Web appendix for
Twin Deficits: Squaring Theory, Evidence and Common Sense

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Contents

1 The Two-country model

This appendix provides a detailed presentation of the model employed in the quantitative analysis presented and discussed in the paper. The model — briefly outlined in Box 1 of the paper — is a variant of Backus, Kehoe, and Kydland (1994) and Heathcote and Perri (2002) — we draw especially on the latter contribution to clarify the role of assets market. We introduce two features specific to our analysis. First, for the reasons discussed in the main text, we assume that government spending falls entirely on domestic intermediate goods. Second, under incomplete markets, we assume that the discount factor is endogenous. This is a technical modification to ensure stationarity of bond holdings under incomplete financial markets.[1] Below we will state the first order conditions of households and firms and provide a complete list of the linearized equilibrium conditions used in our simulations.

1.1 Structure and solution of the model

1.1.1 The economies

The world consists of two countries, each of which is populated by the same number of identical, infinitely lived households. Let c_{it} denote consumption and n_{it} the amount of labor supplied by the representative household in country i. The objective of such household is given by

$$\max E_0 \sum_{t=0}^{\infty} \beta(\{c_{i\tau}\}_{\tau=0}^{t-1}, \{n_{i\tau}\}_{\tau=0}^{t-1}) \frac{1}{1-\gamma}[c_{it}^{\mu}(1-n_{it})^{1-\mu}]^{1-\gamma}, \tag{1}$$

subject to a budget constraint discussed below. The parameter γ measures the degree of risk aversion and the parameter μ measures the weight of consumption in the utility function relative to leisure. The rate of time preference may depend on the sequence of consumption and labor decision. Households supply labor and rent capital to i-firms which produce intermediate goods. Labor and capital are internationally immobile; households in each country own the capital stock k_{it} of that country. I-firms in country 1 produce good a; i-firms in country 2 produce good b on the basis of the production function:

$$y_{it} = e^{z_{it}} k_{it}^{\theta} n_{it}^{1-\theta}, \tag{2}$$

where z_{it} is an exogenous technology shock. Letting w_{it} and r_{it} denote the wage and rental rate on capital (in terms of intermediate goods), an i-firm's problem is given by

$$\max_{k_{it}, n_{it}} (y_{it} - w_{it}n_{it} - r_{it}k_{it}). \tag{3}$$

The law of one price holds for intermediate goods a and b. I-firms sell intermediate goods to domestically located f-firms and to the government. Households use the domestic final good for either

[1] Heathcote and Perri (2002) assume portfolio costs.

consumption or investment. Investment, x_{it}, increases the existing capital stock in the following way:

$$k_{it+1} = (1-\delta)k_{it} + x_{it}, \tag{4}$$

where δ is the depreciation rate. F-firms produce final goods, f_{it}, by combining the intermediate goods a and b:

$$f_{1t} = \left[\omega^{1/\sigma} a_{1t}^{(\sigma-1)/\sigma} + (1-\omega)^{1/\sigma}\, b_{1t}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{5}$$

$$f_{2t} = \left[(1-\omega)^{1/\sigma}\, a_{2t}^{(\sigma-1)/\sigma} + \omega^{1/\sigma} b_{2t}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{6}$$

where σ is the elasticity of substitution between goods a and b and $\omega > 0.5$ determines the extent to which there is a home bias in private expenditures on consumption and investment. F-firm's objective is given by

$$\max_{a_{it}, b_{it}} \left(f_{it} - q_{it}^a a_{it} - q_{it}^b b_{it} \right), \tag{7}$$

where q_{it}^a and q_{it}^b are the prices of goods a and b in country i in units of the final good produced in country i.

Government purchases, g_{it}, fall entirely on domestic intermediate goods. The government also makes lump-sum transfers to households, T_{it}, in terms of intermediate goods in order to balance its budget in each period.[2] Purchases and transfers are financed by taxing labor and capital income at rate τ_{it}. The (balanced) budget constraint of the government (in terms of intermediate goods) is thus given by

$$T_{it} = \tau_{it}(w_{it}n_{it} + r_{it}k_{it}) - g_{it}, \tag{8}$$

where tax rates and government spending follow an exogenous process specified below.

Bond economy In our model with incomplete markets, we assume that agents can lend and borrow across borders by issuing a bond in zero net supply. We follow Mendoza (1991) assuming that preferences imply an endogenous rate of time preference. This ensures the stationarity of bond holdings in case asset markets allow for trade in non-contingent bonds only. Specifically, we assume

$$\beta(\{c_{i\tau}\}_{\tau=0}^{t-1}, \{n_{i\tau}\}_{\tau=0}^{t-1}) = \exp\left[\sum_{\tau=0}^{t-1} -\nu(c_{i\tau}, n_{i\tau})\right],$$

where

$$\nu(c_{it}, n_{it}) = \ln(1 + \psi[c_{it}^{\mu}(1-n_{it})^{1-\mu}]. \tag{9}$$

The parameter ψ determines how the discount factor responds to the level of consumption and leisure; it pins down the steady state discount factor.

[2] Baxter (1995) stresses that these transfers can also be interpreted as a fiscal surplus in a world where government debt is Ricardian.

