Abstract

This paper studies the strategic interaction between a country issuing debt to smooth its consumption in low productivity periods, and a country with a stable and high productivity and investors willing to diversify their investments and seeking higher yields.

The academic literature is wide and deep in the study of countries issuing debt being treated as small open economies confronted to a "rest of the world" with an infinite capacity of financing its budget deficit. As opposed to this approach, this paper intends to couple the optimization problem of the indebted country with that of the creditor country.

A two-country DSGE model is built for this purpose, and response to productivity shocks in the indebted country are analyzed.

Key words: External debt, Two-country model

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1. The baseline model

1.1. Country S optimization problem

Country S households maximize their utility function as follows

$$\max \sum_{t=0}^{\infty} \beta^{t} u(c_t^i)$$

Control Variables: $c_t^i, b_{t+1}^i, k_{t+1}^i$

Budget constraint:

$c_t^i + b_{t+1}^i + k_{t+1}^i = (1 + r_t^i - \delta)k_t^i + w_t^i + R_t b_t^i$

As we see, country S issues debt b to finance further spendings in case of need.

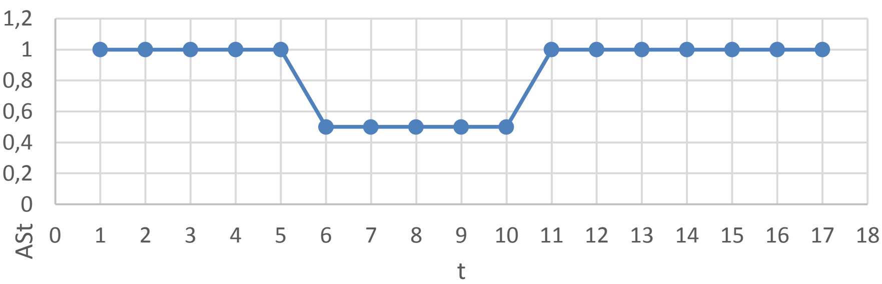
- Firm problem: Cobb-Douglas $\quad y_t^i = A_t^i {k_t^i}^{\alpha} {l_t^i}^{(1-\alpha)}$

  With perfect competition, l=1:

$$w_t^i = (1-\alpha) A_t^i {k_t^i}^{\alpha}$$

$$r_t^i = \alpha A_t^i {k_t^i}^{\alpha - 1} \qquad \text{i=S}$$

- A_t^S subject to deterministic shocks:



- Or stochastic shocks of the form

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AB = 1+ z;
z=rho*z(-1)-e;
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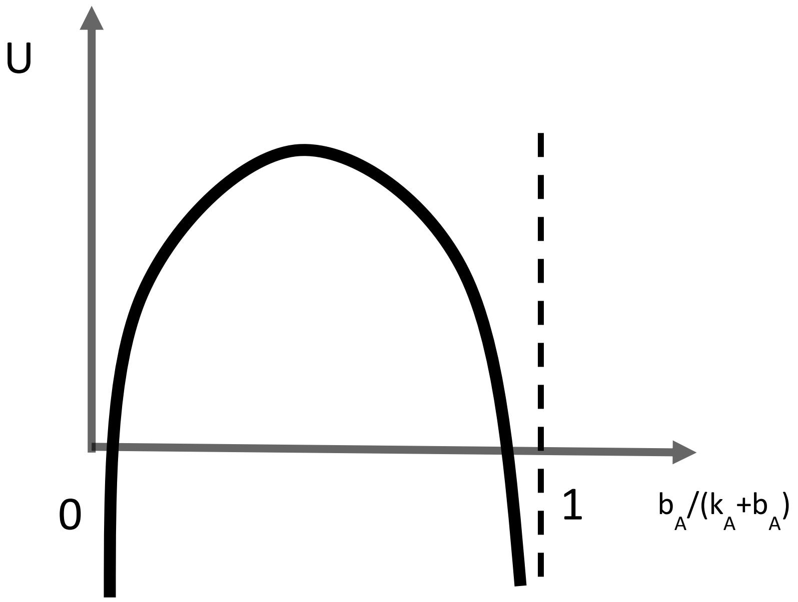
where z is an autoregressive scalar (z0=0, rho=0.9) and e is white noise (variance=`0.0001`).

