rate equation. Note that by plugging (22) into (19) and (20), we can now express welfare costs simply in terms of the pair $(\gamma, E[r^f])$.

From the data I obtain a measure for $E[r^f]$. On the basis of the historical average of the real return of 3-Month T-Bills in the US, from 1950:Q2 to 1990:Q1, I fix the annual mean of the risk free rate at 1.1%. In this way, welfare costs become simply a function of the RRA. In Fig. 2 I show that *when the restriction in equation (22) is imposed, welfare costs decrease with risk aversion.* In particular, the top panel of Fig. 2 illustrates the link between the subjective discount factor and the RRA implied by (22). According to this link, economies with higher risk aversion are also economies in which the low level of the risk free rate can be matched only with a lower degree of patience. A lower discount

---

[17] Tallarini(2000) interprets uncertainty of the economy as risk; Anderson et al. (2003) and Hansen-Sargent (2007) refer to model uncertainty.

[18] See Barillas-Hansen-Sargent (20007) to understand this parameter in relation to the implied detection error probabilities.

factor limits the role of permanent uncertainty and reduces welfare costs despite higher risk aversion.

In order to understand which RRA coefficient to adopt, I focus on the expression for the equity premium:

$$E[r_{t+1}^{d}-r_{t}^{f}]+\frac{1}{2}V_t(r_{t+1}^{ex}) \quad = \quad \gamma\phi_c\sigma^2+(\gamma-1)\kappa_d\frac{\delta}{1-\rho\delta}\frac{\phi_x-1}{1-\rho\kappa_d}\sigma_x^2 \qquad (23)$$

When no long-run risk is present in the economy, $\rho=\sigma_x=0$, the equity premium is determined simply by $\gamma\phi_c\sigma^2=\gamma cov_t(\Delta d_{t+1},\Delta c_{t+1})$. One empirical fact is that the contemporaneous covariance between dividend and consumption growth is very small in the data,[19], implying that a very high risk aversion coefficient is required in order to match the post-war annual US equity premium of about 6%.[20] On the basis of equation (22), *explaining the equity premium only by means of a high risk aversion coefficient actually produces lower welfare costs.*

Conversely, when long-run risk is present in the economy, the second term in equation (23) shows that it is possible to have a high equity premium with low risk aversion, provided that $\rho\delta\approx 1^{-}$. Introducing long-run risk increases the welfare costs for two reasons: (1) it increases the "amount of risk"; (2) according to eq. (22), it increases the effective discount factor required to match the risk free rate level and hence, amplifies the impact of uncertainty.

Given these relationships, it is interesting to compare an economy with long-run risk - à la Bansal-Yaron (2004) - and one in which there is no long-run uncertainty - à la Tallarini (2000). Panel A of Table 2 reports my alternative calibrations. Panel B compares basic statistics obtained by simulating the model under the two alternative calibrations and shows also their value in the data.[21] The model matches the level of the equity premium both in the economy with long-run risk and in that one with only short-run risk, but very different levels of risk aversion must be assumed in the two cases. In an economy without long-run risk, the high equity premium is matched with a very high risk aversion. In an economy with long-run risk, the risk aversion is 17, a more moderate level.[22] Panel C

---

[19] The estimate of this moment depends heavily on the sample and the frequency of the data. See for example, Bansal and Yaron (2004), Bansal Dittmar and Lundbland (2005), Colacito and Croce (2005) and Lettau and Ludvigson (2005). I now calibrate the quarterly correlation between dividend and consumption growth to a conservative value of 10%.

[20] See Appendix A. for more details on the data.

[21] See Appendix A. for details on the data.

[22] In the economy with long-run risk, I impose $\phi_c=0$ and this implies that in order to match the contemporaneous correlation between consumption and dividend growth, a high long-run dividend

confirms what is shown in Fig. 22. This figure shows that despite the lower risk aversion coefficient, total welfare costs are about 24 times larger. This is primarily due to the fact that there is now an additional source of risk. However, note also that welfare costs associated with short-run risk are also higher, by a factor of about three. This is because in an economy with long-run risk, I calibrate the subjective discount factor to a higher value in an attempt to keep the implied risk free rate mean close to the value observed in the data. In the calibration associated with long-run risk, however, it is impossible to obtain values for the annual mean of the risk free rate smaller than 1.6%. This problem, nevertheless, can be solved. The long-run risk literature, in fact, shows that an IES larger than 1 helps to keep the risk free rate at low levels. For this reason, in the next section I examine the model in the general case in which $\Psi \neq 1$, and I explore both asset prices and welfare cost implications produced in such a framework.

3.5.2 Welfare costs with $\Psi \neq 1$

As shown in the preceding section, the IES is a crucial variable for the risk free rate and can have important implications on the measurement of welfare costs. More generally, examining the role played by the intertemporal elasticity of substitution is interesting for at least two other reasons: (1) from an empirical point of view, whether the IES is larger or smaller than 1 is still debatable;[23] and (2) while long-run risk models are typically calibrated with an IES>1, standard business cycle models with time-additive CRRA preferences usually adopt an IES$\leq$1.[24]

The top panel of Fig. 3 shows welfare costs obtained in (19)-(20) as a function of the IES. For simplicity, I fix the risk aversion at 16 and the annualized subjective discount factor,δ^{12}, at .993. Costs are increasing in the IES since the latter implicitly makes the agent more patient. As it is possible to see in the bottom panel of Fig. 3, when the intertemporal elasticity of substitution is higher, the effective discount rates converge closer to 1.

It is interesting that also *the composition of the total cost changes* with the IES. In particular, note that as the IES increases, welfare costs of the short-run risk become more and more important. The bottom panel of Fig. 3 shows that this is simply due to the fact

---

leverage is needed, $\phi_x = 6.4$. The high relative exposure of dividends to long-run risk produces a high equity premium despite the moderate risk aversion.

[23] See, for example, Hall (1988), Lustig and Von Nieuwerburgh (2005), Guvenen (2005), Attanasio and Weber (1989), Attanasio and Vissing (2005).

[24] In standard RBC models $\Psi = 1/\gamma \leq 1$.

that the effective discount rate in economies in which there is no long-term risk, δ^{srr}, is more sensitive to the IES than is the effective discount factor in economies in which there is long-run risk.

Let me now focus on the total welfare costs and their link with the risk free rate. Fig. 4 shows both total welfare costs and risk free rate mean as a function of the IES and confirms the observations described in the previous sections: in economies in which there are permanent sources of uncertainty, welfare costs are driven by the effective degree of patience, which is tightly related to the risk free rate. In particular, targeting lower levels of the risk free rates produces higher welfare costs.

Table 3 shows in detail the moments produced by the model when the IES is .8[25] and when it is 1.5. Focusing on panel C, under both calibrations, the same equity premium is achieved by adjusting slightly the relative risk aversion. While the equity premium is the same, the level of welfare costs is significantly different. Calibrating the IES to 1.5, I match the empirical average of the risk free rate and obtain extremely high costs, in the order of 1512%. Calibrating the IES to .8, the risk free rate mean produced by the model is almost two times bigger then the empirical one and costs are about 15 times lower than before. These results confirm that the measurement of welfare cost is actually far more sensitive to the restrictions imposed in reference to the risk free rate than to those obtained from the equity premium. I now examine the link between welfare cost and risk free rate with standard CRRA preferences.

3.6 Preferences or Long-Run Risk? Both.

Finally in this section, I examine implied welfare costs in an economy with long-run risk and standard time-additive CRRA preferences by imposing the following restriction: $\Psi = 1/\gamma$. In Fig. 5, I plot implied welfare costs and risk free rate as a function of the risk aversion. It is important to note that the welfare costs are *lower* than before by approximately two orders of magnitude. Furthermore, welfare costs are no longer increasing in the risk aversion coefficient. These two facts are strongly correlated and are explained as follows. A *higher risk aversion* implies a *lower effective discount factor* and, hence, *lower welfare costs.* An economy with standard CRRA preferences might underestimate welfare costs because of the implicit negative relationship between the risk aversion coefficient and the effective discount factor. The inability of these preferences to keep low the risk free rate with higher levels of risk aversion significantly affects their

---

[25] This is the estimate provided by Favero(2005).

implications for welfare costs. Epstein-Zin-Weil(1989) preferences, however, solve the risk free rate puzzle and their inclusion is then essential in disentangling risk aversion from the degree of patience, which is one of the main determinants of welfare costs.

Notice, however, that the high welfare costs obtained in the previous sections depend also on the *presence of long-run risk*, not simply on the choice of the preferences. It is the introduction of the long-run risk that allows the tradeoff between degree of patience and RRA - induced by the restriction on the mean of the risk free rate - to play such a crucial role. Economies in which the growth rates are perfectly *i.i.d.* are economies in which the RRA must be high to match the equity premium, in turn implying lower welfare costs. Only after the introduction of long-run risk does attention shift toward economies in which the risk aversion is moderate and the degree of patience is higher. Looking, for example, at Fig. 2, it is the introduction of long-run risk that forces us to move from the righthand side of the panels - where we have economies in the manner of Tallarini (2000) - to the lefthand side of the panels - where we have economies as in Bansal and Yaron (2004) with lower RRA but higher welfare costs.

4 The production economy

4.1 The role of investment

The endowment economy presented in section 3 showed that welfare costs can be quite high even with moderate levels of risk aversion. Working with an exogenous endowment helps to directly connect this result to the restrictions implied by asset price data. However, an endowment economy is silent about the role that endogenous movements of capital and investment can have on asset prices and, consequently, on welfare costs. The economy presented in the previous section, in fact, can be seen as a special production economy in which labor is fixed, there is no capital depreciation and, at the same time, there are infinite capital adjustment costs. Under these restrictions, the representative agent consumes the entire output in every period and his consumption growth equals the exogenous productivity growth rate. The optimal investment is, then, zero and capital is constant over time. In the data, however, the investment growth rate is about five times more volatile than the consumption growth rate. The investment-output ratio, on average, is about 20%.[26] Introducing to the model our information about investment fluctuations could potentially have important implications. For this reason, in this section I relax the

---

[26] See Appendix A. for details about the data.

assumption of infinite adjustment costs and explicitly assume that the primary source of long-run risk lies in the productivity growth rate. There are three important questions to answer in order to interpret implied welfare costs in this scenario:
(1) Can long-run productivity uncertainty generate long-run consumption risk when investment is free to move?
(2) Do investment fluctuations change the properties of market returns and, hence, the preferences calibration?
(3) Is an IES>1 consistent with standard understanding of business cycles?

If the agent is able to use investment in order to decrease the exposure of his consumption process to long-run risk, implied welfare costs decrease. Furthermore, we know that welfare costs depend not only on the amount of long-run consumption risk but also on the specific calibration of the preference parameters. For a given degree of long-run consumption risk, if investment fluctuations tend to reduce the volatility of returns, a higher risk aversion is required in order to get high equity premiums. As shown in Fig. 2, a higher risk aversion can actually produce a reduction in welfare costs when the interest rate is kept fixed.

These arguments give us good reasons to expect lower welfare costs in a production economy. However, there is a third channel through which investment can affect the measure of welfare costs and it is important since it could actually increase welfare costs. This additional channel is related to the introduction of concave capital adjustment costs as in Jerman (1998). Barlevy (2000) shows that in economies with capital adjustment costs, welfare costs produced by the volatility in productivity are amplified. In this kind of environment, the volatility of the productivity shocks affects negatively the unconditional mean of the consumption growth. Let $G(I/K)$ denote the concave adjustment cost function, then:

$$\begin{aligned} \mu_{\Delta c} &\equiv \mathrm{E}\left[\frac{C_{t+1}}{C_t}\right] = \mathrm{E}\left[\frac{K_{t+1}}{K_t}\right] \\ &= \mu + .5\underbrace{\overline{G}''}_{<0}\mathrm{Var}[I/K] \end{aligned} \tag{24}$$

When uncertainty is completely removed, the representative agent experiences higher welfare benefits for two reasons: 1) *her consumption process is perfectly smooth*, and 2) *her consumption is growing faster*. In a world with uncertainty, in fact, the representative agent must frequently adjust the investment-capital ratio. This adjustment is costly and produces a negative income effect that has a depressing impact not only on the level of consumption, but also on its growth rate. This negative effect disappears once all the uncertainty is removed. For this reason, movements in investment might actually produce higher welfare costs.

In the next section I describe the model used to investigate the effect that investment has on welfare costs.

4.2 The model

This section presents the model used to examine the business cycle and to evaluate welfare costs. In order to keep the analysis as simple as possible, I focus only on the representative agent consumption-saving problem and I keep constant the labor supply. The representative agent has preferences defined only by aggregate consumption:

$$\begin{aligned} U_t &= \left[(1-\delta) C_t^{1-\frac{1}{\Psi}} + \delta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\Psi}}} \\ 0 &\leq C_t \end{aligned}$$

The consumption good is produced according to a constant returns-to-scale neoclassical production function:

$$Y_t = K_t^{\alpha} \left[A_t n_t \right]^{1-\alpha}$$

where K_t is the fixed stock of capital carried into date t, n_t is the labor input at t and A_t is an aggregate productivity shock. The productivity growth rate, $\Delta a_{t+1} \equiv log(A_{t+1}/A_t)$, has a long-run risk component and evolves as described below:

$$\begin{aligned} \Delta a_{t+1} &= \mu + x_t + \sigma \epsilon_{a,t+1} \\ x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\ \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \end{aligned}$$

The resource constraint of this economy is:

$$C_t + I_t \leq Y_t$$

The capital stock evolves according to:

$$\begin{aligned} K_{t+1} &= (1-\delta_k)K_t + G\left(\frac{I_t}{K_t}\right)K_t \\ &\text{where} \\ G\Big(\frac{I_t}{K_t}\Big) &= \left[\frac{a_1}{1-\frac{1}{\tau}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\tau}} + a_2\right] \end{aligned}$$

The rate of depreciation of capital is denoted by δ_k and the function $G(.)$ transforms investment in new capital as in Jerman(l998). The agent is endowed with $\overline{n}$ units of time that she can devote to leisure (denoted by l_t) or labor according to the following constraint:

$$n_t + l_t \leq \overline{n}$$

Since leisure does not appear in the utility function, the representative agent will always find it optimal to offer $n_t = \overline{n}$ units of labor.[27]

4.2.1 Equilibrium

In this economy, the allocation that solves the planner's problem can be decentralized by means of competitive markets.[28] It is then possible to find the competitive equilibrium allocation by solving the planner's problem. Let us define the following stationary variables:

$$\{c_t, i_t, y_t, k_t, u_t\} \equiv \left\{\frac{C_t}{A_{t-1}}, \frac{I_t}{A_{t-1}}, \frac{Y_t}{A_{t-1}}, \frac{K_t}{A_{t-1}}, \frac{U_t}{A_{t-1}}\right\} \tag{25}$$

Let $s_t \equiv [\Delta a_t, x_t, k_t]$ denote the vector of the states of the economy. Let $u(s)$ be the value of the planner's problem evaluated at the optimum for given state s. The planner's

---

[27] I impose $\overline{n} = .18$. As Tallarini(2000), I consider total employment multiplied by average weekly hours worked divided by the civilian non-istitutional population 16 years and older.

[28] See Sargent-Ljungqvist (2004).

problem can be rewritten in the following recursive way:

$$\begin{aligned}
u(s) &= \max_{c,k'} \left[(1-\delta)c^{1-\frac{1}{\Psi}} + \delta e^{(1-\frac{1}{\Psi})\Delta a}\left(E_s[u(s')^{1-\gamma}]^{\frac{1-\frac{1}{\Psi}}{1-\gamma}}\right)\right]^{\frac{1}{1-\frac{1}{\Psi}}} \\
& s.t. \\
c &\geq 0, k' \geq 0 \\
c+i &= y \equiv e^{(1-\alpha)\Delta a} k^{\alpha} \bar{n}^{(1-\alpha)} \\
k' e^{\Delta a} &= (1-\delta)k + G\left(\frac{i}{k}\right)k \\
x' &= \rho x + \sigma_x \epsilon_a' \\
\Delta a' &= \mu + x + \sigma \epsilon_a'
\end{aligned}$$

I solve this problem numerically, I describe my computational methods in Appendix C. Once the planner's allocation is found, prices and returns can be derived from the solution to the planner's problem as follows. The stochastic discount factor takes exactly the same form as that derived in (2). The risk free rate is:

$$R_t^f = E_t\left[\delta\left(e^{\Delta a}\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\Psi}}\left(\frac{u_{t+1}}{E_t[u_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma}\right]^{-1}$$

The marginal value of standardized capital is equal to the marginal rate of transformation between new capital and consumption:

$$q_t = \frac{1}{G'(\frac{i_t}{k_t})}$$

The returns per unit of normalized capital are:

$$\begin{aligned}
R_{t+1} &\equiv \frac{q_{t+1} + d_{t+1}}{q_t} \\
& \text{where} \\
d_{t+1} &\equiv \alpha\frac{y_{t+1}}{k_{t+1}} - \delta_k q_{t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1}G\left(\frac{i_{t+1}}{k_{t+1}}\right)
\end{aligned}$$

In accordance with Boldrin-Christiano-Fisher (1999), I introduce financial leverage. The leveraged excess return is:

$$R_{t+1}^d - R_t^f \equiv (R_{t+1} - R_t^f)\left(1 + \frac{\overline{B}}{S}\right)$$

where $\overline{B/S}$ is the average debt-share ratio of the firm. As in Boldrin-Christiano-Fisher (1999), I keep leverage constant and assume $\overline{B/S} = 2/3$. Finally, wages equate the marginal productivity of labor.

