1. **DSGE Methodology**

4.1 Core DSGE Model without financial frictions

Our departure point is an opened economy version of the standard New Keynesian DSGE model (Smets and Wouters, 2007, and Christiano et al, 2005) appended with a financial frictions. Such a model economy is populated by households, producers, as well as fiscal and monetary authorities. Households consume, invest in physical capital, and provide labor and capital for the production firms. Also, they are shareholders of firms that have access to the international markets. Monopolistically competitive retail firms are price-setters in the goods market, and households are wage-setters in the labor market. Nominal rigidities are introduced through staggered price and wage-setting as in Calvo (1983) and through indexation of prices and wages to past inflation. Fiscal authorities use lump sum taxes to finance government expenditure and monetary authorities set the short-term interest rate according to a Taylor rule. Financial intermediaries in our model accept deposits from households and lend to domestic entrepreneurs in order to finance their investment.

4.1.1 Households

The economy is populated by a continuum of homogenous households, infinitely long living which sum up to one, i.e. indexed by $j \in [0,1]$. The j-th household makes a sequence of decisions during each period in order to maximize the expected lifetime utility. First, it makes a consumption decision and a capital accumulation decision, and it decides how many units of capital services to supply. Second, it purchases securities, whose payoffs are contingent on whether it can re-optimize its wage decision. The representative households seek to maximize the following inter-temporal sum of utility:

$$U = E_0^j \sum_{t=0}^{\infty} \beta^t \left\{ \zeta_t^c \frac{\left(C_t(j) - \gamma C(j)_{t-1}\right)^{1-\delta_c}}{1+\delta_c} - \zeta_t^h \frac{h_t(\mathrm{j})^{1+\delta_h}}{1+\delta_h} \right\} \quad \text{-1-}$$

Here, E_0^j is the expectation operator conditional on aggregate and household j's idiosyncratic information up to, and including, time t; $C_t(j)$ denotes time t consumption and $h_t(j)$ denotes time t hours worked. γ is the consumption habit parameter, δ_c and δ_h denotes the inverse Frisch elasticity for consumption and labor. ζ_t^c and ζ_t^h denotes shocks for consumption preferences and labor supply respectively which follow AR(1) processes characterized by

$$\begin{aligned} \ln\left(\zeta_t^c\right) &= (1-\rho_c)\ln\left(\overline{\zeta_c}\right) + \rho_c \ln\left(\zeta_{t-1}^c\right) + \eta_t^c \\ \ln\left(\zeta_t^h\right) &= (1-\rho_l)\ln\left(\overline{\zeta_h}\right) + \rho_l \ln\left(\zeta_{t-1}^h\right) + \eta_t^l \end{aligned} \quad \text{-2-}$$

where η_t^c and η_t^h are vectors of shocks innovation which are white noise processes.

Each household uses labor and capital income to finance its consumption and investment expenditures and lump sump taxes $T_t(k)$. As in Chari and al. (2002), we assume househods have access to contingent bonds $B_t(k)$ traded at price $(1+i_t)^{-1}$.

$$\frac{B_t(j)}{P_t(1+i_t)}+C_t(j)+I_t \leq \frac{B_{t-1}(j)}{P_t}+\frac{(1-\tau_t^w)W_t(j)h_t(j)}{P_t}+\Pi_t(j)+T_t(j)+\left[r_t^k u_t(j)-\Phi(u_t(j))\right]K_t(j) \quad \text{-3-}$$

where $u_t(j)$ denotes the utilization rate of capital, which we assume is set by the household. $\left(r_t^k u_t(j) K_t(j)\right)-\Phi\left(u_t(j)\right)$ represents the household's earnings from supplying capital services. The term $\Phi\left(u_t(j)\right)$ represents the costs of changing capital utilization. The assumption that households make the capital accumulation and utilization decisions is a matter of convenience (Christiano, Eichenbaum and Evans, 2005). τ_t^w is a stochastic tax on labor income given by

$$\ln\left(\tau_t^w\right)=\left(1-\rho_{\tau^w}\right)\ln\left(\overline{\tau^w}\right)+\rho_{\tau^w}\ln\left(\tau_{t-1}^w\right)+\eta_t^{\tau^w} \quad \text{-4-}$$