1.2.Country N optimization problem, exponential utility function

The optimization problem is similar in country N, but in this case we introduce some variations in the utility function to highlight the adversity to invest all the savings in one single asset. Country N does not issue debt but may hold assets of country S debt.

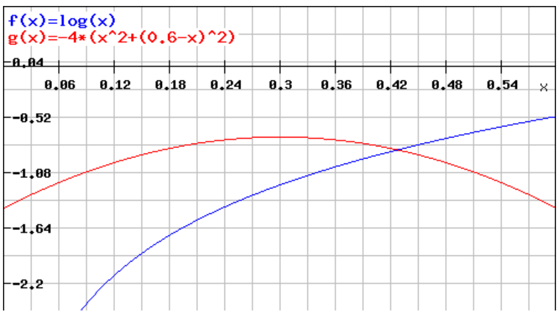


Country N households maximize their utility function as follows

$$\max \sum_{t=0}^{\infty} \beta^{t} \left[\ln c_t^i + \psi_1 {k_{t+1}^i}^{\varphi 1} + \psi_2 {b_{t+1}^i}^{\varphi 2} + \psi_3 k_{t+1}^i\right]$$

Or any other function that penalizes the utility of b=0 and k=0

Control Variables: $c_t^i, b_{t+1}^i, k_{t+1}^i$



We choose the following parameter values for psi and phi so diversifications between k and b are preferred. The graph shows the evolution of k/(k+b), in red, and ln(c) in blue.

Note: I am currently searching other utility functions that better match the purpose of the thesis.

Budget constraint:

$c_t^i + b_{t+1}^i + k_{t+1}^i = (1 + r_t^i - \delta)k_t^i + w_t^i + R_t b_t^i$

- Firm problem: Cobb-Douglas, perfect competition, l=1 $y_t^i = A_t^i {k_t^i}^{\alpha} {l_t^i}^{(1-\alpha)}$

$w_t^i = (1-\alpha) A_t^i {k_t^i}^{\alpha}$

$r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$ i=N

$A_t^N \ constant$

1.2.1. Equilibrium equations

Therefore we have the following 11 equations:

(1) Budget constraint for S: $c_t^i + b_{t+1}^i + k_{t+1}^i = (1 + r_t^i - \delta)k_t^i + w_t^i + R_t b_t^i$ i=S

(2) Budget constraint for N: $c_t^i + b_{t+1}^i + k_{t+1}^i = (1 + r_t^i - \delta)k_t^i + w_t^i + R_t b_t^i$ i=N

(3) Interest rate for S: $r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$ i=S

(4) Interest rate for N: $r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$ i=N

(5) Salary for S: $w_t^i = (1-\alpha) A_t^i {k_t^i}^{\alpha}$ i=S

(6) Salary for N: $w_t^i = (1-\alpha) A_t^i {k_t^i}^{\alpha}$ i=N

(7) Euler equation for N: $\psi_1 \varphi_1 {k_{t+1}^i}^{\varphi 1 - 1} + \psi_3 + \frac{\beta r_{t+1}^i}{c_{t+1}^i} + \frac{1}{c_t^i} = 0$ i=N

(8) Euler equation for N: $\psi_2 \varphi_2 {b_{t+1}^i}^{\varphi 2 - 1} + \psi_3 + \frac{\beta R_{t+1}}{c_{t+1}^i} + \frac{1}{c_t^i} = 0$ i=N

(9) Euler equation for S: $c_{t+1}^i = \beta R_{t+1} c_t^i$ i=S

(10) Euler equation for S: $r_{t+1}^i = R_{t+1}$

(11) Debt market closes: $b_t^S + b_t^N = 0$

Note: Chi3 is chosen to have a steady state in equilibrium (namely, rN=R in steady state). This should not happen in the final version of the utility function.