4.2.2 The definition of welfare costs

I define the cost of uncertainty similarly to that in section 3.2. Given any calibration for $\{\mu, \sigma, \sigma_x\}$, there are three different endogenous consumption processes to examine:

$$\begin{array}{lll} C^* & \text{obtained when} & \Delta a^*_{t+1} = \mu* \\ C^{**} & \text{obtained when} & \Delta a^{**}_{t+1} = \mu^{**} + \sigma\epsilon^a_{t+1} \\ C & \text{obtained when} & \Delta a_{t+1} = \mu + \sigma\epsilon^a_{t+1} + x_t \end{array}$$

The unconditional drifts of these processes, μ^*, μ^{**} and μ, differ from each other because they are adjusted in order to keep constant the unconditional growth rate of TFP in raw units.

Finally, I define the welfare costs generated by the adjustment costs as:

$$\lambda^{adj} = u(c|0 < \tau < \infty) - u(c|\tau = \infty) \tag{26}$$

Eq. (26) compares the utility-consumption ratios of two economies that have the same short-run and long-run degree of uncertainty, but different elasticities for their supplies of capital.

4.3 Model Predictions on Quantities and Prices

First, I solve the planner's problem for different values of the IES. In particular, in order to be able to compare my new results with those obtained in an exchange economy, I focus on the case in which $IES \in \{.8; 1; 1.5\}$. In Panel A of Table 4, I detail the entire calibration. I calibrate the RRA to 16 and fix it in order to better isolate the role of the IES. I choose the subjective discount factor so as to match the same level of the risk free rate I obtained in the exchange economy with IES=1.5. The elasticity of the supply curve of capital is equal to .98, a value in line with empirical evidence.[29] The parameter α is calibrated to match the capital income share. The annualized capital depreciation rate is 6%. The annualized mean of the productivity growth rate is 2%. The volatility of the short-run productivity shock is about 1.4 times the volatility in the exchange economy for

---

[29] Abel (1980) reports estimates that range between .5 and 1.14; Eberly (1997) has a 95% confidence interval for the US of $[1.08, 1.36]$.

consumption. This is required in order to match the actual volatility of the consumption growth rate. I fix the persistence of the long-run component to .98 in order to stay as close as possible to what happens in an exchange economy. I scale the volatility of the long-run component so that it explains only 7% of the total volatility of the productivity growth rate. This allows the retention of the first order autocorrelation of the quarterly consumption growth rate in line with the data.

Before analyzing the moments produced by the model, it is important to study the policy functions of the representative agent. In Fig. 6, I plot the optimal consumption-productivity ratio, C_t/A_{t-1}, and the optimal investment-productivity ratio, I_t/A_{t-1}, as a function of the exogenous states. The panels on the left describe consumption, the panels on the right refer to investment. The top panels refer to an economy in which the IES is 1.5, the middle panels are obtained when the IES is 1 and the bottom two panels are computed for an IES of .8. In each panel, there are two curves: the solid curve shows the slope of the given policy function with respect to variations in the long-run component; the dashed curve shows the slope of the policy function with respect to the short-run shock. Capital is fixed at its steady state value.

For every value of the IES considered in the figure, the agent finds it optimal to respond to positive short-run shocks by simultaneously increasing consumption and investment (all the dashed lines are upward sloping).

The behavior of consumption and investment is instead very different with respect to the long-run component. This is due to the fact that long-run news simultaneously generates both an income and a substitution effect that each work in opposite directions. The higher expected long-run productivity generates a substitution effect that increases the opportunity cost of consumption and that tends to stimulate investment. Since output is predetermined, an increase in investment generates a drop in consumption.

At the same time, a positive long-run shock allows the agent to feel much richer and to desire higher consumption. The long-run component, in fact, is highly persistent and, for this reason, a single long-run shock is able to affect the flow of expected future utility over a very long time-horizon. A positive long-run shock translates into a remarkable increase in the continuation value of the agent.

When the IES is low, current consumption and continuation value are complements: the income effect dominates and the agent finds it optimal to increase consumption by reducing investment. However, when the IES is high, the degree of substitutability between continuation value and current consumption is high. In this case, the substitution effect dominates and the agent finds it optimal to decrease consumption in order to increase investment and accumulate capital. Finally, when the IES is equal to 1 - the log-case - the income effect and the substitution effect offset each other and the demand for new capital tends not to move.[30]

It is interesting to examine what are the quantitative implications for the growth rates of consumption, C_t, and investment, I_t. Fig. 7 and Fig. 9 show the percentage deviations from the steady state of the monthly growth rates of consumption, investment and output realized after a single positive pulse shock to both the short-run component (left panels) and the long-run risk (right panels). The shocks materialize only at time 2 and they are normalized according to their standard deviation ($\epsilon_{a,2} = \sigma$ and $\epsilon_{x,2} = \sigma_x$). A shock to the long-run component is able to produce persistent movements in the growth rate of consumption, investment and output. These movements are small and allow the autocorrelation function of the quarterly consumption growth to be maintained at .35 as shown in Panel B of Table 4. The annualized volatility of quarterly consumption growth is about 1.35% when the IES is different from 1 and about 1.2% when the IES is 1.

In Panel B of Table 5, I report the volatility ratios produced by the model for consumption, output and investment. The figures show two important features: (1) the IES has only a modest impact on volatility ratios, implying that even with an IES> 1 investment is more volatile than consumption; (2) the model under-predicts the volatility of investment. This problem is mostly due to adjustment costs and disappears when the supply curve of capital is perfectly elastic, i.e., when $\tau = \infty$. In this case, the investment growth rate becomes six times more volatile than the consumption growth rate. The fact that investment is not very volatile implies that output growth is also less volatile than what is implied in the data. In the data, the volatility ratio for output and consumption is about 1.5, while the model prediction is 1.0. In Panel A of Table 5, I report contemporaneous correlations of quarterly growth rates of consumption, investment, output and market returns.[31] Let us focus now on the quantities. In the data, consumption growth has a correlation of about 83% with output and 43% with investment. These correlations are typically significantly higher in standard real business cycle models that ignore the presence of long-run uncertainty in productivity. In standard models, in fact, all the

---

[30] The response of the investment growth rate depends in reality on the interaction between the elasticity of the supply curve of capital and the IES. Since I calibrate the elasticity of capital to a value very close to 1 ($\tau = .98$), this functions as a useful normalization that allows an exclusive focus on IES.

[31] Given that I keep fixed the supply of labor, in this economy the measured productivity growth rate ($\Delta TFP_t \equiv \Delta y_t - (1-\alpha)\Delta n_t$) is exactly equal to the output growth. In the data, however, these two series have a correlation of about .7 and show different degrees of persistence. In particular, $\text{ACF}_1[\Delta TFP] = .15$ while $\text{ACF}_1[\Delta y] = .4$. The model cannot account for these differences.

variables move together in response to short-run news. This is exactly what happens in this model when the IES is 1. Even if there is long-run uncertainty, quantities do not react to long-run news because the income effect and the substitution effect offset each other. Hence, the correlations of the quantities are explained by the co-movements with short-run shocks. When the IES varies from 1, the model instead produces lower contemporaneous correlations since the long-run news moves consumption and investment in opposite directions, introducing a negative effect on their covariance. In order to better match the correlations of the quantities, the IES must be other than 1. On the other hand, taking consideration of prices implies that the IES should be calibrated at a value above 1. Panel B of Table 4, in fact, shows that a higher IES helps to obtain higher excess returns and lower risk free rates. When IES=.8, the implied equity premium predicted by the model is zero, while the risk free rate is about 3.18%. When the IES=1.5, the risk free rate declines to 1.4% and the excess return is 2%. These results show that the market return is not particularly sensitive to IES and, for this reason, a reduction in the risk free rate produces almost a one-to-one increase in the excess return. This result can be better understood by focusing on the conditional covariance between the returns of capital,r_{t+1}, and the intertemporal marginal rate of substitution (IMRS), m_{t+1}. The Euler Equation implies that $E_t[r_{t+1} - r_{f,t}] \approx -cov_t(m_{t+1}, r_{t+1})$, meaning that capital must offer a higher equity premium when its returns move in the opposite direction to that of the IMRS. In order to characterize the conditional response of the prices, I show the impulse response functions of the IMRS, the return to capital and the capital gain component (q_{t+1}/q_t). Fig. 10 focuses on the case IES= .8, while Fig. 8 shows the case IES= 1.5. In both figures, positive short-run news implies a drop in the IMRS and, at the same time, an increase in capital returns induced by the realization of both higher dividends and capital gains. The increase in the price of capital is induced by the positive shift of the demand for new capital produced by the positive short-run news. The situation is different when long-run news materializes. In this case, the demand for investment shifts upward only if the substitution effect is stronger than the income effect, i.e., only if the IES is above 1. In particular, when the IES is above 1, a long-run shock produces both a positive response in returns and a very strong drop in IMRS.[32] When IES$<$ 1, the capital claim offers insurance against long-run shocks since good news about long-run productivity growth produces a reduction in investment and a fall in the price of capital. The implied capital loss produces

---

[32] The IMRS percentage deviation from its steady state is about -9% after short-run news and -24% when a long-run shock materializes. This is a key feature of the Epstein-Zin-Weil (1989) Preferences. They are very sensitive to long-run shocks and they allow the IMRS to be very volatile in response to even small long-run fluctuations. Among others, see Croce-Lettau-Ludvigson(2005).

low returns associated with lower IMRS. In this case, the long-run uncertainty reduces the equity premium.

Overall, however, the highest equity premium that the model is able to produce is only 2%. This is primarily because, under this calibration, the endogenous exposure of the growth to consumption risk is lower than what was assumed in an exchange economy. In an exchange economy, $E_t[\Delta d_{t+j}] = \phi_x E_t[\Delta c_{t+j}]$, $j = 1, 2, ...$ and I calibrated $\phi_x = 6.4$. With endogenous production, however, the expected dividend growth is only 3 times larger than that of consumption, implying that the price-dividend ratio is less sensitive to long-run shocks. Note also that after a short-run shock to productivity, the expected dividend growth is negative and its absolute value is 5 times larger than that of consumption, implying a negative (but small) drop in the price–dividend ratio.[33] It is because of these two elements that the returns are less risky than those produced in an exchange economy.

Finally, examining the second moments of the returns, it is possible to see that the model predicts a low volatility of market excess returns. In particular, the problem is that the volatility of the dividend growth rate is about 6%, half of that observed in the data.[34] On the other hand, the model is successfully able to keep the volatility of the risk free rate at low levels.[35]

4.4 Properties of the consumption growth rate

Before analyzing the welfare costs implied by the model, it is important to analyze the properties of the consumption growth rate. If we let $\widehat{c}_t \equiv C_t/A_t$ denote the consumption-productivity ratio, the growth rate of consumption can be rewritten as:

$$\Delta c_{t+1} = \Delta a_{t+1} + \Delta \widehat{c}_{t+1}$$

---

[33] When a short-run shock materializes, the agent responds by increasing investment. After the shock, the agent begins to sustain consumption growth above its unconditional mean by consuming the extra units of capital just accumulated. This implies that the dividend-productivity ratio declines, in turn pushing the growth rate of dividends below its unconditional average.

[34] In order to better match the observed correlation between market returns and aggregate quantities, dividends should have an extra component that is volatile but uncorrelated with the productivity shocks. This suggests that a more sophisticated leverage process that is able to introduce dividend-specific shocks is required.

[35] Models with habits typically produce volatile risk free rates, see for example Jerman(1998) and Boldrin-Christiano-Fisher (2000). Epstein and Zin (1989) preferences allow, instead, the IMRS to have a small conditional volatility and, at the same time, a high unconditional volatility.

The first term shows movements in consumption growth induced by exogenous productivity growth fluctuations. The second term, however, captures endogenous movements induced by the optimal response of the agent to the exogenous shocks. If the agent keeps constant the consumption-productivity ratio over time ($\Delta\widehat{c}_{t+1} = 0$ for any t), his consumption process has the same properties as the productivity process and therefore it has the same exposure to both long- and short-run risk. However, this is not what the model predicts. The previous section, in fact, shows that consumption growth is smoother than productivity growth ($\sigma_{\Delta C} \approx .67\sigma$), implying that the agent moves $\widehat{c}_t$ counter-cyclically and let the consumption process be less exposed to productivity shocks. Fig. 11 plots the impulse response functions of both the productivity growth and the consumption-productivity ratios with respect to both short-run and long-run news when the IES is 1.5 and all other parameters are calibrated as in Table 4. When a positive short-run shock materializes (at time $t = 2$), the consumption-productivity ratio drops since the agent responds by increasing also investment. After the first period, the growth rate of productivity goes back to precisely zero while that of consumption remains positive, even if very close to zero. The agent, in fact, uses the extra capital accumulated in the first period in order to keep the growth rate of consumption above its steady state over a longer time-horizon. This effect is both very persistent - the decay rate of Δc_t after the initial shock is about .9953 - and very small, almost invisible in the figure. In order to capture it, in the bottom-left panel of Fig. 11, I plot the spectral density of consumption growth generated by short-run risk. The same panel shows the theoretical spectral density of the productivity short-run risk.[36] The spectrum of short-run productivity shock is perfectly flat, while that of consumption growth has a spike at low frequencies. This shows that the model introduces a moderate amount of endogenous persistence in the growth rate of consumption even after a simple *iid* shock. Something similar happens when long-run news materializes. When the shock arrives, the agent responds immediately by increasing investment and letting the consumpion-productivity ratio decline. After the initial fall, consumption growth becomes positive but remains smaller than the growth rate of productivity for the remaining plotted periods. The growth rate of productivity, however, declines more quickly than the consumption growth rate and, after about 150 months, it becomes smaller than the consumption growth rate. In particular, after the initial shock, consumption growth decays at a rate of about .9838, while the productivity growth rate's decay rate is $\rho = .98$. This is engendered by endogenous capital accumulation dynamics: the agent accumulates extra units of capital for many periods and, in a second moment

---

[36] In Appendix D.1, I show in detail how these spectral densities are computed.

(150 months in this case), begins to use them to sustain consumption growth over a longer time-horizon. The implied spectral density of consumption growth (bottom-right panel of Fig. 11) now has a pronounced spike at low frequencies, showing that long-run risk in productivity can generate long-run risk in consumption. Fig. 14 shows the theoretical spectral density of the exogenous consumption process used in the exchange economy, the theoretical spectral density of the productivity growth rate, and the spectral density of the endogenous consumption process generated by the model with production when $\Psi = 1.5$. The consumption growth spectral density produced by the model with production is very close to that of the process assumed in an endowment economy. However, the model with production reallocates part of the volatility from the higher to the lower frequencies, producing more long-run consumption risk than that in an exchange economy. Note that such change in spectral density has only a modest effect on the autocorrelation of the quarterly consumption growth rate. This autocorrelation, in fact, is now .36 (it was .32 in an exchange economy), a value still close to what is observed in the data.