Let B_{it+1} denote the quantity and Q_t the prices (in units of good a) of bonds bought by households in country i. The bond pays one unit of good a in period $t+1$ irrespectively of the state in $t+1$. The budget constraint of a representative household in country 1 is given by

$$q^a_{1t}[(1-\tau_{1t})(w_{1t}n_{1t}+r_{1t}k_{1t})+T_{1t}+B_{1t}]=c_{1t}+x_{1t}+q^a_{1t}Q_tB_{1t+1}. \tag{10}$$

The budget constraint in country 2 is analogous:

$$q^b_{2t}[(1-\tau_{2t})(w_{2t}n_{2t}+r_{2t}k_{2t})+T_{2t}+\frac{q^a_{2t}}{q^b_{2t}}B_{2t}]=c_{2t}+x_{2t}+q^a_{2t}Q_tB_{2t+1}$$

Complete markets Alternatively we consider that case that asset markets are complete by allowing for trade in a complete set of state-contingent securities denominated in units of good a. For convenience, in this case we drop the assumption that the discount factor is endogenous:

$$\beta(\{c_{i\tau}\}^{t-1}_{\tau=0},\{n_{i\tau}\}^{t-1}_{\tau=0})=\beta^t.$$

after verifying that it makes no difference in our numerical experiments. Letting $Q_{t,t+1}$ denote the stochastic discount factor used to price the portfolio of securities in period t, A_{t+1}, the budget constraint of a representative household in country 1 is given by

$$q^a_{1t}[(1-\tau_{1t})(w_{1t}n_{1t}+r_{1t}k_{1t})+T_{1t}]+q^a_{1t}A_{1t}=c_{1t}+x_{1t}+q^a_{1t}E_t[Q_{t,t+1}A_{1t+1}]. \tag{11}$$

The budget constraint in country 2 is analogous:

$$q^b_{2t}[(1-\tau_{2t})(w_{2t}n_{2t}+r_{2t}k_{2t})+T_{2t}]+q^a_{2t}A_{2t}=c_{2t}+x_{2t}+q^a_{2t}E_t[Q_{t,t+1}A_{2t+1}].$$

Definition of equilibrium An equilibrium is a set of prices for all $t\geq 0$ such that when i-firms, f-firms and households take these prices as given, households solve (1) subject to either constraint (11) or constraint (10) and firms solve their static problems (3) and (7) and all markets clear. Market clearing for intermediate goods requires that

$$y_{1t} = a_{1t}+a_{2t}+g_{1t} \tag{12}$$

$$y_{2t} = b_{1t}+b_{2t}+g_{2t} \tag{13}$$

Market clearing for final goods requires that

$$f_{it}=c_{it}+x_{it},\quad i=1,2. \tag{14}$$

In the bond economy, bond market clearing requires that

$$B_{1t}=B_{2t}. \tag{15}$$

Additional variables of interest We define the terms of trade as the price of imports relative to the price of exports. Thus

$$p_t = q_{1t}^b / q_{1t}^a \tag{16}$$

denotes the terms of trade for country 1. Its trade balance is defined as the ratio of net exports to output

$$nx_t = \frac{a_{2t} - p_t b_{1t}}{y_{1t}} \tag{17}$$

Finally, we define domestic investment relative to foreign investment:

$$x_t = \frac{x_{1t}}{x_{2t}} \tag{18}$$

1.1.2 First order conditions

Households Consider the households' problem in the *bond economy*. Substituting for investment in the budget constraint (10) using the law of motion for capital (4) and letting λ_t denote the multiplier on the budget constraint we obtain the first order conditions for the household's problem in the home country (the index 'i' is dropped to simplify the exposition):

$$u_c(c_t, n_t) = \lambda_t \tag{19}$$

$$u_n(c_t, n_t) = -\lambda_t q_t^a (1 - \tau_t) w_t \tag{20}$$

$$\lambda_t q_t^a Q_t = \exp(-v(c_t, n_t)) E_{t+1} \left[\lambda_{t+1} q_{t+1}^a\right] \tag{21}$$

$$\lambda_t = \exp(-v(c_t, n_t)) E_{t+1} \left[\lambda_{t+1} q_{t+1}^a (1 - \tau_{t+1}) r_{t+1} + \lambda_{t+1}(1 - \delta)\right] \tag{22}$$

Note here that

$$\frac{\beta^{t+1}(c_{t+1}, n_{t+1})}{\beta^t(c_t, n_t)} = \frac{\exp\left[\sum_{\tau=0}^{t} -\nu(c_{i,\tau}, n_{i,\tau})\right]}{\exp\left[\sum_{\tau=0}^{t-1} -\nu(c_{i,\tau}, n_{i,\tau})\right]} = \exp(-\nu(c_t, n_t))$$

It is convenient to define

$$\beta_t \equiv \exp(-\nu(c_t, n_t)) \tag{23}$$

Now, consider the households' problem under *complete markets*. The first order conditions for consumption, labor supply and investment are the same (except for the rate of time preference being constant: $\exp(-\nu(c_t, n_t)) = \beta$). Combining the first order condition for state-contingent securities in country 1 and 2 and iterating backwards gives the risk sharing condition (see, for instance, Chari, Kehoe and McGrattan, 2002):

$$u_c(c_{1,t}, n_{1,t}) q_{1,t}^a = u_c(c_{2,t}, n_{2,t}) q_{2,t}^a. \tag{24}$$

I-firms The first order conditions to (3) define the wage and the rental rate of capital (in terms of intermediate goods)

$$w_{it} = (1-\theta)\frac{y_{it}}{n_{it}} \quad (25)$$

$$r_{it} = \theta\frac{y_{it}}{k_{it}} \quad (26)$$

F-firms The first order conditions to (7) give the demand functions for intermediate goods

$$a_1 : \frac{\partial f_1}{\partial a_1} = q_1^a \quad \Leftrightarrow \quad a_1 = (q_1^a)^{-\sigma}\omega f_1 \quad (27)$$

$$b_1 : \frac{\partial f_1}{\partial b_1} = q_1^b \quad \Leftrightarrow \quad b_1 = (q_1^b)^{-\sigma}(1-\omega) f_1 \quad (28)$$

$$b_2 : \frac{\partial f_2}{\partial b_2} = q_2^b \quad \Leftrightarrow \quad b_2 = (q_2^b)^{-\sigma}\omega f_2 \quad (29)$$

$$a_2 : \frac{\partial f_2}{\partial a_2} = q_2^a \quad \Leftrightarrow \quad a_2 = (q_2^a)^{-\sigma}(1-\omega) f_2 \quad (30)$$

