Table 1 Calibration of the parameters

Parameter	Definition	Value	Reference
σ	Intertemporal elasticity of substitution in private consumption	5.00	Nunes and Portugal (2009)
α	Sensitivity of output gap to the debt	0.20	Pires (2008)
κ	Sensitivity of inflation rate to the output gap	0.50	Gouvea (2007), Walsh (2003)
β	Sensitivity of agents to the inflation rate	0.99	Cavallari (2003), Pires (2008)
i^*	Natural rate of interest	0.07	Barcelos Neto and Portugal (2009)
$\overline{b}$	Steady state debt value	0.20	Kirsanova et al. (2005), Nunes and Portugal (2009)
ϖ	Tax rate	0.26	Kirsanova et al. (2005), Nunes and Portugal (2009)

In the cooperative solution, as in the Nash equilibrium one, the exogenous processes are assumed to follow stationary $AR(1)$ representations, where each ε_j is independent and identically distributed with zero mean and constant variance σ_j^2. The same calibration described in Table 1 is applied here. The simulation was carried on in Dynare for MATLAB.

5 Numerical results

In order to evaluate the performance of the alternative regime of coordination, we simulate the models encompassing the Phillips curve, IS curve, government budget constraint, and optimal monetary and fiscal rules. Additionally, we provide an overview on the social losses generated by the distinct monetary and fiscal policy arrangements, and compute impulse response functions. The calibration exercise is meant for the Brazilian economy in the period after the implementation of the Real Plan.[9] Following most of the literature, we assume that each period corresponds to one quarter of a year. The calibrated parameters, along with the respective sources, are reported in Table 1.

One of the major goals of the simulation is to obtain variances of the variables under the optimal trajectories, allowing for the computation of the expected social loss associated to each regime of coordination. As a robustness check, we calculate and plot social losses generated by alternative monetary and fiscal policy decisions, i.e. by varying the weights placed on the target variables. We also compute impulse response functions to analyze how the dynamics of the model behave under shocks of demand, supply, debt, monetary policy, and fiscal policy. Therefore, the analysis will focus on efficient aspects for macroeconomic stabilization.

5.1 Social loss analysis

The social loss is defined as the sum of the authorities' expected individual losses, which can be easily obtained by computing the unconditional variance.[10] Taking, for instance, the

[9]The Real Plan was edited in June 1994.

[10]See Woodford (2003) for details.

Table 2 Loss values for different coefficients under the Nash solution

σ	κ	$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$ $L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$ Variance of					L^M	L^F	L^S
		π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$			
0.50	0.10	12.7452	0.7057	19.1851	27.5664	3.4317	12.9905	4.2008	17.1913
	0.50	5.2119	0.2723	7.9440	11.3445	1.4225	5.3084	1.7033	7.0117
	0.90	2.7197	0.1369	4.1761	5.9437	0.7486	2.7688	0.8842	3.6500
2.50	0.10	9.6978	2.1892	211.2660	16.3928	11.3685	10.2861	5.6368	15.9229
	0.50	2.3218	0.4530	52.7042	4.0432	2.6607	2.4452	1.2729	3.7181
	0.90	0.9891	0.1819	22.8264	1.7428	1.1210	1.0389	0.5301	1.5690
5.00	0.10	8.1027	2.9793	414.0482	11.6572	18.6392	8.8767	6.6826	15.5593
	0.50	1.5190	0.4980	81.3970	2.2583	3.4490	1.6492	1.1882	2.8374
	0.90	0.6059	0.1908	33.0275	0.9111	1.3681	0.6559	0.4654	1.1213

monetary authority period loss function, $L_t^M = \gamma_\pi \pi_t^2 + \gamma_x \hat{x}_t^2 + \gamma_i(\hat{i}_t - i^*)^2$, it is easy to calculate the expected loss for the monetary authority, given by:[11]

$$L^M = \gamma_\pi^2 \operatorname{var}(\pi_t) + \gamma_x^2 \operatorname{var}(\hat{x}_t) + \gamma_i^2 \operatorname{var}(\hat{i}_t - i^*). \tag{30}$$

Thus, the social loss is given by $L^S = L^M + L^F$.

