Cross-Country Co-movement in Long-Term Interest Rates: A DSGE Approach*

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July 10, 2015

Model Appendix

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England. We would like to thank Paul Beaudry, Charles Engel, Peter Karadi, Marco Del Negro, Kalin Nikolov, Frank Schorfheide, Eric Swanson, Tao Zha as well as workshop and conference participants at Aarhus University, Bank of England and University of Glasgow for helpful discussions and comments.

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A Theoretical Model

A.1 Domestic Economy

The description of all structural parameters used in this model and their prior moments are provided by Tables 1-3.

Table 1: Calibrated Parameters

Mnemonic	Description	Value
$100(\beta^{-1}-1)$	Time Discount Factor	1.00
$\frac{r^{h,*}}{r^*}$	Steady-State Spread Between the Foreign Long-Term and Consumer Rates	1.00
$\frac{r^{L,*}}{r^*}$	Steady-State Slope of the Foreign Yield Curve	1.00
h^*	Steady-State Foreign Hours	1.00
z^*	Steady-State Foreign TFP	1.00
g^*	Steady-State Foreign Government Spending to GDP Ratio	0.18
λ_{y^*}	Steady-State Foreign Prices Markup	1.20
λ_{y^*}	Steady-State Foreign Wages Markup	1.05
$\frac{b^*+\bar{b}^{L,*}}{y^*}$	Steady-State Total Foreing Debt to GDP Ratio	0.52
$\frac{r^h}{r}$	Steady-State Spread Between the Domestic Long-Term and Consumer Rates	1.00
$\frac{r^L}{r}$	Steady-State Slope of the Domestic Yield Curve	1.00
h	Steady-State Domestic Hours	1.00
z	Steady-State Domestic TFP	1.00
g	Steady-State Domestic Government Spending to GDP Ratio	0.17
λ_m	Steady-State Import Prices Markup	1.20
λ_x	Steady-State Export Prices Markup	1.20
λ_y	Steady-State Domestic Prices Markup	1.20
λ_w	Steady-State Domestic Wages Markup	1.05
$\frac{b+\bar{b}^L}{y}$	Steady-State Total Domestic Debt to GDP Ratio	0.41

Table 2: Description of the Foreign Economy Estimated Parameter & Prior Moments

Mnemonic	Description	Density	Mean	STD
$\frac{\bar{b}^{L,*}}{y^*}$	Steady-State Foreign Long-Term Debt to GDP Ratio	Normal	0.20	0.01
θ^*	Foreign Transfer Policy Reaction to Government Debt	Normal	0.75	0.25
$100\chi^*$	Foreign Liquidity Adjustment Cost	Normal	10.00	1.00
ϕ_{π^*}	Foreign Policy Reaction to Inflation	Normal	1.50	0.10
ϕ_{y^*}	Foreign Policy Reaction to Output	Normal	0.13	0.05
ϕ_{R^*}	Foreign Taylor Rule Smoothing	Beta	0.75	0.10
σ_{C^*}	Foreign Inverse Intertemporal Substitution	Normal	1.50	0.10
φ^*	Foreign Inverse Labour Supply Elasticity	Normal	2.00	0.10
b^*	Foreign Consumption Habit	Beta	0.70	0.10
κ_{y^*}	Foreign Prices Indexation	Beta	0.50	0.15
ξ_{y^*}	Foreign Prices Reset Probability	Beta	0.50	0.10
κ_{w^*}	Foreign Wages Indexation	Beta	0.50	0.15
ξ_{w^*}	Foreign Wages Reset Probability	Beta	0.50	0.10
ρ_{z^*}	Persistence of Foreign Productivity Shock	Beta	0.75	0.10
$\rho_{\bar{b}^{L,*}}$	Persistence of Foreign Long-Term Debt Risk Premium Shock	Beta	0.75	0.10
ρ_{d^*}	Persistence of Foreign Discount Factor Shock	Beta	0.75	0.10
$\rho_{b^{S,*}}$	Persistence of Foreign Short-Term Debt Risk Premium Shock	Beta	0.75	0.10
ρ_{g^*}	Persistence of Foreign Government Spending Shock	Beta	0.75	0.10
$100\sigma_{z^*}$	Uncertainty of Foreign Productivity Shock	Inv-Gamma	0.50	0.20
$100\sigma_{\bar{b}^{L,*}}$	Uncertainty of Foreign Long-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_{d^*}$	Uncertainty of Foreign Discount Factor Shock	Inv-Gamma	0.50	0.20
$100\sigma_{b^{S,*}}$	Uncertainty of Foreign Short-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_{R^*}$	Uncertainty of Foreign Policy Shock	Inv-Gamma	0.50	0.20
$100\sigma_{g^*}$	Uncertainty of Foreign Government Spending Shock	Inv-Gamma	0.50	0.20

Notes: STD denotes the prior standard deviation moment of the estimated parameter.

Table 3: Description of the Domestic Economy Estimated Parameter & Prior Moments

Mnemonic	Description	Density	Mean	STD
$\frac{b+\bar{b}^L+\bar{b}^{L,*}}{y}$	Steady-State Domestic Total Debt to GDP Ratio	Normal	0.40	0.10
$\frac{\bar{b}^L}{y}$	Steady-State Domestic Long-Term Debt to GDP Ratio	Normal	0.20	0.01
θ	Domestic Transfer Policy Reaction to Government Debt	Normal	0.75	0.25
100χ	Domestic Liquidity Adjustment Cost	Normal	10.00	1.00
η	Substitution Elasticity Consumption	Normal	1.50	0.10
ϕ_π	Domestic Policy Reaction to Inflation	Normal	1.50	0.10
ϕ_y	Domestic Policy Reaction to Output	Normal	0.13	0.05
ϕ_R	Domestic Taylor Rule Smoothing	Beta	0.75	0.05
σ_C	Domestic Inverse Intertemporal Substitution	Normal	1.50	0.10
φ	Domestic Inverse Labour Supply Elasticity	Normal	2.00	0.10
b	Domestic Consumption Habit	Beta	0.70	0.10
κ_y	Domestic Prices Indexation	Beta	0.50	0.15
ξ_y	Domestic Prices Reset Probability	Beta	0.50	0.10
κ_m	Domestic Import Prices Indexation	Beta	0.50	0.15
ξ_m	Domestic Import Prices Reset Probability	Beta	0.50	0.05
κ_w	Domestic Wages Indexation	Beta	0.50	0.15
ξ_w	Domestic Wages Reset Probability	Beta	0.50	0.10
ρ_z	Persistence of Domestic Productivity Shock	Beta	0.75	0.10
$\rho_{\bar{b}^L}$	Persistence of Domestic Long-Term Debt Risk Premium Shock	Beta	0.75	0.10
ρ_d	Persistence of Domestic Discount Factor Shock	Beta	0.75	0.10
ρ_{b^S}	Persistence of Domestic Short-Term Debt Risk Premium Shock	Beta	0.75	0.10
ρ_g	Persistence of Domestic Government Spending Shock	Beta	0.75	0.10
$100\sigma_z$	Uncertainty of Domestic Productivity Shock	Inv-Gamma	0.50	0.20
$100\sigma_{\bar{b}^L}$	Uncertainty of Domestic Long-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_d$	Uncertainty of Domestic Discount Factor Shock	Inv-Gamma	0.50	0.20
$100\sigma_{b^S}$	Uncertainty of Domestic Short-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_g$	Uncertainty of Domestic Government Spending Shock	Inv-Gamma	0.50	0.20
$100\sigma_R$	Uncertainty of Domestic Policy Shock	Inv-Gamma	0.50	0.20

Notes: STD denotes the prior standard deviation moment of the estimated parameter.

A.1.1 Firms

Three types of firms are operated in the domestic economy. The intermediate monopolistically competitive domestic firms use labour supplied by households to produce a differentiated good that is sold to a final good producer who employs a continuum of these differentiated goods in her constant elasticity of substitution – CES – production to deliver the final good. The monopolistically competitive importing firms use a costless technology and turn a homogenous good – bought in the world market – into a differentiated good, which is then sold to the domestic consumers. The exporting monopolistically competitive firms use similar 'brand naming' technology and transform the domestic final good into a differentiated product that is sold to foreign households.

Domestic Firms This sector consists of three firms, the 'labour packer' who hires labour from households and transforms it into a homogenous input good – h_t^d, a continuum of monopolistically competitive firms that buys h_t^d and produces an intermediate $y_{i,t}$ and the final good producer who combines all these intermediate products into a single good consumed by households. The final good producer's CES production function is given by

$$y_t^d = \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{y,t}}} di \right]^{\lambda_{y,t}} \tag{1}$$

where

$$\lambda_{y,t} = \left(1 - \rho_{\lambda_y}\right) \lambda_y + \rho_{\lambda_y} \lambda_{y,t-1} + \sigma_{\lambda_y} \omega_{\lambda_y,t} \tag{2}$$

denotes the time-varying mark-up in the domestic good market. The final good producer's demand curve for $y_{i,t}$ arises from the profit minimisation problem – $\max_{y_{i,t}} \left\{ p_t \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{y,t}}} di \right]^{\lambda_{y,t}} - \int_0^1 p_{i,t} y_{i,t} \right\}$

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} y_t^d \tag{3}$$

The final good price index is obtained by combining 1 and 3

$$p_t = \left[\int_0^1 p_{i,t}^{\frac{1}{1-\lambda_{y,t}}} di \right]^{1-\lambda_{y,t}} \tag{4}$$

Intermediate good producers use the following production function

$$y_{i,t} = z_t h_{i,t}^d \tag{5}$$

where

$$z_t = (1 - \rho_z) z + \rho_z z_{t-1} + \sigma_z \omega_{z,t} \tag{6}$$

is a stationary exogenous technological process and $h^d_{i,t}$ is the amount of homogeneous labour rented by the firm i^{ih}. The intermediate firm select $h^d_{i,t}$ in order to minimise its production cost

$$\min_{h^d_{i,t}} w_t h^d_{i,t} + mc_t p_t \left[y_{i,t} - z_t h^d_{i,t} \right] \tag{7}$$

The real marginal cost for the intermediate firms is given by the first order condition of (7) with respect to $h^d_{i,t}$ is

$$mc_t = \frac{w_t}{\tilde{p}_t z_t} \tag{8}$$

where $w_t \equiv \frac{\tilde{w}_t}{p^c_t}$ is the real wage.