1.1.3 Derivation of the equilibrium condition in the capital market

In this subsection we consider in more detail the first order condition for investment (22) — as this condition is central to our argument in the main text. To complement the analysis in the main text, in the following we allow for the possibility that the composition of investment goods differs from that of consumption goods. The price of investment goods in terms of consumption goods, q_{1t}^x, will be generally different from one. In this case, the first order condition for investment (22) is replaced by[3]

$$q_{1t}^x = \beta_{1t} E_t \left[\frac{\lambda_{1t+1}}{\lambda_{1t}} q_{1t+1}^a \left(1-\tau_{1t+1}\right) r_{1t+1} + \frac{\lambda_{1t+1}}{\lambda_{1t}} \left(1-\delta\right) \right]$$

we can write

$$\begin{aligned} q_{1t}^x &= \beta_{1t} E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} \cdot E_t \left[\left(1-\tau_{1t+1}\right) r_{1t+1} q_{1t+1}^a \right] + \\ &\quad + Cov\left[\beta_{1t}\frac{\lambda_{1t+1}}{\lambda_{1t}}, \left(1-\tau_{1t+1}\right) r_{1t+1} q_{1t+1}^a \right] + \left(1-\delta\right)\beta_{1t} E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} \end{aligned}$$

The consumption-based interest rate (r_{1t}^C) is conventionally defined as the rate of growth of marginal utility of consumption

$$\beta_{1t} E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} \equiv \frac{1}{1+r_{1t}^C} \quad (31)$$

Using this rate, we can rewrite the above as

$$\begin{aligned} q_{1t}^x &= \frac{1}{1+r_{1t}^C} E_t \left[\left(1-\tau_{1t+1}\right) r_{1t+1} q_{1t+1}^a \right] + \\ &\quad Cov\left[\frac{1}{1+r_{1t}^C}, \left(1-\tau_{1t+1}\right) r_{1t+1} q_{1t+1}^a \right] + \left(1-\delta\right)\frac{1}{1+r_{1t}^C} \end{aligned}$$

[3]Note that in the following $\beta_t = \beta$ under complete markets.

In the main text we focus on the first term on the right hand side — which illustrates the core mechanism of interest. Without loss of generality, we abstract from taxes and depreciation (setting $\delta = 1$ and $\tau_t = 0$). We also ignore the covariance between the consumption-based real rate of return, and the marginal revenue from investment. In terms of the notation of the main paper we have: $q^a_{1t+1} = \frac{P_{dt+1}}{P_{t+1}}$ for country 1. Together with (26) this implies

$$1 + r^C_{1t} = E_t \underbrace{\frac{P_{dt+1}}{P_{t+1}} \theta \frac{y_{1t+1}}{k_{1t+1}} \frac{1}{q^x_{1t}}}_{\text{real return to investment}} . \tag{32}$$

Setting $q^x_{1t} = 1$ gives the condition discussed in the main text. This condition holds both under complete and incomplete markets. Clearly, prices (including r^C and q^a) will behave differently across allocations with different degrees of consumption risk sharing.

Generally, the expression for the rate of return is multiplied by the inverse of the lagged price of investment in terms of consumption. When the import content of investment is larger than consumption, as is in the data, this price is lower than one as a result of the terms of trade appreciation. Therefore investment goods are cheaper than consumption goods. Hence, the positive effect of a terms of trade appreciation on the real rate of return is magnified. Conversely, if the import content of investment is counterfactually low, the improvement in the rate of return from terms of trade movements becomes smaller, or may even change sign.

1.1.4 Steady state

Home Bias and Openness We consider a symmetric steady state with balanced trade such that $a_2 = b_1$. For simplicity we focus our analysis on country 1 (symmetric expressions hold for country 2). First we relate the home bias parameter ω to openness, i.e. the share of imports in GDP. Divide the FOC for a_1, equation (27), by the FOC for b_1, equation (28), and note that because of symmetry the prices for intermediate goods a and b are equal in steady state such that $q^a_1 = q^b_1$ and thus

$$\frac{a_1}{b_1} = \frac{\omega}{1-\omega} \tag{33}$$

Letting w_d denote the share of net output ($y' = y - g$) not exported (=not imported) in steady state we have

$$a_1 = w_d y'_1 \quad b_1 = (1 - w_d)\, y'_1 . \tag{34}$$

Substituting into (33) gives $\omega = w_d$. Hence, the home bias parameter ω measures the share of net output which is not exported, and $1 - \omega$ is a measure for openness, as it measures imports (=exports) as a share of net output in steady state. Let g_y denote the steady state share of government spending in GDP and assume that government spending falls on domestic goods only. Such that

$$y' = y - g = (1 - g_y)y$$

We can then pin down ω on the basis of (34) using the share of imports in total output (which is observable):

$$\omega = 1 - \frac{b_1}{y_1'} \Leftrightarrow \omega = 1 - \underbrace{\frac{1}{1-g_y}\frac{b_1}{y_1}}_{\text{first moments of data}}. \tag{35}$$

Total final goods, f equal net output in steady state y', which can be seen by substituting (34) into the production function for final goods, i.e. the Armington aggregator given by (5). Relative to the specification of the weights in the Armington aggregator in Backus et al. (1994), we thus impose a priori that $y' = f$.[4] This in turn implies

$$c_1 + x_1 = f_1 = y_1' = y_1 - g_1 \Leftrightarrow y_1 = c_1 + x_1 + g_1;$$

in steady state consumption, investment and government spending add up to total GDP.