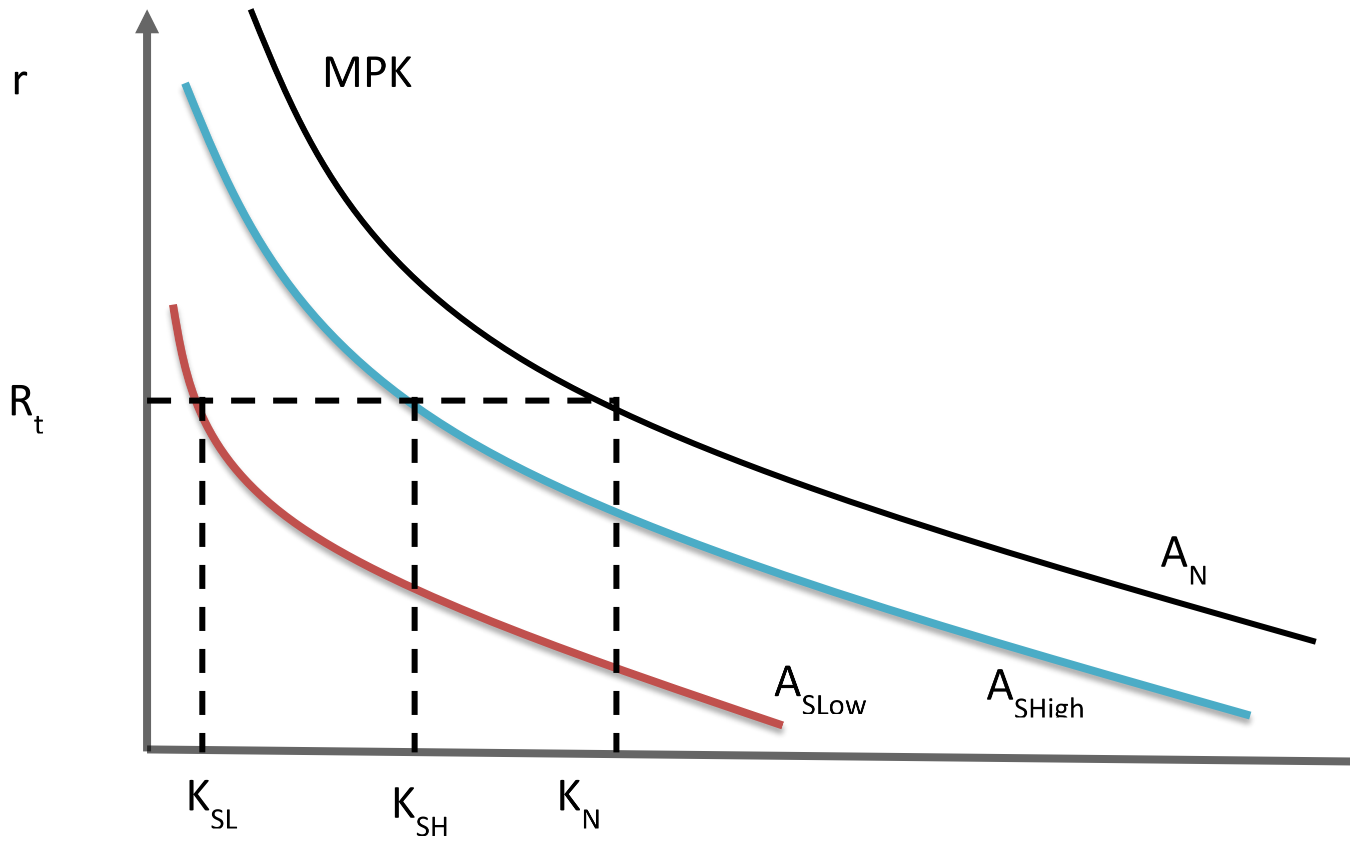
1.2.2. Model dynamics under productivity shocks

As stated in the previous section, country S productivity will be subject to a deterministic shock. The investment markets of both countries are linked through the debt market, with the following effects:

-In low productivity times, country S may be tempted to issue debt to minimize the consumption reduction.