A fraction – $\left(1 - \xi_y\right)$ – of intermediate firms receive a random signal and they are allowed to optimally reset their prices – $p^{new}_{i,t}$. The proportion – ξ_y – of firms that cannot reoptimise prices will set p_t based on backward-looking rule

$$p_t = \pi^{\kappa_y}_{t-1} p_{t-1} \tag{9}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation and κ_y is the indexation parameter. The pricing problem of firm i is then

$$\max_{p^{new}_{i,t}} E_t \sum_{j=0}^{\infty} \left(\beta \xi_y\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \pi^{\kappa_y}_{t+s-1} \frac{p^{new}_{i,t}}{p_{t+j}} - mc_{t+j} \right) y_{i,t+j} \right\} \tag{10}$$

subject to

$$y_{i,t+j} = \left(\prod_{s=1}^{j} \pi^{\kappa_y}_{t+s-1} \frac{p^{new}_{i,t}}{p_{t+j}} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} y^d_{t+j} \tag{11}$$

The first-order condition is expressed as system of difference equations

$$f_{1,t} = \lambda_t mc_t y^d_t + \beta \xi_y E_t \left(\frac{\pi^{\kappa_y}_t}{\pi_{t+1}} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} f_{1,t+1} \tag{12}$$

$$f_{2,t} = \lambda_t \bar{\pi}_t y^d_t + \beta \xi_d E_t \left(\frac{\pi^{\kappa_y}_t}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_{y,t}-1}} \left(\frac{\bar{\pi}_t}{\bar{\pi}_{t+1}} \right) f_{2,t+1} \tag{13}$$

$$0 = \lambda_{y,t} f_{1,t} - f_{2,t} \tag{14}$$

$$1 = \xi_y \left(\frac{\pi^{\kappa_y}_{t-1}}{\pi_t} \right)^{-\frac{1}{\lambda_{y,t}-1}} + \left(1 - \xi_y\right) \bar{\pi}_t^{-\frac{1}{\lambda_{y,t}-1}} \tag{15}$$

where $\bar{\pi}_t \equiv \frac{p^{new}_t}{p_t}$.

Market clearing condition in the domestic sector

$$y_t = \int_0^1 y_{i,t} di = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} di y^d_t = \upsilon^p_t y^d_t \tag{16}$$

where $\upsilon_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t-1}}} d_i$ is the price dispersion term and it is given by

$$\upsilon_t^p = \xi_y \left(\frac{\pi_{t-1}^{\kappa_y}}{\pi_t}\right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} \upsilon_{t-1}^p + \left(1-\xi_y\right)\bar{\pi}_t^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} \tag{17}$$

Importing firms The import sector consists of a continuum of monopolistically competitive firms that buy a homogenous good c_t^m in the world market at price p_t^*. These firms have access to a costless technology and transform the homogenous good into a differentiated product – $c_{i,t}^m$ – consumed by domestic households. Similar to Adolfson et al. (2007) and Burgess et al. (2013) we assume local currency in order to allow for incomplete exchange rate pass-through to the import prices. To be precise, the importing firms follow the Calvo price-setting scheme, meaning that a fraction – $1-\xi_m$ – of them is allowed to reset their price optimally – $p_{m,t}^{new}$ – only when they receive a random price change signal, while those firms that missed this signal can only index their prices by past inflation – $p_{m,t} = \pi_{m,t-1}^{\kappa_m} p_{m,t-1}$. The pricing problem of the firm becomes

$$\max_{p_{i,t}^{m,new}} E_t \sum_{j=0}^{\infty} (\beta\xi_m)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m\right)^{\kappa_m} \frac{p_{i,t}^{m,new}}{p_{t+j}^m} - mc_t^m \right) c_{i,t}^m \right\} \tag{18}$$

where $mc_t^m \equiv \frac{s_t p_t^*}{p_t^m}$ is the real marginal cost of the importing firm and s_t is the nominal exchange rate. The final import good is a composite of a continuum of these differentiated imported good and it is given by the following CES production function

$$c_t^{m,d} = \left[\int_0^1 \left(c_{i,t}^m\right)^{\frac{1}{\lambda_m}} di\right]^{\lambda_m} \tag{19}$$

Taking p_t^m and $p_{i,t}^m$ as given the final import good producer's demand curve for $C_{i,t}^m$ can be derived form the profit minimisation problem – $\max_{c_{i,t}^m} \left\{ p_t^m \left[\int_0^1 \left(c_{i,t}^m\right)^{\frac{1}{\lambda_m}} di\right]^{\lambda_m} - \int_0^1 p_{i,t}^m c_{i,t}^m \right\}$

$$c_{i,t}^m = \left(\frac{p_{i,t}^m}{p_t^m}\right)^{-\frac{\lambda_m}{\lambda_m - 1}} c_t^{m,d} \tag{20}$$

Finally, total amount of imported goods is obtained by integrating out over all differentiated imported goods

$$c_t^m = \int_0^1 c_{i,t}^m di \tag{21}$$

$p_{i,t}^{m,new}$ is derived by maximising 18 subject to

$$c_{i,t+j}^m = \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m\right)^{\kappa_m} \frac{p_{i.t}^{m,new}}{p_{t+j}^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} c_{t+j}^m \tag{22}$$

and the first-order condition – expressed as a system of first-order difference equations – is

$$
\begin{aligned}
g_{1,t} &= \lambda_t mc_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{(\pi_t^m)^{\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} g_{1,t+1} && (23) \\
g_{2,t} &= \lambda_t \bar{\pi}_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{(\pi_t^m)^{\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{1}{\lambda_m - 1}} \left(\frac{\bar{\pi}_t^m}{\bar{\pi}_{t+1}^m} \right) g_{2,t+1} && (24) \\
0 &= \lambda_{m,t} g_{1,t} - g_{2,t} && (25) \\
1 &= \xi_m \left(\frac{\left(\pi_{t-1}^m\right)^{\kappa_m}}{\pi_t^m} \right)^{-\frac{1}{\lambda_m - 1}} + (1 - \xi_m) \left(\bar{\pi}_t^m\right)^{-\frac{1}{\lambda_m - 1}} && (26)
\end{aligned}
$$

where $\bar{\pi}_t^m = \frac{p_t^{m,new}}{p_t^m}$.

The market clearing condition in the import sector

$$c_t^m = \int_0^1 c_{i,t}^m di = \upsilon_t^m c_t^{m,d} \tag{27}$$

where $\upsilon_t^m = \int_0^1 \left(\frac{p_{i,t}^m}{p_t^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} d_i$ is the import price dispersion term with its law of motion

$$\upsilon_t^m = \xi_m \left(\frac{\left(\pi_{t-1}^m\right)^{\kappa_m}}{\pi_t^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} \upsilon_{t-1}^m + (1 - \xi_m) \left(\bar{\pi}_t^m\right)^{-\frac{\lambda_m}{\lambda_m - 1}} \tag{28}$$

Exporting firms Again there is a continuum of exporting firms indexed by i on the unit interval. Each firm i buys a homogenous final domestic good in the domestic market c_t^x and differentiates it by using costless banding technology. They next sell the differentiated goods to the rest of the world. Foreign households' demand schedule is given by

$$c_{i,t}^x = \left(\frac{p_{i,t}^x}{p_t^x} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} c_t^{x,d} \tag{29}$$

where $\frac{\lambda_x}{\lambda_x - 1}$ denotes the elasticity of substitution between differentiated exporting goods, The exported price index is

$$p_t^x = \left[\int_0^1 \left(p_{i,t}^x\right)^{\frac{1}{1-\lambda_x}} di \right]^{1-\lambda_x} \tag{30}$$