Relative Prices Next, we consider relative prices in steady state. Applying the Euler theorem to the Armington aggregator allows to write

$$\begin{aligned} f_1 &= \left(\left[\omega^{1/\sigma} a_{1t}^{(\sigma-1)/\sigma} + (1-\omega)^{1/\sigma} b_{1t}^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)} \omega^{1/\sigma} a_{1t}^{-1/\sigma}\right) a_1 \\ &\quad + \left(\left[\omega^{1/\sigma} a_{1t}^{(\sigma-1)/\sigma} + (1-\omega)^{1/\sigma} b_{1t}^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)} (1-\omega)^{1/\sigma} b_{1t}^{-1/\sigma}\right) b_1. \\ &= q_1^a a_1 + q_2^b b_1 \end{aligned}$$

Note again that in steady state $q_1^a = q_1^b$ and exploiting symmetry ($a_2 = b_1$) yields

$$f_1 = q_1^a\,(a_1 + a_2) = q_1^a y_1' => q_1^a = q_1^b = 1. \tag{36}$$

Discount factor It is convenient to define

$$\bar\beta = \exp(-v(c,n)) = (1 + \psi(c^\mu(1-n)^{1-\mu})^{-1}$$

Then the FOC for bond holds evaluated in steady state gives:

$$Q = \bar\beta$$

Great Ratios The above allows us to evaluate the FOC for k_{t+1} in steady state and to derive the capital-GDP-ratio in steady state:

$$k_y = \frac{\theta\bar\beta(1-\tau)}{1-\bar\beta\,(1-\delta)}$$

The law of motion for capital implies that $x_y = \delta k_y$. For consumption this implies

$$c_y + x_y = f_y = \frac{y_1'}{y} = (1-g_y) => c_y = 1 - g_y - \delta k_y$$

[4] In Backus et al. (1994) the weights in the Armington aggregator are set such as intermediate output is equal to final goods in steady state, see also Ravn (1997).

Hours Combining the FOC for hours and consumption in steady state implies:

$$n = \frac{\mu(1-\tau)(1-\theta)y_c}{1+\mu((1-\tau)(1-\theta)y_c - 1)}$$

such that, given c_y, the parameters θ, τ and μ, pin down hours in steady state.

Government budget In steady state we have from equation (8)

$$T_y = \tau - g_y. \tag{37}$$

1.1.5 The Linearized model near steady state

In the following, unless noted otherwise all variables denote percentage deviations from steady state.
Market clearing intermediate goods (12) is approximated as

$$y_{1t} = \omega(1-g_y)a_{1t} + (1-\omega)(1-g_y)a_{2t} + g_y g_{1t} \tag{A-1}$$
$$y_{2t} = \omega(1-g_y)b_{2t} + (1-\omega)(1-g_y)b_{1t} + g_y g_{2t} \tag{A-2}$$

Market clearing final goods (14) is approximated as

$$(1-g_y)f_{1t} = c_y c_{1t} + x_y x_{1t} \tag{A-3}$$
$$(1-g_y)f_{2t} = c_y c_{2t} + x_y x_{2t} \tag{A-4}$$

Production function intermediate goods (2) is approximated as

$$y_{1t} = z_{1t} + \theta k_{1t} + (1-\theta)n_{1t} \tag{A-5}$$
$$y_{2t} = z_{2t} + \theta k_{2t} + (1-\theta)n_{2t} \tag{A-6}$$

Production function final goods (5) and (6) is approximated as

$$f_{1t} = \omega a_{1t} + (1-\omega)b_{1t} \tag{A-7}$$
$$f_{2t} = \omega b_{2t} + (1-\omega)a_{2t} \tag{A-8}$$

Demand for intermediate goods (27)-(30) is approximated as

$$a_{1t} = -\sigma q^a_{1t} + f_{1t} \tag{A-9}$$
$$b_{1t} = -\sigma q^b_{1t} + f_{1t} \tag{A-10}$$
$$b_{2t} = -\sigma q^b_{2t} + f_{2t} \tag{A-11}$$
$$a_{2t} = -\sigma q^a_{2t} + f_{2t} \tag{A-12}$$

Approximating the discount factor (23) gives:

$$\hat{\beta}_{1t} = -\mu(1-\bar{\beta})c_{1t} + \frac{(1-\mu)n}{1-n}(1-\bar{\beta})n_{1t} \tag{A-13}$$

$$\hat{\beta}_{2t} = -\mu(1-\bar{\beta})c_{2t} + \frac{(1-\mu)n}{1-n}(1-\bar{\beta})n_{2t} \tag{A-14}$$

FOC Consumption

$$\lambda_{1t} = (\mu(1-\gamma)-1)c_{1t} - \frac{n}{1-n}(1-\mu)(1-\gamma)n_{1t} \tag{A-15}$$

$$\lambda_{2t} = (\mu(1-\gamma)-1)c_{2t} - \frac{n}{1-n}(1-\mu)(1-\gamma)n_{2t} \tag{A-16}$$

FOC Labor

$$\lambda_{1t} + q^a_{1t} - \frac{\tau}{1-\tau}\tau_{1t} + w_{1t} = \mu(1-\gamma)c_{1t} + \left\{\frac{n}{1-n}[\gamma(1-\mu)+\mu]\right\} n_{1t} \tag{A-17}$$

$$\lambda_{2t} + q^b_{2t} - \frac{\tau}{1-\tau}\tau_{2t} + w_{2t} = \mu(1-\gamma)c_{2t} + \left\{\frac{n}{1-n}[\gamma(1-\mu)+\mu]\right\} n_{2t} \tag{A-18}$$

FOC Capital

$$\lambda_{1t} - \hat{\beta}_{1t} = (1-\bar{\beta}(1-\delta))E_t(q^a_{1t+1} - \frac{\tau}{1-\tau}\tau_{1t+1} + r_{1t+1}) + E_t\lambda_{1t+1} \tag{A-19}$$

$$\lambda_{2t} - \hat{\beta}_{2t} = (1-\bar{\beta}(1-\delta))E_t(q^a_{2t+1} - \frac{\tau}{1-\tau}\tau_{2t+1} + r_{2t+1}) + E_t\lambda_{2t+1} \tag{A-20}$$

