1 The model

Consider the following model.

Households allocate their resources between consumption C_t, investments I_t and government-issued bonds B_t. They receive income from labor services $W_t h_t$, from profits D_t, from renting capital services K_t at the rate R_t^k and from holding government bonds.

Households' utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

Households' budget constraint (in nominal terms) is:

$$P_t C_t + P_t I_t + B_t = R_{t-1} B_{t-1} + W_t h_t + D_t + R_t^k K_{t-1} - P_t T_t \tag{2}$$

Capital accumulation equation:

$$K_t = (1-\delta) K_{t-1} + I_t \tag{3}$$

P_t is the consumption price index R_t is the (gross) nominal interest rate, K_t is the physical capital stock and T_t are lump-sum taxes.

Labor decisions are made by a central authority within the household, a union, which supplies labor monopolistically to a continuum of labor markets. W_t is an index of nominal wages prevailing in the economy. In each particular labor market, the union takes W_t as exogenous. The union is assumed to supply enough labor to satisfy demand. Each agent provides each possible type of labor input. This assumption avoid heterogeneity across households in the number of hours worked. This specification gives rise to a wage-inflation Phillips curve with a larger coefficient on the wage-markup gap that the model with employment heterogeneity across households.

We assume that each period only a fraction $1-\xi_w$ of unions, drawn randomly from the population, reoptimize their nominal wage. All unions resetting their wage in any given period will choose the same wage, since they face an identical problem.

The final good Y_t is produced under perfect competition. Intermediate firms are monopolistically competitive and use as inputs capital and labor services, K_t and h_t respectively. The production technology is:

$$Y_t = K_{t-1}^{\alpha} h_t^{1-\alpha}$$

Firms maximize their (nominal) profits $D_t = P_t Y_t - W_t h_t - R_t^k K_{t-1}$ subject to the technology constraint, choosing how much inputs K_t and h_t to use.

Following Calvo (1983), each firm may reset its price only with probability $1-\xi_p$ in any given period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\xi_p$ of producers reset their prices, while a fraction keep their prices unchanged. As a result, the average duration of a price is given by $\left(1-\xi_p\right)^{-1}$. In this context, ξ_p becomes a natural index of price stickiness.

The government budget constraint is:

$$R_{t-1}B_{t-1} = B_t + P_t T_t$$

The monetary authority sets the nominal interest rate according to a Taylor rule.

2 First order conditions

The first order conditions obtained from this model are the following.

From the households' problem, we obtain:

$$C_t^{-\sigma} = \lambda_t \tag{4}$$

$$R_t = \pi_{t+1}^{p} \frac{\lambda_t}{\beta \lambda_{t+1}} \tag{5}$$

$$\frac{\lambda_{t+1}}{\lambda_t} \beta \left[r_{t+1}^{k} + (1-\delta) \right] = 1 \tag{6}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{7}$$

$$MRS_t \equiv -\frac{U_h\left(C_t, h_t\right)}{U_C\left(C_t, h_t\right)} = C_t^{\sigma} h_t^{\varphi} = \frac{h_t^{\varphi}}{\lambda_t} \tag{8}$$

The following 4 equations define wage dynamics:

$$1 = \xi_w \left(\frac{1}{\pi_t^w} \right)^{1-\varepsilon_w} + (1-\xi_w) \left(\frac{W_t^*}{W_t} \right)^{1-\varepsilon_w} \tag{9}$$

$$W_t^* = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\psi_t^w}{\phi_t^w} \tag{10}$$

$$\psi_t^w = \lambda_t MRS_t h_t + \beta \xi_w E_t \psi_{t+1}^w \tag{11}$$

$$\phi_t^w = \lambda_t \frac{h_t}{P_t} + \beta \xi_w E_t \phi_{t+1}^w \tag{12}$$

From the firms' problem, we obtain:

$$\frac{K_{t-1}}{h_t} = \frac{\alpha}{(1-\alpha)} \frac{w_t}{r_t^k} \tag{13}$$

$$mc_t = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \left(r_t^k\right)^{\alpha} w_t^{1-\alpha} \tag{14}$$