Finally, in Fig. 13 I show the contribution of both the short- and long-run component to the long-horizon variances in the productivity growth rate (right panel) and the consumption growth rate (left panel). I define the variance of the productivity growth rate at the horizon h as: $Var[\Delta a_{t+h|t}]/h$. Since I assume that long- and short-run news is uncorrelated, the previous long-horizon variance is easily decomposed into two subcomponents:

$$Var[\Delta a_{t+h|t}]/h = \underbrace{Var\left[\sum_{k=1}^{h-1} x_{t+k-1}\right]/h}_{Var_h^{lrr}(\Delta a)} + \underbrace{Var\left[\sum_{k=1}^{h} \epsilon_{a,t+k}\right]/h}_{Var_h^{srr}(\Delta a)}$$

In the right panel of Fig. 13, the solid line shows that the contribution of short-run risk to the long-horizon variance is constant across horizons, because the short-run risk is *i.i.d.* The dashed line, however, shows that the variance of consumption attributable to the long-run component increases over longer time horizons and actually becomes dominant after about 15 periods. This is due to the fact that the long-run component is persistent and its auto-covariances play an important role in amplifying the long-horizon productivity variance. I proceed in an analogous way to examine the long-horizon variance of the consumption growth rate.[37] The left panel of the figure confirms that, for any horizon h, the variances of both the long- and short-run components of consumption growth are smaller

---

[37] I describe in more detail the computations for the long-horizon variance of consumption in Appendix D.2.

than those of productivity growth. Consumption is then smoother than productivity over both shorter and longer horizons. Note that the variance induced by the short-run shock is not flat across time-horizons. In accordance with prior observation, variance actually increases slightly because of the small endogenous persistence in consumption growth introduced by the model. Finally, the long-run productivity component is able to generate a strong increase in the variance of consumption over longer time-horizons similar to what occurs in the productivity growth process. In the next section, I study the impact of this effect on welfare costs.

4.5 Welfare costs, IES and risk free rate.

In the previous section we have seen that the endogenous consumption growth rate has a spectral density similar to that observed in an endowment economy, but also a higher level of long-run uncertainty. For this reason, when the preference parameters are kept unchanged, welfare costs in a production economy are higher than those observed in an exchange economy. In particular, when the risk aversion, the IES, and the monthly subjective discount factors are calibrated as in Table 3, the total welfare costs generated by the productivity uncertainty range from 2512% (when IES=1.5 and the RRA=16) to 560% (when the IES is .8 and the RRA is 20). As in an exchange economy, the IES and the implied effective discount factor continue to play a very important role.

At the same time, however, the higher exposure of consumption to long-run risk implies that the risk free rate mean is lower than that obtained in an endowment economy. In order to offset this effect, I calibrate the subjective discount factor to .99923, a value that allows to match the risk free rate mean in an exchange economy in the case of IES=1.5.[38] This adjustment, even if apparently small, has a strong negative effect on the level of welfare costs. As is apparent in Panel C of Table 4, the total welfare costs range now from 404% (IES=1.5) to 218% (IES=.8). *The welfare costs are still increasing in the IES, but the lower subjective discount factor allows them to be much smaller.*[39] Although this

---

[38] Another way to decrease the risk free rate would be to decrease the persistence of the long-rung component in productivity. I prefer to lower the subjective discount factor since this helps to better match both the risk free rate and the mean of the investment-output ratio observed in the data.

[39] Notice that now the moving average representation of the consumption process is slightly different from that used in an exchange economy. When $IES \neq 1$, the consumption growth process is very sensitive to contemporaneous innovations in the long-run component. In section B.4.1 of the appendix, I show that when the IES is below 1, this response tends to increase welfare costs. This explains why the welfare costs with IES=.8 are much larger in a production economy

modification of the calibration allows the matching of the level of the risk free rate when the IES is 1.5, the equity premium is far from what is observed in the data. As in an exchange economy, I adjust my relative risk aversion in order to match a higher level of equity premium. In the next section, I describe how I change the calibration and I show the implications on welfare costs, prices, and quantities. Before concluding this section, let us briefly examine the welfare costs produced by the introduction of the adjustment cost (λ^{adj}). In Panel C, we can see that λ^{adj} ranges from 5% to 7%, consistent with Barlevy's (2004) estimates that go from 7% to 10%. These costs are remarkable in absolute terms and are quite large if compared to Lucas' results. Note, however, that they represent about 2% of the total welfare costs and do not significantly change the basic intuitions developed in the previous sections.

4.6 Welfare costs and equity premium

In order to better match the equity premium, the relative risk aversion premium is increased to a value of 30. This value is still reasonably low given the well known difficulties in producing high equity premiums in production economies. Tallarini (2000), for example, using a risk aversion of 100, obtains an annualized equity premium of just .04%, about 100 times smaller than in the data. A risk aversion of 30 is a plausible value given the recent empirical findings in Attanasio and Vissing (2005).

In my benchmark calibration, I fix the IES at a value of 2. This value is consistent with what is estimated by Basal-Gallant-Tauchen (2004), Colacito-Croce (2005), Bansal-Kiku-Yaron (2006) and Attanasio and Vissing (2005). As shown before, this helps to better match the unconditional correlation between investment and consumption growth and the maintenance of the risk free rate at low levels.

Given higher risk aversion, a higher IES and a higher degree of long-run risk, the risk free rate tends to be lower than what was reported in the previous tables. In order to keep the risk free rate in line with my previous calibrations, I lower the subjective discount factor and I calibrate $\delta = .98^{\frac{1}{12}}$. This helps to keep the investment-output ratio low.[40] All other parameters remain unchanged.

I report the main results produced by this calibration in Table 6. The implied equity premium is now 4.8%. This value is slightly below the historical mean of the CRPS stock market excess returns.[41] However, it is in line with the estimates reported by Fama-French

---

than in an exchange economy.

[40] Under this calibration, at the steady state, this ratio is 25% (in the data, it is about 20%).

[41] See Appendix A. for more details about the data.

(2000).

The risk free rate has an annual mean of 1.2% and is smoother than the excess returns. Similar to what is observed in the data, its volatility is about 10 times smaller than the volatility of market returns.

Increasing the IES from 1.5 to 2 does not change the main properties of consumption. The annualized volatility of quarterly consumption growth is still 1.4% and its autocorrelation function is .35. In order to better compare the consumption process generated by the model to the data, in Fig. 14 I plot the spectrum of the *quarterly* growth rate of consumption. In particular, the dashed line shows the spectral density estimated from the data, while the other lines show the bottom 2.5%, the median, and the top 97.5% percentile of the distribution of quarterly spectral densities generated by simulating the model with production.[42] If we focus on the business cycle frequencies, [.2 - .8], we see that the median spectral density produced by the model is reasonably close to its empirical counterpart. More generally, the model is able to capture the decline over higher frequencies of the empirical density. Note also that the spectrum estimated from the data strays outside the confidence interval generated by the model only for frequencies higher than .75, equivalent to cycles with duration shorter than 8 quarters.

The contemporaneous correlation with investment growth is about 50%, a value that is close to what is observed in the data. Investment is 1.3 times more volatile than consumption.

The welfare costs associated with this calibration are about 200%. They are two times smaller than those obtained in the previous section matching a lower equity premium. This is primarily due to the fact that the higher risk aversion required to match the equity premium allows me to keep low the risk free rate with a lower subjective discount factor inducing a drastic reduction in welfare costs.

As seen in section 3, the trade-off between RRA and subjective discount factor produced by the risk free rate mean is crucial for welfare costs. Examining a production economy suggests that the calibration of the RRA has to be higher than what is usually assumed in the long-run risk literature. The implied total welfare costs are still significantly bigger than what is usually obtained with models that focus only on quantities.

---

[42] In order to generate these distributions, I simulate the model with a production economy over a sample of 600 months, time-aggregate the quantities to quarterly frequencies, and compute the implied growth rates for consumption. I repeat this procedure 500 times in order to generate a sample of spectral densities.

5 Conclusion

This paper measures and characterizes the welfare costs of consumption uncertainty when a representative agent has generalized recursive preferences and is exposed to long-run consumption risk. Working with an endowment economy, I show that: first, welfare costs can be high even when risk aversion is moderate and short-run volatility of consumption growth is low; second, welfare costs are far more sensitive to the intertemporal elasticity of substitution in consumption (IES) than to risk aversion. In particular, welfare costs are increasing sharply in the IES when that parameter exceeds unity. I then solve a stochastic growth model with long-run risk in productivity growth and show that: (1) assuming an IES bigger than 1 does not alter the basic properties of consumption and investment; (2) long-run risk in productivity may propagate in consumption even when investment is free to adjust; (3) the main asset price implications of the model remain unchanged; and (4) the main implications for welfare costs are preserved. In a production economy, the total welfare costs of aggregate consumption fluctuations are on the order of 200%, a number that is about 2000 times larger than that obtained by Lucas(1987).

There is a number of directions in which this work could be fruitfully extended.

First, in this paper I assume a constant supply of labor in order to keep the analysis as simple as possible. However, labor fluctuations can have an important impact on welfare costs and should be taken into account in future research. In particular, it will be especially important to study the implications that the model has on the relationship between labor income, aggregate variables and capital returns.

Second, as we have seen, the model predicts excessively smooth investment growth rates. This is induced by the presence of concave adjustment costs. Boldrin-Christiano-Fisher(2004) show that a model with habits and real rigidities can partially solve this problem and simultaneously produce volatile returns and investment. It would be important to reexamine the relation between investment, returns and welfare costs in a model with long-run risk, Epstein-Zin-Weil preferences and pre-determined investment as in Boldrin-Christiano-Fisher.

Finally, this paper remains silent about the effective role that policymakers can adopt to stabilize consumption fluctuations. A recent paper by Panageas and Yu (2006) shows that long-run movements in consumption can be generated by technological innovations embodied in new vintages of capital stocks. Given the high welfare costs of long-run fluctuations, I believe that one of the next steps for this field of research should be the examination of the effective role of policies directed toward stabilizing long-run technological adoption processes and R&D activity.

Appendix A. Data.

In order to compute the spectral densities of consumption I use data on real per capita consumption from the Campbell(2003). Data are quarterly observations on private consumption of nondurables and services from 1970.Q1 to 1998.Q8. When I produce joint statistics about consumption and investment, I use the following quarterly data from BEA:

(1) Real Private Fixed Investment, seasonally adjusted in billions of chained 2000 dollars, from 1947:Q1 to 2005:Q2;

(2) Real Personal Consumption of Nondurables and Services, seasonally adjusted in billions of chained 2000 dollars, from 1947:Q1 to 2005:Q2;

I compute output as sum of consumption and investment. I exclude the government expenditure.

I measure inflation by the CPI index, from 1947:Q1 to 2005:Q2. In order to take into account seasonality, I smooth inflation by computing in every quarter the average of the last four quarterly inflation rates.

The risk free rate is measured by the return of the 3-Months t-Bills minus the realized inflation, from 1947:Q1 to 2003:Q2.

The market returns (from 1947:Q2 to 2003:Q2), the annualized price-dividend ratio (from 1947:Q1 to 2002:Q4) and the dividends(from 1947:Q1 to 2003:Q3) are from CRSP.

From the BLS I obtain both a time-series for the Civilian Employment-Population Ratio (from 1948:Q1 to 2005:Q2) and data on Average Weekly Hours (Total Private Industies, from 1964:Q1 to 2005:Q2). I measure labor by multiplying the average weekly hours worked by the civilian employment-population ratio. I then normalize the series dividing it by 7*16 (a measure of the total number of workable hours in a week).

Finally, I compute the total factor productivity in log units as:

$$tfp_t = output_t - (1-\alpha) labor_t$$

Appendix B. A Log-Linear Approximation

In this section of the appendix I derive the main results that regard the exchange economy. I characterize first the asset pricing implications of the model. I finally link the utility-consumption ratio to the price-consumption ratio.

B.1 The Economy

The representative agent has the following preferences:

$$U_t = \left[(1-\delta)C_t^{1-\frac{1}{\Psi}} + \delta E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1-\frac{1}{\Psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\Psi}}} \tag{B.1}$$

The endowment process is:

$$\begin{aligned}\Delta c_{t+1} &= \mu + x_t + \sigma\epsilon_{t+1}^c \\ x_t &= \rho x_{t-1} + \sigma_x\epsilon_t^x \\ \begin{bmatrix}\epsilon^c \\ \epsilon^x\end{bmatrix} &\sim iidN\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\right)\end{aligned} \tag{B.2}$$

There is a redundant asset paying a dividend process with the following growth rate:

$$\Delta d_{t+1} = \mu_d + \phi x_t + \phi_c\sigma\epsilon_{t+1}^c + \sigma_d\epsilon_{t+1}^d \tag{B.3}$$

where the shock $\epsilon_{t+1}^d \sim i.i.d.N(0,1)$ is uncorrelated with all the other shocks. Let C denote the variance-covariance matrix of the vector of shocks $\epsilon_{t+1} \equiv \begin{bmatrix}\epsilon_{t+1}^c & \epsilon_{t+1}^d & \epsilon_{t+1}^x\end{bmatrix}$, then:

$$C \equiv \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix} \tag{B.4}$$

B.2 The Asset Pricing Results.

B.2.1 Risk Free Rate and Consumption Returns.

The case $\Psi \neq 1$.

Let P_t^C denote the ex-dividend price of a claim to an asset that pays a consumption stream

C_t measured at the end of time t. Define:

$$\begin{aligned} W_{c,t} &\equiv P_t^C / C_t \\ R_{C,t+1} &\equiv (P_{t+1}^C + C_{t+1}) / P_t^C \end{aligned}$$

Let log variables be denoted by small letters. The Campbell-Shiller log linearization of the returns implies:

$$\begin{aligned} w_{c,t} &= \overline{w_c} + \sum_{i=0}^{\infty} \kappa_c E_t[\Delta c_{t+1+i}] - \sum_{i=0}^{\infty} \kappa_c^i E_t[r_{c,t+1+i}] \\ \kappa_c &\equiv \frac{\overline{exp(w_c)}}{1 + \overline{exp(w_c)}} \end{aligned} \tag{B.5}$$

When Epstein-Zin preferences are adopted, $\gamma \neq 1/\Psi$, and (B.15) holds, a log-linearization of the first order conditions of the representative agent implies:

$$\begin{aligned} m_{t+1} &= \overline{m} - \frac{1}{\Psi} x_{c,t} - \kappa_c \frac{\gamma - 1/\Psi}{1 - \rho\kappa_c} \sigma_x \epsilon_{x,t+1} - \gamma\sigma\epsilon_{c,t+1} \qquad \text{(B.6)} \\ r_{c,t+1} &= \overline{r_c} + \frac{1}{\Psi} x_t + \kappa_c \frac{1 - 1/\Psi}{1 - \rho\kappa_c} \sigma_x \epsilon_{x,t+1} + \sigma\epsilon_{c,t+1} \\ r_{f,t} &= \overline{r_f} + \frac{1}{\Psi} x_t \\ w_{c,t} &= \overline{w_c} + \frac{1 - 1/\Psi}{1 - \kappa_c \rho} x_t \end{aligned}$$

Given the results above, the Euler Equation for the asset that pays consumption - evaluated at x_t=0 - provides the following non linear equation in κ_c:

$$\kappa_c = \delta e^{(1-\frac{1}{\Psi})\left(\mu - .5(\gamma-1)Var[\sigma\epsilon_{c,t+1} + \kappa_c(1-\rho\kappa_c)^{-1}\sigma_x\epsilon_{x,t+1}]\right)} \tag{B.7}$$

Rewrite the stochastic discount factor m_{t+1} and the return in vector form:

$$\begin{aligned} m_{t+1} &= \overline{m} - \frac{1}{\Psi} x_{c,t} + \Gamma_m \epsilon_{t+1} \\ \Gamma_m &\equiv \left[\begin{array}{ccc} -\gamma\sigma & 0 & -\kappa_c \frac{\gamma - 1/\Psi}{1-\rho\kappa_c} \sigma_x \end{array} \right] \\ r_{c,t+1} &= \overline{r_c} + \frac{1}{\Psi} x_t + \Gamma_c \epsilon_{t+1} \\ \Gamma_c &\equiv \left[\begin{array}{ccc} \sigma & 0 & \kappa_c \frac{1-1/\Psi}{1-\rho\kappa_c} \sigma_x \end{array} \right] \end{aligned}$$

Remember that:

$$E_t[r^{ex}_{c,t+1}] = -cov\left(m_{t+1} - E_t[m_{t+1}], r_{c,t+1} - E_t[r_{c,t+1}]\right) - .5V(r_{c,t+1} - E_t[r_{c,t+1}])$$

Then the following holds:

$$E_t[r^{ex}_{c,t+1}] = -\Gamma_m C \Gamma_c' - .5\Gamma_c C \Gamma_c'$$

By rearranging the definition of the stochastic discount factor with Epstein-Zin(1989) preferences, I get:

$$\begin{aligned} E[r_f] &= -\log(\delta) + \frac{1}{\Psi}E(\Delta c) + \frac{(1-\theta)}{\theta}E_t[r^{ex}_{c,t+1}] - \frac{1}{2\theta}V_t[m_{t+1}] \\ \theta &\equiv \frac{1-\gamma}{1-1/\Psi} \end{aligned}$$

So, it follows that:

$$E[r_f] = -\log(\delta) + \frac{1}{\Psi}\mu + \frac{(1-\theta)}{\theta}(-\Gamma_m C \Gamma_c' - .5\Gamma_c C \Gamma_c') - \frac{1}{2\theta}\Gamma_m C \Gamma_m'$$

We are now able to find the intercept of the log stochastic discount factor:

$$\overline{m} = \theta \log \delta - \frac{\theta}{\Psi}\mu + (\theta - 1)\left(E[r^{ex}_c] + E[r_f]\right)$$

The special case $\Psi = 1$.