FOC i-firm (25) and (26)

$$r_{1t} = y_{1t} - k_{1t} \tag{A-21}$$

$$r_{2t} = y_{2t} - k_{2t} \tag{A-22}$$

$$w_{1t} = y_{1t} - n_{1t} \tag{A-23}$$

$$w_{2t} = y_{2t} - n_{2t} \tag{A-24}$$

Law of motion of capital

$$k_{1t+1} = (1-\delta)k_{1t} + \delta x_{1t} \tag{A-25}$$

$$k_{2t+1} = (1-\delta)k_{2t} + \delta x_{2t} \tag{A-26}$$

In the bonds only economy: FOC for bonds

$$E_t\lambda_{1t+1} - \lambda_{1t} = q^a_{1t} + Q_t - E_t q^a_{1t+1} - \hat{\beta}_{1t} \tag{A-27}$$

$$E_t\lambda_{2t+1} - \lambda_{2t} = q^b_{2t} + Q_t - E_t q^a_{2t+1} - \hat{\beta}_{2,t} \tag{A-28}$$

Budget Constraint

$$(1-\tau)(q^a_{1t} + y_{1t}) - \tau\tau_{1t} + \hat{B}_{1t} + T_{1t} = c_y c_{1t} + x_y x_{1t} + \bar{\beta}\hat{B}_{1t+1} \tag{A-29}$$

$$(1-\tau)(q^b_{2t} + y_{2t}) - \tau\tau_{2t} + \hat{B}_{2t} + T_{1t} = c_y c_{2t} + x_y x_{2t} + \bar{\beta}\hat{B}_{2t+1} \tag{A-30}$$

where $\hat{B}_{it} = \frac{B_{it}}{y_i}$, i.e. it denotes bonds as a fraction of steady state output.
Bond market clearing [5]

$$\hat{B}_{1t} = \hat{B}_{2t} \tag{A-31}$$

Under complete markets bonds are redundant. We thus impose

$$\hat{B}_{1t} = 0 \tag{A*-27}$$

$$\hat{B}_{2t} = 0 \tag{A*-28}$$

$$Q_t = 0 \tag{A*-29}$$

And linearize the risk sharing condition (24)

$$\lambda_{1t} + q^a_{1t} = \lambda_{2t} + q^a_{2t}. \tag{A*-30}$$

Arbitrage condition. To close the economy, we require that the law of one price holds, i.e. the relative price of goods is equal across countries. Therefore we define the real exchange rate, rx_t, and impose two conditions:

$$rx_t = q^b_{1t} - q^b_{2t} \tag{A-32}$$

$$rx_t = q^a_{1t} - q^a_{2t} \tag{A-33}$$

Government. Approximating (8) gives

$$g_y g_{1t} = \tau\tau_{1t} + \tau y_{1t} - T_y T_{1t} \tag{A-34}$$

$$g_y g_{2t} = \tau\tau_{2t} + \tau y_{2t} - T_y T_{2t}, \tag{A-35}$$

Additional definitions: Terms of trade (16)

$$p_t = q^b_{1,t} - q^a_{1,t} \tag{A-36}$$

Trade balance (17)

$$nx_t = (1-\omega)(1-g_y)(a_{2,t} - b_{1,t} - p_t) \tag{A-37}$$

Relative investment (18)

$$x_t = x_{1t} - x_{2t} \tag{A-38}$$

[5] In fact, by Walras' law it is sufficient to impose good markets clearing and one budget constraint. In other words given that the demand for resources meets supply, and one agent (out of two) satisfies his budget constraint, the other one holds as well. Thus we only keep this equation to check consistency.

Exogenous shock processes: for technology, government spending and tax rate, both in county 1 and 2:

$$z_{1t+1} = \rho_z z_{1t} \tag{A-39}$$

$$z_{2t+1} = \rho_z z_{2t} \tag{A-40}$$

$$g_{1t+1} = \rho_g g_{1t} \tag{A-41}$$

$$g_{2t+1} = \rho_g g_{2t} \tag{A-42}$$

$$\tau_{1t+1} = \rho_\tau \tau_{1t} \tag{A-43}$$

$$\tau_{2t+1} = \rho_\tau \tau_{2t} \tag{A-44}$$

1.1.6 Numerical implementation

We write these 44 expectational difference equations in matrix form:

$$AE_t x_{t+1} = Bx_t,$$

where x_t is a vector containing all the 44 variables defined by the above equations. We follow Klein (2000) to obtain the state space representation of the economy (in particular, we use his matlab function solab.m).

1.2 Numerical experiments

In the following we briefly comment on the numerical experiments underlying figure 3 and 4 in the main text. In addition, we provide results from a sensitivity analysis of our main results with respect to variations in the value of the intratemporal elasticity of substitution between home and foreign goods. We also briefly report results from an experiment on the effects of tax cuts to which we briefly refer in the main paper.

Table 1 displays the parameter values used in the various simulations of the model. The values are standard and taken from Backus et al. (1994). Recall that the parameter ω - capturing home bias in private spending - is pinned down by the import share in GDP (for any given g_y), see equation (35). We will explore the role of openness (import share) for the transmission of fiscal shocks below - by varying ω.

1.2.1 Figure 3 - The locus of 'no-relative investment changes'

This experiment establishes numerically the locus of parameters values for which relative investment does not directly respond to a domestic shock to government spending in the import-share-shock-persistence-plane.[6] The matlab program contourplots.m carries out a grid-search over ω and ρ: it

[6] In an earlier version of the paper we established this relationship analytically for the linearized economy under complete markets and with fixed labor supply.

Table 1: Parameter values (period = 1 quarter)

Discount factor (steady state)	$\beta = 0.99$
Consumption share	$\mu = 0.34$
Risk aversion	$\gamma = 2$
Capital share	$\theta = 0.36$
Depreciation rate	$\delta = 0.025$
Government spending (steady state)	$g_y = 0.2$
Tax rate (baseline)	$\tau = 0$
Intratemporal elasticity of substitution (baseline)	$\sigma = 1.5$
Persistence of government spending shock (baseline)	$\rho_g = 0.9$

calls model_function.m which provides the state-space representation of the model. We find the locus 'no-relative investment changes' as those combinations of ω and ρ for which the coefficient on government spending in the policy rule for relative investment is zero. An indirect response of relative investment to spending shocks - via changes in other state variables - is possible in the periods after the impact of the shock.