The total amount of exported goods is obtained by integrating over all goods

$$c_t^x = \int_0^1 c_{i,t}^x di \tag{31}$$

Under Calvo pricing contract and indexation $p_{i,t}^{x,new}$ is derived by maximising

$$\max_{p_{i,t}^{x,new}} E_t \sum_{j=0}^{\infty} (\beta \xi_x)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^x\right)^{\kappa_m} \frac{p_{i,t}^{x,new}}{p_{t+j}^x} - mc_t^x \right) c_{i,t}^x \right\} \tag{32}$$

subject to

$$c^x_{i,t+j} = \left(\prod_{s=1}^{j} \left(\pi^*_{t+s-1}\right)^{\kappa_m} \frac{p^{x,new}_{i.t}}{p^x_{t+j}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} c^x_{t+j} \tag{33}$$

where $mc^x_t = \frac{p_t}{s_t p^x_t}$. The first-order condition can be expressed as

$$u_{1,t} = \lambda_t mc^x_t c^x_{t+j} + \beta \xi_x E_t \left(\frac{\left(\pi^*_t\right)^{\kappa_x}}{\pi^x_{t+1}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} u_{1,t+1} \tag{34}$$

$$u_{2,t} = \lambda_t \bar{\pi}^x_t c^x_{t+j} + \beta \xi_x E_t \left(\frac{\left(\pi^*_t\right)^{\kappa_x}}{\pi^x_{t+1}} \right)^{-\frac{1}{\lambda_x - 1}} \left(\frac{\bar{\pi}^x_t}{\bar{\pi}^x_{t+1}} \right) u_{2,t+1} \tag{35}$$

$$0 = \lambda_{x,t} u_{1,t} - u_{2,t} \tag{36}$$

$$1 = \xi_x \left(\frac{\left(\pi^*_{t-1}\right)^{\kappa_x}}{\pi^x_t} \right)^{-\frac{1}{\lambda_x - 1}} + \left(1 - \xi_x\right) \left(\bar{\pi}^x_t\right)^{-\frac{1}{\lambda_x - 1}} \tag{37}$$

where $\bar{\pi}^x_t = \frac{p^{x,new}_t}{p^x_t}$.

The market clearing condition in the export sector is given

$$c^x_t = \upsilon^x_t c^{x,d}_t \tag{38}$$

where $\upsilon^x_t = \int_0^1 \left(\frac{p^x_{i,t}}{p^x_t} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} d_i$ is the export price dispersion term with

$$\upsilon^x_t = \xi_x \left(\frac{\left(\pi^*_{t-1}\right)^{\kappa_x}}{\pi^x_t} \right)^{-\frac{1}{\lambda_x - 1}} \upsilon^x_{t-1} + \left(1 - \xi_x\right) \left(\bar{\pi}^x_t\right)^{-\frac{1}{\lambda_x - 1}} \tag{39}$$

A.1.2 Households

The domestic economy is populated by a continuum of households that attain utility from consumption – $c_{\kappa,t+j}$ – and leisure – $h_{\kappa,t+j}$. Household's preferences are separable

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ d_{t+j} \frac{\left(c_{\kappa,t+j} - b c_{\kappa,t+j-1}\right)^{1-\sigma_c}}{1 - \sigma_c} - \psi \frac{h^{1+\varphi}_{\kappa,t+j}}{1+\varphi} d\kappa \right\} \tag{40}$$

where d_t is a discount factor shock

$$d_t = \left(1 - \rho_d\right) d + \rho_d d_{t-1} + \sigma_d \omega_d \tag{41}$$

β is the discount factor, φ the inverse of the Frisch elasticity, σ_c the inverse of intertemporal elasticity of substitution and b the habit formation parameter. Aggregate consumption is function of domestically produced – c^d_t – and imported good – c^m_t

$$c_t = \left[\left(1 - \alpha\right)^{\frac{1}{\eta_c}} \left(c^{dp,d}_t\right)^{\frac{\eta_c - 1}{\eta_c}} + \alpha^{\frac{1}{\eta_c}} \left(c^{m,d}_t\right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}} \tag{42}$$

The elasticity of substitution between domestic and foreign goods in given by η_c and α measures the 'trade openness'. The maximisation of 42 subject to the budget constraint $p_t^c c_t = p_t c_t^{dp,d} + p_t^m c_t^{m,d}$ delivers the following demand functions

$$c_t^{dp} = (1-\alpha)\,\tilde{p}_t^{-\eta_c} c_t \tag{43}$$

$$c_t^{m} = \alpha\,(\tilde{p}_t^{m})^{-\eta_c}\, c_t \tag{44}$$

where $\tilde{p}_t \equiv \frac{p_t}{p_t^c}$ and $\tilde{p}_t^m \equiv \frac{p_t^m}{p_t^c}$ is the relative domestic consumption and import price, respectively. Plugging 43 and 44 into the budget constraint we obtain the definition of the consumer price index – CPI

$$(p_t^c)^{1-\eta_c} = (1-\alpha)\,p_t^{1-\eta_c} + \alpha\,(p_t^m)^{1-\eta_c} \tag{45}$$

alternatively

$$1 = (1-\alpha)\,\tilde{p}_t^{1-\eta_c} + \alpha\,(\tilde{p}_t^m)^{1-\eta_c} \tag{46}$$

Another interesting relation that links the CPI inflation – $\pi_t^c = \frac{p_t^c}{p_{t-1}^c}$ – with the home produced inflation – π_t, the imported inflation – π_t^m – and the terms of trade is the following one

$$\left(\frac{\pi_t^c}{\tilde{p}_{t-1}}\right)^{1-\eta_c} = (1-\alpha)\,\pi_t^{1-\eta_c} + \alpha\,(\pi_t^m T_{t-1})^{1-\eta_c} \tag{47}$$

Household's real budget constraint is given by

$$D_{\kappa,t}^h + c_{\kappa,t} + \Upsilon_{\kappa,t} = \frac{r_{t-1}^h}{\pi_t^c} D_{\kappa,t-1}^h + w_{\kappa,t} h_{\kappa,t} + F_t - T_t \tag{48}$$

The household κ uses its labour income – $w_{\kappa,t}h_{\kappa,t}$, gross interest rate financial intermediary deposits – $\frac{r_{t-1}}{\pi_t^c} D_{\kappa,t-1}^h$, government transfers – T_t – and profits – F_t – to finance consumption and new purchases of financial assets – $c_{\kappa,t} + D_{\kappa,t} + \Upsilon_{\kappa,t}$. The household maximises 40 with respect to $c_{\kappa,t}$, and $D_{\kappa,t}^h$ subject to 48

$$\frac{d_t}{(c_{\kappa,t} - bc_{\kappa,t-1})^{\sigma_c}} - E_t \frac{\beta b d_{t+1}}{(c_{\kappa,t+1} - hc_{\kappa,t})^{\sigma_c}} = \lambda_{\kappa,t} \tag{49}$$

$$\lambda_{\kappa,t} = \beta E_t \left\{ \lambda_{\kappa,t+1} \frac{r_t^h}{\pi_{t+1}^c} \right\} \tag{50}$$

A.1.3 Financial intermediary

The financial intermediary firm issues deposits to households paying a gross interest rate r_t^h. The firm then purchases a portfolio of short and long term government issued bonds paying interest r_t^S and r_t^L as well as a small fraction of long term debt issued by the foreign government $\left(r_t^{L,*}\right)$.

Similar to Andres et al. (2004), Chen et al. (2012), Harrison (2012) and Liu et al. (2014) we follow the formulation in Woodford (2001) and long-term bonds are perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t+s+1$ where $0 < \kappa \leq 1$. As it is explained in

Woodford (2001) and Chen et al. (2012) the advantage of this formulation is that the price in period t of a bond issued s periods ago $p_{L-s,t}$ is a function of the coupon the current price $p_{L,t}$

$$p_{L-s,t} = \kappa^s p_{L,t} \tag{51}$$

This relation allows to express the balance sheet equation and government budget constraint (below) in a familiar form that it is easy to work with it (see the discussion in Chen et al. (2012)). Furthermore, in order to keep things simple, we rule out the possibility of a secondary market for long-term bonds, meaning that agents who buy long-term debt must hold it until maturity.[1] Finally, for simplicity we assume that all government bonds issued are purchased by this firm.

The intermediary's balance sheet is given

$$b^h_{\kappa,t} = \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} + \frac{p_{L,t} b^L_{\kappa,t}}{\varepsilon^{b^L}_t} + \frac{q_t \left(1-\varrho\right) p^*_{L,t} b^{L,*}_{\kappa,t}}{\varepsilon^{b^{L,*}}_t}$$

or

$$b^h_{\kappa,t} = \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} + \frac{\bar{b}^L_{\kappa,t}}{\varepsilon^{b^L}_t} + \frac{\bar{b}^{L,*}_{\kappa,t}}{\varepsilon^{b^{L,*}}_t} \tag{52}$$

where

$$\begin{aligned} p_{L,t} &= \frac{1}{r^L_t - \kappa} \\ p^*_{L,t} &= \frac{1}{r^{L,*}_t - \kappa^*} \end{aligned}$$

and the term $1-\varrho$ reflects the fraction of the foreign long-term

debt held by the domestic financial intermediary.

Motivated by the work of Smets and Wouters (2007) we assume the balance sheet equation is subject to two 'financial' shocks: a short and a long-term risk premium shocks dented by $\varepsilon^{b^S}_t$, $\varepsilon^{\bar{b}^L}_t$ and $\varepsilon^{\bar{b}^{L,*}}_t$, respectively.