The welfare criterion defines a function which depends upon both monetary and fiscal social losses. We make use of that criterion to analyze the cooperative solution, which occurs indirectly when both authorities associate a positive weight on their instrumental variables. The mechanism permits a direct adjustment to ongoing actions taken by the other authority. Basically, the problem is to maximize a social utility (welfare) or, on the other hand, to minimize the social loss function L^S, which is defined by $L^S = L^M + L^F$, that is, the sum of the authorities' individual losses.

The results reported in Tables 2, 3, 4 and 5 show the variance of each time series and the losses of each authority for different values of σ and κ. The former parameter is the intertemporal elasticity of substitution in private consumption and the latter one measures the sensitivity of the inflation rate to the output gap in the Phillips curve. These parameters came from Eqs. (1) and (2). The reason for choosing these parameters is that they play a crucial role in both structural equations and policy rules.

According to Table 2, keeping κ unchanged, the increases in σ tend to reduce the loss for the monetary but not for the fiscal authority. A high intertemporal elasticity of substitution in private spending means a preference for future consumption, namely, the agents are willing to postpone consumption. Under a higher interest rate, aggregate demand experiences a shrinkage, reducing the output gap and inflation. The monetary policy is more effective, leading to smaller monetary loss under a higher σ. The fiscal policy also experiences a similar decrease in loss, but not for all parameter combinations.

Turning now to the parameter κ, when it increases for a given σ, both fiscal and monetary losses decrease. The idea behind a rise in parameter κ is a steeper Phillips curve. Thus, a higher value of κ tends to increase the sensitivity of inflation to the output gap, yielding a negative effect on the loss. It is interesting to notice that when $\sigma = 5.00$ and $\kappa = 0.90$, we

[11] In order to simplify the notation, we will not distinguish between social loss and expected social loss.

Table 3 Loss values for different coefficients under the Stackelberg solution: Fiscal leadership

σ	κ	$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$ $L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$ Variance of					L^M	L^F	L^S
		π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$			
0.50	0.10	0.0355	0.0077	0.0318	0.0612	0.0034	0.0376	0.0169	0.0545
	0.50	0.0219	0.0066	0.0183	0.0365	0.0036	0.0236	0.0124	0.0360
	0.90	0.0149	0.0052	0.0120	0.0241	0.0035	0.0163	0.0092	0.0255
2.50	0.10	0.0311	0.0677	0.1868	0.0213	0.0087	0.0481	0.0763	0.1244
	0.50	0.0142	0.0311	0.0238	0.0086	0.0034	0.0220	0.0350	0.0570
	0.90	0.0080	0.0173	0.0071	0.0047	0.0017	0.0123	0.0195	0.0318
5.00	0.10	0.0296	0.1147	0.1973	0.0095	0.0072	0.0583	0.1227	0.1810
	0.50	0.0121	0.0427	0.0148	0.0038	0.0020	0.0228	0.0459	0.0687
	0.90	0.0064	0.0217	0.0057	0.0021	0.0010	0.0118	0.0234	0.0352

Table 4 Loss values for different coefficients under the Stackelberg solution: Monetary leadership

σ	κ	$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$ $L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$ Variance of					L^M	L^F	L^S
		π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$			
0.50	0.10	0.0325	0.0051	1.1690	0.0001	0.0486	0.0338	0.0176	0.0514
	0.50	0.0143	0.0119	0.3080	0.0018	0.0337	0.0173	0.0185	0.0358
	0.90	0.0073	0.0103	0.1555	0.0050	0.0196	0.0099	0.0139	0.0238
2.50	0.10	0.0325	0.0050	1.1446	0.0013	0.0491	0.0338	0.0175	0.0513
	0.50	0.0132	0.0138	0.2759	0.0009	0.0353	0.0167	0.0203	0.0370
	0.90	0.0055	0.0132	0.1352	0.0001	0.0212	0.0088	0.0165	0.0253
5.00	0.10	0.0326	0.0050	1.1389	0.0015	0.0490	0.0339	0.0176	0.0515
	0.50	0.0133	0.0137	0.2690	0.0018	0.0353	0.0167	0.0202	0.0369
	0.90	0.0055	0.0133	0.1305	0.0006	0.0213	0.0088	0.0166	0.0254

obtain the lowest loss ($L^S = 1.1213$), meaning that social welfare is maximized under that parameter combination.