$$Y_t = K_{t-1}^{\alpha} h_t^{1-\alpha} \tag{15}$$

The following 4 equations define price dynamics:

$$1 = \xi_p \left(\frac{1}{\pi_t^p}\right)^{1-\varepsilon_p} + \left(1-\xi_p\right)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon_p} \tag{16}$$

$$\frac{P_t^*}{P_t} = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{\psi_t^p}{\phi_t^p} \tag{17}$$

$$\psi_t^p = \lambda_t Y_t mc_t + \beta \xi_p E_t \pi_{t+1}^{\varepsilon_p} \psi_{t+1}^p \tag{18}$$

$$\phi_t^p = \lambda_t Y_t + \beta \xi_p E_t \pi_{t+1}^{\varepsilon_p - 1} \phi_{t+1}^p \tag{19}$$

Market clearing:

$$Y_t = C_t + I_t \tag{20}$$

Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_t^p}{\pi^p}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\phi_R} \tag{21}$$

where λ_t is the Lagrange multiplier for the problem of households (real terms budget constraint), π_t^w is gross wage inflation, π_t^p is gross price inflation, w_t is the real wage and r_t^k is th real price of capital. ε_w is the elasticity of substitution between labour services and ε_p is the elasticity of substitution between goods.

3 Steady state

The steady state of the model is derived as follows:

$$\pi^p = 1$$

$$R = \frac{\pi^p}{\beta}$$

$$r^k = \frac{1}{\beta} - (1-\delta)$$

$$\frac{I}{K} = \delta$$

$$mc = \frac{\varepsilon_p - 1}{\varepsilon_p}$$

$$\frac{Y}{K} = \frac{r^k}{\alpha} \frac{1}{mc}$$

$$\frac{I}{Y} = \frac{\frac{I}{K}}{\frac{Y}{K}}$$

$$\frac{C}{Y} = 1 - \frac{I}{Y}$$

$$P^* = P$$

$$w = \left[\frac{\varepsilon_p - 1}{\varepsilon_p} \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(r^k)^\alpha} \right]^{\frac{1}{1-\alpha}}$$

$$\frac{K}{h} = \frac{\alpha}{1-\alpha} \frac{w}{r^k}$$

$$\frac{Y}{K} = \left(\frac{K}{h} \right)^{-(1-\alpha)}$$

$$MRS = w \frac{\varepsilon_w - 1}{\varepsilon_w}$$

$$\frac{C}{h} = \frac{Y}{K} \frac{K}{h} - \delta \frac{K}{h} = \left(\frac{K}{h} \right)^\alpha - \delta \frac{K}{h}$$

$$h = \left[MRS \left(\frac{C}{h} \right)^{-\sigma} \right]^{\frac{1}{\sigma+\varphi}}$$

4 Log-linearized model

The set of log-linearized equations is the following:

$$-\sigma \hat{C}_t = \hat{\lambda}_t$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \hat{R}_t - \hat{\pi}^p_{t+1}$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + [1 - \beta (1-\delta)] \hat{r}^k_{t+1}$$

$$\hat{K}_t = (1-\delta)\,\hat{K}_{t-1} + \delta \hat{I}_t$$

$$\widehat{MRS}_t = \sigma \hat{C}_t + \varphi \hat{h}_t$$

$$\hat{\pi}_t^w = \beta \hat{\pi}_{t+1}^w - \frac{(1-\xi_w)\,(1-\beta\xi_w)}{\xi_w}\left(\hat{w}_t - \widehat{MRS}_t\right)$$

$$\hat{K}_{t-1} - \hat{N}_t = \hat{w}_t - \hat{r}_t^k$$

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1-\alpha)\,\hat{w}_t$$

$$\hat{Y}_t = \alpha \hat{K}_{t-1} + (1-\alpha)\,\hat{h}_t$$

$$\hat{\pi}_t^p = \beta \hat{\pi}_{t+1}^p + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\widehat{mc}_t$$

$$\frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t = \hat{Y}_t$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1-\phi_R)\left(\phi_\pi \hat{\pi}_t^p + \phi_y \hat{Y}_t\right)$$