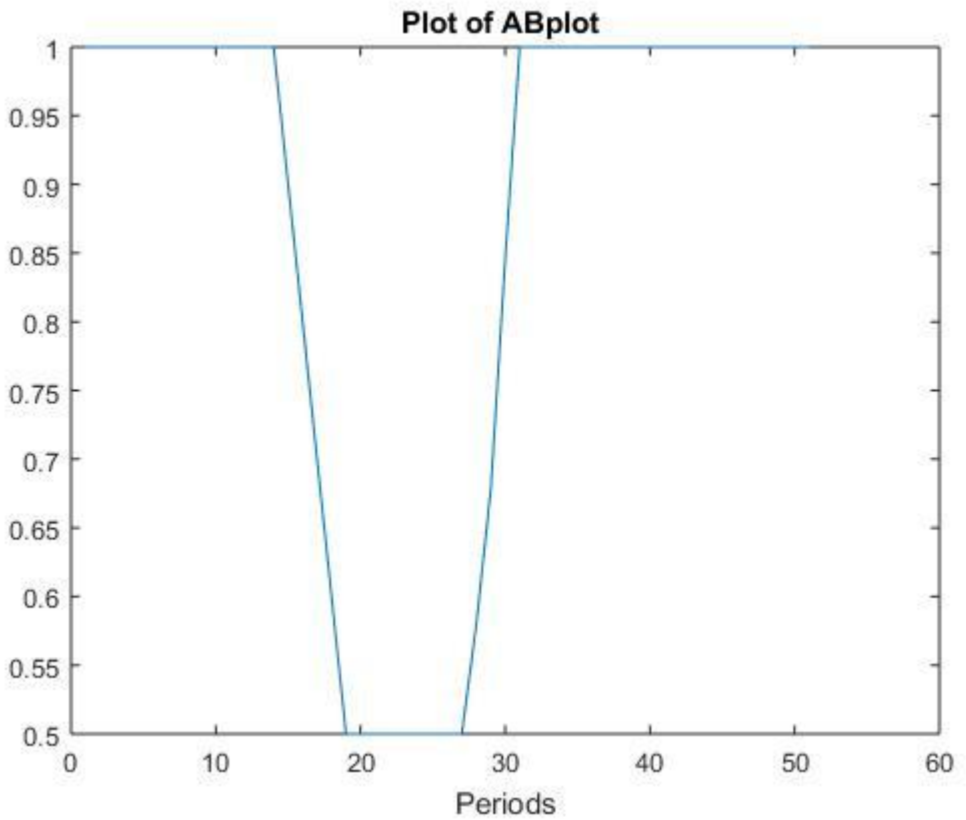
-At the same time, to keep pace with "international" interest rates, capital accumulation may be reduced ($K_{SL}<K_{SH}$)

-In the case of country N, capital will accumulate till a certain point, when diminishing MPK makes bonds in S attractive.



The equilibrium equations are introduced in the software package Dynare, which linearizes the equations around the steady state and calculates their response to shocks.