Intermediary's profit function is then given by

$$\begin{aligned} \xi_t \;=\; & \underbrace{b^h_{\kappa,t} + \frac{r^S_{t-1}}{\pi^c_t} b^S_{\kappa,t-1} + \frac{r^L_t}{\pi_t} p_{L,t} b^L_{\kappa,t-1} + \left(1-\varrho\right) \frac{r^{L,*}_t}{\pi^*_t} p^*_{L,t} b^{L,*}_{\kappa,t-1}}_{\text{revenues}} \\ & \underbrace{- \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} - \frac{p_{L,t} b^L_{\kappa,t}}{\varepsilon^{\bar{b}^L}_t} - \frac{q_t \left(1-\varrho\right) p^*_{L,t} b^{L,*}_{\kappa,t}}{\varepsilon^{\bar{b}^{L,*}}_t} - \frac{r^h_{t-1}}{\pi^c_t} b^h_{\kappa,t-1} - \frac{x}{2} \left(\delta \frac{b^S_{\kappa,t-1}}{b^L_{\kappa,t-1} + \left(1-\varrho\right) b^{L,*}_{\kappa,t-1}} - 1 \right)^2 \frac{1}{\pi^c_t}}_{\text{expenditures}} \end{aligned}$$

Intermediary's profits are subject to two adjustment costs. The fist one captures the idea that altering the foreign debt held by domestic intermediaries to GDP ratio is costly.[2] The second term reflects the situation where although intermediaries prefer to hold more long than short-term debt that decreases 'liquidity'.

[1] See the discussion in Andres et al. (2004) for the advantages of that assumption.

[2] In our model this term is not required to make the net foreign asset position of the model stationary (see the discussion in Schmitt-Grohe and Uribe (2003)) as we properly model long-term debt in the foreign economy.

Using the balance sheet equation the profit function becomes

$$\begin{aligned} E_t\xi_{t+1} &= \frac{r_t^S}{\pi_{t+1}^c} b_{\kappa,t}^S + E_t\left\{\frac{r_{t+1}^L}{\pi_{t+1}^c}\frac{p_{L,t+1}}{p_{L,t}}\right\}\bar{b}_{\kappa,t}^L + E_t\left\{\frac{r_{t+1}^{L,*}}{\pi_{t+1}^*}\frac{p_{L,t+1}^*}{p_{L,t}^*}\frac{q_{t+1}}{q_t}\right\}\bar{b}_{\kappa,t}^{L,*} \\ &\quad -E_t\left\{\frac{r_t^h}{\pi_{t+1}^c}\right\} b_{\kappa,t}^h - \frac{x}{2}\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)^2 \frac{1}{E_t\pi_{t+1}^c} \end{aligned}$$

Profit maximisation with respect to short, domestic and foreign long-term debt and subject to the balance sheet condition delivers an expression for the effective rate faced by the household, long-term interest rate and exchange rate

Short-term debt

$$\begin{aligned} 0 &= \frac{r_t^S}{E_t\pi_{t+1}^c} - \frac{r_t^h}{\varepsilon_t^{b^S} E_t\pi_{t+1}^c} - x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta}{b_{\kappa,t}^L + b_{\kappa,t}^{L,*}}\frac{1}{E_t\pi_{t+1}^c} \\ \frac{r_t^h}{\varepsilon_t^{b^S}} &= r_t^S - x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} \end{aligned} \tag{53}$$

Long-term domestic debt

$$\begin{aligned} 0 &= E_t\left\{\frac{r_{t+1}^L}{\pi_{t+1}^c}\frac{p_{L,t+1}}{p_{L,t}}\right\} p_{L,t} - \frac{r_t^h}{\varepsilon_t^{\bar{b}^L} E_t\pi_{t+1}^c} p_{L,t} + x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta b_{\kappa,t}^S}{\left(b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}\right)^2} \\ E_t\left\{r_{t+1}^L\frac{p_{L,t+1}}{p_{L,t}}\right\} &= \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}} - x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta b_{\kappa,t}^S}{p_{L,t}\left(b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}\right)^2} \end{aligned}$$

Long-term foreign debt

$$0 = E_t\left\{\frac{r_{t+1}^{L,*}}{\pi_{t+1}^*}\frac{p_{L,t+1}^*}{p_{L,t}^*}\frac{q_{t+1}}{q_t}\right\}(1-\varrho)\,\frac{r_t^{L,*}}{\pi_t^*}p_{L,t}^* - \frac{r_t^h\,(1-\varrho)\,\frac{r_t^{L,*}}{\pi_t^*}p_{L,t}^*}{\varepsilon_t^{\bar{b}^{L,*}} E_t\pi_{t+1}^c} + x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta b_{\kappa,t}^S\,(1-\varrho)}{\left(b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}\right)^2}$$

$$E_t\left\{\frac{r_{t+1}^{L,*}}{\pi_{t+1}^*}\frac{p_{L,t+1}^*}{p_{L,t}^*}\frac{q_{t+1}}{q_t}\right\} = \frac{r_t^h}{\varepsilon_t^{\bar{b}^{L,*}} E_t\pi_{t+1}^c} - x\left(\delta\frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}} - 1\right)\frac{\delta b_{\kappa,t}^S}{\frac{r_t^{L,*}}{\pi_t^*}p_{L,t}^*\left(b_{\kappa,t}^L + (1-\varrho)\, b_{\kappa,t}^{L,*}\right)^2 E_t\pi_{t+1}^c} \tag{55}$$

It is important to stress here the Uncover Interest Rate Parity (UIP) condition (55) is defined in terms of long and not short-term real interest rate differentials as it is typically done in the SoE literature (see Adolfson et al. (2007), Christiano et al. (2011) and Burgess et al. (2013) among others). This is because *i*) we restrict households to invest on domestic or/and foreign assets directly and *ii*) intermediaries can hold only foreign long and not short-term debt. This is definitely a 'shortcoming' as the exchange rate in our model is not the exchange observed in the real world, however, our approach has trivial advantages. To be precise, the first restriction allows domestic households in our model to own multiple assets and we are able to pin down their prices without employing complicated portfolio asset solution techniques. The model can be linearised, estimated and, furthermore, to use regime-switching techniques (similar to Liu et al. (2011)) to study how changes in the domestic or/and foreign macroeconomic uncertainty affect asset prices and what this means for policy makers. The second assumption restricts the number of the UIP condition to one and makes the domestic long-term interest rate a direct function of the foreign long-term debt and rates via the term-premia.

A.1.4 Wages

We follow Erceg et al. (2000) and assume that each monopolistically competitive household supplies a differentiated labour service to the production section. They set their nominal wage and supply any amount of labour demanded by the firms at that wage rate. For convenience, we assume that there exist a representative firm that combines households' labour inputs into a homogenous input hood - h_t^d - using a CES production function

$$h_t^d = \left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_w}} d\kappa \right]^{\lambda_w} \tag{56}$$

where λ_w is the wage mark-up. Taking w_t and $w_{\kappa,t}$ as given the aggregator's demand for the labour hours of household κ results its profit maximisation $\max_{h_{\kappa,t}} \left\{ w_t \left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_w}} d\kappa \right]^{\lambda_w} - \int_0^1 w_{\kappa,t} h_{\kappa,t} \right\}$

$$h_{\kappa,t} = \left(\frac{w_{\kappa,t}}{w_t} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_t^d \tag{57}$$

The aggregate wage arise from the profit condition and the demand curve

$$w_t = \left[\int_0^1 w_{\kappa,t}^{\frac{1}{1-\lambda_w}} d\kappa \right]^{1-\lambda_w} \tag{58}$$

In each period, a function $- \ 1 - \xi_w \ -$ of households receive a random signal and they are allowed to reset wages optimally $- \ w_t^{new}$. All other households can only partially index their wages by past inflation. The problem of setting wages can be described as follows

$$\max_{w_t^{new}} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ -\psi_{t+j} \frac{h_{\kappa,t+j}^{1+\varphi}}{1+\varphi} + \lambda_{t+j} \prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa_w}}{\pi_{t+s}^c} w_{\kappa,t} h_{\kappa,t} \right\} \tag{59}$$

subject to

$$h_{\kappa,t} = \left(\prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa w}}{\pi_{t+s}^c} \frac{w_{\kappa,t}}{w_t} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_t^d \tag{60}$$

The first order is summarised by the following recursive equations

$$v_{1,t} = \frac{1}{\lambda_{w,t}} \left(w_t^{new}\right)^{\frac{1}{1-\lambda_{w,t}}} \lambda_t w_t^{\frac{\lambda_w}{\lambda_w - 1}} h_t^d + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\kappa_w}}{\pi_{t+1}^c}\right)^{\frac{1}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}}\right)^{\frac{1}{\lambda_w - 1}} v_{1,t+1} \tag{61}$$

$$v_{1,t} = \psi_t \left(\frac{w_t}{w_t^{new}}\right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} \left(h_t^d\right)^{1+\varphi} + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\kappa_w}}{\pi_{t+1}^c}\right)^{\frac{(1+\varphi)\lambda_w}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}}\right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} v_{1,t+1} \tag{62}$$

$$w_t^{\frac{1}{1-\lambda_w}} = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w}}{\pi_t^c}\right)^{\frac{1}{1-\lambda_w}} w_{t-1}^{\frac{1}{1-\lambda_w}} + (1-\xi_w)\left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}} \tag{63}$$

The market clearing condition in the labour market is

$$h_t = \int_0^1 h_{\kappa,t} d\kappa = \upsilon_t^w h_t^d \tag{64}$$

where $\upsilon_t^w = \int_0^1 \left(\frac{w_{i,t}}{w_t}\right)^{-\frac{\lambda_w}{\lambda_w - 1}} d_i$ is the wage dispersion term and its evolution is described

$$\upsilon_t^w = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w}}{\pi_t^c}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{w_{t-1}}{w_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} \upsilon_{t-1}^w + (1-\xi_w)\left(\frac{w_t^{new}}{w_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} \tag{65}$$