The results reported in Table 3 resembles the Nash equilibrium case when we consider variations in κ. On the other hand, variations in σ do not have clear effects, given that there are decreases and increases in the monetary loss depending on the value of κ. The combination of $\sigma = 0.50$ and $\kappa = 0.90$ provides the lowest loss ($L^S = 0.0255$). In addition, the losses for the fiscal leadership are lower than the losses for the Nash equilibrium, suggesting that the former is more efficient.

Table 4 shows losses similar to what was observed under the fiscal leadership. However, increases in σ have lower impacts on the fiscal loss. Once again, the pair of values $\sigma = 0.50$ and $\kappa = 0.90$ provides the lowest loss ($L^S = 0.0238$). Comparing all tables, that value is the global minimum, which was obtained under a monetary leadership solution.

The coordination scheme presented in Table 5 has characteristics similar to the Nash equilibrium outcome. Thereby, the same analysis can be employed here. The combination of

Table 5 Loss values for different coefficients under the cooperative solution

σ	κ	$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$ $L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$ Variance of					L^M	L^F	L^S
		π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$			
0.50	0.10	12.1107	0.7355	29.9229	30.4118	4.0704	12.3706	4.1296	16.5002
	0.50	4.4734	0.2630	11.2059	11.3251	1.5229	4.5675	1.5184	6.0859
	0.90	2.2562	0.1300	5.7000	5.7413	0.7742	2.3030	0.7637	3.0667
2.50	0.10	7.8141	1.6191	364.3679	15.3638	11.8087	8.2573	4.6354	12.8927
	0.50	0.1146	0.1094	0.5639	0.1531	0.4110	0.1423	0.1750	0.3173
	0.90	1.6526	0.3492	72.2684	3.1152	2.4879	1.7477	0.9862	2.7339
5.00	0.10	5.6878	1.9734	769.0163	9.0333	17.8931	6.2037	5.0057	11.2094
	0.50	1.0476	0.3632	112.2076	1.4691	3.0751	1.1421	0.9019	2.0440
	0.90	0.4451	0.1524	44.5084	0.6046	1.2813	0.4847	0.3790	0.8637

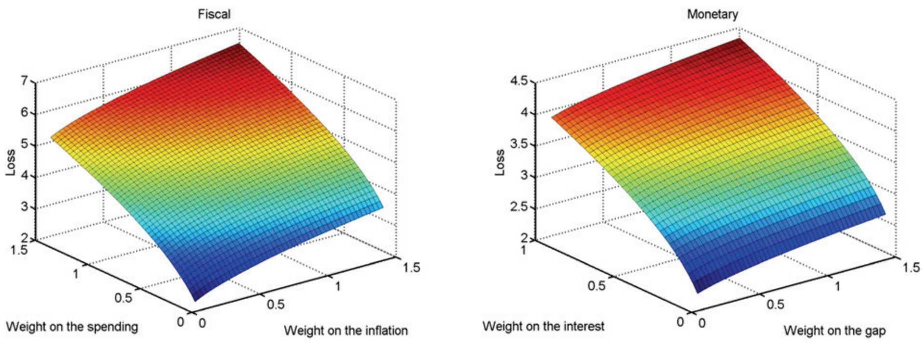

Fig. 1 Social losses for different weights under the Nash solution

$\sigma = 2.50$ and $\kappa = 0.50$ leads to the minimum value for the loss function ($L^S = 0.3173$). This performance, however, is well above the smallest loss obtained under a monetary leadership in the Stackelberg game.

According to the smallest social loss criterion, the policy regimes might be ordered as (1) monetary leadership, (2) fiscal leadership, (3) cooperative solution, and (4) Nash equilibrium solution. Thus, when the monetary authority moves first as a Stackelberg leader we get the best scheme of coordination between the authorities. In addition, both Stackelberg solutions are superior to the remaining ones. Finally, comparing the cooperative and the Nash equilibrium solutions, we can note that the former regime is more efficient in minimizing the social loss.

5.2 Sensitivity analysis

As a robustness check, we evaluated social losses generated by the three mechanisms of coordination discussed in the previous section. In each case, it is assumed that the economy is hit by a supply shock and the weights placed in output gap, inflation, and government spending vary from 0.10 to 1.50, and in interest rate from 0.05 to 1.00. The resulting losses are shown in Figs. 1, 2, 3 and 4.