The recursive representation of

$$\kappa_t = \sum_{n=0}^{\infty} \left(a^n \prod_{k=0}^{n} x_{t+k} \right)$$

which is the old equation can be obtained by

$$\begin{aligned}
\kappa_t &= a^0 x_{t+0} + a^1 x_{t+0} x_{t+1} + a^2 x_{t+0} x_{t+1} x_{t+2} + ... \\
&= x_{t+0}(a^0 + a^1 x_{t+1} + a^2 x_{t+1} x_{t+2} + ...) \\
&= x_{t+0}(1 + a^1(a^0 x_{t+1} + a^1 x_{t+1} x_{t+2} + ...)) \\
&= x_{t+0}(1 + a^1(a^0 x_{t+1} + a^1 x_{t+1} x_{t+2} + ...)) \\
&= x_t(1 + a\kappa_{t+1})
\end{aligned}$$

The new problem, however, is more complicated and reads

$$\kappa_t = \sum_{n=0}^{\infty} \left(a^n \frac{\mathrm{E}_t[\prod_{k=0}^{n} R_{t+k}^{-1} C_{t+n}]}{\mathrm{E}_t[\prod_{k=0}^{n} R_{t+k} C_{t+n}^{-1}]} \right)$$