1.2.2 Figure 4 - selected impulse response to government spending shock

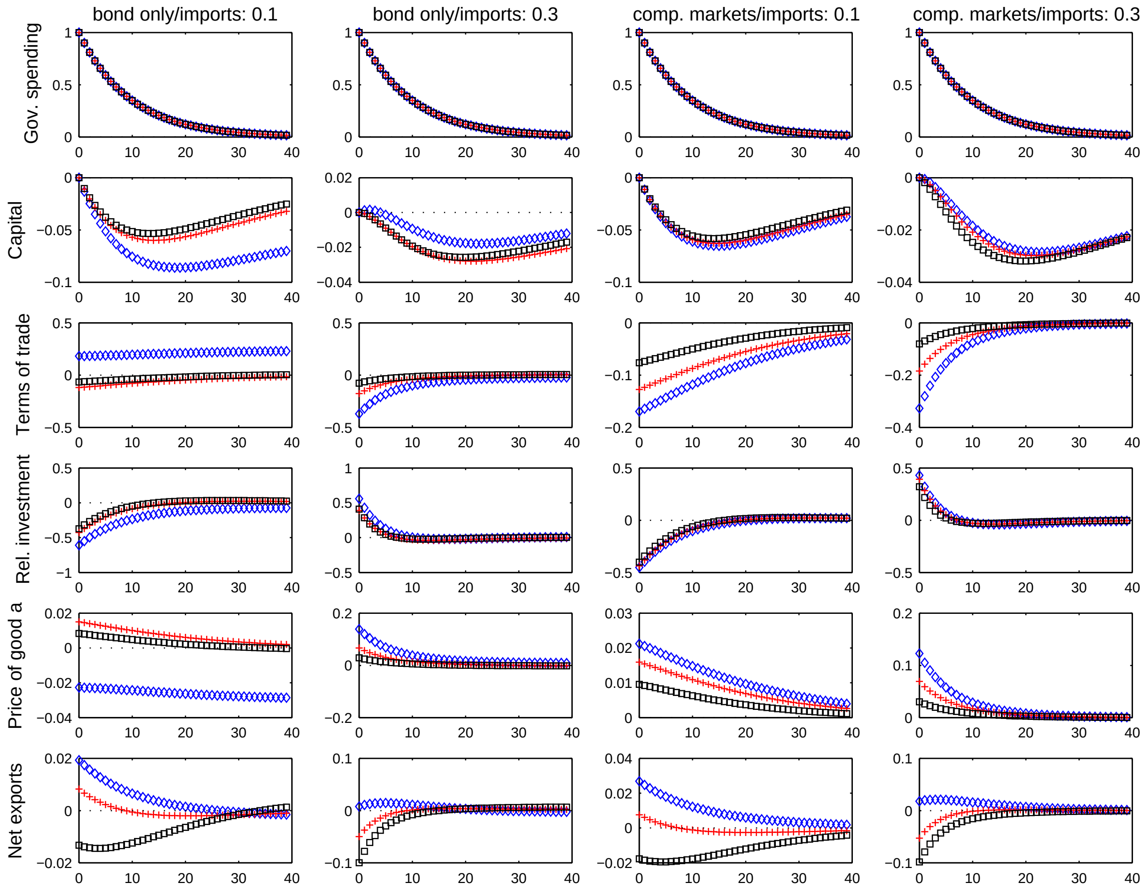
This experiment computes selected impulse response functions to a shock to government spending using the matlab function model_IRF.m. The value capturing shock persistence, ρ_g, is at its baseline value of 0.9. Two values for the import share are considered: 0.1 and 0.3. These imply values for ω of 0.875 and 0.625, respectively.

1.2.3 Sensitivity of main results with respect to the intratemporal elasticity of substitution

Above, we compute the zero-investment locus for two different possible asset market structures: complete markets and bonds only. It appears that the asset market structure has little bearing on the results. A possible explanation is that, for a values of σ close to 1, it is well known that the movements of the terms of trade provide insurance against country-specific productivity risks — even in the absence of complete financial markets and formal risk-sharing arrangements, see Cole and Obstfeld (1991).

We thus explore whether our main results are robust with respect to variations of this parameter value. We consider three different values for the intratemporal elasticity of substitution, $\sigma = \{0.3, 1, 3\}$ - varying both the assets market structures and import shares (model_IRF_sensitivity.m). Figure 1 displays the results. The main result of our analysis is confirmed: the sign of the relative investment response (fourth row) depends on the degree of openness and is hardly affected by changes

Figure 1: Selected impulse responses to government spending shock



Notes: Each column displays the responses for different combinations of openness (steady state share of imports) and asset market structure. In each column different values of the intratemporal elasticity of substitution between home and foreign goods are considered; $\diamond : \sigma = 0.3$; $+ : \sigma = 1$; $\Box : \sigma = 3$. Vertical axes indicate percentage deviations from steady state; horizontal axes indicate quarters.

in σ. However, σ determines whether the response of the trade balance has the same sign as the response of relative investment. For a high value of σ, the trade balance may decline even though one observes a strong fall in relative investment. This is because, for high σ, the composition of private goods changes substantially in response to changes in the terms of trade, see Müller (2006).