The following shock in the productivity of country S is introduced:

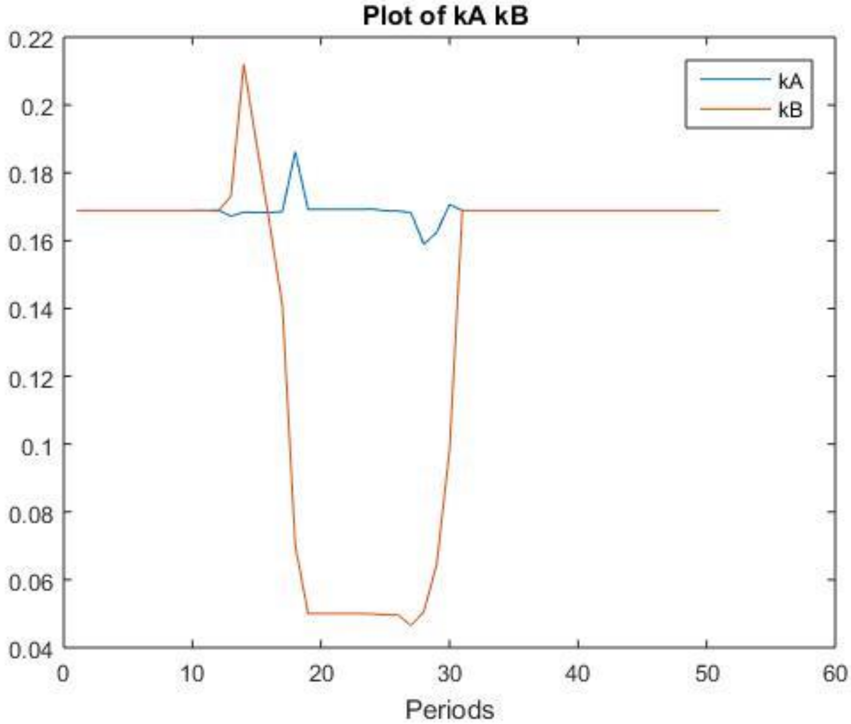


And the results are studied hereafter.

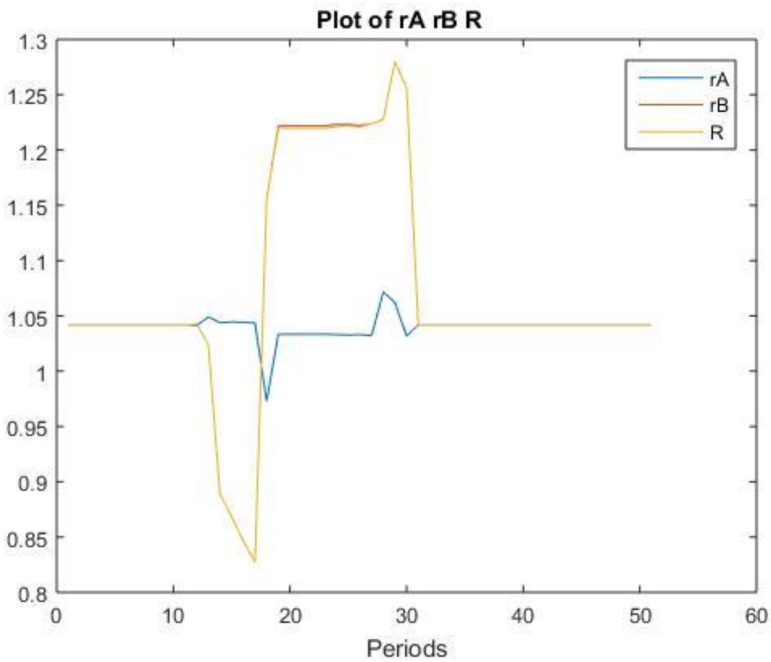
Note: The behaviour in the first periods of the shock is counterintuitive and is still left unexplained. Further versions of utility function of N may correct this.

In the graphs we find: A=country N, B=country S.