When $\Psi = 1$ it's possible to find a closed form solution for both the stochastic discount factor and the risk free rate. In particular:

$$\begin{aligned} m_{t+1} &= \overline{m} - x_{c,t} + \Gamma_m \epsilon_{t+1} \\ \Gamma_m &\equiv \left[\begin{array}{ccc} -\gamma\sigma & 0 & -\delta\frac{\gamma-1}{1-\rho\delta}\sigma_x \end{array}\right] \\ \overline{m} &\equiv \log(\delta) - \mu - .5(\gamma-1)^2\Gamma_u C \Gamma_u' \\ \Gamma_u &\equiv \left[\begin{array}{ccc} \sigma & 0 & \frac{\delta}{1-\rho\delta}\sigma_x \end{array}\right] \\ r_{f,t} &= \overline{r_f} + x_t \\ \overline{r_f} &= -\log(\delta) + \mu + .5\left((\gamma-1)^2\Gamma_u C \Gamma_u' - \Gamma_m C \Gamma_m'\right) \end{aligned} \tag{B.8}$$

B.2.2 Dividend Returns.

Let P_t^D denote the ex-dividend price of a claim to an asset that pays a consumption stream D_t measured at the end of time t.Define:

$$\begin{aligned} W_{d,t} &\equiv P_t^D / D_t \\ R_{d,t+1} &\equiv (P_{t+1}^D + D_{t+1}) / P_t^D \end{aligned}$$

A log-linearization of the Euler Equation implies:

$$w_{d,t} = \overline{w_d} + \frac{\phi - 1/\Psi}{1 - \kappa_d \rho} x_t \tag{B.9}$$

$$r_{d,t+1} = \overline{r_d} + \frac{1}{\Psi} x_t + \kappa_d \frac{\phi - 1/\Psi}{1 - \rho \kappa_d} \epsilon_{x,t+1} + \sigma_d \epsilon_{d,t+1} + \phi_c \sigma \epsilon_{c,t+1} \tag{B.10}$$

Now rewrite the returns of the asset paying dividends in vector form:

$$\begin{aligned} r_{d,t+1} &= \overline{r_d} + \frac{1}{\Psi} x_{c,t} + \Gamma_d \epsilon_{t+1} \\ \Gamma_d &\equiv \left[\begin{array}{ccc} \phi_c \sigma & \sigma_d & \kappa_d \frac{\phi - 1/\Psi}{1 - \rho \kappa_d} \sigma_x \end{array} \right] \end{aligned}$$

Given the results above, the Euler Equation for the asset that pays dividend - evaluated at $x_t = 0$ - provides the following non linear equation in κ_d:

$$\kappa_d = e^{\overline{m} + \mu + .5 Var[(\Gamma_m + \Gamma_d) \epsilon_{t+1}]}$$

The equity premium in this case is:

$$E_t[r_{d,t+1}^{ex}] = -\Gamma_m C \Gamma_d' - .5 \Gamma_d C \Gamma_d'$$

B.3 The Utility-Consumption Ratio

The case $\Psi \neq 1$
Epstein-Zin-Weil(1989) preferences are homogeneous-homothetic, this implies that:

$$\frac{U_t}{C_t} = \left[(1-\delta)\frac{W_t}{C_t}\right]^{\frac{1}{1-\frac{1}{\Psi}}}$$

Where $\frac{W}{C}_t$ is the wealth-consumption ratio at time t defined by the budget constraint of the representative household:

$$W_{t+1} = R_{w,t+1}\left(W_t - C_t\right)$$

where $R_{w,t+1}$ is the return of wealth. In this economy there is just one asset providing total consumption, so $R_{w,t+1} = R_{c,t+1}$. In order to satisfy feasibility, $W_t = P_t^C + C_t$. Hence:

$$\frac{U_t}{C_t} = \left[(1-\delta)(W_{c,t}+1)\right]^{\frac{1}{1-\frac{1}{\Psi}}} \tag{B.11}$$

The utility-consumption ratio at the steady state is derived as:

$$\frac{\overline{U}}{C} = \left[\frac{1-\delta}{1-\kappa_c}\right]^{\frac{1}{1-\frac{1}{\Psi}}}$$

When no long-run risk is introduced in the model ($\rho = \sigma_x = 0$) this formula is exact. Finally, notice that $\delta^{lrr} = \kappa_c$ and that in order to recover δ^{srr} it is enough to impose $\rho = \sigma_x = 0$ in (B.7). If one log-linearizes (B.11) the following is true:

$$u_t = \overline{u} + \frac{\kappa_c}{1-\rho\kappa_c}x_t$$

The special case $\Psi = 1$
In this case the utility-consumption ratio is exactly:

$$\begin{aligned} u_t &= \mu_u + \frac{\delta}{1-\rho\delta}x_t \\ \mu_u &= \frac{\delta}{1-\delta}\left[\mu + .5(1-\gamma)\frac{\delta^2}{(1-\rho\delta)^2}\sigma_x^2 + .5(1-\gamma)\sigma^2\right] \end{aligned} \tag{B.12}$$

B.4 The Welfare Costs

Remember that in order to keep constant the growth rate of consumption in levels the following restriction is imposed:

$$\begin{aligned} \mu &= \mu^L - .5(\sigma^2 - \frac{\sigma_x^2}{1-\rho^2}) \\ \mu^L &\equiv E[C_{t+1}/C_t] \end{aligned}$$

The special case $\Psi = 1$

Just apply the definition of cost and use μ_u as it is derived in (B.12) in order to get:

$$\lambda^{srr} = .5\gamma\sigma^2 \frac{\delta}{1-\delta} \tag{B.13}$$

$$\lambda^{lrr} = .5\left[\frac{1}{1-\rho^2} + (\gamma-1)\frac{\delta^2}{(1-\rho\delta)^2}\right]\sigma_x^2 \frac{\delta}{1-\delta} \tag{B.14}$$

The case $\Psi \neq 1$

In this case the welfare costs are:

$$\begin{aligned} \lambda^{srr} &= \frac{1}{1-\frac{1}{\Psi}}(\log(1-\delta^{srr}) - \log(1-\delta^{nr})) \\ \lambda^{lrr} &= \frac{1}{1-\frac{1}{\Psi}}(\log(1-\delta^{lrr}) - \log(1-\delta^{srr})) \end{aligned}$$

where:

$$\begin{aligned} \delta^{nr} &\equiv \kappa_c \text{ and } \sigma_x = \sigma = \rho = 0 \\ \delta^{srr} &\equiv \kappa_c \text{ and } \sigma_x = \rho = 0, \sigma \neq 0 \\ \delta^{lrr} &\equiv \kappa_c \text{ and } \sigma_x \neq 0, \rho \neq 0, \sigma \neq 0 \end{aligned}$$

A log-linearization of $\log(1-\delta^{srr})$ with respect to σ^2 around $\overline{\sigma} = 0$ implies:

$$\lambda^{srr} \approx .5\gamma\sigma^2 \frac{\delta^{srr}}{1-\delta^{srr}}$$

A log-linearization of $\log(1-\delta^{lrr})$ with respect to σ_x^2 around $\overline{\sigma_x} = 0$ implies:

$$\lambda^{lrr} \approx .5\left[\frac{1}{1-\rho^2} + (\gamma-1)\frac{(\delta^{lrr})^2}{(1-\rho\delta^{lrr})^2}\right]\sigma_x^2 \frac{\delta^{lrr}}{1-\delta^{lrr}}$$

B.4.1 A special case

Assume that:

$$\begin{aligned}
\Delta c_{t+1} &= \mu + x_t + \sigma\epsilon^c_{t+1} + \theta\epsilon^x_{t+1} \qquad \text{(B.15)}\\
x_t &= \rho x_{t-1} + \sigma_x \epsilon^x_t \\
\begin{bmatrix} \epsilon^c \\ \epsilon^x \end{bmatrix} &\sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)
\end{aligned}$$

The implied welfare costs of the long-run component is:

$$\lambda^{lrr} = .5\left[\frac{1}{1-\rho^2} + (\gamma-1)\left(\theta + \frac{\delta^{lrr}}{1-\rho\delta^{lrr}}\right)^2\right]\sigma_x^2 \frac{\delta^{lrr}}{1-\delta^{lrr}}$$

If $\theta = 0$, the costs are as big as those ones obtained in the previous section. If $\theta > 0$ - this is, for example, true in production economy when $\Psi < 1$ - the welfare costs are even bigger because future consumption and future continuation value co-move together. Finally when $\theta < 0$ - this is, for example, true in production economy when $\Psi > 1$ - the welfare costs are smaller since future consumption and future utility-consumption ratio move in opposite directions.

Appendix C. Computational methods for the endowment economy

All the results produced in the tables regarding the exchange economy are obtained by employing computational methods. The following sections explain which methods I use.

C.1 Value Function

The case $\Psi \neq 1, \sigma_x \neq 0, \rho \neq 0$

In order to compute the welfare costs I need to solve for the utility-consumption ratios. Given a consumption process as in (B.15) the associated utility-consumption ratio can be found solving the following functional equation:

$$\frac{U}{C}(x) = \left[(1-\delta) + \delta E_x\left[\left(\frac{U}{C}(\rho x + \sigma_x \epsilon'_x)\right)^{1-\gamma} e^{(1-\gamma)(\mu + x + \sigma\epsilon'_C)}\right]^{\frac{1}{\theta}}\right]^{\frac{1}{1-\frac{1}{\Psi}}}$$

The contraction mapping theorem applies. All I have to do is to discretize the problem.I discretize the support of x using J equidistant points on the interval $[-5\frac{\sigma_x^2}{1-\rho^2} \quad +5\frac{\sigma_x^2}{1-\rho^2}]$ and I discretize the distribution of a shock $\epsilon \sim N(0,1)$ generating a grid over the interval $[-5 \quad +5]$ with I equidistant points $\epsilon_1, \ldots, \epsilon_I$. The probability that I assign to a generic point ϵ_i is:

$$\phi_i = \frac{e^{-.5\epsilon_i^2}}{\sum_{i=1}^{I} e^{-.5\epsilon_i^2}}$$

I interpolate the utility-consumption ratio using a cubical polynomial:

$$\frac{U}{C}(x') = [1 \quad x' \quad x'^2 \quad x'^3]\beta$$

where β is a vector of coefficients that I recover by OLS, so that $\beta = (X'X)^{-1}X'\frac{U}{C}(X)$ and X is the associated collocation matrix. I define:

$$s_{ij} \equiv \left[1 \quad (\rho x_j + \sigma_x \epsilon_i) \quad (\rho x_j + \sigma_x \epsilon_i)^2 \quad (\rho x_j + \sigma_x \epsilon_i)^3\right]$$

At this point I guess β_0 and I iterate until convergence the following operator:

$$\begin{aligned}
\frac{U}{C}(x_j)_{n+1} &= \left[(1-\delta) + \delta \left(\sum_{i}^{I} \sum_{k}^{I} (s_{ji}\beta_n e^{\mu + x_j + \sigma\epsilon_k})^{(1-\gamma)} \phi_i \phi_k \right)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\Psi}}} \\
j &= 1, 2, \ldots J \\
\beta_{n+1} &= (X'X)^{-1}X'\frac{U}{C}(X)_{n+1}
\end{aligned}$$

All the other cases

When $\Psi = 1$ and when $\Psi \neq 1$ but at the same time $\sigma_x = \rho = 0$, it is possible to obtain exact close form solutions for the utility-consumption ratio as explained in the previous section.

C.2 Price-Dividend Ratio

The special case $\Psi = 1$

I compute the price-dividend ratio schedules by minimizing the Euler Equation errors.

When there is the long-run risk, I iterate until convergence the following recursion:

$$\begin{aligned}
V^d(x_j)_{n+1} &= \sum_{i=1}^{I}\sum_{k=1}^{I}\sum_{l=1}^{I} e^{\mu_m - x_j - \gamma\sigma\epsilon_k + (1-\gamma)U_u\sigma_x\epsilon_i}(1+s_{ji}\beta_n^d)e^{\mu_d+\phi x_j+\phi_c\sigma\epsilon_k+\sigma_d\epsilon_l}\phi_l\phi_i\phi_k \\
\beta_n^d &= (X'X)^{-1}X'V^d(X)_n
\end{aligned}$$

I construct the grids for the state and the shocks exactly as described in the previous section. I interpolate the price-dividend ratio using the same cubical polynomial. When dividends and consumption growth are both *iid*, I compute the level of the price dividend as:

$$\begin{aligned}
V^d &= \frac{\Omega}{1-\Omega} \\
\Omega &= \sum_{k=1}^{I}\sum_{l=1}^{I} e^{\mu_m+\mu_d+(\phi_c-\gamma)\sigma\epsilon_k+\sigma_d\epsilon_l}\phi_l\phi_k
\end{aligned}$$

The case $\Psi \neq 1$

When $\Psi \neq 1$ and there is long-run Risk in consumption and dividends, first of all I solve for the utility-consumption ratio and I find β. Then I iterate until convergence the following recursion:

$$\begin{aligned}
V^d(x_j)_{n+1} &= \frac{C_{1j}C_2C_3}{S_j}\sum_{i=1}^{I}(s_{ji}\beta)^{\frac{1}{\Psi}-\gamma}(1+s_{ji}\beta_n^d)\phi_i \\
&\text{where} \\
C_{1j} &= \delta e^{\mu_d-\gamma\mu+(\phi-\gamma)x_j} \\
C_2 &= \sum_{k=1}^{I} e^{(\phi_c-\gamma)\sigma\epsilon_k}\phi_k \\
C_3 &= \sum_{k=1}^{I} e^{\sigma_d\epsilon_k}\phi_k \\
S_j &= \left(\sum_{i=1}^{I}\sum_{k=1}^{I}(s_{ji}\beta)^{1-\gamma}e^{(1-\gamma)(\mu+x_j+\sigma\epsilon_k)}\phi_i\phi_k\right)^{\frac{\frac{1}{\Psi}-\gamma}{1-\gamma}} \\
\beta_n^d &= (X'X)^{-1}X'V^d(X)_n
\end{aligned}$$

I construct the grids and I interpolate the price-dividend function exactly as shown before for the utility-consumption ratio.