Technology for capital accumulation. The law of motion of the stock of physical capital takes into account investment adjustment costs as introduced by Christiano and al (2005). The capital utilization decision involves Keynes' notion of "user cost". That is, a higher utilization rate causes a faster depreciation of the capital stock, either because wear and tear increase with use or because less time can be devoted to maintenance.

$$K_{t+1}=\left(1-\delta\left(u_t\right)\right)K_t+\Upsilon_t\left(1-S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \quad \text{-5-}$$

Where the non-negative depreciation function δ satisfies $0<\delta<1$, $\delta'>0$ and $\delta''>0$. The adjustment cost function S satisfies the following properties: $S(1)=S'(1)=0$ and $S''(1)=\upsilon>0$.

Gross investment, as corresponding to the national income accounts, is I_t. Its contribution to the production capacity in *t+1*, however, depends on the technical shift factor Υ_t affecting the productivity of the new capital goods. The productivity of the already installed capital stock K_t is not directly affected by the new technology. This technological disturbance is very different from the usual technological shock, related to the production function, used in the real business cycle models. Υ_t works as a shift in the marginal efficiency of capital produced in period t which comes on line in t+1.

$$\ln\left(\Upsilon_t\right)=\left(1-\rho_\Upsilon\right)\ln\left(\overline{\Upsilon}\right)+\rho_\Upsilon\ln\left(\Upsilon_{t-1}\right)+\eta_t^\Upsilon \quad \text{-6-}$$

The parametrization of the depreciation function which specifies only the steady state level, slope, and curvature of the depreciation function, is consistent with the following functional form

$$\delta\left(u_t\right)=\delta+\frac{1}{1+\varsigma}u_t^{1+\varsigma} \text{ -7-}$$

where ς represents the elasticity of marginal depreciation with respect to the utilization rate:

$$\varsigma=\frac{u\delta''(u)}{\delta'(u)}>0$$

Household consumption and investment decisions. The first order condition for consumption and bond are

$$\lambda_t=\zeta_t^c\left(C_t-\gamma C_{t-1}\right)^{-\delta_c}-\beta\gamma E_t\zeta_{t+1}^c\left(C_{t+1}-\gamma C_t\right)^{-\delta_c} \text{ -8-}$$

$$\left(1-i_t\right)^{-1}=\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right] \text{ -9-}$$

By differentiating the Lagrange representation of the household`s problem with the respect to investment, I_t, Capital services, K_t and capital utilization, u_t, first order conditions are

$$\vartheta_t=E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\vartheta_{t+1}\left(1-\delta\left(u_t\right)\right)+r_{t+1}^k-\Phi(\mathrm{u}_{t+1})\right)\right] \text{ -10-}$$

$$1=\vartheta_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)-\frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right]\Upsilon_t+\beta E_t\left[\vartheta_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{I_{t+1}}{I_t}\right)^2S'\left(\frac{I_{t+1}}{I_t}\right)\Upsilon_{t+1}\right] \text{ -11-}$$

$$r_t^k=\Phi'_t\left(u_t\right) \text{ -12-}$$

ϑ_t is the multiplier on the household`s period t capital accumulation constraint. We adopt the following functional for capital utilization, Φ:

$$\Phi(\mathrm{u}_t)=0.5\sigma_b\sigma_a u_t^2+\sigma_b(1-\sigma_a)\mathrm{u}_t+\sigma_b((\sigma_a/2)-1) \text{ -13-}$$

Where σ_a and σ_b are the parameters of this function.