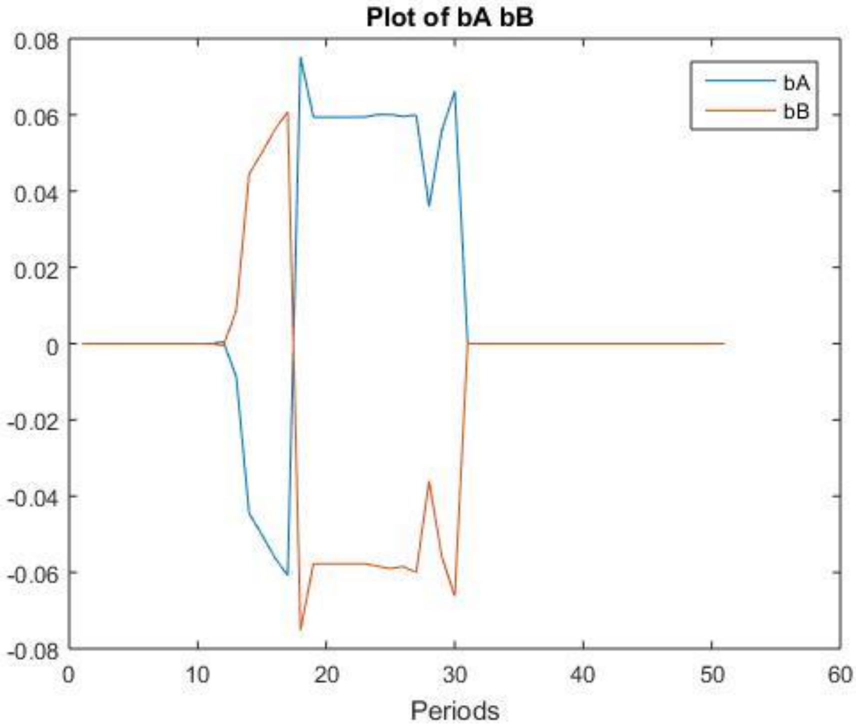
-Capital accumulation: as expected, country S suffers a big drop in capital accumulation, since higher MPK are sought after.



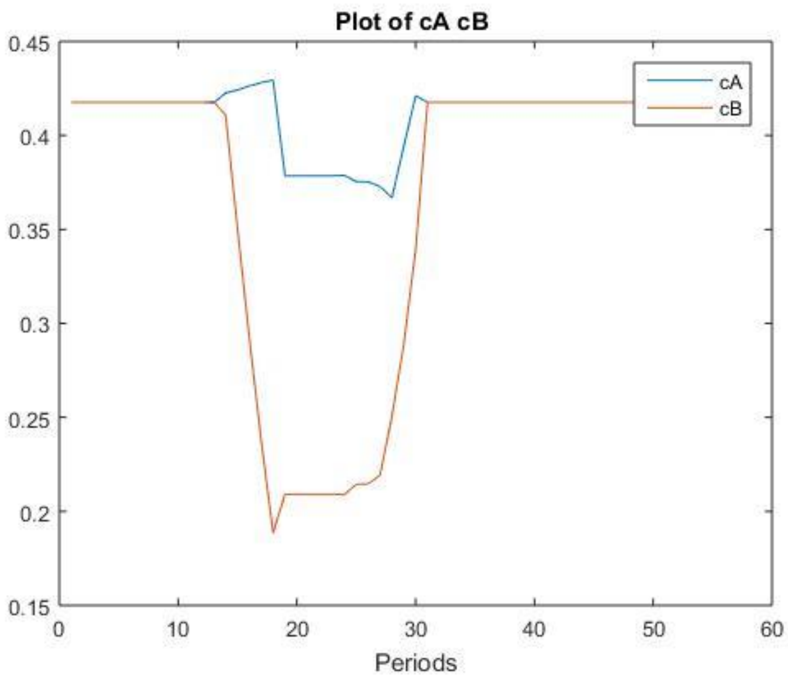
-Interest rates: the rates remain roughtly constant in country N, whereas in country S the interest rate of the bonds raises opening the way to the captation of investments from country N. No arbitrage condition in S implies the consequent rise in interest rates on capital, rS.



-Debt: as expected, we find that during the productivity drop period country S issues debt which is bought by country N.