Appendix D. Production Economy

The problem that I have to solve is:

$$
\begin{array}{rcl}
s & \equiv & [\ \Delta a \quad x \quad k\ ] \\
u(s) & = & \max\limits_{c,k'} \left[(1-\delta)c^{1-\frac{1}{\Psi}} + \delta e^{(1-\frac{1}{\Psi})\Delta a} \left(E_s[u(s')^{1-\gamma}]^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right) \right]^{\frac{1}{1-\frac{1}{\Psi}}} \\
 & s.t. & \\
c & \geq & 0, k' \geq 0 \\
c+i & = & y \equiv e^{(1-\alpha)\Delta a} k^{\alpha} \bar{n}^{(1-\alpha)} \\
k' e^{\Delta a} & = & (1-\delta)k + G\left(\dfrac{i}{k}\right) k \\
x' & = & \rho x + \sigma_x \epsilon'_a \\
\Delta a' & = & \mu + x + \sigma \epsilon'_a \\
\epsilon'_a & \perp & \epsilon'_x
\end{array}
$$

This problem can be solved by standard value function iterations. I program my algorithm in Fortran, Compaq Visualizer 6.a-professional, by HP.
The first thing I do is to discretize the state space. I use: (1) N_a equidistant points for Δa on the interval $[-2std[\Delta a]\ +2std[\Delta a]]$; (2) N_x equidistant points for x on the interval $[-2std[x]\ +2std[x]]$; (3) N_k equidistant points for k on the interval $[.1k^{ss}\ 1.9k^{ss}]$. Where k^{ss} is the value of capital at the deterministic steady state. At this point I guess a value function $u(\Delta a, x, k)$, that is an array $N_a \times N_x \times N_k$. From now one, similarly to what done in Fortran, I use ':' to indicate all the elements in one dimension. So, for example, $u(:, x_i, k_l)$ indicates all the values of the utility function with respect to the state Δa, given $x = x_i$ and $k = k_l$. Now I am ready to start an iteration, I list below the required steps:
STEP 1: Computing $Q(\Delta a, x, k) \equiv E_s[u(\Delta a', x', k)^{1-\gamma}]$.
I discretize a shock distributed according to a standard normal as in Appendix C. I use N_s points. Looking at the definition of $E_s[u^{'(1-\gamma)}]$, it is possible to see that this object is in reality invariant with respect to Δa. The only two variables that have predictive power and really matter for the conditional expected value are k and x. So, in order to make

fast this step, for every (x_i, k_l) in the grid, I compute the following expression:

$$H(x_i, k_l) = \sum_{h=1}^{N_s} \sum_{f=1}^{N_s} u(\mu + x_i + \sigma\epsilon_h, \rho x + \sigma_x \epsilon_f, k_l)^{1-\gamma} \phi_f \phi_h$$

I compute $H(x_i, k_l)$ only $N_k \times N_x$ times and then impose $Q(:, x_i, k_l) = H(x_i, k_l)$. In order to compute $H(\cdot, \cdot)$ I have to be able to compute $u(\mu + x_i + \sigma\epsilon_h, \rho x + \sigma_x \epsilon_f, k_l)$. The initial guess gives information only on $u(\Delta a_j, x_i, k_l)$, so I need to be able to approximate and interpolate $u(\cdot, \cdot, k_l)$ on points that are outside the grid of the states. For given k_l, $\log(u(:, :, k_l))$ gives me a set of points - over a two-dimensional surface - which I approximate using a *tensor product of Chebyshev polynomials in* Δa *and* x. This surface is very close to be linear, so I don't need many polynomials to produce a good approximation. In particular, the number of polynomial is smaller than $N_{\Delta a} \times N_x$ and for this reason I compute the polynomials coefficients by a standard projection method (for every k_l in the grid, I apply OLS on $\log(u(:, :, k_l))$, using as regressors all the polynomials I have in Δa and x). Once I have the Chebyshev coefficients, I interpolate $\log(u)$ in correspondence of the generic points $(\mu + x_i + \sigma\epsilon_h, \rho x + \sigma_x \epsilon_f, k_l)$. Once this is done, it is immediate to compute H.

STEP 2: iterpolating $Q(\Delta a, x, k)$ with respect to capital.

I want to solve the maximization problem by applying a standard Newton-algorithm to the control variable k'. In order to apply this algorithm I need to be able to evaluate the right hand side of my recursion for every admissible value of the control variable k'. This means that I have to be able to evaluate Q for every feasible value of capital k'. For every x_i in my grid, I project through OLS-method $\log(H(x_i, :))$ on a constant, $\log(\overrightarrow{k})$ and $\log(\overrightarrow{k})^2$, where: $\overrightarrow{k}$ indicates the vector that collects the log-values of capital in my grid and $\log(\overrightarrow{k})^2$ is a vector collecting the square of log-values of capital. This transformation is very useful since $\log(H(x_i, :))$ is very close to be linear in log-capital and allows me to have very small interpolation residuals (in the order of 1.e-6, log units). For every x_i, I run this OLS interpolation and I get an OLS coefficients-vector that I denote as β_i. At this point, using the interpolation implies the following fast way to evaluate the expected value of future utility at k': $H(x_i, k') = exp([1 \ k' \ (k')^2]\beta_i)$.

STEP 3: maximizing

Given a the state, $s = [\ \Delta a_j \quad x_i \quad k_l\ ]$, I solve the following maximization problem:

$$\begin{array}{rcl} \max\limits_{c \geq 0} & & \left[(1-\delta)c^{1-\frac{1}{\Psi}} + \delta e^{(1-\frac{1}{\Psi})\Delta a_j}\left(exp([1\ k'\ (k')^2]\beta_i)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}}\right)\right]^{\frac{1}{1-\frac{1}{\Psi}}} \\ & s.t. & \\ i & = & e^{(1-\alpha)\Delta a_j} k_l^{\alpha} \bar{n}^{(1-\alpha)} - c \\ k' & = & \dfrac{(1-\delta) + G\left(\frac{i}{k_l}\right)}{e^{\Delta a_j}} k_l \\ k' & \geq & 0 \end{array}$$

I solve this problem using the Fortran subroutine 'DUVMIF'. The subroutine evaluates also the objective function at the optimum and allows me to update the value function.
STEP 4: stopping rule.
Once done with step 3, I measure the distance between the new value function, u', and the initial one using a standard sup-norm. If $\frac{||(u')^{1-\frac{1}{\Psi}} - (u)^{1-\frac{1}{\Psi}}||}{1-\beta} < 1.e-8$ I stop. Otherwise I use the new value function as initial guess and I go back to STEP 1.

D.1 Spectrum

Given a stochastic process X, its spectral density evaluated at the frequency f is:

$$S_X(f) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k cos(kf)$$

where

$$\gamma_k = cov(X_t, X_{t-k})$$

Given the following moving average representation for X:

$$X_t = \sum_{j=0}^{\infty} \theta_j^X \epsilon_{t-j}^X$$

the auto-covariances can be computed in the following way:

$$\gamma_k = \sum_{j=0}^{\infty} \theta_j^X \theta_{j-k}^X (\sigma_X)^2$$

Notice also that given two *orthogonal* stochastic processes X and Y, the spectrum of their sum is equal to the sum of their spectral densities:

$$S_{(X+Y)}(f) = S_X(f) + S_Y(f)$$

If we neglect non linear terms, the consumption growth rate obtained by the model can be represented as:

$$\Delta c_{t+1} - \overline{\Delta c} = \underbrace{\sum_{j=0}^{\infty} \theta_j^x \epsilon_{x,t-j}}_{\Delta c^{lrr}} + \underbrace{\sum_{j=0}^{\infty} \theta_j^a \epsilon_{a,t-j}}_{\Delta c^{srr}}$$

The IRF produced by giving a one-pulse positive shock to the short-run component gives me the sequence $\{\theta_j^a \sigma\}$, which allows me to compute the auto-covariances and the spectral density of Δc^{lrr}. The IRF produced by giving a one-pulse positive shock to the long-run component gives me the sequence $\{\theta_j^x \sigma_x\}$, which allows to compute the auto-covariances and the spectral density of Δc^{srr}. Since the long-run component news are orthogonal to the short-run news, the spectral density of the consumption growth is equal to the sum of the spectral densities of the consumption growth sub components that I denoted as Δc^{srr} and Δc^{lrr}. Finally, note that:
(1) when I compute the monthly spectral density, for computational reasons I truncate the moving average and I consider only the first 200 coefficients (i.e. I compute the IRF.s only for 200 periods);
(2) in order to have a consistent estimate of the spectrum of the quarterly consumption data I actually compute $S_X(f) = \frac{1}{2\pi}\sum_{k=1}^{8} \gamma_k cos(kf)(1-\frac{k}{9})$;
(3) when I compute the quarterly spectral density produced by the model I use the same formula that I apply to the data: $S_X(f) = \frac{1}{2\pi}\sum_{k=1}^{8} \gamma_k cos(kf)(1-\frac{k}{9})$.

D.2 Long Horizon Variance

Let me define the long-run variance of the monthly productivity growth rate at the horizon h as: $Var[\Delta a_{t+h|t}]/h$. Since I assume that the long-run and the short-run news are uncorrelated, I can easily decompose the previous long horizon variance in two subcomponents:

$$Var[\Delta a_{t+h|t}]/h = \underbrace{Var\left[\sum_{k=1}^{h-1} x_{t+k-1}\right]/h}_{Var_h^{lrr}(\Delta a)} + \underbrace{Var\left[\sum_{k=1}^{h} \epsilon_{a,t+k}\right]/h}_{Var_h^{srr}(\Delta a)}$$

Since the short-run risk in productivity is *i.i.d.*, $Var_h^{srr}(\Delta a) = \sigma$. The long-run component is, instead, persistent over time and for this reason:

$$Var_h^{lrr}(\Delta a) = Var(x) + \frac{2}{j}\sum_{k=1}^{h-1}(h-k)\rho^k V(x)$$

If we neglect non linear terms, the consumption growth rate obtained in the production economy can be represented as:

$$\Delta c_{t+1} - \overline{\Delta c} = \underbrace{\sum_{j=0}^{\infty}\theta_j^x \epsilon_{x,t-j}}_{\Delta c^{lrr}} + \underbrace{\sum_{j=0}^{\infty}\theta_j^a \epsilon_{a,t-j}}_{\Delta c^{srr}}$$

The coefficients $\{\theta_j^a, \theta_j^x\}$ can be recovered from the IRFs of the model as shown in the previous section. The auto-covariances generated by the short-run news can be computed in the following way:

$$\gamma_k^a \equiv \sum_{j=0}^{\infty}\theta_j^a\theta_{j-k}^a\sigma^2$$

The implied long-run variance is computed as follows:

$$Var_h^{srr}(\Delta c) = \gamma_0^a + \frac{2}{j}\sum_{k=1}^{h-1}(h-k)\gamma_k^a$$

Similarly, the auto-covariances generated by the long-run news can be computed as:

$$\gamma_k^x \equiv \sum_{j=0}^{\infty}\theta_j^x\theta_{j-k}^x(\sigma_x)^2$$

The implied long-run variance is computed as follows:

$$Var_h^{lrr}(\Delta c) = \gamma_0^x + \frac{2}{j}\sum_{k=1}^{h-1}(h-k)\gamma_k^x$$

For computational reasons I truncate the moving average and I consider only the first 300 coefficients: $\{\theta_j^a, \theta_j^x\}_{j=1}^{300}$.

References

[1] Abel, Andrew, 1980. Empirical investment Equations: An Integrative Framework, Carnegie-Rochester Conference Series on Public Policy, 12, p39-91.

[2] Anderson, E., L.P. Hansen, and T.J. Sargent, 2003. A Quartet of Semigroups for Model Specification, Robustness, Price of Risk, and Model Detection, Journal of the European Economic Association 1 (1):68-123.

[3] Alvarez, Fernando and Urban J . Jermann, 2004. Using Asset Prices to Measure the Cost of Business Cycles, Journal of Political Economy, University of Chicago Press, vol. 112(6), pages 1223-1256.

[4] Attanasio, O.P. and G. Weber (1989). Intertemporal substitution, risk aversion and the euler equation for consumption, The Economic Journal, Supplement: Conference Papers. 99, 59-73.

[5] -, - and A. Vissing-Jorgensen (2003). Stock Market Partecipation, Intertemporal Substitution and Risk Aversion. American Economic Review Papers and Proceedings, May 2003.

[6] Bansal, Ravi, and Amir Yaron, 2004. Risk for the long-run: a potential resolution of asset pricing puzzles, The Journal of Finance, Vol. LIX, No. 4, August 2004, pages 1481-1509.

[7] -, A.R. Gallant, and G. Tauchen, 2004. Rational Pessimism and Exuberance. Working Paper.

[8] -, Robert F. Dittmar and Christian T. Lundbland 2006. Consumption, Dividends, and the Cross-Section of Equity Returns, The Journal of Finance, forthcoming.

[9] -, Dana Kiku, and Amir Yaron, 2006. Risk For the long-run: Estimation and Inference.

[10] Barillas, Francisco, L.P. Hansen, and T.J.Sargent (2006). Reinterpreting a graph (and a parameter) of Tallarini. New York University.

[11] Barlevy, Gadi. 2004. The Cost of Business Cycles under Endogenous Growth, American Economic Review, American Economic Association. vol. 94(4). pages 964-990.

[12] Boldrin, Michele, Lawrence J. Christiano, and Jonas D. I. Fisher, 1995. Asset pricing Lessons for modeling Business Cycles, NBER Working paper 5262.

[13] -, -, -, and -, 2001. Habit Persis tence, Asset Returns, and the Business Cycle, American Economic Review, American Economic Association, vol. 91(1), pages 149-166.

[14] Colacito, Riccardo and Mariano M. Croce, 2005. Risk for the long-run and the real exchange rate, New York University.

[15] Colacito, Riccardo, 2006. Six Puzzles looking for a model. A consumption based explanation of International Finance Puzzles. UNC, Kenan-Flagler Business School.

[16] Croce, Mariano M., Martin Lettau and Sydney C. Ludvigson, 2005. Investor Information, Long-Run Risk, and the Duration of Risky Flows. New York University.

[17] Eberly, Janice, 1997. International Evidence on investment and Fundamentals, European Economic Review, 41(6), June, p1055-78.

[18] Epstein, Larry and Stan Zin, 1989. Substitution Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. Econometrica. 1989, 57, 937-965.

[19] Fama, Eugene F., and K.R. French, 2001. The Equity Premium. Working Paper.

[20] Guvenen, F. (2005). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. Journal of Monetary Economics, forthcoming.

[21] Hall, Robert, 1988, Intertemporal Substitution in Consumption, Journal of Political Economy 96, 339-357.

[22] Hansen, Lars P., John C. Heaton and Nan Li, 2005. Consumption strikes back? Measuring Long-Run Risk, University of Chicago.

[23] - , Thomas J. Sargent, 1995. Discounted linear exponential quadratic gaussian control, University of Chicago.

[24] - , Thomas J. Sargent, 2006. Fragile beliefs and the price of model uncertainty.

[25] Jermann, Urban J. 1998. Asset pricing in production economies. Journal of Monetary Economics, Elsevier, vol. 41(2), pages 257-275.

[26] Kiku, Dana, 2005. Is the value premium a puzzle? Duke University.

[27] Lettau, Martin and Harald Uhlig 2000. Can Habit Formation be Reconciled with Business Cycle Facts?, Review of Economic Dynamics, 3(l) , 79-99, February 2000.

[28] -,- and Sydney C. Ludvigson 2005. Expected returns and expected dividend growth, Journal of Financial Economics 76(2005) 583-626.

[29] Lucas, Robert, 1987. Models of Business Cycles. Oxford: Basil Blackwell.

[30] Lustig, H., and S. Van Nieuwerburgh, 2005. The Returns on Human Capital: good news on wall street is bad news on main street.

[31] Obstfeld, Maurice. 1994. Evaluating risky consumption paths: The role of intertern-poral substitutability. European Economic Review, August. Vol. 38, No 7, pp. 1471-1486.

[32] Panageas, Stavros, and Jianfeng Yu, 2006. Technological Growth, Asset pricing, Consumption Risk. The Wharton School, University of Pennsylvania

[33] Parker, Jonathan 2001. The Consumption Risk of the Stock Market, Brookings papers on Economic Activity, 2001, 2, 279-348.

[34] - and Christian Julliard. Consumption Risk and the Cross-Section of Expected Returns: Journal of Political Economy, February 2004, 113(1), 185-222.

[35] Reis, Ricardo, 2005. The time-series properties of aggregate consumption: implications for the cost of fluctuations. Princeton University.

[36] Sargent, Thomas J., and Lars Ljungqvist (2004). Recursive macroeconomics theory. The MIT Press, 2004.

[37] Tallarini, Thomas D. Jr., 2000. Risk-sensitive real business cycles, Journal of Monetary Economics, Elsevier, vol. 45(2000), pages 507-532.

[38] Weil, Philippe 1989. The Equity Premium Puzzle and the Risk-Free Rate Puzzle, Journal of Monetary Economics. 1989, 24(3), 401-421.

TABLE 1
CALIBRATION WITH LONG RUN CONSUMPTION RISK

Parameter	Value	Moment	Model
μ^L	.1645%	$\mathrm{E}[C_{t+1}/C_t]$	1.99%
σ	.44e-2	$\mathrm{std}[C_{t+1}/C_t]$	1.39%
σ_x	$4.4\%\sigma$	$\mathrm{ACF}_1[C_{t+1}/C_t]$	.32
ρ	.98	$\mathrm{ACF}_1[p_t - d_t]$	.97

All the statistics are based on quarterly data and are annualized. The entries for the model are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency.