A.1.5 Government

Government's budget constraint adjusted for long-term debt is given by

$$\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{p_{L,t} b_t^L}{\varepsilon_t^{\bar{b}^L}} + T_t = \frac{r_{t-1}^S}{\pi_t^c} b_{t-1}^S + \frac{r_t^L}{\pi_t^c} \frac{p_{L,t}}{p_{L,t-1}} p_{L,t-1} b_{t-1}^L + \tilde{p}_t G_t$$

$$\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}} + T_t = \frac{r_{t-1}^S}{\pi_t^c} b_{t-1}^S + \frac{r_t^L}{\pi_t^c} \frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{t-1}^L + \tilde{p}_t G_t$$

$$T_t = \frac{r_{t-1}^S}{\pi_t^c} b_{t-1}^S + \frac{r_t^L}{\pi_t^c} \frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{t-1}^L + \tilde{p}_t G_t - \left(\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}}\right)$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t. While the right hand side reflects the total deficit at time t. Similar to Chen et al. (2012) we assume that the government controls the supply of long-term bonds and this is fixed

$$\frac{b_t^L}{b^L} = 1 \tag{66}$$

Government consumption is a fraction (g_t) of GDP

$$G_t = g_t y_t \tag{67}$$

$$\frac{g_t}{g} = \left(\frac{g_t}{g}\right)^{\rho_g} e^{\sigma_g \omega g,t} \tag{68}$$

Finally, transfers are adjusted according to the following rule

$$T_t = \left(b^S + \bar{b}^L\right)^{1-\theta_b} \left(b_{t-1}^S + \bar{b}_{t-1}^L\right)^{\theta_b} \tag{69}$$

A.1.6 Monetary policy

The monetary authority sets its instrument short-term interest rate according to a Taylor rule

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\phi_R} (\pi_t^c)^{(1-\phi_R)\phi_\pi} \left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_y} e^{\sigma_R \omega_{R,t}} \tag{70}$$

In other words, the policymaker adjusts the nominal interest rate in response to its lag value, to inflation deviations from the target – $\pi = 1$ – and to output deviations from the long-run equilibrium – y.

A.1.7 Exports

As in our model the evolution of the foreign debt is determined in the foreign economy, we use this section to derive an expression for exports that ensures consistency between domestic economy's debt and the evolution of the foreign long-term debt given imports. In other words, exports become the residuals in the net foreign asset position accumulation equation.

To derive the latter expression we start from the clearing conditions in the markets for domestic intermediate goods

$$y_t = \int y_{i,t} = \int c_{i,t}^{dp} + \int c_{i,t}^{x} = c_t^{dp} + c_t^{x} = \upsilon_t^p c_t^{dp,d} + \upsilon_t^x c_t^{x,d}$$

and household's integrated budget constraint

$$D_t^h + c_t = \frac{r_{t-1}^h}{\pi_t^c} D_{t-1}^h + \int w_{\kappa,t} h_{\kappa,t} + F_t - T_t$$

Furthermore, the resource constraint implies that

$$y_t^d = c_t^{dp,d} + c_t^{x,d} + G_t \tag{71}$$

Substituting (43) into (71) we obtain

$$(1-\alpha)\,\tilde{p}_t^{-\eta_c} c_t + c_t^{x,d} = \frac{z_t h_t^d}{\upsilon_t^p} = \frac{z_t h_t}{\upsilon_t^w \upsilon_t^p} \tag{72}$$

Domestic profits are given

$$\begin{aligned} F_t &= F_{c,t} + \tilde{p}_t F_{dp,t} + \frac{p_t^m F_{m,t}}{p_t^c} + \frac{s_t p_t^x}{p_t^c} F_{x,t} + \xi_t \\ &= c_t + \tilde{p}_t G_t - mc_t \upsilon_t^p \tilde{p}_t y_t^d - q_t \upsilon_t^m c_t^{m,d} + \left(q_t \tilde{p}_t^x - \tilde{p}_t\left(\upsilon_t^x - 1\right)\right) c_t^{x,d} \\ &\quad + \frac{r_{t-1}^S}{\pi_t^c} b_{\kappa,t-1}^S + \frac{r_t^L}{\pi_t^c} \frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{\kappa,t-1}^L + \frac{r_t^{L,*}}{\pi_t^*} \frac{p_{L,t}^*}{p_{L,t-1}^*} \frac{q_t}{q_{t-1}} \bar{b}_{\kappa,t-1}^{L,*} \\ &\quad - \frac{r_{t-1}^h}{\pi_t^c} b_{t-1}^h - \frac{x}{2}\left(\delta \frac{b_{t-1}^S}{\bar{b}_{t-1}^L + \bar{b}_{t-1}^{L,*}} - 1\right)^2 \frac{1}{\pi_t^c} \end{aligned}$$

Next we use government's budget constraint to obtain an expression for $F_t - T_t$

$$\begin{aligned} F_t - T_t &= c_t - mc_t \upsilon_t^p \tilde{p}_t y_t^d - q_t \upsilon_t^m c_t^{m,d} + \left(q_t \tilde{p}_t^x - \tilde{p}_t \left(\upsilon_t^x - 1\right)\right) c_t^{x,d} \\ &+ \frac{r_t^{L,*}}{\pi_t^*} \frac{p_{L,t}^*}{p_{L,t-1}^*} \frac{q_t}{q_{t-1}} \bar{b}_{\kappa,t-1}^{L,*} + b_t^h - \frac{\bar{b}_t^{L,*}}{\varepsilon_t^{\bar{b}^L}} \\ &- \frac{r_{t-1}^h}{\pi_t} b_{t-1}^h - \frac{x}{2} \left(\delta \frac{b_{t-1}^S}{\bar{b}_{t-1}^L + \bar{b}_{t-1}^{L,*}} - 1\right)^2 \frac{1}{\pi_t^c} \end{aligned}$$

We substitute the latter expression, the definition of the balance sheet equation and labour market condition into household's integrated budget constraint

$$\begin{aligned} D_t^h + c_t &= \frac{r_{t-1}^h}{\pi_t^c} D_{t-1}^h + \int w_{\kappa,t} h_{\kappa,t} \\ &+ c_t - mc_t \upsilon_t^p \tilde{p}_t y_t^d - q_t \upsilon_t^m c_t^{m,d} + \left(q_t \tilde{p}_t^x - \tilde{p}_t \left(\upsilon_t^x - 1\right)\right) c_t^{x,d} \\ &+ \frac{r_t^{L,*}}{\pi_t^*} \frac{p_{L,t}^*}{p_{L,t-1}^*} \frac{q_t}{q_{t-1}} \bar{b}_{\kappa,t-1}^{L,*} + b_t^h - \frac{\bar{b}_t^{L,*}}{\varepsilon_t^{\bar{b}^L}} \\ &- \frac{r_{t-1}^h}{\pi_t^c} b_{t-1}^h - \frac{x}{2} \left(\delta \frac{b_{t-1}^S}{\bar{b}_{t-1}^L + \bar{b}_{t-1}^{L,*}} - 1\right)^2 \frac{1}{\pi_t^c} \end{aligned}$$

and this leads to

$$c_t^{x,d} = q_t \upsilon_t^m c_t^{m,d} + \frac{\bar{b}_t^{L,*}}{\varepsilon_t^{\bar{b}^L}} - \frac{r_t^{L,*}}{\pi_t^*} \frac{p_{L,t}^*}{p_{L,t-1}^*} \frac{q_t}{q_{t-1}} \bar{b}_{\kappa,t-1}^{L,*} + \frac{x}{2} \left(\delta \frac{b_{t-1}^S}{\bar{b}_{t-1}^L + \bar{b}_{t-1}^{L,*}} - 1\right)^2 \frac{1}{\pi_t^c} \tag{73}$$

A.2 Foreign economy

The foreign agents' decision problems are very similar to those discussed in the previous section. To avoid repeating ourselves and to save some space we just list in this section the first-order conditions required for the solution of the model. We keep the same notation with the domestic economy and we add a star symbol – $*$ – to separate the foreign from the domestic variables.

A.2.1 Supply

The production of the intermediate good production is given by

$$y_{i,t}^* = z_t^* h_{i,t}^{d,*} \tag{74}$$

where – similar to the domestic economy – the productivity shock is conditionally heteroscedastic

$$z_t^* = \left(1 - \rho_{z^*}\right) z^* + \rho_{z^*} z_{t-1}^* + \sigma_{z^*} \omega_{z^*,t} \tag{75}$$

The marginal cost is given

$$mc_t^* = \frac{w_t^*}{z_t^*} \tag{76}$$

while the following equations describe firm's pricing first-order conditions

$$f_{1,t}^{*} = \lambda_t^{*} mc_t^{*} y_t^{d,*} + \beta \xi_d^{*} E_t \left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_d}}{\pi_{t+1}^{c,*}} \right)^{-\frac{\lambda_{y,t}^{*}}{\lambda_{y,t}^{*}-1}} f_{1,t+1}^{*} \tag{77}$$

$$f_{2,t}^{*} = \lambda_t^{*} \bar{\pi}_t^{*} y_t^{d,*} + \beta \xi_d^{*} E_t \left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_d}}{\pi_{t+1}^{c,*}} \right)^{-\frac{1}{\lambda_{y,t}^{*}-1}} \left(\frac{\bar{\pi}_t^{*}}{\bar{\pi}_{t+1}^{*}} \right) f_{2,t+1}^{*} \tag{78}$$

$$0 = \lambda_{y,t}^{*} f_{1,t}^{*} - f_{2,t}^{*} \tag{79}$$

$$1 = \xi_d^{*} \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_d}}{\pi_t^{c,*}} \right)^{-\frac{1}{\lambda_{y,t}^{*}-1}} + (1-\xi_d)\left(\bar{\pi}_t^{*}\right)^{-\frac{1}{\lambda_{y,t}^{*}-1}} \tag{80}$$

where $\bar{\pi}_t^{*} \equiv \frac{p_t^{new,*}}{p_t^{*}}$.