-Consumption: consumption in country S is drastically reduced as a result of the productivity drop. The small drop in consumption for country N should be compensated with a medium term increase when debt is paid back, but this effect is not apparent in the results TO BE STUDIED WITH BETTER UTILITY FUNCTIONS FOR N



This paper studies the strategic trade off of investors in N which will invest a part in kA and another in bonds in B.

For exogenous paths of AA, AB, the model should give optimal paths of borrowing-lending, investing and consumption for A and B.

If we use a utility function of the form u(c)= ln(c), then the no arbitrage condition on investments will lock the interest rate for the bonds b, making the problem trivial.

What would be the easiest way to include some risk aversion in A investors so even if returns to investment kA, b are not exactly equal, they still diversify their investments to diversify risk.

Or any other suggestion.

1.3.Country N optimization (deterministic, log utility for k, b)

Here we change the utility function used in paragraph 1.2

Country N households maximize their utility function as follows

$$\max \sum_{t=0}^{\infty} \beta^{t} [\ln c_t^i + \psi_1 * \log(k_{t+1}^i) \quad + \psi_2 * \log(b_{t+1}^i) \quad + \psi_3 k_{t+1}^i + \psi_4 b_{t+1}^i]$$

Control Variables: $c_t^i, b_{t+1}^i, k_{t+1}^i$

We choose the following parameter values for psi and phi so diversifications between k and b are preferred. The graph shows the evolution of k/(k+b), in red, and ln(c) in blue.

Note: I am currently searching other utility functions that better match the purpose of the thesis.

Budget constraint:

c_t^i+$b_{t+1}^i + k_{t+1}^i$=$\left(1 + r_t^i - \delta\right)k_t^i + w_t^i + R_t b_t^i$

- Firm problem: Cobb-Douglas, perfect competition, l=1 $y_t^i = A_t^i {k_t^i}^{\alpha} {l_t^i}^{(1-\alpha)}$

$w_t^i = (1-\alpha)A_t^i {k_t^i}^{\alpha}$

$r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$ i=N

$A_t^N \; constant$

1.3.1. Equilibrium equations

Therefore we have the following 11 equations:

(12)Budget constraint for S:	c_t^i+$b_{t+1}^i + k_{t+1}^i$=$\left(1 + r_t^i - \delta\right)k_t^i + w_t^i + R_t b_t^i$	i=S
(13)Budget constraint for N:	c_t^i+$b_{t+1}^i + k_{t+1}^i$=$\left(1 + r_t^i - \delta\right)k_t^i + w_t^i + R_t b_t^i$	i=N
(14)Interest rate for S:	$r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$	i=S
(15)Interest rate for N:	$r_t^i = \alpha A_t^i {k_t^i}^{\alpha-1}$	i=N

(16)Salary for S:	$w_t^i = (1-\alpha)A_t^i {k_t^i}^\alpha$	i=S
(17)Salary for N:	$w_t^i = (1-\alpha)A_t^i {k_t^i}^\alpha$	i=N
(18)Euler equation for N:	$\psi_1/k_{t+1}^i \quad + \psi_3 + \frac{\beta r_{t+1}^i}{c_{t+1}^i} + \frac{1}{c_t^i} = 0$	i=N
(19)Euler equation for N:	$\psi_2/b_{t+1}^i \quad + \psi_4 + \frac{\beta R_{t+1}}{c_{t+1}^i} + \frac{1}{c_t^i} = 0$	i=N
(20)Euler equation for S:	$c_{t+1}^i = \beta R_{t+1} c_t^i$	i=S
(21)Euler equation for S:	$r_{t+1}^i = R_{t+1}$	
(22)Debt market closes:	$b_t^S + b_t^N = 0$	

***Results in model0806.mod*

Note: Chi3 is chosen to have a steady state in equilibrium (namely, rN=R in steady state). This should not happen in the final version of the utility function.