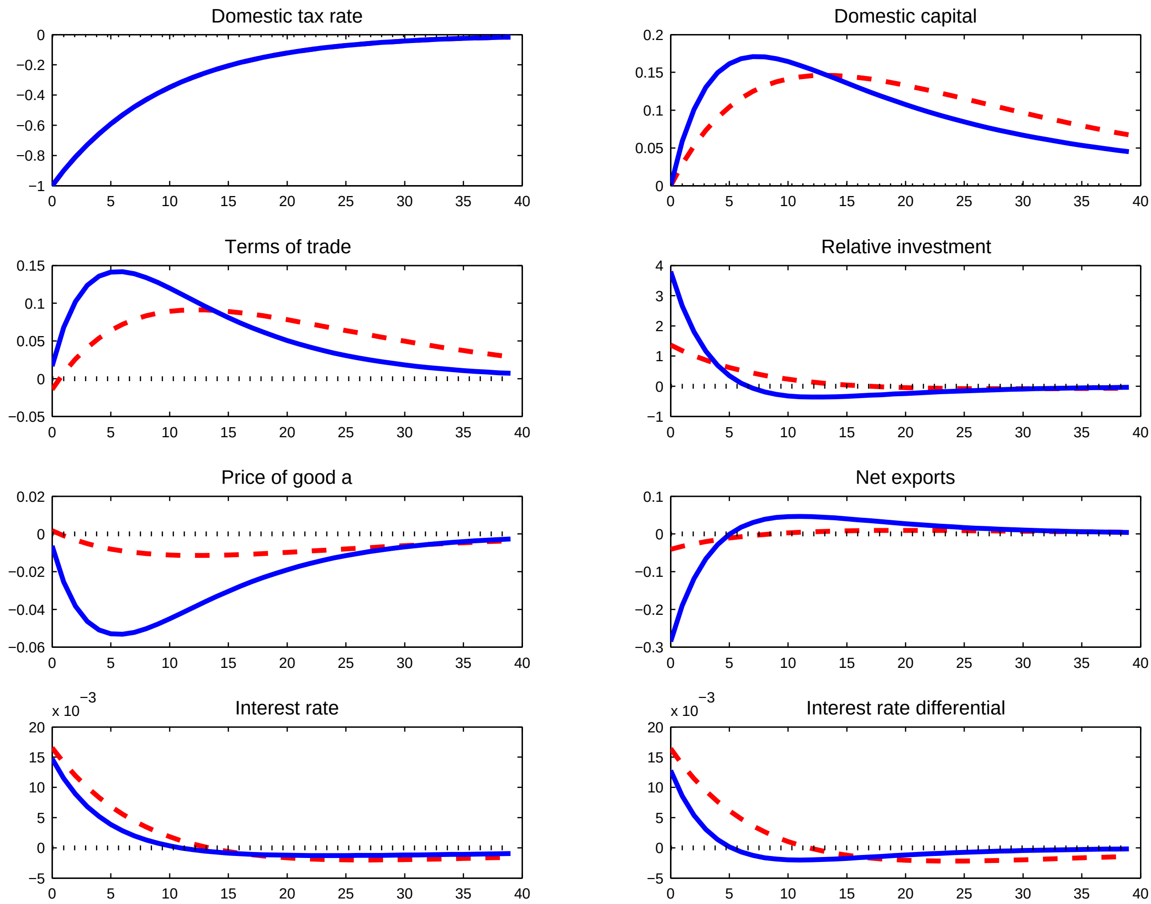
Note that in the fifth row we also display the response of the variable q^a_{1t}, i.e. the price of good a in units of country 1's final good, because of its role for the real return to investment, see equation (32).

A further point worth noting is that the terms of trade depreciate (increase) for a low value of σ if the import share is low and financial markets are incomplete (left panel, third row). An intuition for this result can be found in Corsetti, Dedola, and Leduc (2004).

1.2.4 Cut in the tax rate

In section 3.5 of the main paper, we briefly discuss the international transmission of fiscal shocks in the form of temporary cuts in the tax rate. For this experiment (model_IRF_taxcut.m) we assume that the steady state tax rate is 25 percent of GDP, such that transfers account for 5 percent of GDP in steady state.[7] Figure 2 displays the result: openness also matters in this case. However, it turns out that the key variable driving the transmission of a tax cut is the consumption-based real interest rate (left panel, fourth row), defined above. To illustrate this point, after linearizing (31), we compute the difference between the domestic and the foreign consumption-based interest rates. The interest rate differential is displayed in the right panel of the last row.

Figure 2: Selected impulse responses to tax shock



Notes: Cut of tax rate by one percent. Straight line gives the responses of economy with import share of 30 percent. Dashed line gives responses of economy with import share of 10 percent. Vertical axes indicate percentage deviations from steady state; horizontal axes indicate quarters.

[7] If transfers were assumed to be zero in steady state, one would need to scale them by steady state GDP outside steady state.

1.3 Non-Tradables: The case of government employment

In this subsection we briefly consider the possibility that government spending also falls on nontradable goods. Distinguishing between a sector producing tradables and a sector producing nontradable goods requires a modeling choice regarding the capital intensity in both sectors. In the following we consider the extreme case that part of government spending falls only on domestic labor services - thus on a nontradable good which is produced without capital. Our setup follows Finn (1998) as a way to account for the fact that the government wage bill accounts for a large fraction of the government budget.[8]

Model modification To investigate the effects of an exogenous increase in government employment, let n^g_{it} denote labor services employed by the government at the competitive wage rate determined in the private sector which employs n^p_{it}. Let n_{it} denote the total labor supply. Labor market clearing requires:

$$n_{it} = n^p_{it} + n^g_{it} \tag{B-1}$$

In addition the government budget constraint has to be adjusted appropriately:

$$g_{it} + w_{it}n^g_{it} + T_{it} = \tau_{it}w_{it}n_{it} + \tau_{it}r_{it}k_{it} \tag{B-2}$$

Finally, we also allow for capital adjustment cost is the modified economy by assuming the following law of motion for capital

$$k_{it+1} = (1-\delta)k_{it} + \phi(x_{it}/k_{it})k_{it}, \tag{B-3}$$

where adjustment costs are captured by $\phi(\cdot)$, as, for instance, in Baxter and Crucini (1993). We allow for mild capital adjustment costs below, because otherwise capital falls very strongly on impact in response to an exogenous increase in government employment.