TABLE 2

THE SPECIAL CASE $\Psi = 1$

Panel A: Calibration

	μ^L	σ	σ_x	ρ	ϕ	ϕ_c	σ_d	δ	γ
LRR	.1645%	.44e-2	4.4%σ	.98	6.4	0	9σ	.9999	17
NO LRR	.1645%	.48e-2	0	0	0	.9	8.5σ	.99435	280

Panel B: Statistics

	DATA	LRR	NO LRR
$\text{E}[C_{t+1}/C_t]$	02.00	02.00	02.00
$\text{std}[C_{t+1}/C_t]$	01.30	01.39	01.39
$\text{ACF}_1[C_{t+1}/C_t]$	32.00	31.00	21.00
$\rho_{\Delta c,\Delta d}$	10.00	11.00	11.00
$\text{std}[D_{t+1}/D_t]$	12.00	12.00	12.00
$\text{E}[r_{t+1} - r_t^f]$	06.00	06.01	06.05
$\text{E}[r_t^f]$	01.10	01.70	01.06
$\text{std}[r_{t+1} - r_t^f]$	16.40	19.20	14.20
$\text{std}[r_t^f]$	01.35	00.53	0
$\text{std}[p_t - d_t]$	40.00	18.00	0
$\text{ACF}_1[p_t - d_t]$	97.00	97.00	-

Panel C: Welfare Costs

	δ	γ	$\text{E}[r^f]$	$\text{E}[r^{ex}]$	λ^{tot}	λ^{lrr}	λ^{srr}
DATA	-	-	1.10	6.00	-	-	-
LRR	.9999	17	1.70	6.01	907	743	164
NO LRR	.99435	280	1.06	6.05	57	-	57

In Panel A, monthly calibrations for the model with and without LRR are provided. In Panel B, all the statistics are annualized and multiplied by 100. The entries for the models are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency. In this table $\mu_d^L = \mu^L$. In Panel C, the welfare costs are multiplied by 100.

Table 3
The Role of $\Psi \neq 1$

Panel A: Calibration

	μ^L	σ	σ_x	ρ	ϕ	ϕ_c	σ_d	δ	γ
$\Psi = 1.5$	.1645%	.44e-2	4.4%σ	.98	6.4	0	9σ	.99945	16
$\Psi = .8$	.1645%	.44e-2	4.4%σ	.98	6.4	0	9σ	.99945	20

Panel B: Statistics

	DATA	$\Psi = .8$	$\Psi = 1.5$
$\mathrm{E}[C_{t+1}/C_t]$	02.00	02.00	02.00
$\mathrm{std}[C_{t+1}/C_t]$	01.30	01.39	01.39
$\mathrm{ACF}_1[C_{t+1}/C_t]$	32.00	31.00	31.00
$\rho_{\Delta c,\Delta d}$	10.00	11.00	11.00
$\mathrm{std}[D_{t+1}/D_t]$	12.00	12.00	12.00
$\mathrm{E}[r_{t+1} - r_t^f]$	06.00	06.17	06.15
$\mathrm{E}[r_t^f]$	01.10	02.88	01.40
$\mathrm{std}[r_{t+1} - r_t^f]$	16.40	19.20	19.80
$\mathrm{std}[r_t^f]$	01.35	00.66	00.35
$\mathrm{std}[p_t - d_t]$	40.00	18.40	20.00
$\mathrm{ACF}_1[p_t - d_t]$	97.00	97.00	97.00

Panel C: Welfare Costs

	δ	γ	$\mathrm{E}[r^f]$	$\mathrm{E}[r^{ex}]$	λ^{tot}	λ^{lrr}	λ^{srr}
DATA	-	-	1.10	6.00	-	-	-
$\Psi = 1.5$	.99945	16	1.40	6.15	1512	498	1014
$\Psi = .8$	.99945	20	2.88	6.17	124	103	21

In Panel A, monthly calibrations for the model with LRR are provided. In Panel B, all the statistics are annualized and multiplied by 100. The entries for the models are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency. In this table $\mu_d^L = \mu^L$. In Panel C, the welfare costs are multiplied by 100.

Table 4

The Role of Ψ with Production

Panel A: Calibration

μ^L	σ	σ_x	ρ	δ_k	α	τ	δ	γ
.165%	.60%	5.5%σ	.98	.5%	.33	.98	.99923	16

Panel B: Statistics

	DATA	$\Psi = .8$	$\Psi = 1$	$\Psi = 1.5$
$\mathrm{E}[C_{t+1}/C_t]$	02.00	01.91	01.92	01.93
$\mathrm{std}[C_{t+1}/C_t]$	01.30	01.34	01.22	01.36
$\mathrm{ACF}_1[C_{t+1}/C_t]$	32.00	35.00	36.00	36.00
$\rho_{\Delta c,\Delta d}$	10.00	95.00	97.31	89.00
$\mathrm{std}[D_{t+1}/D_t]$	12.00	05.20	05.60	06.40
$\mathrm{E}[r^d_{t+1} - r^f_t]$	06.00	00.00	00.85	02.00
$\mathrm{E}[r^f_t]$	01.10	03.18	02.46	01.36
$\mathrm{std}[r^d_{t+1} - r^f_t]$	16.40	02.50	02.40	03.10
$\mathrm{std}[r^f_t]$	01.35	00.72	00.62	00.46
$\mathrm{std}[p_t - d_t]$	40.00	18.70	19.10	20.00
$\mathrm{ACF}_1[p_t - d_t]$	97.00	98.00	98.00	98.00

Panel C: Welfare Costs

	δ	γ	$\mathrm{E}[r^f]$	$\mathrm{E}[r^{ex}]$	λ^{tot}	λ^{lrr}	λ^{srr}	λ^{adj}
DATA	-	-	1.10	6.00	-	-	-	-
$\Psi = 1.5$	.99923	16	1.40	2.00	404	300	104	6.6
$\Psi = 1$	.99923	16	2.46	0.85	254	220	34	5.4
$\Psi = .8$	.99923	16	3.18	0.00	218	196	22	5.3

Panel A shows the benchmark monthly calibration. In Panel B, all the statistics are annualized and multiplied by 100. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table $\bar{l} = .82$. In Panel C, the welfare costs are multiplied by 100.

TABLE 5
QUARTERLY SECOND MOMENTS WITH ADJUSTMENT COSTS

Panel A: Cross Correlations

DATA

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.85		
ΔC_t	.83	.43	
r_t^d	.12	.02	.18

$\Psi = 1.5$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.86		
ΔC_t	.96	.70	
r_t^d	.57	.75	.42

$\Psi = 1$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.99		
ΔC_t	.99	.99	
r_t^d	.70	.70	.70

$\Psi = .8$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.93		
ΔC_t	.99	.87	
r_t^d	.65	.69	.61

Panel B: Volatility Ratios

	$\sigma_{\Delta Y}/\sigma_{\Delta C}$	$\sigma_{\Delta I}/\sigma_{\Delta C}$
DATA	1.5	5
$\Psi = 1.5$	1.01	1.10
$\Psi = 1$	1.00	0.99
$\Psi = .8$	0.98	1.01

Panel A reports contemporaneous cross correlations between quantities for different calibrations of Ψ. All the other parameters of the model are calibrated as in Table 4. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table $\bar{l} = .82$. Panel B reports volatility ratios.

Table 6
Production Economy: Benchmark Calibration

Panel A: Calibration

μ^L	σ	σ_x	ρ	δ_k	α	τ	δ^{12}	γ	Ψ
.165%	.60%	5.5%σ	.98	.5%	.33	.98	.98	30	2

Panel B: General Statistics

	Mean		St.Dev.		ACF_1	
	Model	Data	Model	Data	Model	Data
C_{t+1}/C_t	2.0	2.0	1.4	1.4	.35	.32
I_{t+1}/I_t	2.0	2.4	1.8	7.0	.30	.5
I_t/Y_t	25	20	0.5	2.0	.98	.97
$r^d_{t+1} - r^f_t$	4.8	6.0	4.0	16.0	.05	.01
r^f_t	1.2	1.1	.34	1.5	.97	.90
$p_t - d_t$	3.25	3.2	.11	.35	.98	.96

Panel C: Cross Correlations

DATA

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.85		
ΔC_t	.83	.43	
r^d_t	.12	.02	.18

MODEL

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.75		
ΔC_t	.94	.51	
r^d_t	.50	.76	.29

Panel D: Welfare Costs

	λ^{tot}	λ^{lrr}	λ^{srr}
MODEL	200	147	53

Panel A shows the benchmark monthly calibration. In Panel B, all the statistics are annualized and multiplied by 100. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table $\bar{l} = .82$. In Panel C, the welfare costs are multiplied by 100.

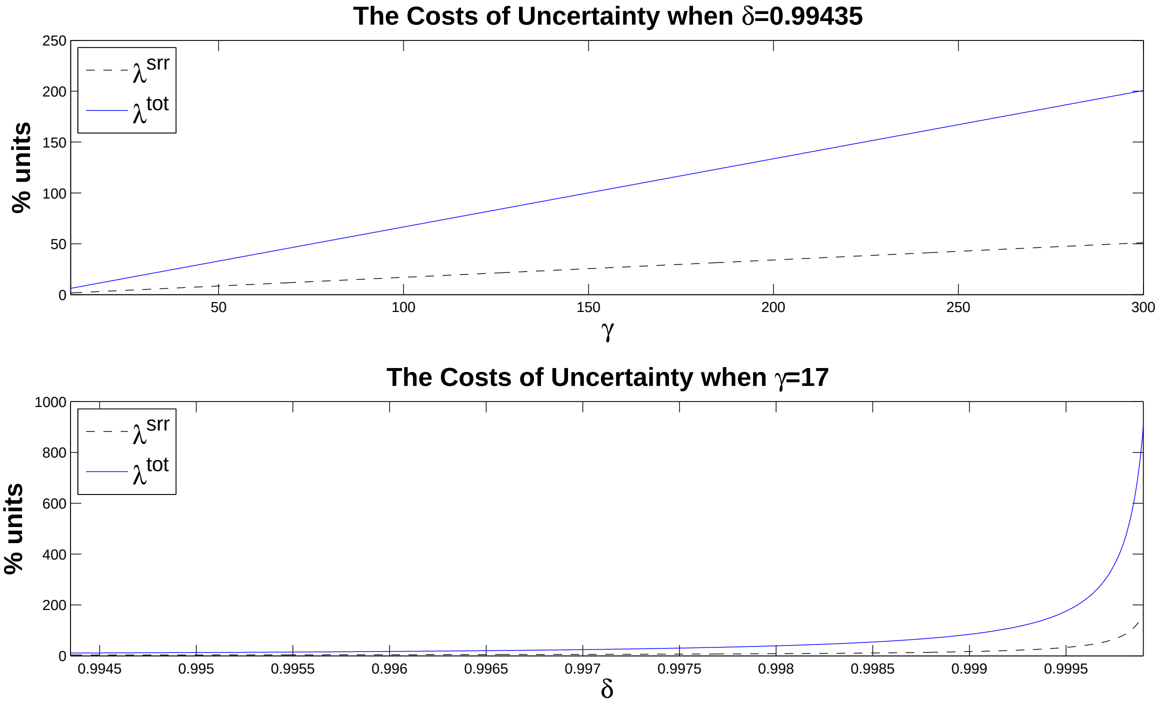


Fig. 1 - The welfare costs as a function of (γ, δ)

The parameters $\{\mu^L, \rho, \sigma, \sigma_x\}$ are calibrated to the values reported in Table 1. The IES=1. The cost functions are multiplied by 100.

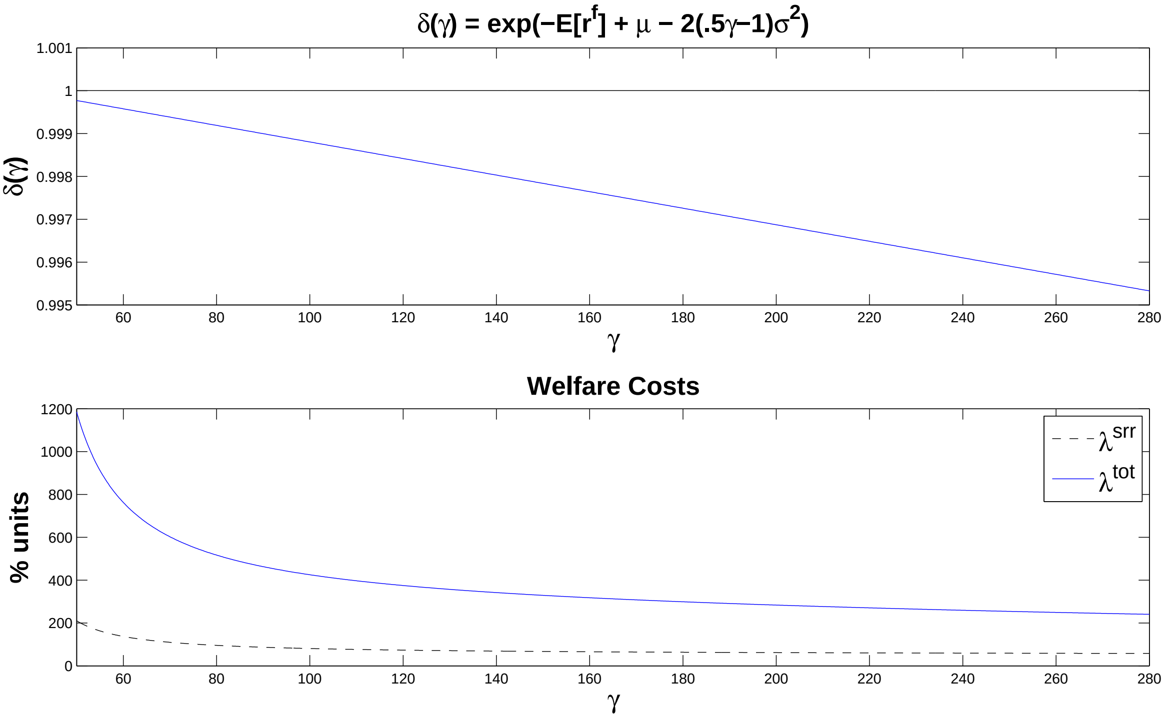


Fig. 2 - The welfare costs as a function of $(\gamma, E[r^f])$

The parameters $\{\mu^L, \rho, \sigma, \sigma_x\}$ are calibrated to the values reported in Table 1. The IES=1, the cost functions are multiplied by 100. In the top panel the annualized mean of the risk free rate is 1.1%.

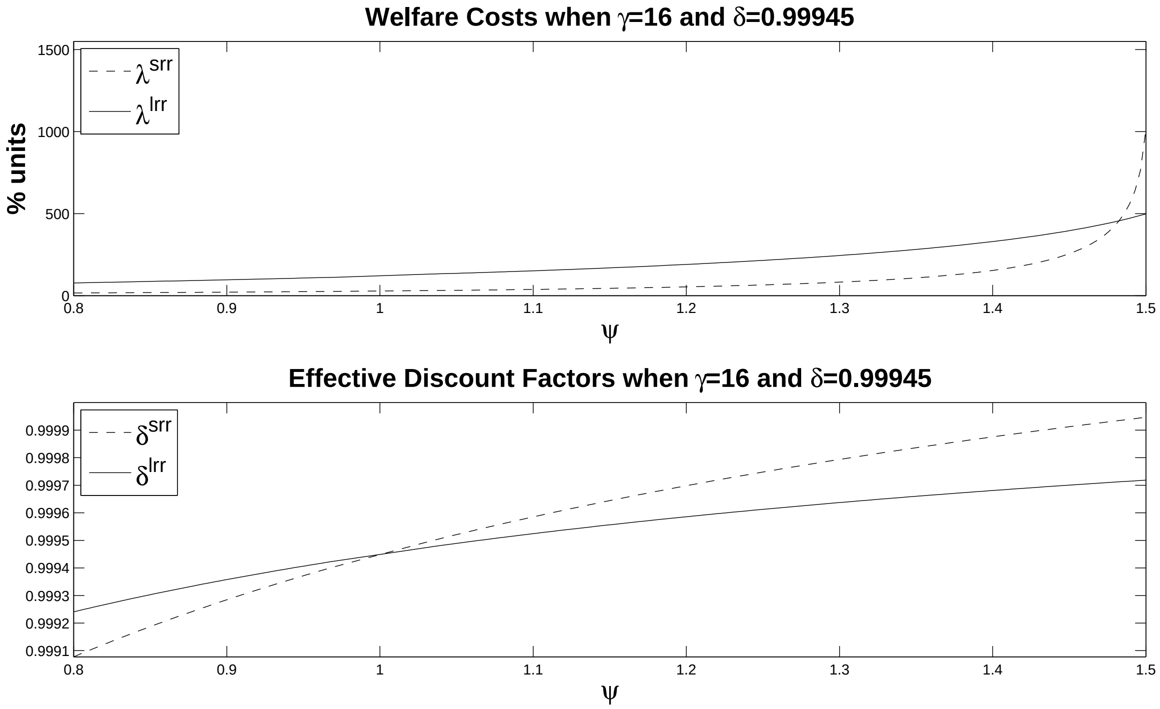


Fig. 3 - The welfare costs as a function of Ψ

The parameters $\{\mu^L, \rho, \sigma, \sigma_x\}$ are calibrated to the values reported in Table 1. In the top panel, the cost functions, are multiplied by 100 and are computed using a log-linearization of the model. The bottom panel shows how the effective discount factors change with the intertemporal elasticity of substitution.