A.2.2 Households

Domestic household's utility maximisation first-order conditions with respect to $c_t^{*}$and D_t^{*} subject to its budget constraint the consumption Euler condition

$$\frac{d_t^{*}}{\left(c_t^{*} - bc_{t-1}^{*}\right)^{\sigma_c}} - E_t \frac{\beta b d_{t+1}^{*}}{\left(c_{t+1}^{*} - hc_t^{*}\right)^{\sigma_c}} = \lambda_t^{*} \tag{81}$$

$$\lambda_t^{*} = \beta E_t \left\{ \lambda_{t+1}^{*} \frac{r_t^{h,*}}{\pi_{t+1}^{c,*}} \right\} \tag{82}$$

Agents' utility function is subject to a discount factor and labour supply shock.

$$d_t^{*} = (1-\rho_{d^{*}})\, d^{*} + \rho_{d^{*}} d_{t-1}^{*} + \sigma_{d^{*}} \omega_{d^{*},t} \tag{83}$$

A.2.3 Financial intermediary

The financial intermediary firm issues deposits households paying a gross interest rate $r_t^{h,*}$. The firm then purchases a portfolio of short and long term government issued bonds paying interest r_t and r_t^{L}. For simplicity we assume that all government bonds issued are purchased by this firm. The interest rates received by this firm depend on the level of bonds the firm issues. The intermediary's balance sheet is

$$b_{\kappa,t}^{h,*} = \frac{b_{\kappa,t}^{S,*}}{\varepsilon_t^{b^{S,*}}} + \frac{\varrho p_{L,t}^{*} b_{\kappa,t}^{L,*}}{\varepsilon_t^{\bar{b}^{L,*}}} \tag{84}$$

where

$$p_{L,t}^{*} = \frac{1}{r_t^{L,*} - \kappa}$$

Intermediary's profit function is given by

$$\xi^*_{t+1} = \frac{r^{S,*}_t}{E_t \pi^{c,*}_{t+1}} b^{S,*}_{\kappa,t} + E_t \left\{ \frac{r^{L,*}_{t+1}}{\pi^{c,*}_{t+1}} \frac{p^*_{L,t+1}}{p^*_{L,t}} \right\} \bar{b}^{L,*}_{\kappa,t} - \frac{r^{h,*}_t}{E_t \pi^{c,*}_{t+1}} b^{h,*}_{\kappa,t} - \frac{x^*}{2} \left(\delta^* \frac{b^{S,*}_{\kappa,t}}{\varrho b^{L,*}_{\kappa,t}} - 1 \right)^2 \frac{1}{E_t \pi^{c,*}_{t+1}} \quad (85)$$

Profit maximisation subject to the balance sheet condition gives

$$\frac{r^{h,*}_t}{\varepsilon^{b^{S,*}}_t} = r^{S,*}_t - x^* \left(\delta^* \frac{b^{S,*}_{\kappa,t}}{\varrho b^{L,*}_{\kappa,t}} - 1 \right) \frac{\delta^*}{\varrho b^{L,*}_{\kappa,t}} \quad (86)$$

$$E_t \left\{ r^{L,*}_{t+1} \frac{p^*_{L,t+1}}{p^*_{L,t}} \right\} \varrho p^*_{L,t} = \frac{r^{h,*}_t}{\varepsilon^{\bar{b}^{L,*}}_t} \varrho p^*_{L,t} - x^* \left(\delta^* \frac{b^{S,*}_{\kappa,t}}{\varrho b^{L,*}_{\kappa,t}} - 1 \right) \frac{\delta^* b^{S,*}_{\kappa,t} \varrho}{\left(\varrho b^{L,*}_{\kappa,t} \right)^2} \quad (87)$$

A.2.4 Wages

The evolution of wages in the foreign economy is described by

$$\begin{aligned} v^*_{1,t} &= \frac{1}{\lambda^*_{w,t}} \left(w^{new,*}_t \right)^{\frac{1}{1-\lambda^*_w}} \lambda^*_t \left(w^*_t \right)^{\frac{\lambda^*_w}{\lambda^*_w - 1}} h^{d,*}_t + \\ & \quad \beta \xi^*_w E_t \left(\frac{\left(\pi^{c,*}_t \right)^{\kappa_w}}{\pi^{c,*}_{t+1}} \right)^{\frac{1}{1-\lambda^*_w}} \left(\frac{w^{new,*}_{t+1}}{w^{new,*}_t} \right)^{\frac{1}{\lambda^*_w - 1}} v^*_{1,t+1} \end{aligned} \quad (88)$$

$$\begin{aligned} v^*_{1,t} &= d^*_t \psi^*_t \left(\frac{w^*_t}{w^{new,*}_t} \right)^{\frac{(1+\varphi)\lambda^*_w}{\lambda^*_w - 1}} \left(h^{d,*}_t \right)^{1+\varphi} + \\ & \quad \beta \xi^*_w E_t \left(\frac{\left(\pi^{c,*}_t \right)^{\kappa_w}}{\pi^{c,*}_{t+1}} \right)^{\frac{(1+\varphi)\lambda^*_w}{1-\lambda^*_w}} \left(\frac{w^{new,*}_{t+1}}{w^{new,*}_t} \right)^{\frac{(1+\varphi)\lambda^*_w}{\lambda^*_w - 1}} v^*_{1,t+1} \end{aligned} \quad (89)$$

$$\left(w^*_t \right)^{\frac{1}{1-\lambda^*_w}} = \xi^*_w \left(\frac{\left(\pi^{c,*}_{t-1} \right)^{\kappa_w}}{\pi^{c,*}_t} \right)^{\frac{1}{1-\lambda^*_w}} \left(w^*_{t-1} \right)^{\frac{1}{1-\lambda^*_w}} + \left(1 - \xi^*_w \right) \left(w^{new,*}_t \right)^{\frac{1}{1-\lambda^*_w}} \quad (90)$$

A.2.5 Monetary policy

Foreign monetary policy authorities follow a similar with the domestic economy Taylor type rule

$$\frac{r^*_t}{r} = \left(\frac{r^*_{t-1}}{r} \right)^{\phi_{R^*}} \left(\pi^{c,*}_t \right)^{(1-\phi_{R^*})\phi_{\pi^*}} \left(\frac{y^{d,*}_t}{y^*} \right)^{(1-\phi_{R^*})\phi_{Y^*}} e^{\sigma_{R^*} R^*,t} \quad (91)$$

A.2.6 Government

Government's budget constraint adjusted for long-term debt is given by

$$b^{S,*}_{\kappa,t} + \frac{1}{r^{L,*}_t - \kappa^*} b^{L,*}_{\kappa,t} = \frac{r^*_{t-1}}{\pi^{c,*}_t} b^{S,*}_{t-1} + \frac{r^{L,*}_t}{\left(r^{L,*}_t - \kappa^* \right) \pi^{c,*}_t} b^{L,*}_{t-1} + G^*_t - T^*_t \quad (92)$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t. While the right hand side reflects the total deficit at time t.

Similar to Chen et al. (2012) we assume that the government controls the supply of long-term bonds and it is fixed

$$\frac{b_t^{L,*}}{b^{L,*}} = 1 \tag{93}$$

Finally, we assume adjust its primary surplus based on debt targeting rule

$$T_t^* = \left(b^{S,*} + \bar{b}^{L,*}\right)^{1-\theta^*} \left(b_{t-1}^{S,*} + \bar{b}_{t-1}^{L,*}\right)^{\theta^*} \tag{94}$$

$$G_t^* = g_t^* y_t^* \tag{95}$$

$$\frac{g_t^*}{g^*} = \left(\frac{g_t^*}{g^*}\right)^{\rho_g} e^{\sigma_{g^*}\omega_{g^*,t}} \tag{96}$$

A.2.7 Market clearing conditions

$$y_t^{d,*} = c_t^* + G_t^* + \frac{x^*}{2}\left(\frac{b_{\kappa,t-1}^{L,*}}{b_{\kappa,t-1}^{S,*}} - \frac{b^{L,*}}{b^{S,*}}\right)^2 \frac{b_{\kappa,t-1}^{L,*}}{\pi_t^{c,*}} \tag{97}$$

$$y_t^{d,*} = \frac{z_t^* h_t^{d,*}}{\upsilon_t^{p,*}} \tag{98}$$

where $\upsilon_t^{p,*}$ is the price dispersion term

$$\upsilon_t^{p,*} = \xi_d^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_d}}{\pi_t^{c,*}}\right)^{-\frac{\lambda_{y,t}^*}{\lambda_{y,t}^*-1}} \upsilon_{t-1}^{p,*} + (1-\xi_d)\left(\tilde{\pi}_t^*\right)^{-\frac{\lambda_{y,t}^*}{\lambda_{y,t}^*-1}} \tag{99}$$

$$h_t^{d,*} = \frac{h_t^*}{\upsilon_t^{w,*}} \tag{100}$$

where $\upsilon_t^{w,*}$ is the wage dispersion term

$$\upsilon_t^{w,*} = \xi_w^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_w}}{\pi_t^{c,*}}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}} \left(\frac{w_{t-1}^*}{w_t^*}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}} + (1-\xi_w^*)\left(\frac{w_t^{new,*}}{w_t^*}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}} \tag{101}$$