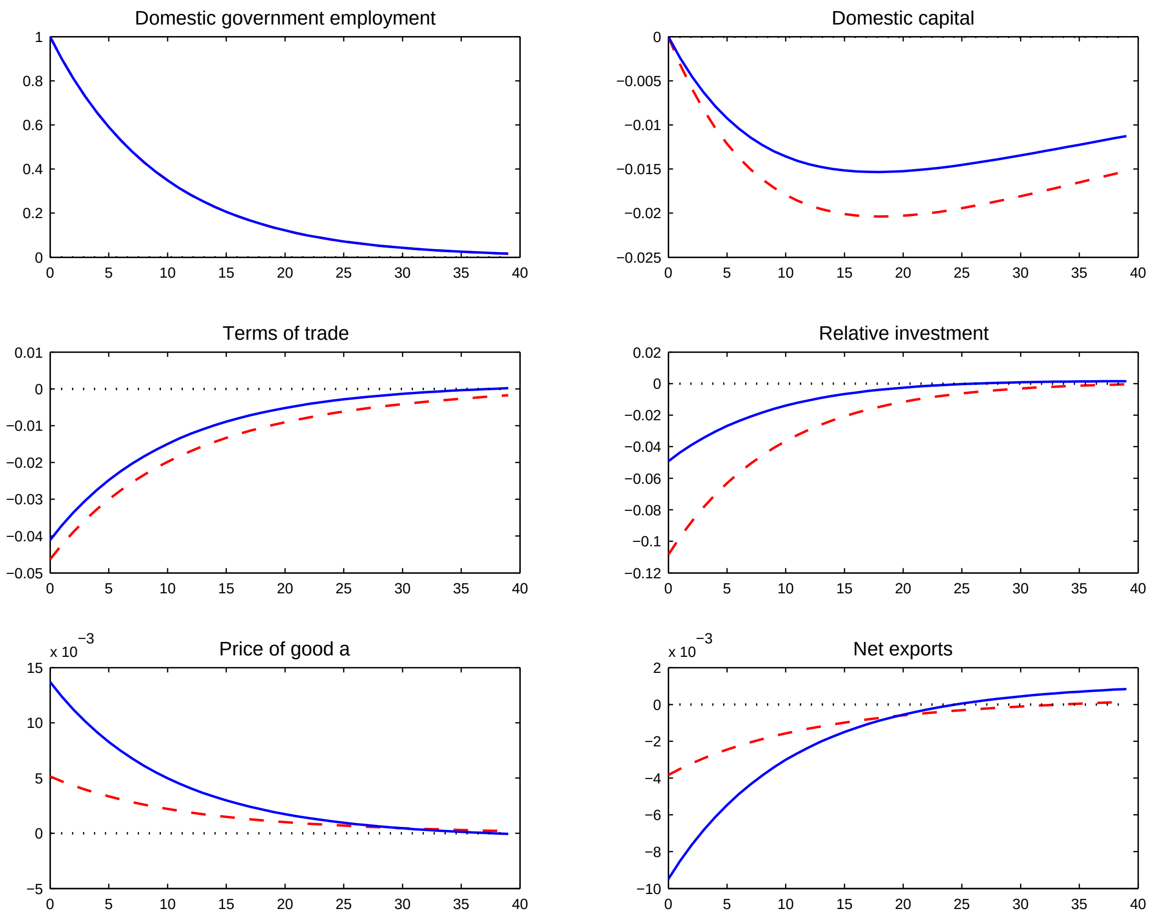
Parametrization We follow Finn an assume that in steady state n^g/n^p=0.18. This implies a government wage bill in steady state of about 10 percent. We set the elasticity of the price of capital with respect to deviations in the investment-capital ratio to 0.25. We then consider the responses to an exogenous increase in government employment. The parameter capturing the degree of autocorrelation is set to 0.9.

Results The matlab function model_IRF_gov_employment.m computes the impulse response functions to a shock to government employment for two economies differing in in their import share.

[8]The average wage consumption expenditure of the government is about 10 percent of GDP in our sample (1980:1-date) for Canada, the UK and the US (OECD Economic Outlook database, CGW) and thus accounts for approximately half of the sum of government consumption and investment expenditure (about 20% of GDP).

Figure 3 displays the results. Relative investment falls in both economies. However, in line with the argument developed in the main text, domestic capital is crowded out more in the relatively closed economy. While further investigation into the transmission of fiscal shocks through the non-tradeable sector is necessary, we note here that the response of q^a_{1t} displayed in the left panel of the last row provides a rationale for the investment response. While the increase in government employment crowds out capital, this effect is mitigated in the more open economy, because in this case the terms of trade appreciation has a stronger off-setting impact on investment decisions.

Figure 3: Selected impulse responses to government employment shock



Notes: Increase in government employment of one percent. Straight line gives the responses of economy with import share of 30 percent. Dashed line gives responses of economy with import share of 10 percent. Vertical axes indicate percentage deviations from steady state; horizontal axes indicate quarters.

1.4 List of matlab codes

In addition to the functions **solab.m**, **qzswitch.m** and **qzdiv.m** available from Paul Klein's webpage (http://www.ssc.uwo.ca/economics/faculty/klein/) and discussed in Klein (2000) we used the following functions to carry out the experiments described above:

contourplots.m	computes the zero relative investment-locus (calls model_function.m)
model_function.m	computes state space representation of model (calls solab.m)
model_IRF	computes state space representation of model and IRF (calls solab.m)
model_IRF_sensitivity.m	computes impulse response functions for various σ (calls solab.m)
model_IRF_taxcut.m	computes impulse response to tax shock (calls solab.m)
model_IRF_gov_employment.m	computes impulse response to government employment shock (calls solab.m)

2 Data and VAR estimation

All data (except those used to calculate the import content, see table 1 and the file table1.xls) are contained in the file OECDdata.xls. We use the following programs to compute the statistics reported in the paper:

transdata.m	loads data from OECDdata.xls and performs basic transformations, including those displayed in figure 1
correlationpattern.m	HP-filters nx and bb and computes correlation function (displayed in figure 2)
VAR_main.m	specifies VAR model for government spending shock
VAR_deficitshock.m	specifies VAR model for deficit shock
mkimp.m	computes impulse response functions (adapted from a code available from the homepage of Larry Christiano)
simdata	simulates data for bootstrap (adapted from a code available from the homepage of Larry Christiano)

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