Labor supply and wage setting. Maximization of equation (1) subject to (3) yields the following first-order conditions with respect to $h_t\left(j\right)$:

$$\zeta_t^h h_t\left(j\right)^{\delta_l}=\frac{\left(1-\tau_t^w\right)W_t\left(j\right)}{P_t} \text{ -14-}$$

Each household has a unique labor type *k*, which is sold to perfectly competitive aggregators, who pool all labor types into one homogeneous labor service with the following function :

$$H_t = \left[\int_0^1 h_t(\mathrm{j})^{\frac{1}{\varphi_L}} dj \right]^{\varphi_l} \quad , 1 \le \varphi_l < \infty. \text{ -15-}$$

Households set their wage rate according the the standard Calvo scheme, i.e. with probability $(1-\theta_w)$ they receive a signal to reoptimize and then set their wage to maximise their utility subject to the demand for their labour services. Denote this wage rate by $\tilde{W}_t$. This is not indexed by *j* because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize.

$$U_t^j = \sum_{i=0}^{\infty} \left(\beta\theta_w\right)^i \left[-\zeta_t^l \frac{h_{t+i}(\mathrm{j})^{1+\delta_l}}{1+\delta_l} + \lambda_{t+i}(1-\tau_t^w) \frac{W_{t+i}(\mathrm{j})\,\mathrm{h}_{t+i}(\mathrm{j})}{P_{t+i}} \right] \quad \text{-16-}$$

where λ_t is the multiplier on the household`s period *t* budget constraint. The demand for for labor services of type *k* conditional on it having optimized in period t and not again since, is :

$$\mathrm{h}_t(\mathrm{j}) = \left[\frac{\tilde{W}_t(\mathrm{j})}{W_t} \right]^{-\frac{\phi_l}{\phi_l - 1}} H_t \quad \text{-17-}$$

where $W_t = \left[\int_0^1 W_t(\mathrm{j})^{-\frac{1}{\phi_l - 1}} dj \right]^{-(\phi_l - 1)}$ is the aggregate wage in the economy.

The equilibrium conditions associated with this problem, i.e wage settings of households that do get to reoptimize, are re given as

$$W_t(\mathrm{j}) = \left[(1-\theta_\omega)(\tilde{W}_t)^{\frac{1}{1-\phi_l}} + \theta_\omega \left(\bar{\pi}^{1-\xi_w} \pi_{t-1}^{\xi_w} W_{t-1}(\mathrm{j}) \right)^{\frac{1}{1-\phi_l}} \right]^{1-\phi_l} \quad \text{-18-}$$

With probability θ_w they do not receive the signal and index their wage according to the following rule :

$$W_t(\mathrm{j}) = \bar{\pi}^{1-\xi_w} \pi_{t-1}^{\xi_w} W_{t-1}(\mathrm{j}) \quad \text{-19-}$$

where $\pi_t = P_t/P_{t-,}$ $\overline{\pi}$ is the steady state inflation rate and $\xi_w \in [0,1]$ denotes degree of indexation.

Final consumption and investment goods

Final consumption good is purchased by households. It is a composite of the home-produced consumption good, $C_t^h(j)$, and imported consumption good, $C_t^f(j)$ as :

$$C_t(j) = \left[\kappa^{\frac{1}{\eta}} C_t^h(j)^{\frac{\eta-1}{\eta}} + (1-\kappa)^{\frac{1}{\eta}} C_t^f(j)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad \text{-20-}$$

where $\eta > 0$ is the elasticity of substitution between home and foreign goods. The parameter κ represents home consumers preference towards domestic and foreign goods, respectively. In equilibrium, the consumption of domestic and foreign goods are given as follows

$$\begin{aligned} C_t^h(j) &= \kappa \left[\frac{P_t^h}{P_t} \right]^{-\eta} C_t(j) \\ C_t^f(j) &= (1-\kappa) \left[\frac{P_t^f}{P_t} \right]^{-\eta} C_t(j) \end{aligned} \quad \text{-21-}$$

where P_t^h and P_t^f are the domestic prices associated with the home-produced and foreign-produced goods respectively. Index price is given as follows:

$$P_t = \left[\kappa \left(P_t^h \right)^{1-\eta} + (1-\kappa) \left(P_t^f \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad \text{-22-}$$