1.4.Stochastic model (log utility for k, b)

Here we introduce the stochastic shocks on country S productivity factor.

```
AB = 1+ z;    z=rho*z(-1)-e;
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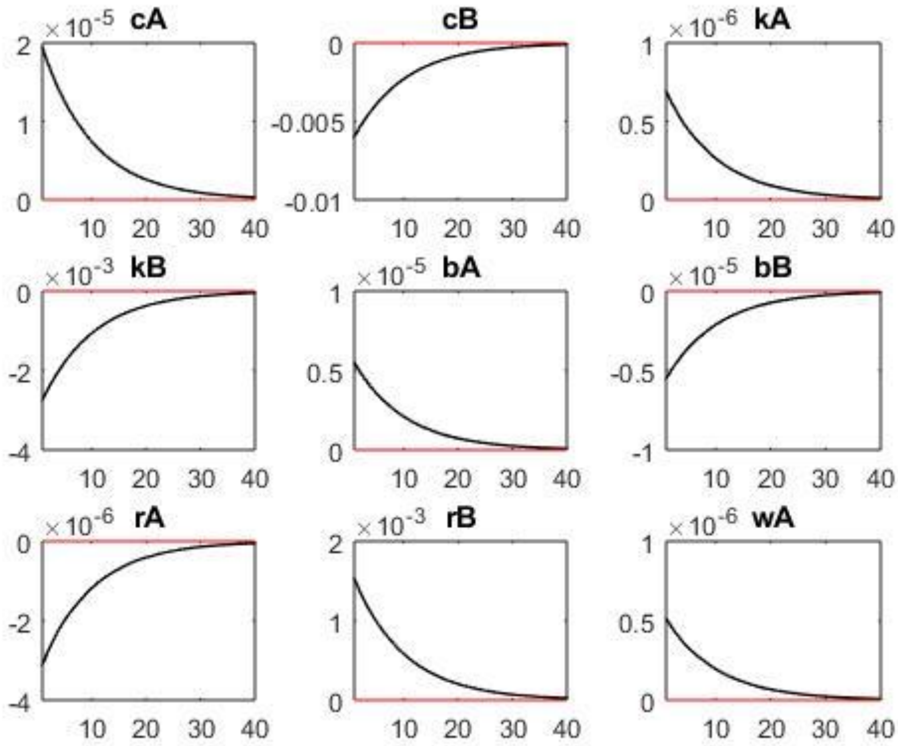
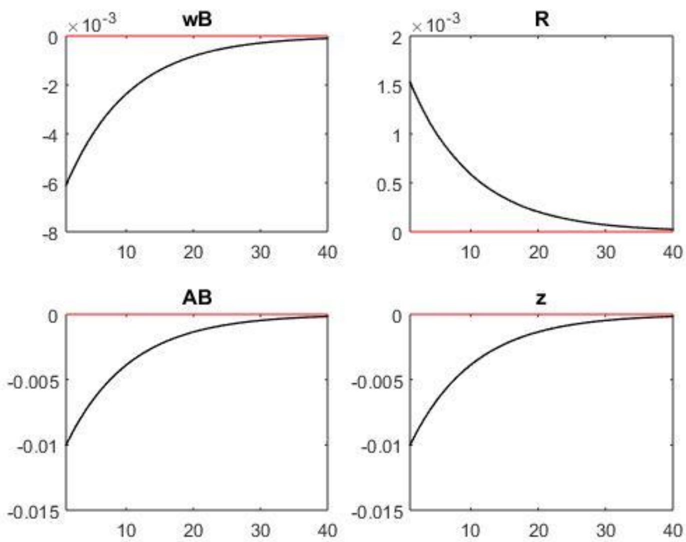
where z is an autoregressive scalar (z0=0, rho=0.9) and e is white noise (variance=0.0001).

The ortogonalized shock to e produces the following answers:





2. Conclusions

Too early for conclusions, further investigation on N Utility function until we find a path for CN that makes sense.