A.3 Steady states

A.3.1 Domestic economy

We assume that

$$\pi^c = \pi = \pi^m = 1 \tag{102}$$

Household's Euler equation implies

$$r^h = \frac{1}{\beta} \tag{103}$$

Household's effective interest rate spread is given by financial intermediary first order condition

$$\frac{r^h}{r^S} = \varepsilon^{b^S}$$

Similarly, the long-term interest rate spread is

$$\begin{aligned} r^L &= \frac{r^h}{\varepsilon^{\bar{b}^L}} = \frac{r^S \varepsilon^{b^S}}{\varepsilon^{\bar{b}^L}} \\ \frac{r^L}{r^S} &= \frac{\varepsilon^{b^S}}{\varepsilon^{\bar{b}^L}} \end{aligned} \tag{104}$$

Assuming that $h = 1/3$ we know from intermediate good producer we know

$$y = zh \tag{105}$$

Imports demand equation

$$c^m = \alpha \left(\tilde{p}^m\right)^{-\eta_c} c$$

Now, from importing and exporting firms we know

$$\begin{aligned} mc^m &\equiv \frac{sp^*}{p^m} = \frac{sp^*}{p^c}\frac{p^c}{p^m} = \frac{q}{\tilde{p}^m} = \frac{1}{\lambda_m} \\ \tilde{p}^m &= \lambda_m \end{aligned} \tag{106}$$

$$\begin{aligned} mc^x &= \frac{p}{sp^x} = \frac{p^c}{sp^*}\frac{p^*}{p^x}\frac{p}{p^c} = \frac{\tilde{p}}{q\tilde{p}^x} = \frac{1}{\lambda_x} \\ \tilde{p} &= \frac{\tilde{p}^x}{\lambda_x} \end{aligned} \tag{107}$$

$$T = \frac{\tilde{p}^m}{\tilde{p}} = \frac{\frac{q\lambda_m}{1}}{\frac{q\tilde{p}^x}{\lambda_x}} = \frac{\lambda_m \lambda_x}{q\tilde{p}^x}$$

If we assume that $q = 1$ and $\tilde{p}^x = 1$ then

$$T = \frac{\tilde{p}^m}{\tilde{p}} = \lambda_m \lambda_x \tag{108}$$

We set

$$b^L = \frac{1-\beta\kappa}{\beta}\frac{\bar{b}^L}{y} \tag{109}$$

and

$$\frac{b^L}{b^L + (1-\varrho)\, b^{L,*}} = \frac{(1-\varrho)\, b^{L,*}}{b^L + (1-\varrho)\, b^{L,*}} \tag{110}$$

This implies

$$(1-\varrho)\, b^{L,*} = b^L$$

$$\varrho = 1 - \frac{b^L}{b^{L,*}} \tag{111}$$

Futhermore, we assume that

$$b^{L,*} = (1+\tau)\, b^L$$

with $\tau > 0$

$$\varrho = 1 - \frac{1}{1+\tau} = \frac{\tau}{1+\tau} \tag{112}$$

This implies that

$$\bar{b}^{L,*} = \frac{\beta}{1-\beta\kappa_*}\,(1+\tau)\, b^L \tag{113}$$

$$\delta = \frac{b^S}{b^L + (1-\varrho)\, b^{L,*}} = \frac{b^S}{2b^L} \tag{114}$$

$$\delta^* = \frac{b^{S,*}}{\varrho b^{L,*}} = \frac{b^{S,*}}{\tau b^L}$$

From good's market clearing condition

$$\begin{aligned} 1 &= \frac{c^{dp}}{y} + \frac{c^x}{y} + \frac{G}{y} \\ 1 &= \frac{c^{dp}}{y} + \frac{c^x}{y} g \end{aligned} \tag{115}$$

given $\frac{c^x}{y}$ and g we obtain

$$\frac{c^{dp}}{y} = 1 - \frac{c^x}{y} - g \tag{116}$$

From domestic consumption demand equation we know

$$c^{dp} = (1-\alpha)\, \tilde{p}^{-\eta_c} c$$

dividing by y

$$\frac{c^{dp}}{y} = (1-\alpha)\, \tilde{p}^{-\eta_c} \frac{c}{y}$$

or

$$\begin{aligned} \frac{c}{y} &= \frac{\tilde{p}^{\eta_c}}{1-\alpha} \frac{c^{dp}}{y} \\ \frac{c}{y} &= \frac{\tilde{p}^{\eta_c}}{1-\alpha} \left(1 - \frac{c^x}{y} - g\right) \end{aligned} \tag{117}$$

From the export equation

$$\begin{aligned} \frac{c^x}{y} &= \frac{c^m}{y} + \left(1 - r^{L,*}\right) \frac{\bar{b}^{L,*}}{y} \\ \frac{c^x}{y} &= \alpha\, (\tilde{p}^m)^{-\eta_c} \frac{c}{y} + \left(1 - r^{L,*}\right) \frac{\bar{b}^{L,*}}{y} \end{aligned}$$

$$
\begin{aligned}
\frac{c}{y} &= \frac{\tilde{p}^{\eta_c}}{1-\alpha}\left(1-\alpha\left(\tilde{p}^m\right)^{-\eta_c}\frac{c}{y}-\left(\frac{1}{\varepsilon^{\bar{b}^L}}-r^{L,*}\right)\frac{\bar{b}^{L,*}}{y}-g\right) \\
\frac{c}{y}\left(1+\frac{\alpha}{1-\alpha}\left(\frac{\tilde{p}}{\tilde{p}^m}\right)^{\eta_c}\right) &= \frac{\tilde{p}^{\eta_c}}{1-\alpha}\left(1-\left(\frac{1}{\varepsilon^{\bar{b}^L}}-r^{L,*}\right)\frac{\bar{b}^{L,*}}{y}-g\right) \\
\frac{c}{y} &= \frac{\frac{\tilde{p}^{\eta_c}}{1-\alpha}\left(1-\left(\frac{1}{\varepsilon^{\bar{b}^L}}-r^{L,*}\right)\frac{\bar{b}^{L,*}}{y}-g\right)}{\left(1+\frac{\alpha}{1-\alpha}\left(\frac{\tilde{p}}{\tilde{p}^m}\right)^{\eta_c}\right)}
\end{aligned}
$$

We also know

$$
\begin{aligned}
1 &= (1-\alpha)\,\tilde{p}^{1-\eta_c}+\alpha\left(\tilde{p}^m\right)^{1-\eta_c} \\
1 &= (1-\alpha)\left(\frac{q\tilde{p}^x}{\lambda_x}\right)^{1-\eta_c}+\alpha\left(q\lambda_m\right)^{1-\eta_c}
\end{aligned}
$$

$$
\begin{aligned}
1 &= (1-\alpha)\left(\frac{1}{\lambda_x}\right)^{1-\eta_c}+\alpha\lambda_m^{1-\eta_c} \\
\frac{1}{\lambda_m^{1-\eta_c}} &= (1-\alpha)\left(\frac{1}{\lambda_x\lambda_m}\right)^{1-\eta_c}+\alpha \\
\alpha &= \frac{\frac{1}{\lambda_m^{1-\eta_c}}-\left(\frac{1}{\lambda_x\lambda_m}\right)^{1-\eta_c}}{1-\left(\frac{1}{\lambda_x\lambda_m}\right)^{1-\eta_c}} \qquad (118)
\end{aligned}
$$

$$
\frac{c^m}{c}=\frac{\frac{1}{\lambda_m^{1-\eta_c}}-\left(\frac{1}{\lambda_x\lambda_m}\right)^{1-\eta_c}}{1-\left(\frac{1}{\lambda_x\lambda_m}\right)^{1-\eta_c}}\left(p^m\right)^{-\eta_c}
$$

We return to consumption share

$$
\frac{c}{y}=\frac{\left(\frac{\tilde{p}^x}{\lambda_x}\right)^{\eta_c}}{1-\alpha}\left(1-\frac{c^x}{y}-g\right)
$$

$$
\begin{aligned}
\frac{b^S}{\varepsilon^{b^S}}+\frac{\bar{b}^L}{\varepsilon^{\bar{b}^L}}+T &= r^S b^S+r^L\bar{b}^L+\tilde{p}G \\
\frac{b^S+\bar{b}^L}{y}+\frac{T}{y} &= \frac{1}{\beta}\frac{b^S+\bar{b}^L}{y}+\tilde{p}g \\
\frac{b^S+\bar{b}^L}{y}\frac{\beta-1}{\beta} &= \tilde{p}g-\frac{T}{y}
\end{aligned}
$$