The rate of inflation of the consumption good is

$$\pi_t = \frac{P_t}{P_{t-1}} = \left[\frac{\kappa \left(P_t^h \right)^{1-\eta} + (1-\kappa) \left(P_t^f \right)^{1-\eta}}{\kappa \left(P_{t-1}^h \right)^{1-\eta} + (1-\kappa) \left(P_{t-1}^f \right)^{1-\eta}} \right]^{\frac{1}{1-\eta}} \quad \text{-23-}$$

Investment goods are produced by arepresentative competitive firm using the following technology

$$I_t + \Phi(u_t) K_t = \left[\chi^{\frac{1}{\varepsilon}} I_t^{h\frac{\varepsilon-1}{\varepsilon}} + (1-\chi)^{\frac{1}{\eta}} I_t^{f\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{-24-}$$

where investment is defined as the sum of investment goods, I_t, used in the accumulation of physical capital plus investment goods used in the capital maintenance, $\Phi(u_t) K_t$.

As in the consumption good sector, the representative investment goods producers take all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs in a form

$$I_t^h(j) = \chi\left[\frac{Q_t^h}{Q_t}\right]^{-\varepsilon}\left(I_t + \Phi(u_t)K_t\right)$$
$$I_t^f(j) = (1-\chi)\left[\frac{Q_t^f}{Q_t}\right]^{-\varepsilon}\left(I_t + \Phi(u_t)K_t\right) \quad \text{-25-}$$

Te price of I_t is related to the price of inputs by

$$Q_t = \left[\chi\left(Q_t^h\right)^{1-\varepsilon} + (1-\chi)\left(Q_t^f\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad \text{-26-}$$

The rate of inflation of the investment good is

$$\pi_t^i = \frac{Q_t}{Q_{t-1}} = \left[\frac{\chi\left(Q_t^h\right)^{1-\varepsilon} + (1-\chi)\left(Q_t^f\right)^{1-\varepsilon}}{\chi\left(Q_{t-1}^h\right)^{1-\varepsilon} + (1-\chi)\left(Q_{t-1}^f\right)^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} \quad \text{-27-}$$

4.1.2 Firm sectors

There are several stages of production in the economy. Intermediate goods firms produce differentiated goods and sell them to aggregators. Aggregators combine those products into a homogeneous final good.

Final good producers

Final good producers play the role of aggregators. They buy differentiated products from intermediate goods producers $Y_t\ (z)$ and aggregate them into a single final good, which they sell in a perfectly competitive market. The final good is produced according to the following technology

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\phi_p - 1}{\phi_p}} dz\right]^{\frac{\phi_p}{\phi_p - 1}} \quad \text{-28-}$$

where ϕ_p is the elasticity of substitution across varieties of goods. Final goods firms solve the problem of choosing $Y_t\ (z)$ to solve the following problem:

$$Min_{Y_t(z)}\int_0^1 P_t(z)Y_t(z)\,dz$$

$$S.t.\ \left[\int_0^1 Y_t(z)^{\frac{\phi_p-1}{\phi_p}}\,dz\right]^{\frac{\phi_p}{\phi_p-1}} \geq \overline{Y} \quad \text{-29-}$$

The problem of the aggregator gives the following demands for differentiated goods

$$Y_t(z)=\left[\frac{P_t(z)}{P_t}\right]^{-\phi_p} Y_t \quad \text{-30-}$$

where P_t is the aggregate wage index. Equations above imply:

$$P_t = \left[\int_0^1 P_t(z)^{1-\phi_p}\,dz\right]^{\frac{1}{1-\phi_p}} \quad \text{-31-}$$

Intermediate goods producers

There is a continuum of intermediate goods producers indexed by z . They rent capital and labor and use a Cobb-Douglas production technology :

$$Y_t(z)=\xi_t(u_t(z)K_t(z))^{\alpha} h_t(z)^{1-\alpha}-\Omega \quad \text{-32-}$$

where ξ_t is the stochastic level of total productivity common to all firms, the log of which follows an exogenous process AR (1) as follow

$$\ln(\xi_t)=(1-\rho_Y)\ln(\overline{\xi})+\rho_Y\ln(\xi_{t-1})+\eta_t^Y \quad \text{-33-}$$

η_t^Y is an iid normal error term. $K_t(z))$ denotes the input of capital and $L_t(z))$ the input of labor used by the intermediate z. Ω represents fixed costs in production.

