$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C_{t+s}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-N_{t+s}) + \eta_x \ln x_{t+s} + \eta_H \ln H_{t+s} \right\} \quad (1)$$

Or use the simple function without x_t and H_t

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C_{t+s}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-N_{t+s}) \right\} \quad (2)$$

We take a second-order approximation to the household's period utility function around the deterministic steady state to obtain

$$\begin{aligned} U_t(C_t, N_t) &\approx U(\tilde{C}, \tilde{N}) + f_{C_t}(\tilde{C}, \tilde{N})(C_t - \tilde{C}) + f_{N_t}(\tilde{C}, \tilde{N})(N_t - \tilde{N}) + \frac{1}{2} f_{C_t C_t}(\tilde{C}, \tilde{N})(C_t - \tilde{C})^2 \\ &\quad + \frac{1}{2} f_{N_t N_t}(\tilde{C}, \tilde{N})(N_t - \tilde{N})^2 + f_{C_t N_t}(\tilde{C}, \tilde{N})(C_t - \tilde{C})(N_t - \tilde{N}) + O_3 \\ &\approx \frac{\tilde{C}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-\tilde{N}) + \tilde{C}^{-\varsigma^{-1}}(C_t - \tilde{C}) + \frac{\eta_N}{\tilde{N}-1}(N_t - \tilde{N}) - \frac{1}{2\varsigma} \tilde{C}^{-\varsigma^{-1}-1}(C_t - \tilde{C})^2 \\ &\quad - \frac{\eta_N}{2(\tilde{N}-1)^2}(N_t - \tilde{N})^2 \\ &= \frac{\tilde{C}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-\tilde{N}) + \tilde{C}^{1-\varsigma^{-1}} \hat{C}_t + \frac{\eta_N \tilde{N}}{\tilde{N}-1} \hat{N}_t - \frac{1}{2\varsigma} \tilde{C}^{1-\varsigma^{-1}} \hat{C}_t^2 - \frac{\eta_N \tilde{N}^2}{2(\tilde{N}-1)^2} \hat{N}_t^2 \end{aligned}$$

where $\hat{C}_t$ and $\hat{N}_t$ denote log-deviations of the variables from their values in the steady state.

The unconditional expectation of the value function, which is the expected infinite discounted sum of the period utilities, is given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t = \frac{1}{1-\beta} \left[\frac{\tilde{C}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-\tilde{N}) - \frac{1}{2\varsigma} \tilde{C}^{1-\varsigma^{-1}} var(\hat{C}_t) - \frac{\eta_N \tilde{N}^2}{2(\tilde{N}-1)^2} var(\hat{N}_t) \right] \quad (3)$$

We have used the fact that the unconditional expectation of log-deviations from steady state is zero, i.e. $E(\hat{X}_t) = 0$.

To express welfare in units of consumption, we divide the value function by the marginal utility of consumption evaluated at the steady state, and denote this welfare measure by V. We have

$$V = \frac{\tilde{C}^{\varsigma^{-1}}}{1-\beta} \left[\frac{\tilde{C}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1-\tilde{N}) - \frac{1}{2\varsigma} \tilde{C}^{1-\varsigma^{-1}} var(\hat{C}_t) - \frac{\eta_N \tilde{N}^2}{2(\tilde{N}-1)^2} var(\hat{N}_t) \right] \quad (4)$$

Thus, to compute welfare under each policy, it is sufficient to compute the unconditional variance of consumption and employment in the model.