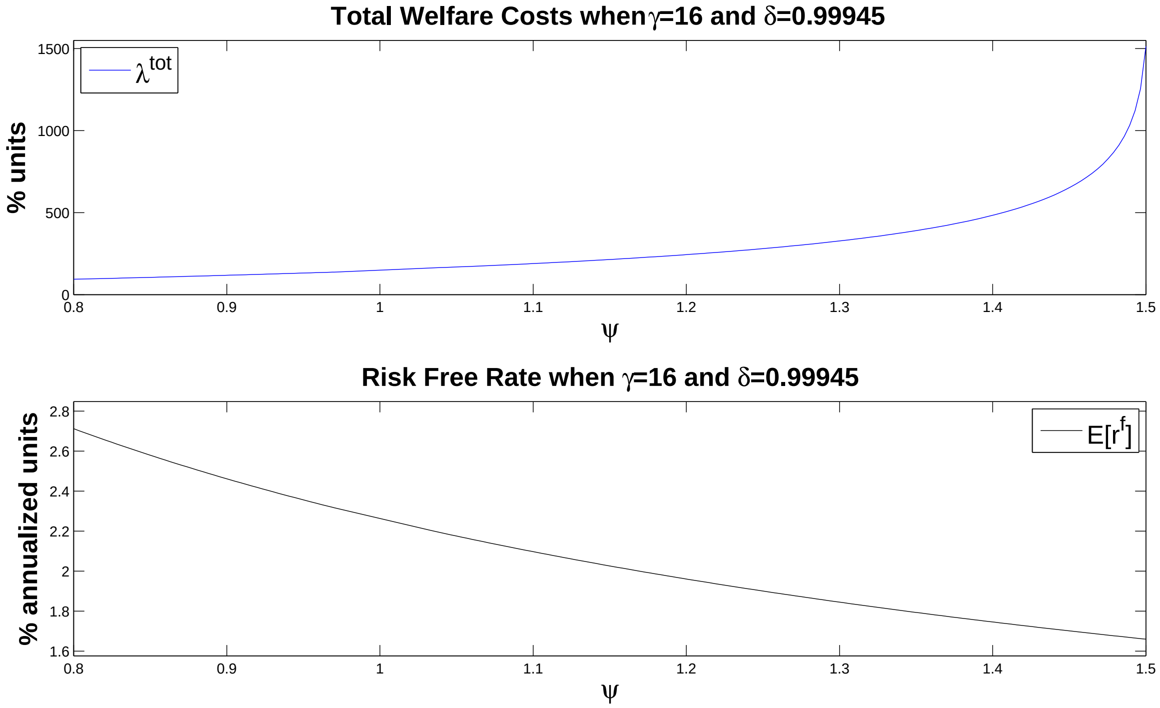


Fig. 4 - The welfare costs and the risk free rate

The parameters $\{\mu^L, \rho, \sigma, \sigma_x\}$ are calibrated to the values reported in Table 1. The cost functions and the annualized risk free rate are multiplied by 100 and are computed using a log-linearization of the model.

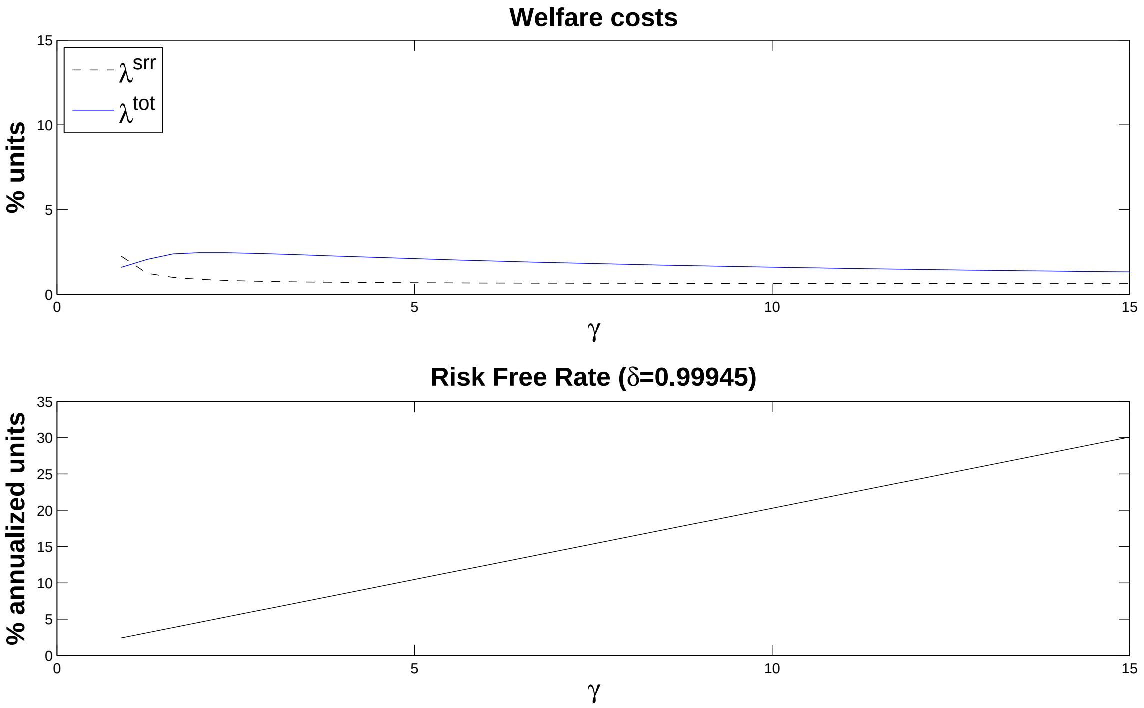


Fig. 5 - The special case $\Psi = 1/\gamma$

The parameters $\{\mu^L, \rho, \sigma, \sigma_x\}$ are calibrated to the values reported in Table 1. The cost functions and the risk free rate are multiplied by 100 and are computed using a log-linearization of the model.

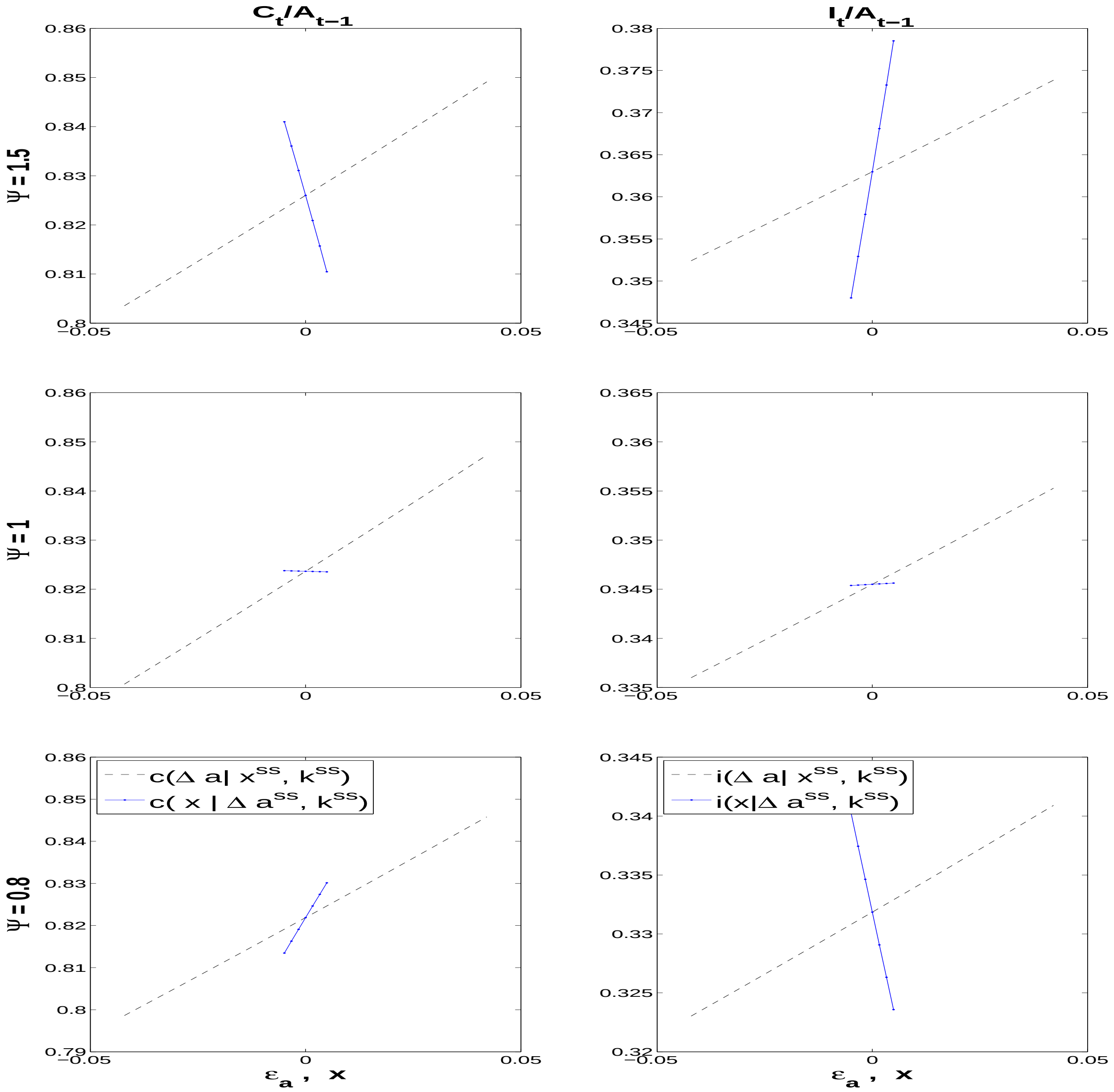


Fig. 6 - The Policy Functions and Ψ

All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically. In every panel, the solid line shows the policy function as a function of x. The dashed line, instead, shows the policy function as a function of ϵ_a. Finally, notice that in this figure $i_t \equiv I_t/A_{t-1}$ and $c_t \equiv C_t/A_{t-1}$.

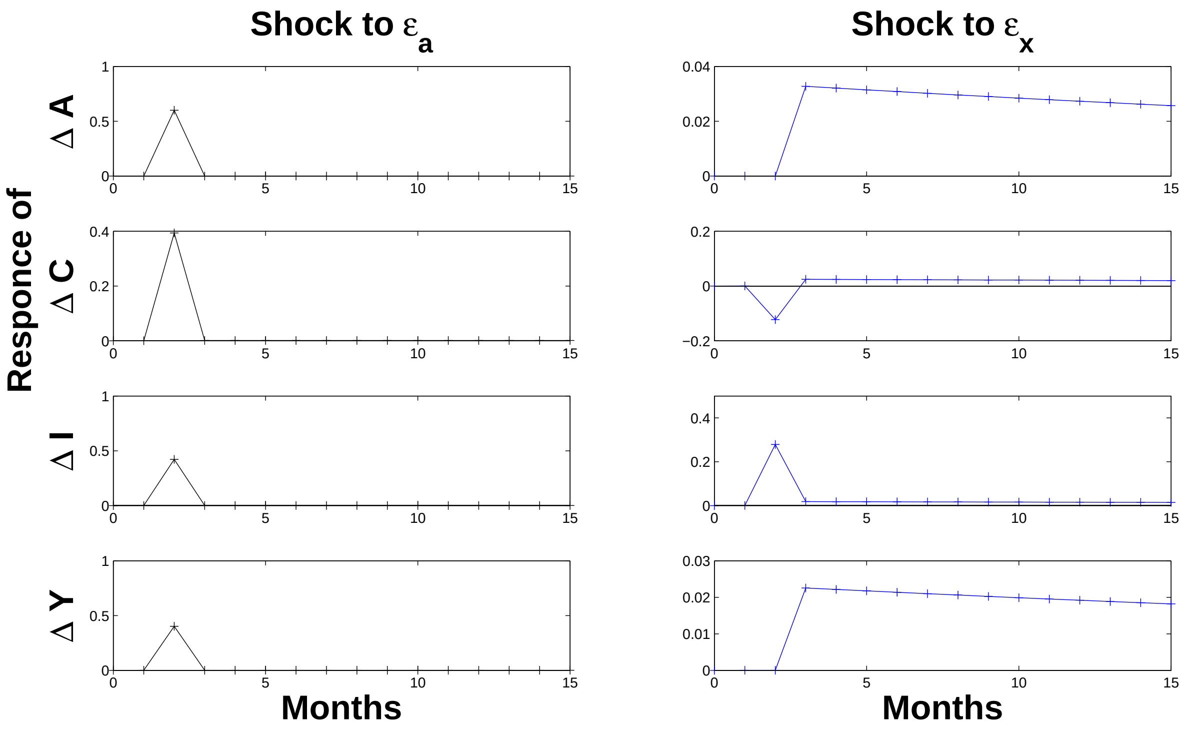


Fig. 7 - Quantities Impulse Response Functions when $\Psi = 1.5$

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.

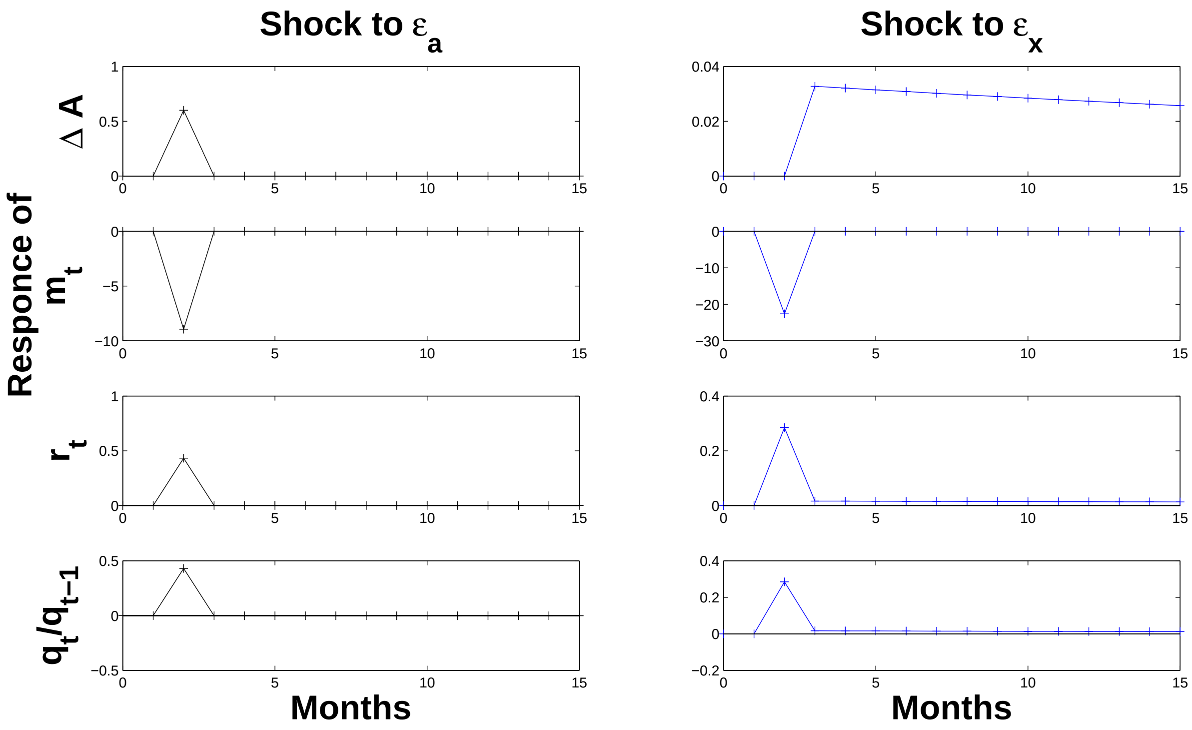


Fig. 8 - Prices Impulse Response Functions when $\Psi = 1.5$

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.