Minimization yields the following first order conditions with respect to capital and labor:

$$mc_t=\tau_t^d\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(r_t^k\right)^{\alpha} W_t^{1-\alpha}\frac{1}{\xi_t} \quad \text{-35-}$$

Retailers simply purchase intermediate goods at a price equal to the marginal cost and differentiate them in a monopolistically competitive market, similarly to labor unions in the labor market. τ_t^d is a tax-like shock which affects marginal cost does not appear in the production function.

Retailers set nominal prices in a staggered fashion à la Calvo (1983). Each retailer resets its price with probability ($1-\theta_p$). For the fraction of retailers that cannot adjust, the price is automatically increased at the aggregate inflation rate.

$$P_t(z) = \overline{\pi}^{1-\xi_p} \pi_{t-1}^{\xi_p} P_{t-1}(z) \quad \text{-36-}$$

With probability ($1-\theta_p$), the firm can change the price. The problem of the *z-th* domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$E_t \sum_{i'=0}^{\infty} \beta^{i'} \lambda_{t+i'} \left(P_{t+i'}(z) Y_{t+i'}(z) - mc_{t+i'} \left(Y_{t+i'}(z) + \Omega \right) \right) \quad \text{-37-}$$

subject to the requirement that production equals demand, $Y_t(\mathrm{j}) = \left[\frac{\tilde{P}_t(\mathrm{j})}{P_t} \right]^{-\frac{\phi_p}{\phi_p - 1}} Y_t$. In the above expression, λ_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits in terms of currency.

The equilibrium conditions associated with this problem, i.e price settings of retailers that do get to reoptimize, are re given as :

$$\mathrm{P}_t(\mathrm{z}) = \left[\left(1-\theta_p\right)(\tilde{\mathrm{P}}_t)^{\frac{1}{1-\phi_p}} + \theta_p \left(\overline{\pi}^{1-\xi_p} \pi_{t-1}^{\xi_p} P_{t-1}(\mathrm{z}) \right)^{\frac{1}{1-\phi_p}} \right]^{1-\phi_p} \quad \text{-38-}$$

4.1.3 Resource Constraint and policies

Output is divided between consumption, investment, government expenditures, and capital maintenance. We suppose further that government expenditures are exogenously fixed at the level ξ_t^g :

$$\ln\left(\xi_t^g\right) = \left(1-\rho_g\right) \ln\left(\overline{\xi^g}\right) + \rho_g \ln\left(\xi_{t-1}^g\right) + \eta_t^g \quad \text{-39-}$$

The economy-wide resource constraint is thus given by

$$Y = C_t + I_t + \xi_t^g + \Phi\left(u_t\right) K_t \quad \text{-40-}$$

Government expenditures, further, are financed by lump taxes and government bonds:

$$\xi_t^g = T_t + \tau_t^w \frac{W_t}{P_t} h_t + \frac{\frac{B_{t+1}}{1+i_t} - B_t}{P_t} \quad \text{-41-}$$

We suppose monetary policy is characterized by a simple Taylor rule with interest-rate smoothing. Let i_t be the net nominal interest rate, *i* is the steady state nominal rate, and t the natural (flexible price equilibrium) level of output. Then,

$$R_t = (1-\rho_i)\left[\overline{R} - \kappa_\pi \pi_t + \kappa_y (\log(Y^* - Y_t)\right] - \rho_i R_{t-1} + \xi_t^R \quad \text{-42-}$$

where the smoothing parameter ρ_i lies between zero and unity, and where ξ_t^i is an exogenous shock to monetary policy given as,

$$\ln(\xi_t^R) = (1-\rho_R)\ln\left(\overline{\xi^R}\right) + \rho_R \ln(\xi_{t-1}^R) + \eta_t^R \quad \text{-43-}$$

The link between nominal and real interest rates is given by the following Fisher relation,

$$R_t = E_t\left[(1+i_t)\frac{P_{t+1}}{P_t}\right] \text{-44-}$$