We fix $\frac{b^S+\bar{b}^L}{y}$ and we solve for T

$$
\begin{aligned}
T &= \frac{r^h}{\varepsilon^{b^S}}b^S+\frac{r^h}{\varepsilon^{\bar{b}^L}}\bar{b}^L+\tilde{p}G-\left(\frac{b^S}{\varepsilon^{b^S}}+\frac{\bar{b}^L}{\varepsilon^{\bar{b}^L}}\right) \\
T &= \left(r^h-1\right)\frac{b^S}{\varepsilon^{b^S}}+\left(r^h-1\right)\frac{\bar{b}^L}{\varepsilon^{\bar{b}^L}}+\tilde{p}G
\end{aligned}
$$

$$
\frac{T}{y}=\tilde{p}g-\frac{\beta-1}{\beta}\left(\frac{b^S}{y\varepsilon^{b^S}}+\frac{\bar{b}^L}{y\varepsilon^{\bar{b}^L}}\right) \qquad (119)
$$

From domestic firms we know

$$\begin{aligned} mc &= \frac{w}{\tilde{p}z} = \frac{1}{\lambda_y} \\ w &= \frac{\tilde{p}z}{\lambda_y} \end{aligned} \tag{120}$$

From labour packager we can find the value of ψ that ensure that $h = 1/3$

$$\begin{aligned} v_1 &= \frac{1}{\lambda_w} (w)^{\frac{1}{1-\lambda_w}} \lambda w^{\frac{\lambda_w}{\lambda_w - 1}} h + \beta \xi_w v_1 \\ v_1 &= \psi h^{1+\varphi} + \beta \xi_w v_1 \end{aligned}$$

$$v_1 = \frac{\psi}{1 - \beta \xi_w} h^{1+\varphi}$$

$$\begin{aligned} v_1 &= \frac{1}{\lambda_w} w \lambda h + \beta \xi_w v_1 \\ v_1 &= \frac{\frac{1}{\lambda_w} \lambda}{1 - \beta \xi_w} wh \\ \frac{\psi}{1 - \beta \xi_w} h^{1+\varphi} &= \frac{\frac{1}{\lambda_w} \lambda}{1 - \beta \xi_w} wh \\ \psi h^{1+\varphi} &= \frac{1}{\lambda_w} \lambda wh \\ \psi &= \frac{\frac{1}{\lambda_w} \lambda w}{h^{\varphi}} \end{aligned} \tag{121}$$

From the euler equation

$$\frac{1 - \beta b}{(1-b)^{\sigma_c} c^{\sigma_c}} = \lambda$$

That implies

$$(1 - \varrho) \frac{b^{L,*}}{r^{L,*} - \kappa^*} = \frac{y}{1 - r^{L,*}} \left[\frac{c^x}{y} - \alpha (\tilde{p}^m)^{-\eta_c} \frac{c}{y} \right]$$

set $\kappa^* = 1$

$$(1 - \varrho) \frac{b^{L,*}}{r^{L,*} - 1} = \frac{y}{1 - r^{L,*}} \left[\frac{c^x}{y} - \alpha (\tilde{p}^m)^{-\eta_c} \frac{c}{y} \right]$$

or

$$\begin{aligned} 1 - \varrho &= -\frac{y}{b^{L,*}} \left[\frac{c^x}{y} - \alpha (\tilde{p}^m)^{-\eta_c} \frac{c}{y} \right] \\ \varrho &= 1 + \frac{y}{b^{L,*}} \left[\frac{c^x}{y} - \alpha (\tilde{p}^m)^{-\eta_c} \frac{c}{y} \right] \end{aligned} \tag{122}$$

Then

$$\begin{aligned} \bar{b}^L + \bar{b}^{L,*} &= \delta b^S \\ \frac{\bar{b}^L + b^S}{y} + \frac{\bar{b}^{L,*}}{y} &= (1 + \delta) \frac{b^S}{y} \\ b^S &= \frac{y}{1 + \delta} \left[\frac{\bar{b}^L + b^S}{y} + \frac{\bar{b}^{L,*}}{y} \right] \end{aligned}$$

$$\frac{\bar{b}^{L,*}}{y} = \frac{b^S + \bar{b}^L}{y} - \frac{b^S}{y}$$
$$b^L = \left(r^L - 1\right)\bar{b}^L$$

A.3.2 Foreign economy

$$\pi^{c,*} = 1 \tag{123}$$

$$mc^* = \frac{1}{\lambda_d^*} \tag{124}$$

$$w^* = \frac{z^*}{\lambda_d^*} \tag{125}$$

$$\lambda^* = \frac{1}{(c^*)^{\sigma_c}}\frac{(1-\beta b)\,d}{(1-b)^{\sigma_c}} \tag{126}$$

$$h^* = \left[\frac{w\frac{\lambda}{\lambda_w}}{d\psi}\right]^{\frac{1}{\varphi}} \tag{127}$$

$$c^* + g^* z^* h^{d,*} = z^* h^{d,*}$$
$$c^* = (1-g^*)\, z^* h^{d,*}$$

B Term Structure Decomposition

B.1 Domestic Economy

Long-Term Interest Rates

$$\hat{r}_{t+1}^L + \hat{p}_{L,t+1} - \hat{p}_{L,t} = \hat{r}_t^h - \hat{\varepsilon}_t^{b^L} + \tilde{x}\left(r^L - \kappa\right)\left[\frac{b^L}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^L + \frac{(1-\varrho)\,b^{L,*}}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^{L,*} - \hat{b}_t^S\right]$$

Household Effective Rate

$$\hat{r}_t^h = \hat{r}_t^S + \hat{\varepsilon}_t^{b^S} + \tilde{x}\delta\left[\frac{b^L}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^L + \frac{(1-\varrho)\,b^{L,*}}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^{L,*} - \hat{b}_t^S\right]$$

Long-Term Asset Price

$$\hat{p}_{L,t} = -\frac{1}{1-\beta\kappa}\hat{r}_t^L$$

Substituting the last two expression into first equation

$$\hat{r}_{t+1}^L - \frac{1}{1-\beta\kappa}\hat{r}_{t+1}^L + \frac{1}{1-\beta\kappa}\hat{r}_t^L = \hat{r}_t^S + \hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{b^L}$$
$$+\left[\delta + \left(r^L - \kappa\right)\right]\tilde{x}\left[\frac{b^L}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^L + \frac{(1-\varrho)\,b^{L,*}}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^{L,*} - \hat{b}_t^S\right]$$

or

$$\hat{r}_t^L = \beta\kappa\hat{r}_{t+1}^L + (1-\beta\kappa)\,\hat{r}_t^S + (1-\beta\kappa)\left(\hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{b^L}\right) + (1-\beta\kappa)\left[\delta + \left(r^L - \kappa\right)\right]\tilde{x}\left[\frac{b^L}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^L + \frac{(1-\varrho)\,b^{L,*}}{b^L + (1-\varrho)\,b^{L,*}}\hat{b}_t^{L,*} - \hat{b}_t^S\right] \quad (128)$$

Iterating the last equation forwards we obtain

$$\hat{r}_t^L = (1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\hat{r}_{t+i}^S + (1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i\left(E_t\hat{\varepsilon}_{t+i}^{b^S} - E_t\hat{\varepsilon}_{t+i}^{b^L}\right) + (1-\beta\kappa)\left[\delta + \left(r^L - \kappa\right)\right]\tilde{x}\sum_{i=0}^{\infty}(\beta\kappa)^i\left[\frac{b^L}{b^L + (1-\varrho)\,b^{L,*}}E_t\hat{b}_{t+i}^L + \frac{(1-\varrho)\,b^{L,*}}{b^L + (1-\varrho)\,b^{L,*}}E_t\hat{b}_{t+i}^{L,*} - E_t\hat{b}_{t+i}^S\right] \quad (129)$$

B.2 Foreign Economy

Long-Term Interest Rates

$$\hat{r}_{t+1}^{L,*} + \hat{p}_{L,t+1}^* - \hat{p}_{L,t}^* = \hat{r}_t^{h,*} - \hat{\varepsilon}_t^{b^{L,*}} + \tilde{x}^*\left(r^{L,*} - \kappa^*\right)\left[\hat{b}_t^{L,*} - \hat{b}_t^{S,*}\right]$$

Household Effective Rate

$$\hat{r}_t^{h,*} = \hat{r}_t^{S,*} + \hat{\varepsilon}_t^{b^{S,*}} + \tilde{x}^*\delta^*\left[\hat{b}_t^{L,*} - \hat{b}_t^{S,*}\right]$$

Long-Term Asset Price

$$\hat{p}_{L,t}^* = -\frac{1}{1-\beta\kappa^*}\hat{r}_t^{L,*}$$

Following the same steps as before

$$\hat{r}_t^{L,*} = (1-\beta\kappa^*)\sum_{i=0}^{\infty}(\beta\kappa^*)^i E_t\hat{r}_{t+i}^{S,*} + (1-\beta\kappa^*)\sum_{i=0}^{\infty}(\beta\kappa^*)^i\left(E_t\hat{\varepsilon}_{t+i}^{b^{S,*}} - E_t\hat{\varepsilon}_{t+i}^{b^{L,*}}\right) + (1-\beta\kappa^*)\left[\delta + \left(r^L - \kappa^*\right)\right]\tilde{x}^*\sum_{i=0}^{\infty}(\beta\kappa^*)^i\left[E_t\hat{b}_{t+i}^{L,*} - E_t\hat{b}_{t+i}^{S,*}\right] \quad (130)$$

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