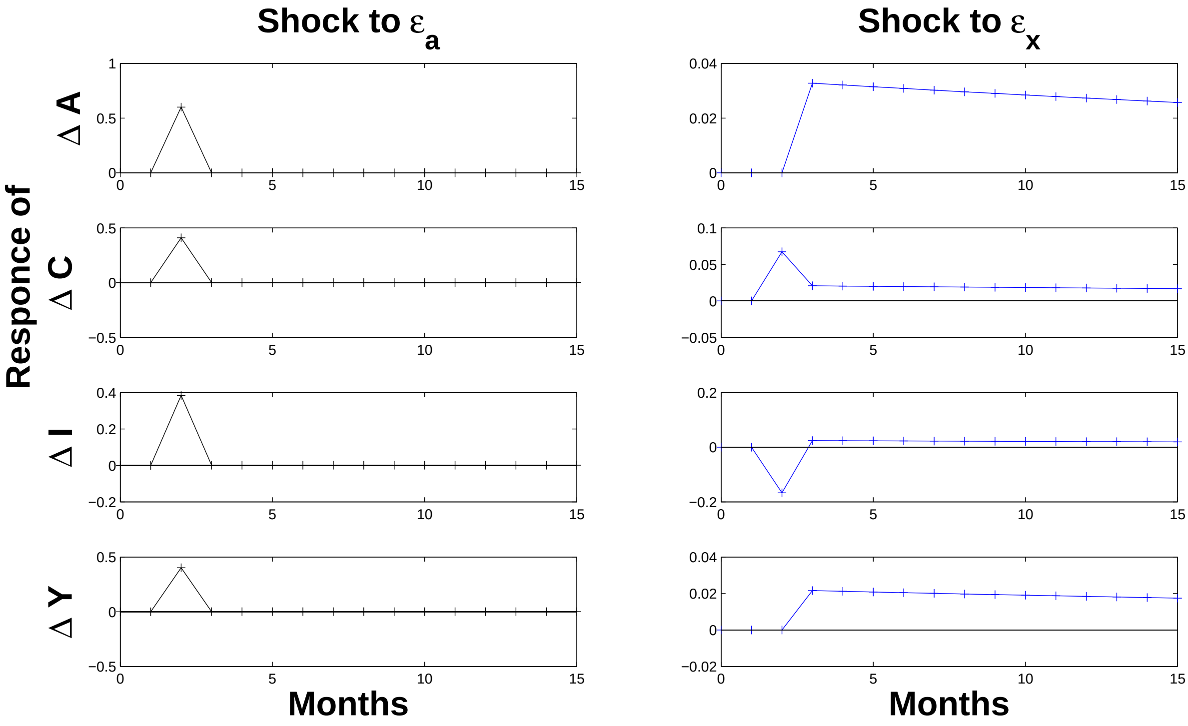


Fig. 9 - Quantities Impulse Response Functions when $\Psi = .8$

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.

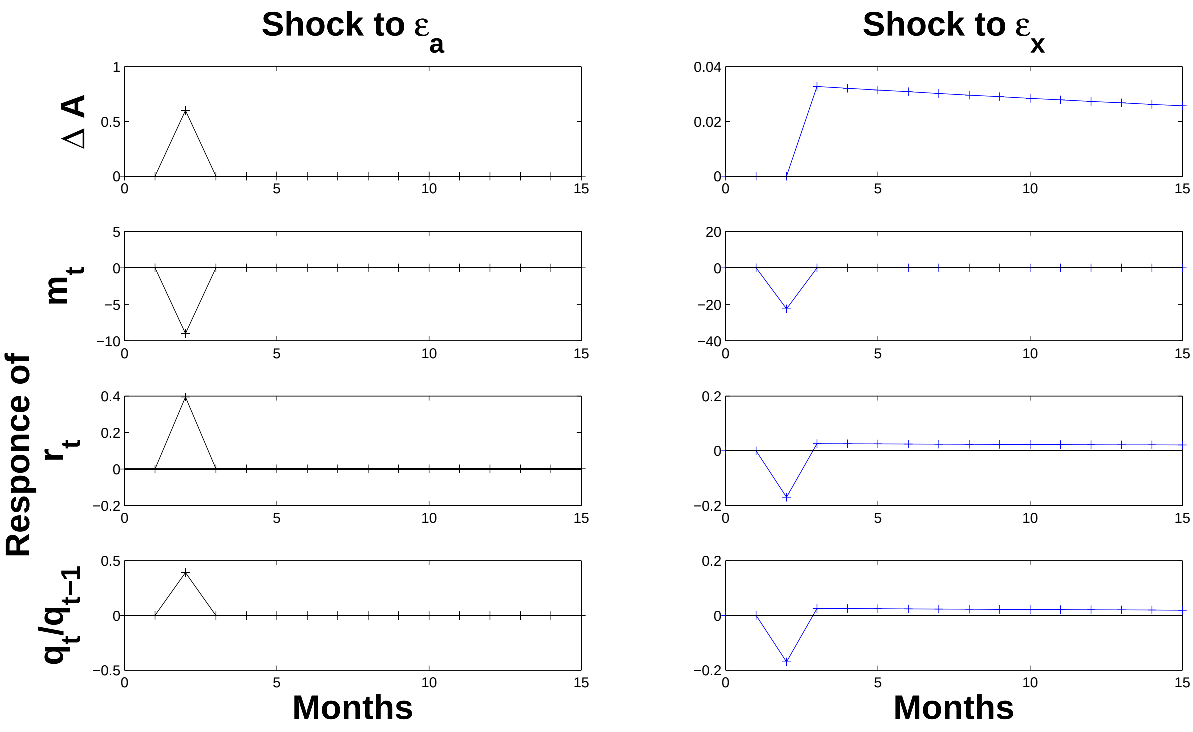
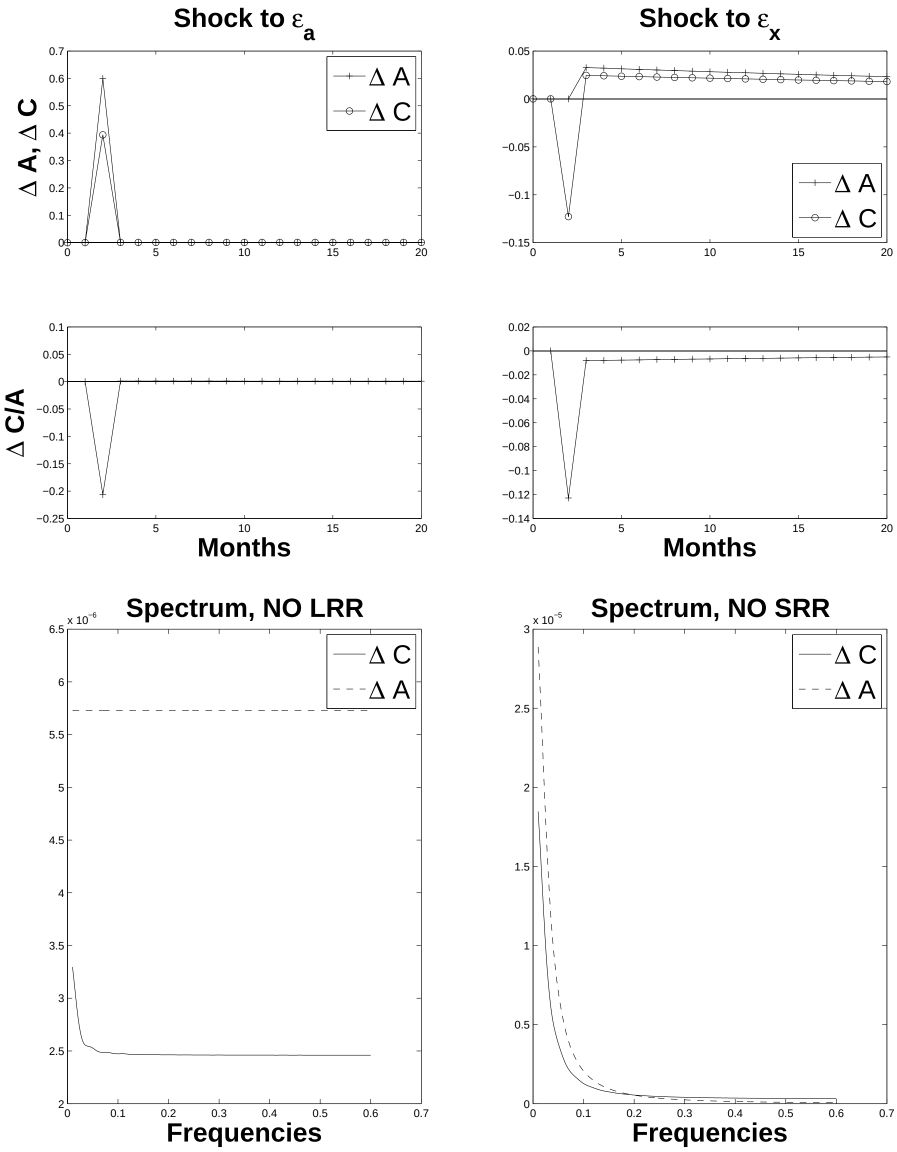


Fig. 10 - Prices Impulse Response Functions when $\Psi = .8$

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.


Shock to ε_a

Shock to ε_x

Spectrum, NO LRR

Spectrum, NO SRR

Fig. 11 - Quantities IRF.s and Spectral Densities ($\Psi = 1.5$)

This figure shows monthly log-deviations from the steady state of both consumption and productivity growth after the arrival of short-run news (top left panels) and long-run news(top right panels). Units are multiplied by 100. The bottom two panels show the implied spectral densities. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.

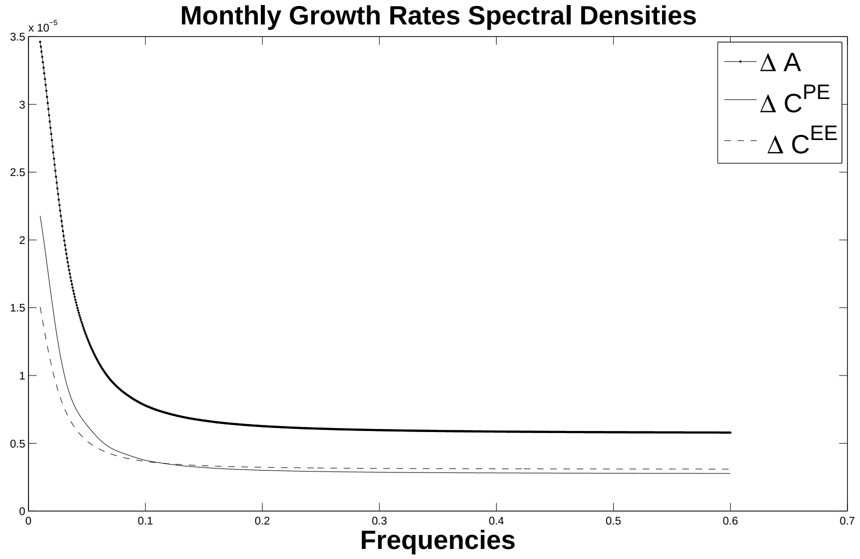


Fig. 12 - Spectral Densities ($\Psi = 1.5$)

This figure shows numerical spectral densities for the monthly growth rates of consumption and productivity. The productivity growth rate ΔA is calibrated as in Table 4. The consumption growth rate ΔC^{PE} ('PE' = Production Economy) is obtained by solving the model with production according to the calibration reported in Table 4. The consumption growth rate ΔC^{EE} ('EE' = Endowment Economy) is obtaining from the endowment economy calibrated as in Table 3.

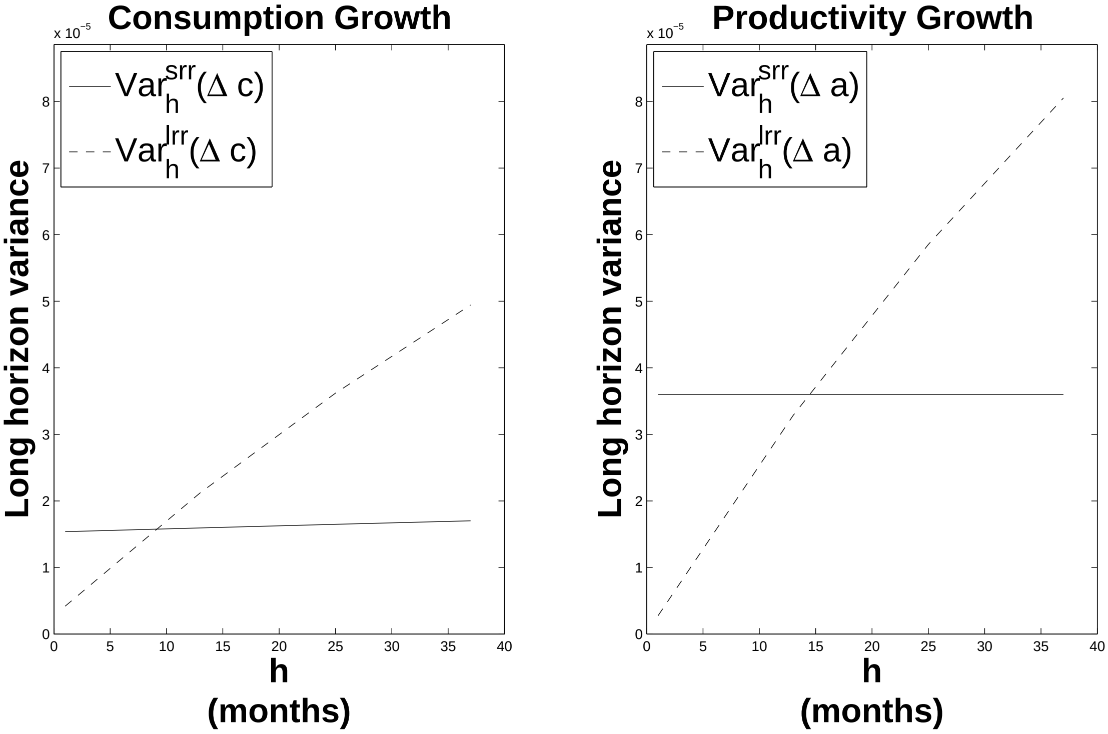


FIG. 13 - LONG HORIZON VARIANCES ($\Psi = 1.5$)

This figure shows long horizon variances for the monthly growth rates of consumption and productivity. In the right panel, $Var_h^{lrr}(\Delta a) \equiv Var\left[\sum_{k=1}^{h-1} x_{t+k-1}\right]/h$ and $Var_h^{srr}(\Delta a) \equiv Var\left[\sum_{k=1}^{h} \epsilon_{a,t+k}\right]/h$. I adopt analogous definitions for $Var_h^{lrr}(\Delta c)$ and $Var_h^{srr}(\Delta c)$. The productivity growth rate Δa is calibrated as in Table 4. The consumption growth rate Δc is obtained by solving the model with production according to the calibration reported in Table 4.

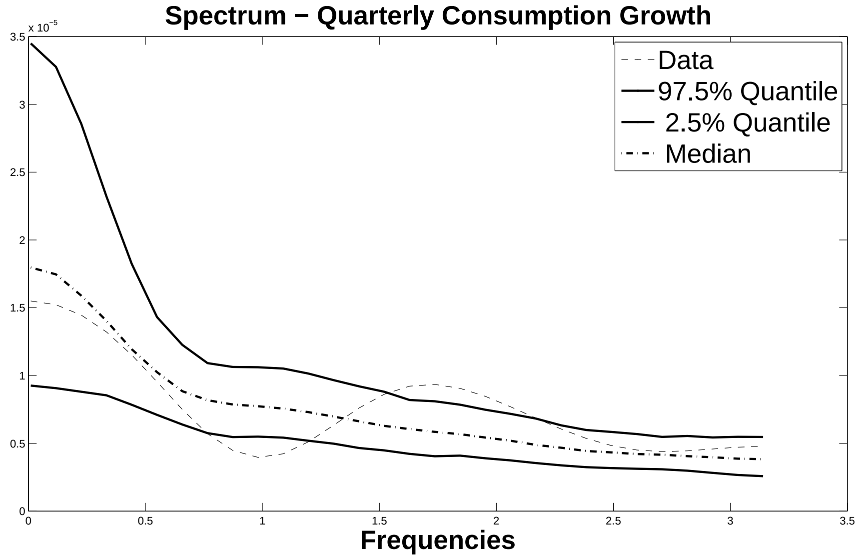


Fig. 14 - Spectrum of Quarterly Consumption

The dashed line shows the spectral density of US quarterly consumption of non-durables and services from 1970:q1 to 1998:q2. The solid lines show the 2.5th and 97.5th percentiles of the distribution of the spectral density of time aggregated quarterly consumption growth generated by simulating 500 times samples of 600 months. The remaining line shows the median of the distribution of the spectrum generated by the model with production. The parameters adopted are reported in Table 6.